Neutrino Mass Models

- Why BSM?
- Neutrino mass models decision tree
- Survey of approaches
- TBM, $A_4$, Form Dominance, CSD
- Family symmetry and GUTs
- Mixing Sum Rules
1. There are no right-handed neutrinos $\nu_R$
2. There are only Higgs doublets of $\text{SU}(2)_L$
3. There are only renormalizable terms

In the Standard Model these conditions all apply so neutrinos are massless, with $\nu_e$, $\nu_\mu$, $\nu_\tau$ distinguished by separate lepton numbers $L_e$, $L_\mu$, $L_\tau$

Neutrinos and anti-neutrinos are distinguished by the total conserved lepton number $L = L_e + L_\mu + L_\tau$

To generate neutrino mass we must relax 1 and/or 2 and/or 3
Staying within the SM is not an option – but what direction?
LSND True or False?

MiniBoone does not support LSND result
does support three neutrinos

In this talk we assume that LSND is false
Dirac or Majorana?

Majorana masses

\[
\begin{align*}
&\begin{cases}
m_{LL} \bar{\nu}_L \nu_L^c \\
M_{RR} \bar{\nu}_R \nu_R^c
\end{cases} \\
&\begin{cases}
m_{LR} \bar{\nu}_L \nu_R \\
\end{cases}
\end{align*}
\]

Dirac mass

CP conjugate

Violates L
Violates \(L_e, L_\mu, L_\tau\)
Neutrino=antineutrino

Conserves L
Violates \(L_e, L_\mu, L_\tau\)
Neutrino \(\neq\) antineutrino

Petcov talk
1st Possibility: Dirac

Recall origin of electron mass in SM with

\[ L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad e^-_R, \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \]

\[ \lambda_e \bar{L} H e^-_R = \lambda_e \left< H^0 \right> \bar{e}_L e^-_R \]

Yukawa coupling \( \lambda_e \) must be small since \( \left< H^0 \right> = 175 \text{ GeV} \)

\[ m_e = \lambda_e \left< H^0 \right> \approx 0.5 \text{ MeV} \leftrightarrow \lambda_e \approx 3 \times 10^{-6} \]

Introduce right-handed neutrino \( \nu_{eR} \) with zero Majorana mass

\[ \lambda_v \bar{L} H^c \nu_{eR} = \lambda_v \left< H^0 \right> \bar{v}_{eL} \nu_{eR} \]

then Yukawa coupling generates a Dirac neutrino mass

\[ m^\nu_{LR} = \lambda_v \left< H^0 \right> \approx 0.2 \text{ eV} \leftrightarrow \lambda_v \approx 10^{-12} \]

\begin{flushright}
Why so small? \\
– extra dimensions
\end{flushright}
Flat extra dimensions with RH neutrinos in the bulk

Dienes, Dudas, Gherghetta; Arkhani-Hamed, Dimopoulos, Dvali, March-Russell

For one extra dimension \( y \) the \( \nu_R \) wavefunction spreads out over the extra dimension, leading to a volume suppressed Yukawa coupling at \( y=0 \)

\[
\rightarrow m_{LR}^\nu = \frac{\lambda \langle H^0 \rangle}{\sqrt{V}} = \lambda \langle H^0 \rangle \frac{M_{\text{string}}}{M_{\text{Planck}}}
\]

\[\text{e.g.}\]
\[
\frac{M_{\text{string}}}{M_{\text{Planck}}} = \frac{10^7}{10^{19}} = 10^{-12}
\]
Warped extra dimensions with SM in the bulk

Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

Randall-Sundrum; Rubakov, Gherghetta, Binetruy,…
Aside: some models with warped extra dimensions address the problem of dark energy in the Universe.

Neutrino Telescopes studying neutrinos from GRBs may be able to shed light on Neutrino Mass, Quantum Gravity and Dark Energy.
2nd Possibility: Majorana

Renormalisable
\[ \Delta L = 2 \text{ operator} \quad \lambda_v \, L L \Delta \]
where \( \Delta \) is light Higgs triplet with VEV < 8 GeV from \( \rho \) parameter

Non-renormalisable
\[ \Delta L = 2 \text{ operator} \quad \frac{\lambda_v}{M} \, L L H H = \frac{\lambda_v}{M} \langle H^0 \rangle^2 \, \bar{\nu}_e \nu^c_e \]
Weinberg

This is nice because it gives naturally small Majorana neutrino masses
\[ m_{LL} \sim \langle H^0 \rangle^2 / M \]
where \( M \) is some high energy scale

The high mass scale can be associated with some heavy particle of mass \( M \) being exchanged (can be singlet or triplet)

- Loop models
- RPV SUSY
- See-saw mechanisms
• Type I and II see-saw mechanism

**Type I see-saw mechanism**

\[ m^{I}_{LL} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T} \]

- Minkowski (1977)

**Type II see-saw mechanism**

\[ m^{II}_{LL} \approx \lambda_{\Delta} Y_{\Delta} \frac{V_{u}^{2}}{M_{\Delta}} \]

Type II contribution governs the neutrino mass scale and renders neutrino-less double beta decay observable.

Unit matrix type II contribution from an SO(3) family symmetry.

Hierarchical type I contribution controls the neutrino mixings and mass splittings.

\[ m^V_{LL} = m^{II} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - m_{LR} M_{RR}^{-1} m_{LR}^T \]
Very precise Tri-bimaximal mixing (TBM)?

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
-\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[
\theta_{12} = 35^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ.
\]

\[\text{c.f. data}\]
\[
\theta_{12} = 33.8^\circ \pm 1.4^\circ, \quad \theta_{23} = 45^\circ \pm 3^\circ, \quad \theta_{13} < 12^\circ
\]

\[
\text{• Current data is consistent with TBM}
\]

Harrison, Perkins, Scott

See other talks at this workshop for more up to date values
Consider the TB neutrino mass matrix in the flavour basis 
\(\text{i.e. diagonal charged lepton basis}\)

\[
M_{eff}^{\nu}_{\text{diag}} = U_{\text{TBM}}^{T}(M_{eff}^{\nu})^{TBM}U_{\text{TBM}} = (m_1, m_2, m_3)
\]

\[(M_{eff}^{\nu})^{TBM} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T\]

\[
\Phi_1 \Phi_1^T = \frac{1}{6} \begin{pmatrix}
4 & -2 & -2 \\
-2 & 1 & 1 \\
-2 & 1 & 1
\end{pmatrix}, \quad \Phi_2 \Phi_2^T = \frac{1}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}, \quad \Phi_3 \Phi_3^T = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & -1 \\
0 & -1 & 1
\end{pmatrix}
\]

\[
\Phi_1 = \frac{1}{\sqrt{6}} \begin{pmatrix}
-2 \\
1 \\
1
\end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
1 \\
-1
\end{pmatrix}
\]

Columns of \(U_{\text{TBM}}\)
How to achieve these relations in a model?

The most elegant models involve ≤ 3 parameters which satisfy these relations

Low, Volkas

Such a mass matrix is called form diagonalizable since it is diagonalized by the TBM matrix for all values of a, b, d

hence for all values of neutrino masses
Form Dominance

Form Dominance is a mechanism for achieving a form diagonalizable effective neutrino mass matrix starting from the type I see-saw mechanism.

Work in diagonal $M_{RR}$ basis

$$M_{RR} = \text{diag}(M_A, M_B, M_C)$$

$M_D$ is LR Dirac mass matrix

$$M_D = (A, B, C) \quad \text{A,B,C are column vectors}$$

$$M_{\nu eff} = M_D M_{RR}^{-1} M_D^T \rightarrow M_{\nu eff} = \frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C}$$

Form Dominance assumption: columns of Dirac mass matrix $\propto$ columns of $U_{TBM}$

$$A = a\Phi_1 = \frac{a}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad B = b\Phi_2 = \frac{b}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad C = c\Phi_3 = \frac{c}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\rightarrow (M_{\nu eff})^{TBM} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T$$

with

$$m_1 = \frac{a^2}{M_A}, \quad m_2 = \frac{b^2}{M_B}, \quad m_3 = \frac{c^2}{M_C}$$

N.B. Only three parameter combinations
Basis Invariance and the R matrix

In FD a particular RH neutrino mass eigenstate is associated with a particular light neutrino mass eigenstate

i.e. in FD the basis invariant Casas-Ibarra matrix $R$ is unit matrix

\[
\begin{pmatrix}
A_i M_A^{-1/2} & B_i M_B^{-1/2} & C_i M_C^{-1/2}
\end{pmatrix} =
\begin{pmatrix}
U_{i1} m_1^{1/2} & U_{i2} m_2^{1/2} & U_{i3} m_3^{1/2}
\end{pmatrix}
R^T
\]

This means that FD may be defined in a basis invariant way as $R=1$
Family Symmetry

Clearly TBM suggests a family symmetry, but one that is badly broken in the charged lepton sector.

Diagonal charged lepton basis Lagrangian \( L = L' + L^E \)

\[
L' = \begin{pmatrix} L_e & L_\mu & L_\tau \end{pmatrix} \begin{pmatrix} a & b & b \\ d & (a+b-d) & \cdot \\ \cdot & d & \cdot \end{pmatrix} \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \quad \text{Respects} \quad L_\mu \leftrightarrow L_\tau
\]

\[
L^E = \begin{pmatrix} L_e & L_\mu & L_\tau \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \quad m_e \ll m_\mu \ll m_\tau \quad \text{does not} \quad \text{respect} \quad L_\mu \leftrightarrow L_\tau
\]
Flavons and Vacuum Alignment

To achieve different symmetries in the neutrino and charged lepton sectors we need to align the Higgs fields which break the family symmetry (flavons) along different symmetry preserving directions (vacuum alignment)

e.g. consider $A_4 = \Delta_{12} = Z_3 \otimes Z_2 \times Z_2$ with reps 3,1,1',1''

Note that $Z_2^S$ respects $L_\mu \leftrightarrow L_\tau$ but $Z_3^T$ violates it

\[ T \quad S \quad TST^2 \]

\[
\begin{pmatrix}
-1 & 2 & 2 \\
2 & -1 & 2 \\
2 & 2 & -1
\end{pmatrix},
\begin{pmatrix}
1 & 0 & 0 \\
0 & \omega^2 & 0 \\
0 & 0 & \omega
\end{pmatrix}
\]

Altarelli, Feruglio

$A_4 \rightarrow Z_2^S$ via the triplet flavon $\phi_S$

\[
\frac{\langle \phi_S \rangle}{\Lambda} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \alpha_s \rightarrow \phi_S \text{ only occurs in } L^v
\]

$A_4 \rightarrow Z_3^T$ via the triplet flavon $\phi_T$

\[
\langle \phi_T \rangle = \begin{pmatrix} \nu_T \\ 0 \\ 0 \end{pmatrix} \quad \rightarrow \quad \phi_T \text{ only occurs in } L^E
\]
A$_4$ see-saw models satisfy form dominance

\[ N = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} \sim 3 \quad L = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'' \quad \tau_R \sim 1' \]

Model 1

\[ M_{RR} = \overline{N^c} N (\langle \phi_S \rangle + \langle u \rangle) = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} \Lambda \]

\[ M_D = y H \overline{L} N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v \quad \text{Diagonal RHN basis} \quad \begin{pmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \]

Model 2

\[ M_{RR} = M_R \overline{N^c} N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} M_R \]

\[ M_D = HLN \left( \frac{\langle \phi_S \rangle}{\Lambda} + \frac{\langle u \rangle}{\Lambda} \right) = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} v \rightarrow \begin{pmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \]

Both satisfy Form Dominance \( \Rightarrow \) R=1
Natural Form Dominance

The $A_4$ see-saw models are very economical since the neutrino sector only involves two flavon VEVs

$$\frac{\langle \phi_s \rangle}{\Lambda} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \alpha_s, \quad \frac{\langle u \rangle}{\Lambda} = \alpha_0$$

Model 1

$$\text{diag}(m_1, m_2, m_3) = \left( \frac{1}{3\alpha_s + \alpha_0}, \frac{1}{\alpha_0}, \frac{1}{3\alpha_s - \alpha_0} \right) \frac{y^2 v^2}{\Lambda} \rightarrow \frac{1}{m_1} - \frac{1}{m_3} = \frac{2}{m_2}$$

Model 2

$$(m_1, m_2, m_3) = \left( (3\alpha_s + \alpha_0)^2, \alpha_0^2, (3\alpha_s - \alpha_0)^2 \right) \frac{v^2}{M_R}$$

However some cancellations of VEVs are required to obtain $\Delta m^2_{atm}$ and $\Delta m^2_{sol}$

This suggests natural form dominance in which a different flavon is associated with each physical neutrino mass $\rightarrow$ 3 flavons $\Phi_{1,2,3}$

$$\langle \Phi_1 \rangle \rightarrow m_1$$
$$\langle \Phi_2 \rangle \rightarrow m_2$$
$$\langle \Phi_3 \rangle \rightarrow m_3$$
Constrained Sequential Dominance

A special case of Natural Form Dominance for $|m_1| \ll |m_2| < |m_3|$

\[<\Phi_3> = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \nu \quad <\Phi_2> = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tilde{\nu} \quad <\Phi'> = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \nu\]

\[M_D \sim \begin{pmatrix} 0 & \nu & \tilde{\nu} & 0 \\ \nu & \tilde{\nu} & 0 & V \\ -\nu & \tilde{\nu} & V & 0 \end{pmatrix}\]

Several examples of suitable non-Abelian Family Symmetries:

SFK, Ross; Velasco-Sevilla; Varzelias

\[
SU(3) \quad \Delta_{27} \\
SO(3) \quad A_4
\]

Discrete subgroups preferred by vacuum alignment

Note for negligible $m_1$ the flavon $\Phi_1$ is irrelevant and can be replaced by the flavon $\Phi'$.
Family $\times$ GUT symmetry

e.g. Chen and Mahanthappa $T' \times SU(5)$
Altarelli, Feruglio, Hagedorn $A_4 \times SU(5)$ (in 5d)
SFK, Malinsky $A_4 \times$ Pati-Salam
Varzielas, SFK, Ross $\Delta_{27} \times$ Pati-Salam/SO(10)
$G_{GUT}$

- $SU(5) \times U(1)$
- $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- $SU(3)_C \times SU(2)_L \times U(1)_Y$
- $SO(10)$
- $SU(3)_C \times SU(3)_L \times SU(3)_R$
- $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$
- $E_6$
G
Family

SU(3) is the largest family group usually considered

SU(3)

$\Delta_{54}$

$PSL_2(7)$

$\Delta_{27}$

$Z_2 \times Z_7$

SO(3)

SU(2)

$S_4$

$A_4$

$D_5$

$S_3$

$D_4$

$T'$
GUT relations

Class of models: $\theta_{12}^d \gg \theta_{12}^u$

$\theta_{12}^d \approx \theta_c$

Georgi-Jarlskog

GUT relation

$\theta_{12}^e \approx \frac{\theta_{12}^d}{3} \approx \frac{\theta_c}{3}$

See-saw $\Rightarrow$

$m_\nu = v_{EW}^2 Y_\nu M^{-1}_R Y^T_\nu$

$Y_u$, $V_{CKM}$, $Y_d$, $V_{MNS}$
\[ U_{PMNS} = V^E_L V^{\nu_L} \]

- Cabibbo-like
- Tri-bimaximal

\[ \theta_{13} \approx \frac{\theta_{12}^e}{\sqrt{2}} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^\circ \]

\[ \theta_{12} = 35^\circ + \frac{\theta_C}{3\sqrt{2}} \cos \delta \]

**Mixing Sum Rule**
- Bjorken; Ferrandis, Pakvasa; SFK
- Antusch, SFK, Malinsky, SFK, Boudjemaa

\[ s \approx r \cos \delta + \eta \left( \frac{1}{6} - \frac{1}{3 m_2^\nu} \cos \alpha_2 \right) \]

\[ s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}} (1 + s), \quad s_{23} = \frac{1}{\sqrt{2}} (1 + a) \]

**Oscillation phase**

**RG correction < 1°**

- Antusch, SFK, Malinsky, SFK, Boudjemaa
Conclusion

- **Neutrino mass and mixing requires new physics BSM**
- Many roads for model building, but answers to key experimental questions will provide the signposts
- If TBM is accurately realised this may imply a new symmetry of nature: family symmetry broken by flavons
- **See-saw naturally leads to TBM via Form Dominance**
- **GUTs** \(\times\) family symmetry with see-saw + FD is very attractive framework for TBM \(\rightarrow\) sum rule prediction
- The sum rule underlines the importance of showing that the deviations from TBM \(r,s,a\) are non-zero and measuring them and CP phase \(\delta\)
- Neutrino Telescopes may provide a window into neutrino mass, quantum gravity and dark energy