Venice, 12 March '09

Models of Neutrino Mixings: Recent Developments

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Plan of the talk

Tri-Bimaximal (TB) mixing is indicated by the data

Is it real? Models of (approximate) TB mixing

Discrete symmetry groups: A4, S4......

Different versions of A4

A4 and GUT's G.A, Feruglio, Hagedorn '08

A different route. May be TB is accidental ----> hint:

complementarity holds: $\theta_{12}+\theta_C=\pi/4$ is empirically true

A possibility: Bimaximal (BM) mixing corrected from diagonalisation of charged leptons

A new model based on S4 G.A, Feruglio, Merlo '09

θ_{13} near the present bound

Preliminaries

• After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$

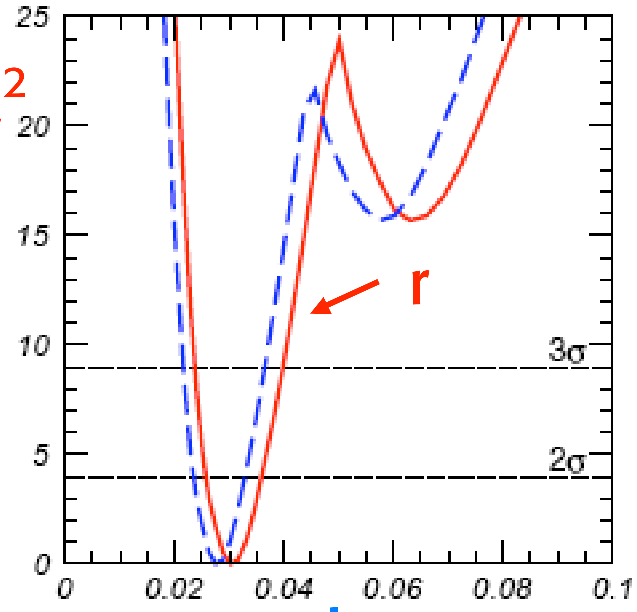
Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08

or

$m_{heaviest} < 0.2$ - 0.7 eV
$m_{next} > \sim 8\ 10^{-3}$ eV



For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) $\longrightarrow$ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.61$

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

  Very small θ_{13} theoretically hard

Fogli et al '08

$\sin^2\theta_{12}$	$0.326^{+0.050}_{-0.040}$	$[2\sigma]$
$\sin^2\theta_{23}$	$0.45^{+0.16}_{-0.09}$	$[2\sigma]$
$\sin^2\theta_{13}$	0.016 ± 0.010	

$\theta_{13} = 0$, $\theta_{23} = \pi/4$ as a possible 1st approximation?

For some time people considered limiting models with $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and θ_{12} generic

The most general mass matrix for $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ is given by

(after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

G.L.Fogli et al'08

At $\sim 1\sigma$: $\boxed{\sin^2\theta_{12} = 0.294\text{-}0.331}$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Harrison, Perkins, Scott '02

Some additional ingredient other than $\mu-\tau$ symmetry needed!

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$m_1 = x - y$
$m_2 = x + 2y$
$m_3 = x - y + 2v$

The 3 remaining parameters are the mass eigenvalues

Tribimaximal Mixing

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \, \mathrm{diag}(m_1, m_2, m_3) \, U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

- For TB mixing all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...;
GA, Feruglio, hep-ph/0504165, hep-ph/0512103
GA, Feruglio, Lin hep-ph/0610165
GA, Feruglio, Hagedorn, 0802.0090
Y. Lin, 0804.2867........

Larger finite groups: T', $\Delta(27)$, S4

Feruglio et al;
Chen, Mahanthappa;
Frampton, Kephart; Lam;
Bazzocchi et al ....

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross
....
King ....

A4

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T as:
with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

x, x′ in same class if

C_1, C_2, C_3, C_4 are equivalence classes $[x' \sim gxg^{-1}]$ g: group element

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$$3,\ 1,\ 1',\ 1''$$

(promising for 3 generations!)

Note:

as many representations as equivalence classes

$\Sigma d_i^2 = 12$ $\quad\quad$ $9+1+1+1=12$

Note: many models tried S3
S3 has no triplets but only 2 , 1, 1′
A4 is better in the lepton sector

Mohapatra, Nasri, Yu
Koide
Kubo et al
Kaneko et al
Caravaglios et al
Morisi; Picariello
Grimus, Lavoura......

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

Recall:
$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: & S=1,\ T=1 \\ 1': & S=1,\ T=\omega \\ 1'': & S=1,\ T=\omega^2 \end{cases}$$

The only irreducible 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

e.g. charged leptons $l \sim 3$
$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

In the S-diag basis consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$, $\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

(under S, 1" is invariant)

In the T-diagonal basis we have:

$VV^\dagger = V^\dagger V = 1$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

Cabibbo '78

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1'' = a_2b_2 + a_1b_3 + a_3b_3$$

We will see that in this basis the charged leptons are diagonal

$$3_{symm} \sim \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_{antisymm} \sim \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

Under A4 the most common classification is:

lepton doublets $l \sim 3$, (in see-saw models $\nu^c \sim 3$)

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ respectively

A4 breaking gauge singlet flavons $\phi_S, \phi_T, \xi \sim 3, 3, 1$

For SUSY version: driving fields $\phi_{0S}, \phi_{0T}, \xi_0 \sim 3, 3, 1$

with the alignment:

$$\boxed{\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle = u \quad &, \quad \langle \tilde{\xi} \rangle = 0 \end{aligned}}$$

!!!

In a serious model the alignment must follow from the symmetries

In all versions there are additional symmetries:

e.g. a broken $U(1)_F$ symmetry and/or discrete symmetries Z_n to ensure hierarchy of charged lepton masses and to restrict allowed couplings

Structure of the model (a 4-dim SUSY version)

GA, Feruglio, hep-ph/0512103

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

In T-diag basis:

with this alignment:

$$\boxed{\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle &= u \quad , \quad \langle \tilde{\xi} \rangle = 0 \end{aligned}}$$

Ch. leptons are diagonal

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

ν's are tri-bimaximal

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

$$a \equiv x_a \frac{u}{\Lambda} \qquad b \equiv x_b \frac{v_T}{\Lambda}$$

recall:

$$m = \begin{pmatrix} x & y & y \\ y & x + v & y - v \\ y & y - v & x + v \end{pmatrix}$$

So, at LO TB mixing is exact

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

The only fine-tuning needed is to account for $r \sim 1/30$

[In most A4 models $r \sim 1$ would be expected as $l, \nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives generically corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we expect:

$\theta_{13} \sim o(\lambda_C^2)$ measurable in next run of exp's

(T2K starts at the end of '09)

Why A4 works?

TB mixing corresponds to m in the basis where charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under SmS = m and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

Invariance under S can be made automatic in A4 while invariance under A_{23} happens if $1'$ and $1''$ flavons are absent.

Charged lepton masses are a generic diagonal matrix, invariant under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

The aligment occurs because is based on A4 group theory:

ϕ_T breaks A4 down to G_T
ϕ_S breaks A4 down to G_S
(G_T, G_S: subgroups generated by T, S)

Note that for TB mixing in A4 it is important that no flavons transforming as 1′ and 1″ exist

Recently Lam claimed that for “a natural” TB model the smallest group is S4 (instead A4 is a subgroup of S4)

This is because he calls “natural” a model only if all possible flavons are introduced

We do not accept this criterium:

In physics we call natural a model if the lagrangian is the most general given the symmetry and the representations of the fields
(for example the SM is natural even if only Higgs doublets are present)

Many versions of A4 models exist by now

- with dim-5 effective operators or with see-saw
- with SUSY or without SUSY
- in 4 dimensions or in extra dimensions

  e.g G.A., Feruglio'05; G.A., Feruglio, Lin '06; Csaki et al '08.....
- with different solutions to the alignment problem

  e.g Hirsch, Morisi, Valle '08
- with sequential (or form) dominance

  e.g King'07 ; Chen, King '09
- with charged lepton hierarchy also following from a special alignment (no $U(1)_{FN}$) Lin'08
- extension to quarks, possibly in a GUT context

Recent directions of research:

- Different (larger) finite groups

Ma;
Kobayashi et al;
Luhn, Nasri, Ramond [$\Delta(3n^2)$];
.....

- Trying to improve the quark mixings

Carr, Frampton
Feruglio et al
Frampton, Kephart.....

- Construct GUT models with approximate tribimaximal mixing

it is indeed possible, also for A4!

GA, Feruglio, Hagedorn 0802.0090

Ma, Sawanaka, Tanimoto; Ma;
Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C x SU(2)_L x SU(2)_R$]; Antusch et al;
Chen, Mahanthappa; Bazzocchi et al [$\Delta(27)$]; .....

SUSY-SU(5) GUT with A4

G.A., Feruglio, Hagedorn 0802.0090

Key ingredients:

- SUSY

  In general SUSY is crucial for hierarchy, coupling unification and p decay
  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective
  Here ED also contribute to produce fermion hierarchies

- Extended flavour symmetry: $A4xU(1)xZ_3xU(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

SUSY-SU(5) GUT with A4

Key ingredients:

- GUT's in 5 dimensions
- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

Froggatt-Nielsen

Reduces to R-parity when SUSY is broken at m_{soft}

Keeps ϕ_S and ϕ_T separate

Field	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_T	φ_S	$\xi,\ \tilde{\xi}$	θ	θ''	φ_0^T	φ_0^S	ξ_0
SU(5)	1	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1
A_4	3	3	$1''$	$1'$	1	1	$1'$	3	3	1	1	$1''$	3	3	1
U(1)	0	0	3	1	0	0	0	0	0	0	-1	-1	0	0	0
Z_3	ω	ω	ω	ω	ω	ω	ω	1	ω	ω	1	1	1	ω	ω
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0	2	2	2

U(1) breaking flavons

driving fields for alignment

● : in bulk

In this model a good description of all quark and lepton masses is obtained.

As for all U(1) models only $o(\lambda^p)$ predictions can be given (modulo $o(1)$ coeff.s)

TB mixing for neutrinos is reproduced in first approximation

Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$ (in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r (nominally of $o(\lambda^2)$ and 1 respectively)

Normal hierarchy is favoured, degenerate ν's are excluded

But agreement with TB mixing could be accidental

If θ_{13} is found near its present bound this would hint that TB is accidental and bimaximal mixing (BM) could be a better first approximation

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$$ Raidal'04

Suggests bimaximal mixing in 1st approximation, corrected by charged lepton diagonalization.

Recall that

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$ $\lambda_C = \sin\theta_C$

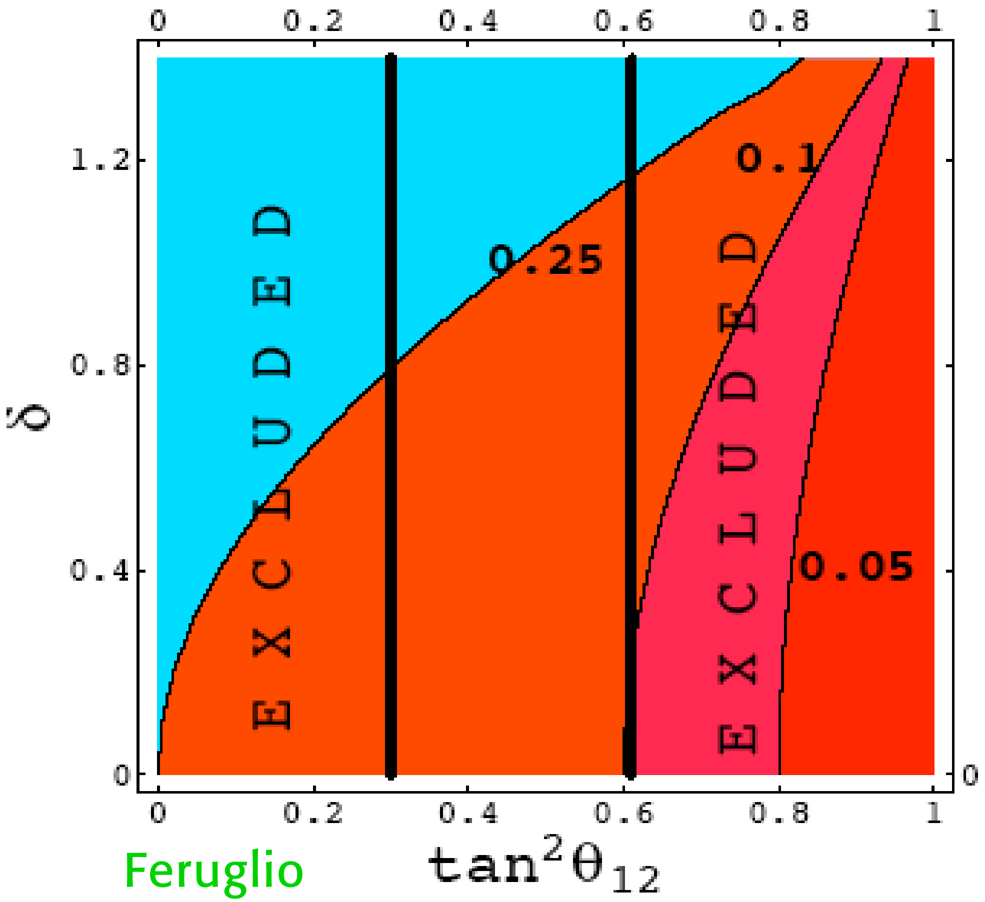
While $\theta_{12} + o(\theta_C) \sim \pi/4$ is easy to realize, exactly $\theta_{12} + \theta_C \sim \pi/4$ is more difficult: no compelling model

Minakata, Smirnov'04

Suggests that deviations from BiMaximal mixing arise from charged lepton diagonalisation (BM: $\theta_{12} = \theta_{23} = \pi/4 \;\; \theta_{13} = 0$)

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina
Frampton et al
Petcov et al
King
Antusch et al........



Feruglio

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12} e^{-i\alpha_2} + s^e_{13} e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12} e^{-i\alpha_2} - s^e_{13} e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2} \frac{1 + s^e_{23} e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

Here we construct a model where BM mixing holds in 1st approximation and is then corrected by terms o(λ_C) from diagonalisation of charged leptons

Revisiting Bimaximal Neutrino Mixing in a Model with S_4 Discrete Symmetry

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just appeared on the web arXiv: 0903.1940

BM mixing

$$\theta_{12} = \theta_{23} = \pi/4,\ \theta_{13} = 0$$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

By adding $\sin^2\theta_{12} \sim 1/2$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Bimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$$m_1 = x + \sqrt{2}y$$

$$m_2 = x - \sqrt{2}y$$

$$m_3 = 2z - x$$

The 3 remaining parameters are the mass eigenvalues

BM corresponds to $\tan^2\theta_{12}=1$
while exp.: $\tan^2\theta_{12}= 0.45 \pm 0.04$
so a large correction is needed

Bimaximal Mixing

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \, \mathrm{diag}(m_1, m_2, m_3) \, U^T$

$$m_{\nu BM} = \left[\frac{m_3}{2} M_3 + \frac{m_2}{4} M_2 + \frac{m_1}{4} M_1 \right]$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \; M_2 = \begin{pmatrix} 2 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & 1 \\ -\sqrt{2} & 1 & 1 \end{pmatrix}, \; M_1 = \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & 1 & 1 \end{pmatrix}$$

Eigenvectors: $(\sqrt{2}, 1, 1)/2, \; (-\sqrt{2}, 1, 1)/2 \; , \; (0, 1, -1)/\sqrt{2}$

BM mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

m is the most general matrix invariant under
SmS = m and $A_{23} m A_{23} = m$ with:

$$S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

Invariance under S can be made automatic in S4 while
invariance under A_{23} happens if the flavon content is suitable

S4: Group of permutations of 4 objects (24 transformations)

Irreducible representations: 1, 1′, 2, 3, 3′

$$S^2 = T^4 = (ST)^3 = (TS)^3 = 1$$

1 $\quad T = 1 \qquad S = 1$

2 $\quad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3 $\quad T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix} \qquad S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

1 <-> 1′ and 3<-> 3′ by changing S, T <-> -S, -T

Symmetry: S4xZ4xU(1)$_{FN}$xU(1)$_R$

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_l	χ_l	ψ_l^0	χ_l^0	ξ_ν	φ_ν	ξ_ν^0	φ_ν^0
S_4	3	1	1′	1	3	1	1	3	3′	2	3′	1	3	1	3
Z_4	1	-1	-i	-i	1	1	1	i	i	-1	-1	1	1	1	1
$U(1)_{FN}$	0	2	1	0	0	0	-1	0	0	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	1	0	0	0	2	2	0	0	2	2

$$\begin{aligned} w_l \;=\;& \frac{y_e^{(1)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2} e^c(l\varphi_l\varphi_l) + \frac{y_e^{(2)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2} e^c(l\chi_l\chi_l) + \frac{y_e^{(3)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2} e^c(l\varphi_l\chi_l) + \\ &+ \frac{y_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c(l\chi_l)' + \frac{y_\tau}{\Lambda}\tau^c(l\varphi_l) + \dots \end{aligned}$$

$$w_\nu \;=\; y(\nu^c l) + M\Lambda(\nu^c\nu^c) + a(\nu^c\nu^c\xi_\nu) + b(\nu^c\nu^c\varphi_\nu) + \dots$$ ← see-saw

$$\frac{\langle\varphi_\nu\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} C \qquad \frac{\langle\xi_\nu\rangle}{\Lambda} = D \qquad \frac{\langle\varphi_l\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} A \qquad \frac{\langle\chi_l\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} B$$

Alignment along minimum of most general potential in LO

In leading order charged leptons are diagonal

$$m_l = \begin{pmatrix} (y_e^{(1)}B^2 - y_e^{(2)}A^2 + y_e^{(3)}AB)t^2 & 0 & 0 \\ 0 & y_\mu Bt & 0 \\ 0 & 0 & y_\tau A \end{pmatrix} v_d$$

$$\frac{\langle\theta\rangle}{\Lambda} = t$$

$U(1)_{FN}$ flavon VEV

and neutrinos show BM mixing

$$m_\nu^D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u \qquad M_N = \begin{pmatrix} 2M+2aD & -2bC & -2bC \\ -2bC & 0 & 2M+2aD \\ -2bC & 2M+2aD & 0 \end{pmatrix} \Lambda$$

Dirac

Majorana

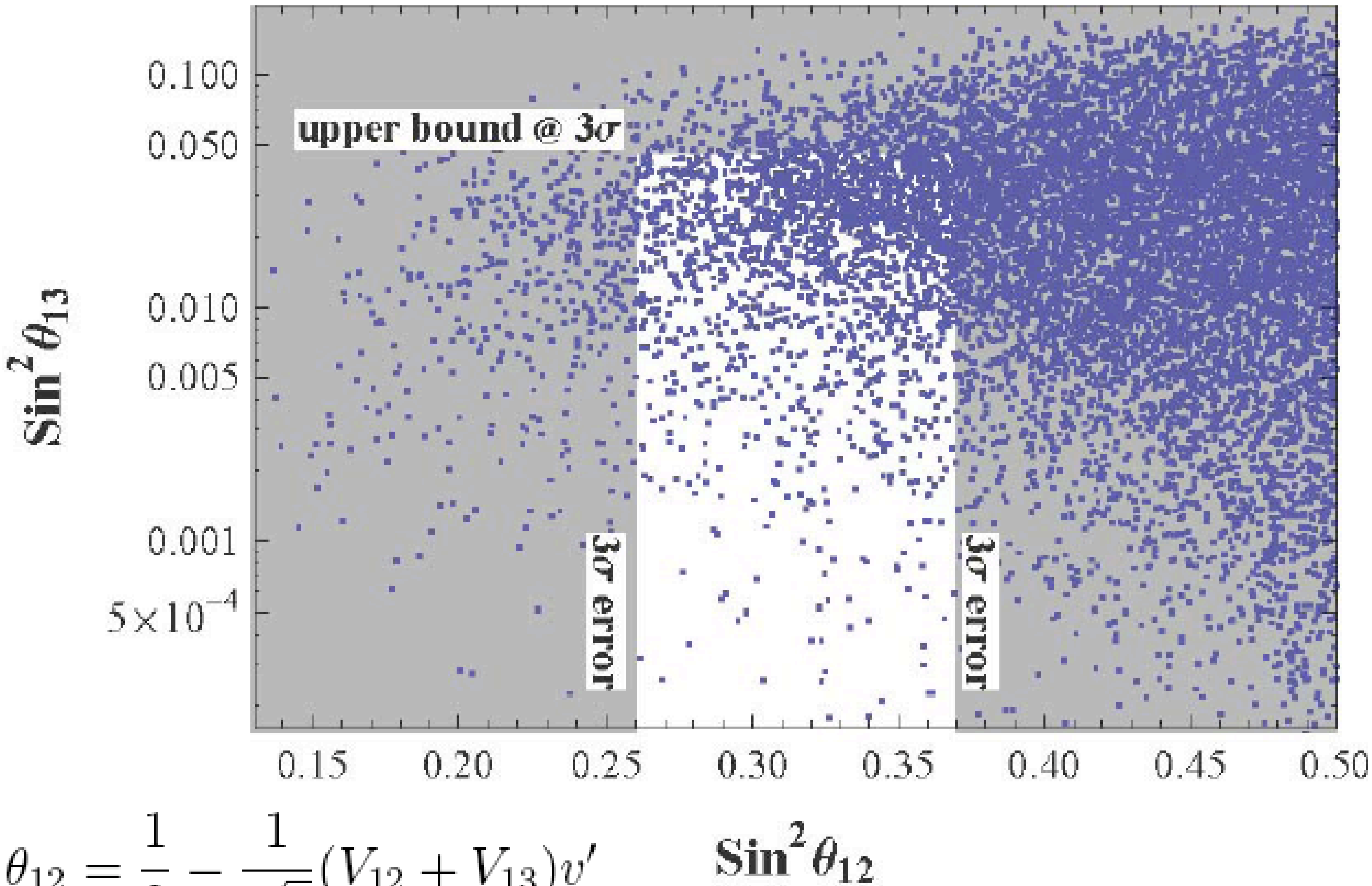
$$|m_1| = \frac{|y^2|v_u^2}{2|M+aD-\sqrt{2}bC|}\frac{1}{\Lambda} \qquad |m_2| = \frac{|y^2|v_u^2}{2|M+aD+\sqrt{2}bC|}\frac{1}{\Lambda} \qquad |m_3| = \frac{|y^2|v_u^2}{2|M+aD|}\frac{1}{\Lambda}$$

$$A \sim B \sim v\,, \qquad C \sim D \sim v'$$

In this model BM mixing is exact at LO

For the special flavon content chosen, at NLO θ_{12} and θ_{13} are corrected only from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections at NNLO(great!)
[for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K, CHOOZ2......



$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}(V_{12}+V_{13})v'$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}(V_{12}-V_{13})v'$$

$|V_{ij}|$ =0-2, v′=0.15

Conclusion

The observed pattern of neutrino masses can be accommodated in different models

For example, TB mixing from A4 with small corrections or BM with large corrections from charged lepton diag.

Quark and lepton mixings can be described together and GUT schemes are also possible

But no compelling illumination about the dynamics of flavour has emerged so far.