

NEUTEL'09, Venice

March 11, 2009

**Core-collapse supernovae:**  
*When neutrinos  
get to know each other*

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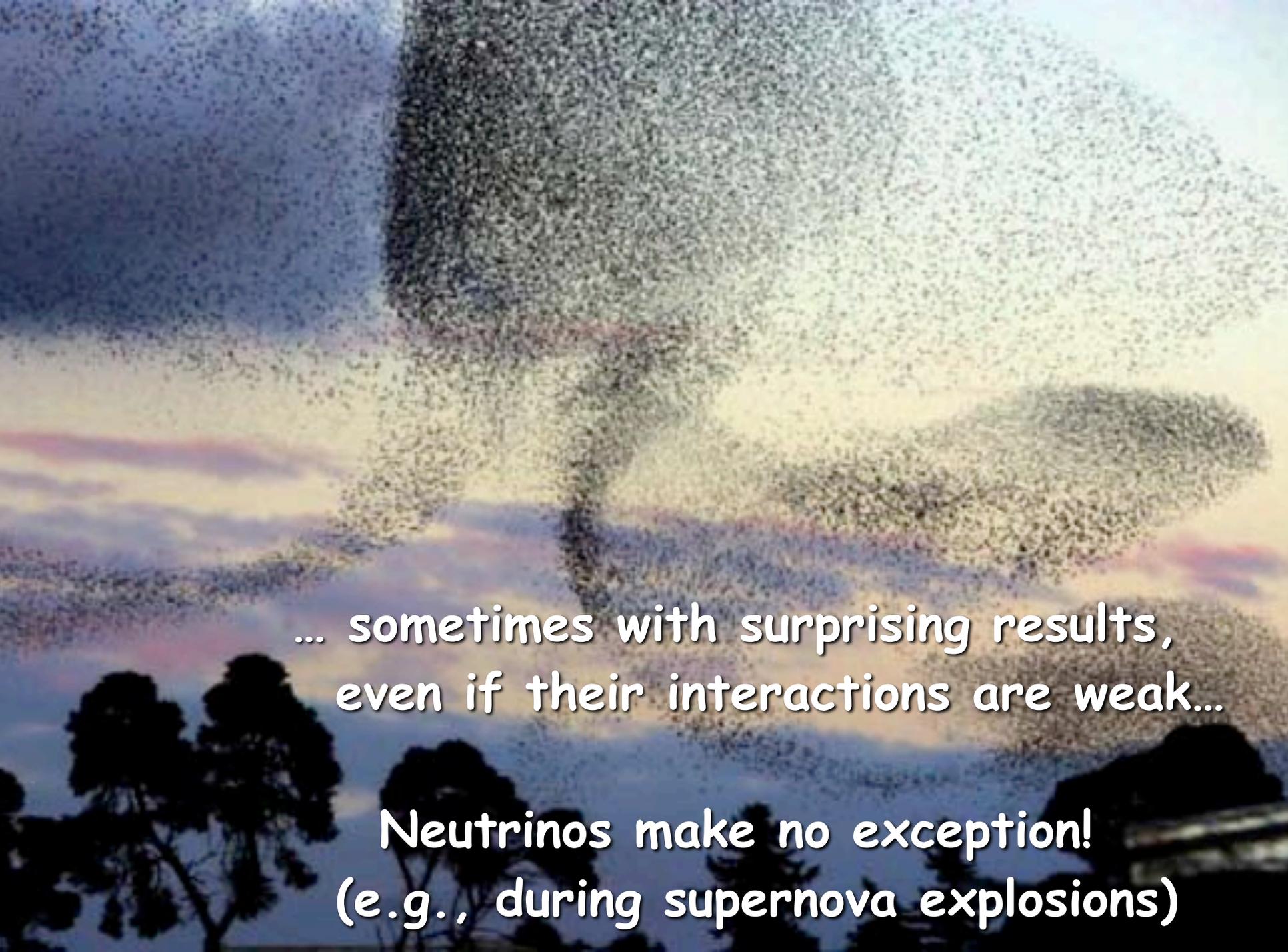
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In low-density environments,  
individuals tend to behave  
independently from each other...





... but they may behave coherently  
when densely packed...

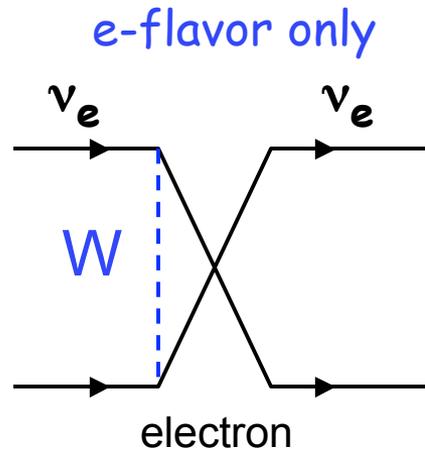
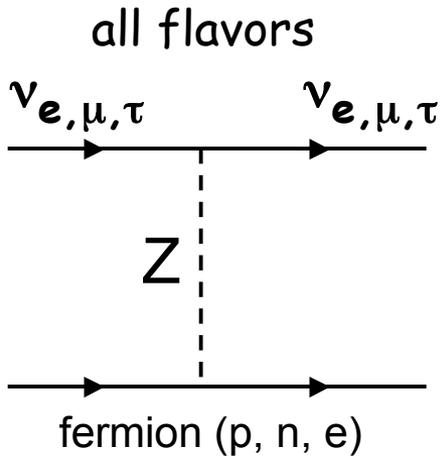
A large flock of birds, likely geese, is captured in flight against a dramatic sunset sky. The birds are arranged in a V-formation, with a dense lead group at the top and a trailing group below. The sky transitions from a deep blue at the top to a bright yellow and orange near the horizon, with soft, wispy clouds. The bottom of the image shows the dark silhouettes of trees and a building, suggesting the birds are flying over a landscape.

... sometimes with surprising results,  
even if their interactions are weak...

Neutrinos make no exception!  
(e.g., during supernova explosions)

## Low neutrino density

In this case, neutrino flavor transitions are mainly sensitive to the difference in  $\nu_e$  forward scattering amplitude ( $\propto G_F$ ) over background fermions (Mikheyev-Smirnov-Wolfenstein effect)



Interaction energy difference depends on electron density  $N_e$ :

$$\lambda = \sqrt{2} G_F N_e$$

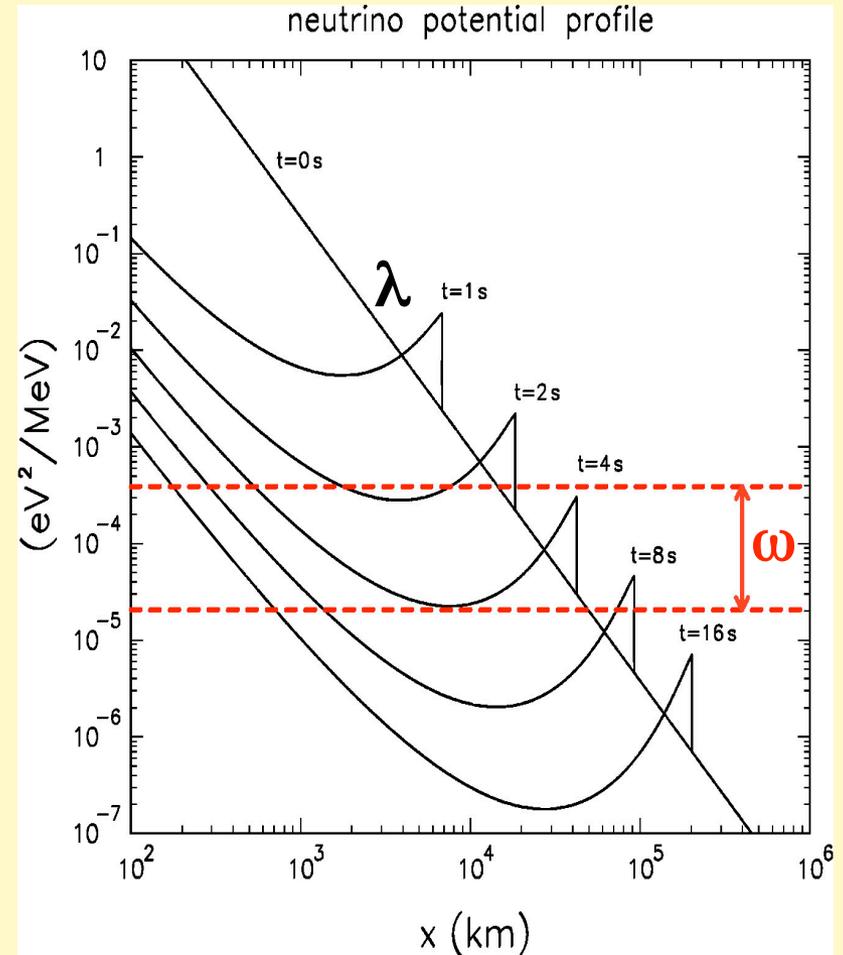
Typical SN  $\nu$  energies,  $E \sim O(10)$  MeV, are below threshold for  $\mu$  and  $\tau$  and production via CC. The  $\nu_\mu$  and  $\nu_\tau$  behave in a similar way during production, propagation, detection, and are often denoted by a common symbol  $\nu_x$ .

# Low neutrino density (cont'd)

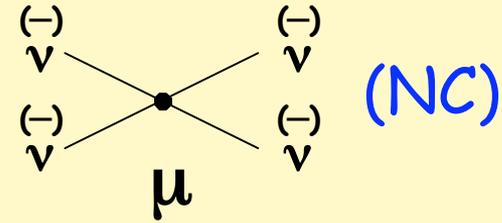
Well-known MSW effects can occur in a SN envelope when the  $\nu$  potential  $\lambda = \sqrt{2} G_F N_e$  is close to osc. frequency  $\omega = \Delta m^2 / 2E$  ( $\Delta m^2 = |m^2_3 - m^2_{1,2}|$ ,  $\theta_{13} \neq 0$ ).

For  $t \sim$  few sec after bounce,  
 $\lambda \sim \omega$  at  $x \gg 10^2$  km (large radii).

What about small radii?  
Popular wisdom:  
 $\lambda \gg \omega$  at  $x < O(10^2)$  km,  
thus flavor transitions suppressed.  
**Incorrect!**



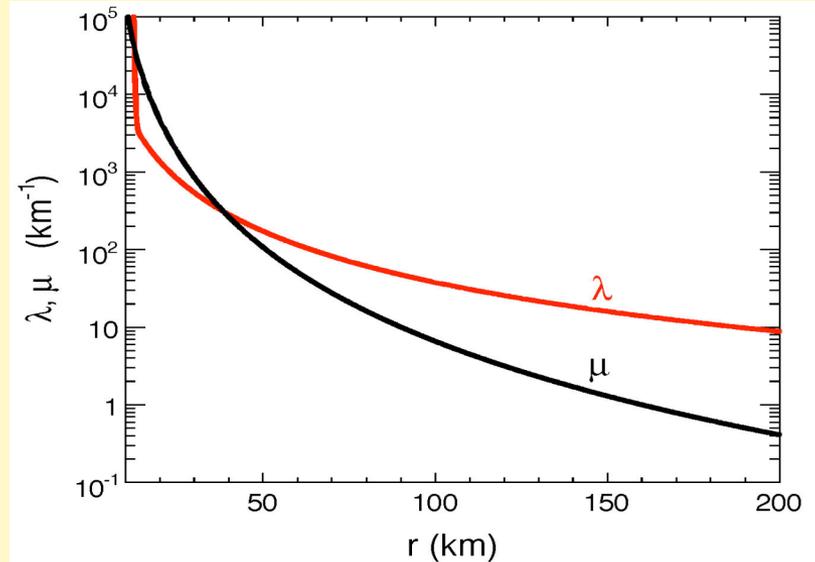
**Large neutrino density:**  
 neutrinos "get to know each other"  
 via neutral currents!



At small  $r$ , neutrino and antineutrino density ( $n$  and  $\bar{n}$ ) high enough to make self-interactions important. Strength:  
 $\mu = \sqrt{2} G_F (n + \bar{n})$

Angular modulation factor:  $(1 - \cos\Theta_{ij})$   
 If averaged: "single-angle" approxim.  
 Otherwise : "multi-angle" (difficult)

Self-interaction effects known for  $\sim 20$  y in SN. But, recent boost of interest after new crucial results by  
 Duan, Fuller, Carlson, Qian '05-'06



**Lesson:** self-interactions ( $\mu$ ) can induce non-MSW, nonlinear flavor evolution at small radii, despite the large matter density  $\lambda$

# Recent “wave” of papers on SN neutrino self interactions (time-ordered):

[01] Fuller & Qian	<a href="#">astro-ph/0505240</a>	
[02] Duan, Fuller & Qian	<a href="#">astro-ph/0511275</a>	<- The “synchronized” and “bipolar” regimes
[03] Duan, Fuller, Carlson & Qian	<a href="#">astro-ph/0606616</a>	<- Large-scale multi-angle calculations
[04] Balantekin & Pehlivan	<a href="#">astro-ph/0607527</a>	
[05] Duan, Fuller, Carlson & Qian	<a href="#">astro-ph/0608050</a>	
[06] Hannestad, Raffelt, Sigl & Wong	<a href="#">astro-ph/0608695</a>	<- The “flavor pendulum” analogy
[07] Raffelt & Sigl	<a href="#">hep-ph/0701182</a>	
[08] Duan, Fuller, Carlson & Qian	<a href="#">astro-ph/0703776</a>	
[09] Raffelt and Smirnov	<a href="#">hep-ph/0705.1830</a>	<- The “spectral split”
[10] Esteban, Pastor, Tomas, Raffelt & Sigl	<a href="#">astro-ph/0706.2498</a>	
[11] Duan, Fuller & Qian	<a href="#">astro-ph/0706.4293</a>	
[12] Duan, Fuller, Carlson & Qian	<a href="#">astro-ph/0707.0290</a>	<- The “spectral split”
[13] Fogli, Lisi, Marrone & Mirizzi	<a href="#">hep-ph/0707.1998</a>	<- Our work ( <b>this talk</b> )
[14] Raffelt & Smirnov	<a href="#">hep-ph/0709.4641</a>	
[15] Duan, Fuller, Carlson & Qian	<a href="#">astro-ph/0710.1271</a>	
[16] Esteban, Pastor, Tomas, Raffelt & Sigl	<a href="#">astro-ph/0712.1137</a>	<- The mu-tau flavor difference
[17] Dasgupta & Dighe	<a href="#">hep-ph/0712.3798</a>	<- The three neutrino formalism
[18] Duan, Fuller & Qian	<a href="#">hep-ph/0801.1363</a>	<- Three-neutrino spectral split
[19] Dasgupta, Dighe, Mirizzi & Raffelt	<a href="#">hep-ph/0801.1660</a>	<- Three-neutrino spectral split
[20] Dasgupta, Dighe & Mirizzi	<a href="#">hep-ph/0802.1481</a>	<- Earth effects + hierarchy
[21] Duan, Fuller & Carlson	<a href="#">astro-ph/0803.3650</a>	
[22] Chakraborty, Choubey, Dasgupta & Kar	<a href="#">hep-ph/0805.3131</a>	
[23] Dasgupta, Dighe, Mirizzi & Raffelt	<a href="#">hep-ph/0805.3300</a>	<- Non-spherical geometry
[24] Gava & Volpe	<a href="#">astro-ph/0807.3418</a>	<- CP-violation effects
[25] Esteban-Pretel & al.	<a href="#">astro-ph/0807.0659</a>	
[26] Duan, Fuller & Qian	<a href="#">astro-ph/0808.2046</a>	
[27] Fogli, Lisi, Marrone, Mirizzi & Tamborra	<a href="#">hep-ph/0808.0807</a>	<- Our work ( <b>this talk</b> )
[28] Dighe	<a href="#">hep-ph/0809.2977</a>	
[29] Raffelt	<a href="#">hep-ph/0810.1407</a>	
[30] Blennow, Mirizzi, Serpico	<a href="#">hep-ph/0810.2297</a>	<- Nonstandard interactions
[31] Fogli, Lisi, Marrone, Tamborra	<a href="#">hep-ph/0812.3031</a>	<- Our work ( <b>this talk</b> )
[32] Sigl & al.	<a href="#">hep-ph/0901.0725</a>	
[33] Gava, Kneller, Volpe, McLaughlin	<a href="#">hep-ph/0902.0317</a>	

# Outline

- Coupled equations of motion
- The "pendulum" and the "split"
- Discussion of our work(\*)
- Conclusions

(\*) [hep-ph/0707.1998](#)  
[hep-ph/0808.0807](#)  
[hep-ph/0812.3031](#)

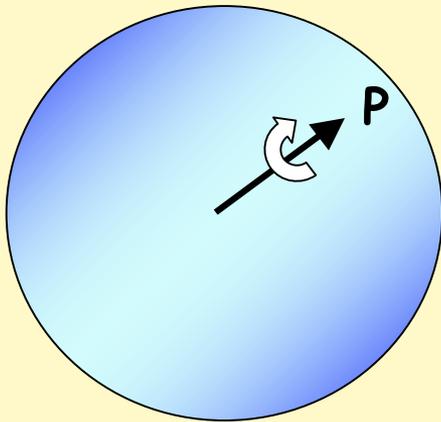
# Coupled equations of motion (for 2 flavors, $e$ and $x=\mu,\tau$ )

Neutrino wavefunction sensitive to neutrino density  $\rightarrow$  use density matrix.

Liouville equations:  $i\partial_t \rho = [H, \rho]$  (for each neutrino mode)

Decompose 2x2 (anti)neutrino density matrix over Pauli matrices to get a "polarization" (Bloch) 3-vector  $\mathbf{P} = (P_1, P_2, P_3) = (P_x, P_y, P_z)$ . [Ditto for  $H$ .]

Bloch equations:  $\partial_t \mathbf{P} = \mathbf{V} \times \mathbf{P}$  (precession-like,  $|\mathbf{P}| = \text{const}$ )



Any mode  $P$  moves on a Bloch sphere (abstract "flavor space").

"up" direction :  $\nu_e$  flavor

"down" direct. :  $\nu_x$  flavor

generic direct. : mixed flavor state

Probability  $P_{ee}$  related to  $P_3 = P_z$

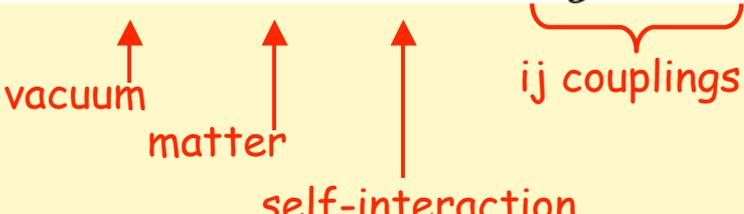
## Coupled equations of motion (cont'd)

The problem is that there are lots of kinematical neutrino modes: continuous distributions over energy and angle(s)  $\rightarrow$  no less than  $\infty^2$  !

Discretize over energy spectrum ( $N_E$  bins), and over angular distribution in multi-angle simulations ( $N_\theta$  bins)  $\rightarrow$  Get discrete index (indices),  $P_i$ .

Evolution governed by  $6 \times N_E \times N_\theta$  coupled Bloch equations of the form (or, in single-angle, by  $6 \times N_E$  coupled equations):

$$\begin{aligned}\dot{\mathbf{P}}_i &= \mathbf{V}_{\text{ector}}[+\omega, \lambda, \mu, \mathbf{P}_j, \bar{\mathbf{P}}_j] \times \mathbf{P}_i \\ \dot{\bar{\mathbf{P}}}_i &= \mathbf{V}_{\text{ector}}[-\omega, \lambda, \mu, \mathbf{P}_j, \bar{\mathbf{P}}_j] \times \bar{\mathbf{P}}_i\end{aligned}$$

  
vacuum      matter      self-interaction      ij couplings

Large, "stiff" set of (strongly) coupled differential equations.

## EOM in single-angle approximation:

$$\begin{aligned}\dot{\mathbf{P}} &= \left[ +\omega\mathbf{B} + \lambda\mathbf{z} + \mu \sum_E (\mathbf{P} - \bar{\mathbf{P}}) \right] \times \mathbf{P} \\ \dot{\bar{\mathbf{P}}} &= \left[ -\omega\mathbf{B} + \lambda\mathbf{z} + \mu \sum_E (\mathbf{P} - \bar{\mathbf{P}}) \right] \times \bar{\mathbf{P}}\end{aligned}$$

$$\begin{aligned}\mathbf{P} &= \mathbf{P}_E \\ \mathbf{z} &= (0, 0, 1)^T \\ \mathbf{B} &= (\sin 2\theta_{13}, 0, \pm \cos 2\theta_{13})\end{aligned}$$

## Subcases and analogies:

- |  |           |
|--|-----------|
| (1) $\dot{\mathbf{P}} = [+ \omega\mathbf{B}] \times \mathbf{P}$  | Vacuum    |
| (2) $\dot{\mathbf{P}} = [+ \omega\mathbf{B} + \lambda\mathbf{z}] \times \mathbf{P}$  | MSW       |
| (3) $\dot{\mathbf{P}} = [+ \omega\mathbf{B} + \lambda\mathbf{z} + \mu \sum_E (\mathbf{P} - \bar{\mathbf{P}})] \times \mathbf{P}$ | Self-int. |

- (1) Precessing "spin"  $\mathbf{P}$  in external "magnetic" field  $\mathbf{B}$  with frequency  $\omega$
- (2) Precession in magnetic field with variable frequency and direction
- (3) Precession in variable m.f. plus additional fields from other spins  
-> collective effects and spin coupling show up

Strong couplings between polarization vectors make the problem difficult, but also make an analytical understanding possible after all !  
 Key tool of "near-alignment" or "strong polarization", e.g.:



In general, bunches of  $P$ 's seem to be "pinned" to some global vector, provided that the global configurations obey net lepton # conservation:

$$(\nu_e \bar{\nu}_e \rightarrow \nu_x \bar{\nu}_x, \text{ but } \#(\nu_e - \bar{\nu}_e) = \text{const})$$

Equations become more complicated in three flavors:

- Need to project  $3 \times 3$  density matrices onto Gell-Mann matrices
- Get 8-dimensional polarization vectors (larger system of eqs.)
- Need to include/check possible subtle 3-neutrino effects

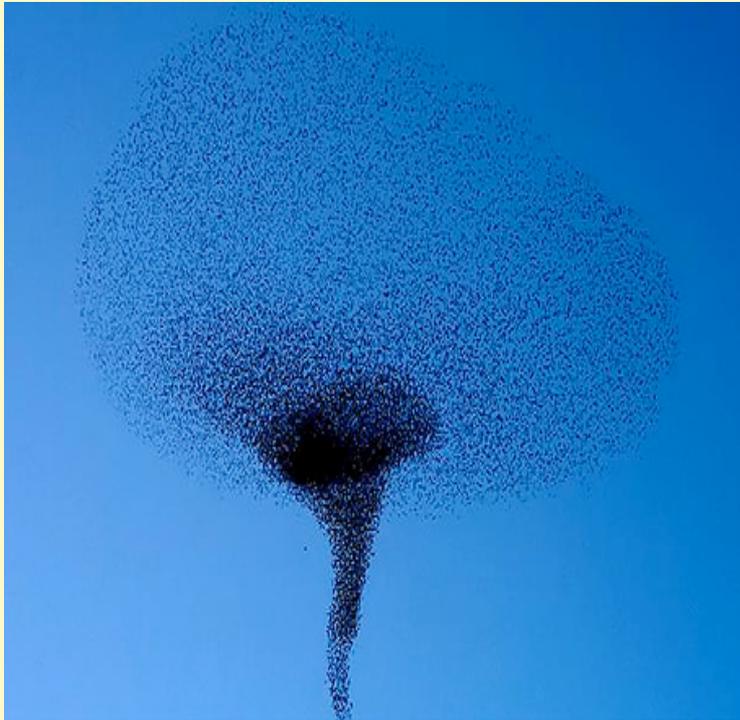
Note that three-family scenarios do not admit exact two-family subcases in the presence of neutrino self-interactions:

e.g., it matters whether the absolute SN neutrino luminosity is shared among three or two families.

[Unlike the “usual” neutrino oscillations at low density, whose description does not depend on the beam luminosity.]

Nevertheless, in both two- and three-flavor scenarios, similar features characterize the most important collective phenomena, such as...

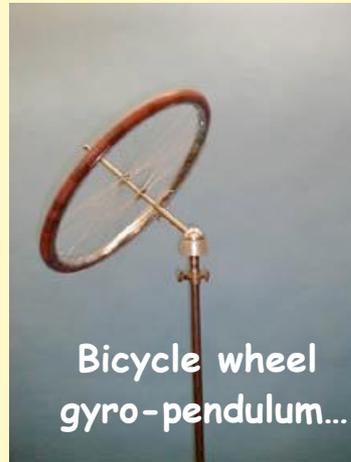
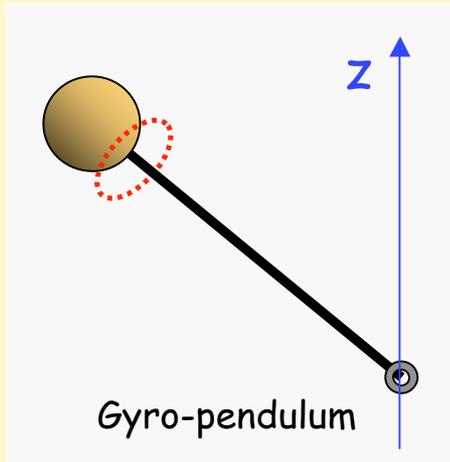
... the flavor pendulum



... and the spectral split



One global polarization vector obeys the eqs. of a gyroscopic pendulum!



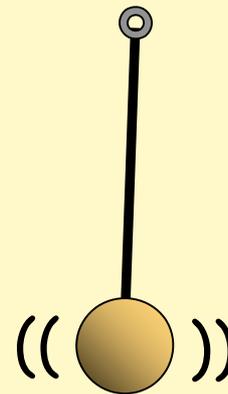
Roughly speaking:

$\text{Mass}^{-1} \sim (\text{anti})\text{neutrino density}$

$\text{Spin} \sim \#\text{neutrino} - \#\text{antineutrino}$

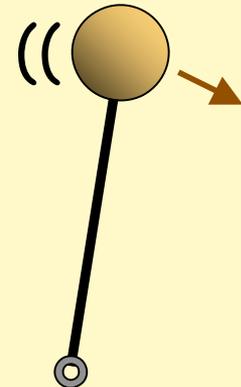
### Normal hierarchy:

Pendulum starts in  $\sim$ downward (stable) position and stays nearby. No significant flavor change.



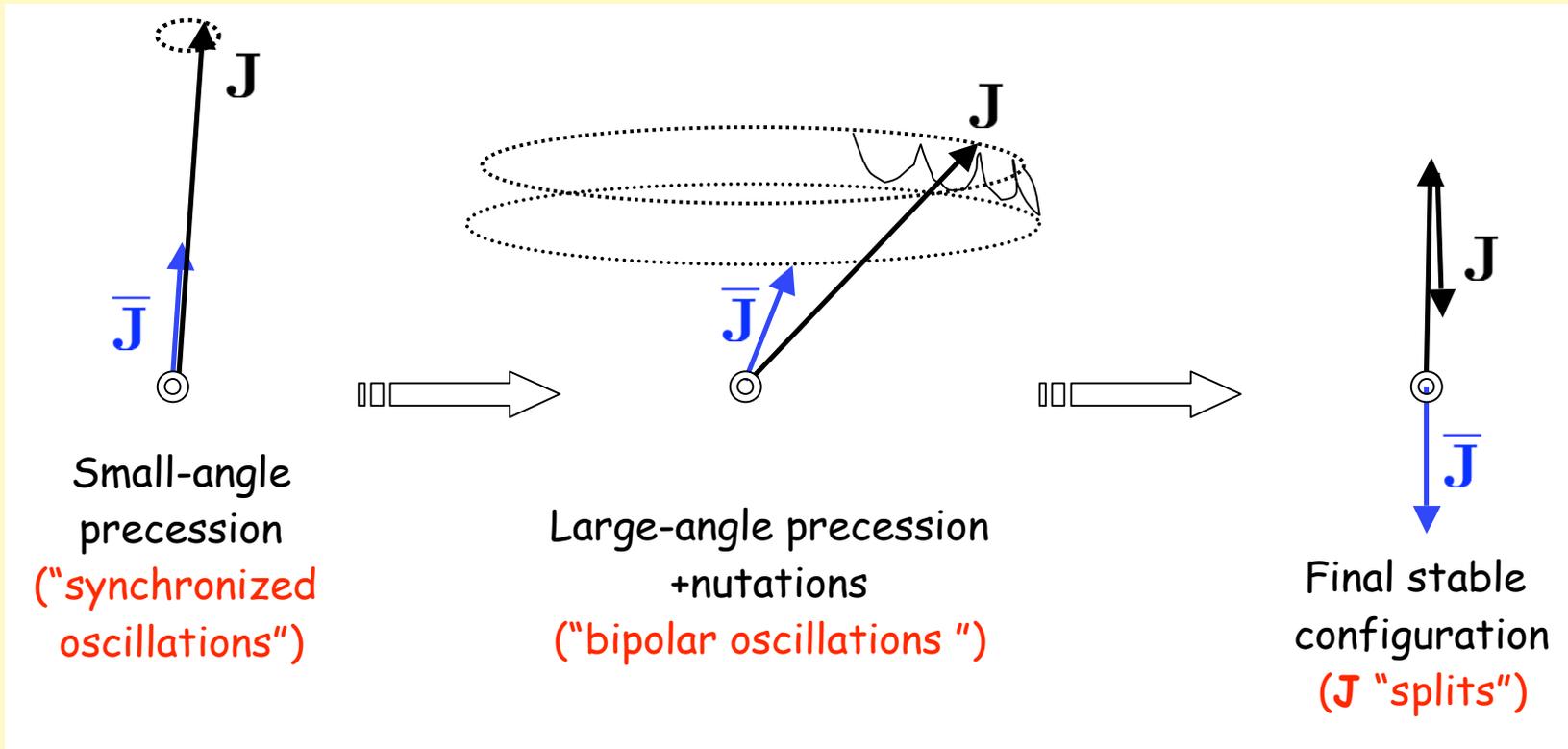
### Inverted hierarchy:

Pendulum starts in  $\sim$ upward (unstable) position and eventually falls down (wiggling) for any  $\theta_{13} > 0$ ! Significant flavor changes.



Hereafter, inv. hierarchy and  $\theta_{13} > 0$  assumed.

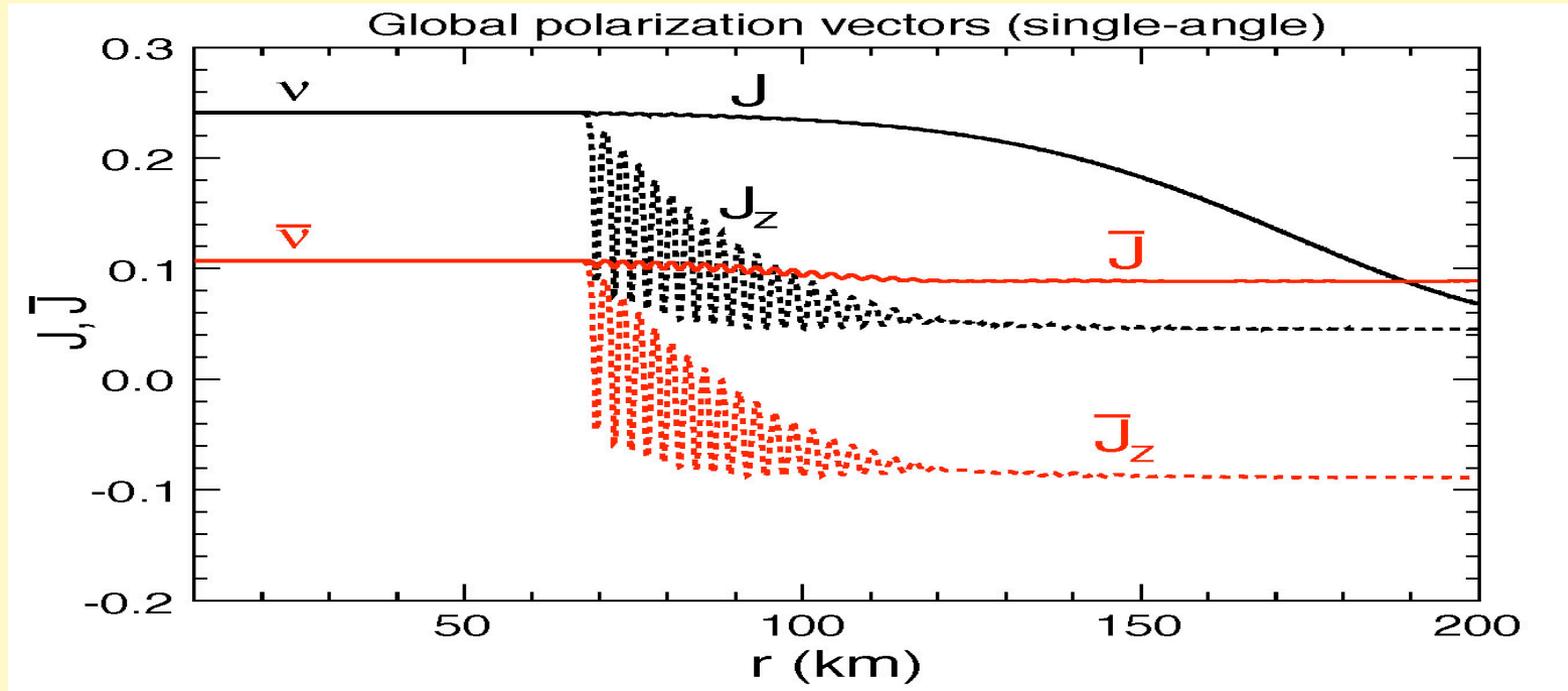
Global polarization vectors of neutrinos and antineutrinos ( $\mathbf{J}$  and  $\underline{\mathbf{J}}$ , with  $|\mathbf{J}| > |\underline{\mathbf{J}}|$ ) follow pendulum motion as far as near-alignment holds. Eventually  $\underline{\mathbf{J}}$  reaches the stable downward position, while  $\mathbf{J}$  can't, to preserve lepton number conservation ( $\sim J_z - \underline{J}_z$ )



Final state: whole  $\underline{\mathbf{J}}$  and high-E part of  $\mathbf{J}$  inverted (**spectral split/swap**)  
(Inversion = complete flavor change)

# Some results from our calculations

## Evolution of global polarization vectors (two flavors)



...alignment



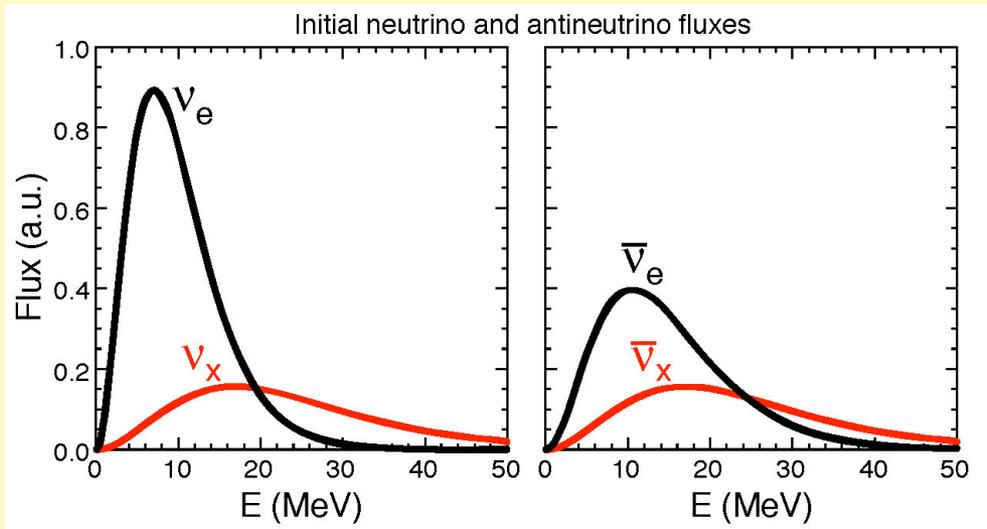
... swings



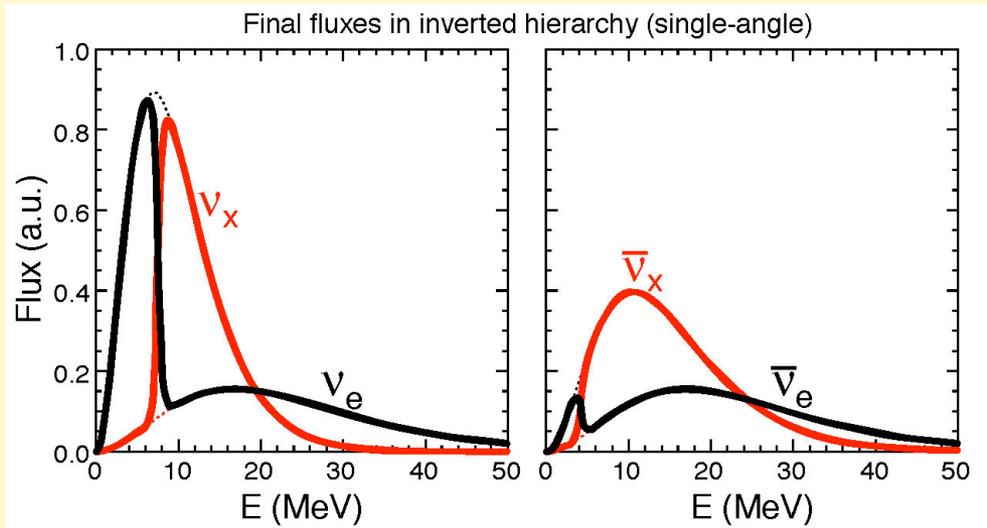
... split



# Energy spectra (two-flavors)



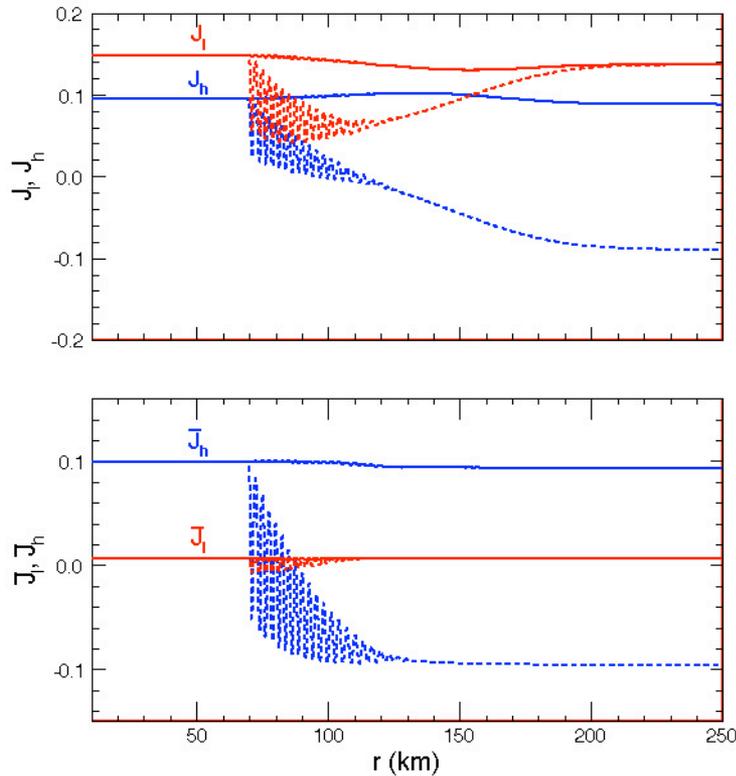
Initial fluxes at the  
neutrinosphere ( $r \sim 10$  km)



Final fluxes at the end of  
collective effects ( $r \sim 200$  km)

Expected neutrino split

+unexpected antineutrino  
split at lower energy



The antineutrino split is related, in a still unclear way, to nonadiabatic features of the flavor evolution.

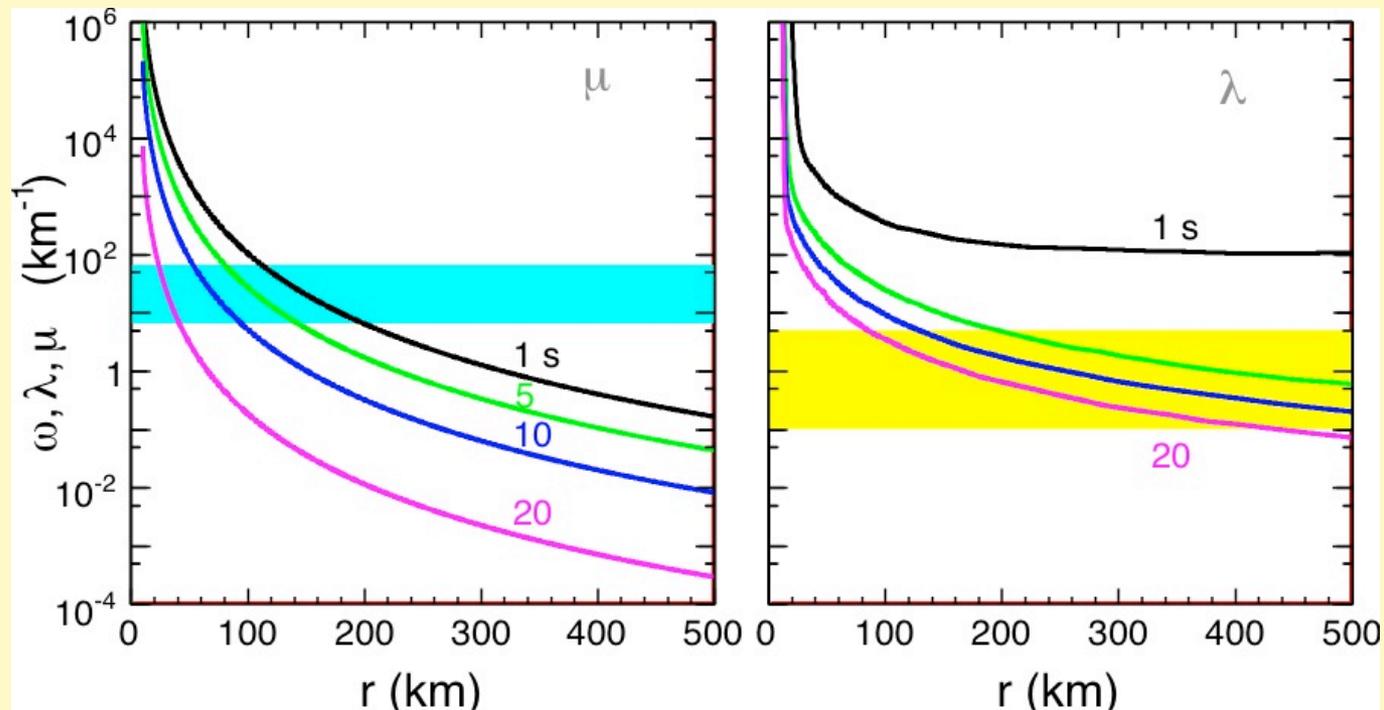
It appears that a description in terms of four global vectors (low/high energy neutrinos and antineutrinos) may capture this behavior.

$$\begin{aligned}\dot{\mathbf{J}}_l &= (+\omega_l \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{J}_l, \\ \dot{\mathbf{J}}_h &= (+\omega_h \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \mathbf{J}_h, \\ \dot{\bar{\mathbf{J}}}_l &= (-\bar{\omega}_l \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \bar{\mathbf{J}}_l, \\ \dot{\bar{\mathbf{J}}}_h &= (-\bar{\omega}_h \mathbf{B} + \lambda \mathbf{z} + \mu \mathbf{D}) \times \bar{\mathbf{J}}_h.\end{aligned}$$

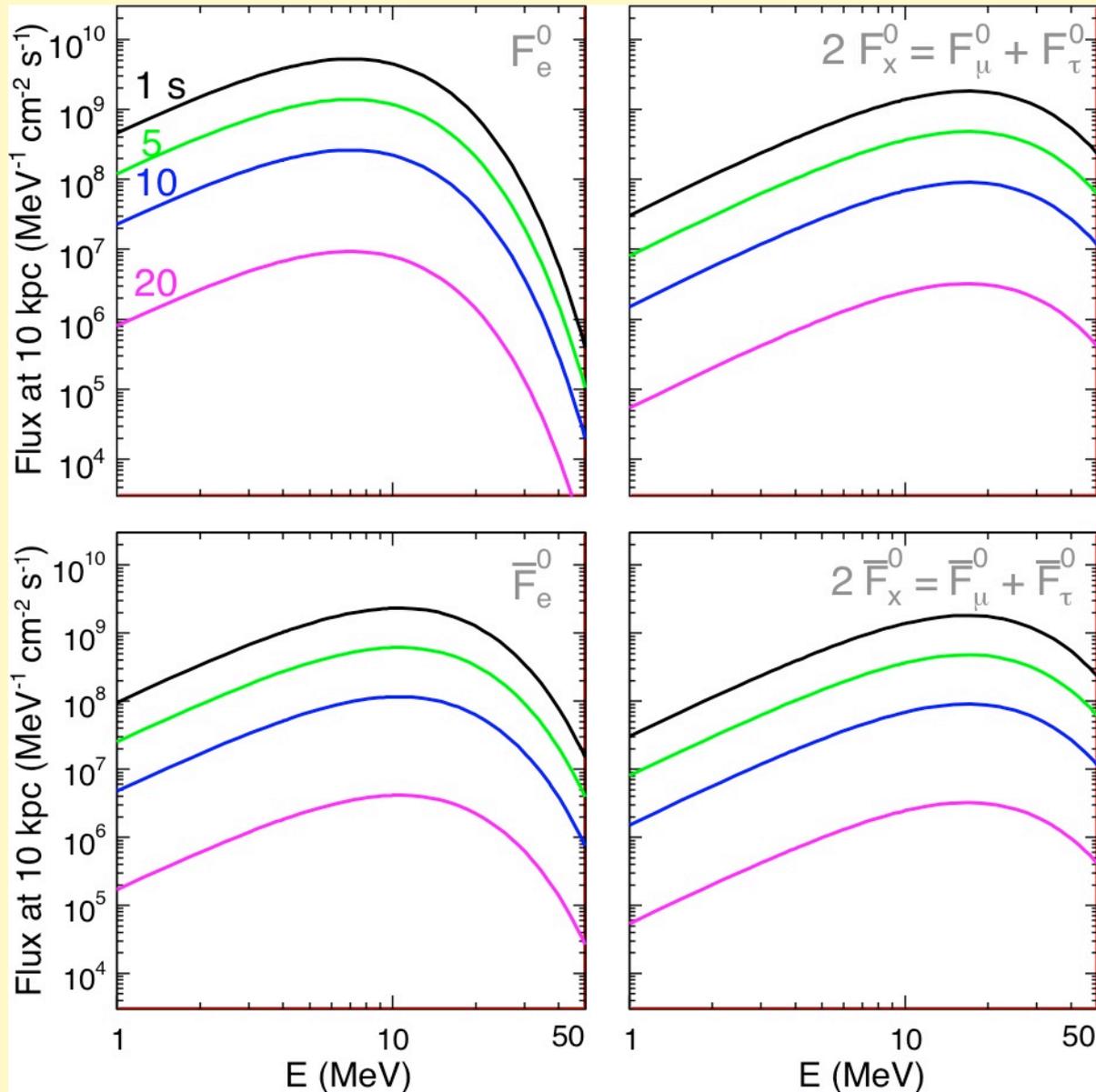
When passing to multi-angle simulations, the neutrino split feature is robust, while the antineutrino one seems to be more fragile. More work is needed to understand the low-energy behavior of antineutrinos, and the conditions for their (observable) split.

Recently, we have carried out a full **three-flavor** calculation in a benchmark supernova model at **different post-bounce times** (in single-angle approximation), in order to provide estimates for **absolute observable fluxes** of (anti)neutrinos at  $d = 10$  kpc. We have taken  $\sin^2\theta_{13} = 10^{-6}$  to suppress later MSW effects.

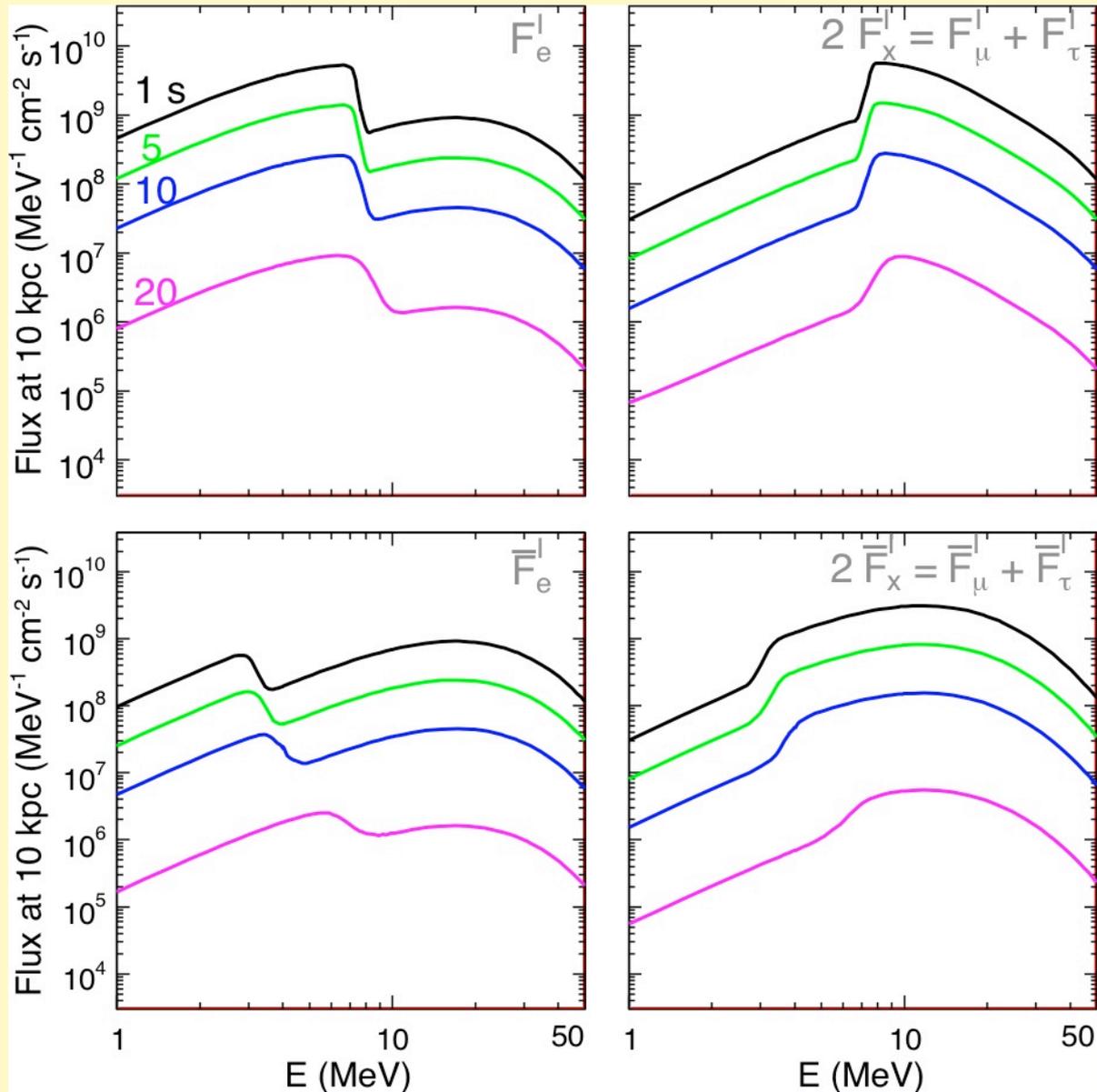
SN self-interaction potential and matter potential:



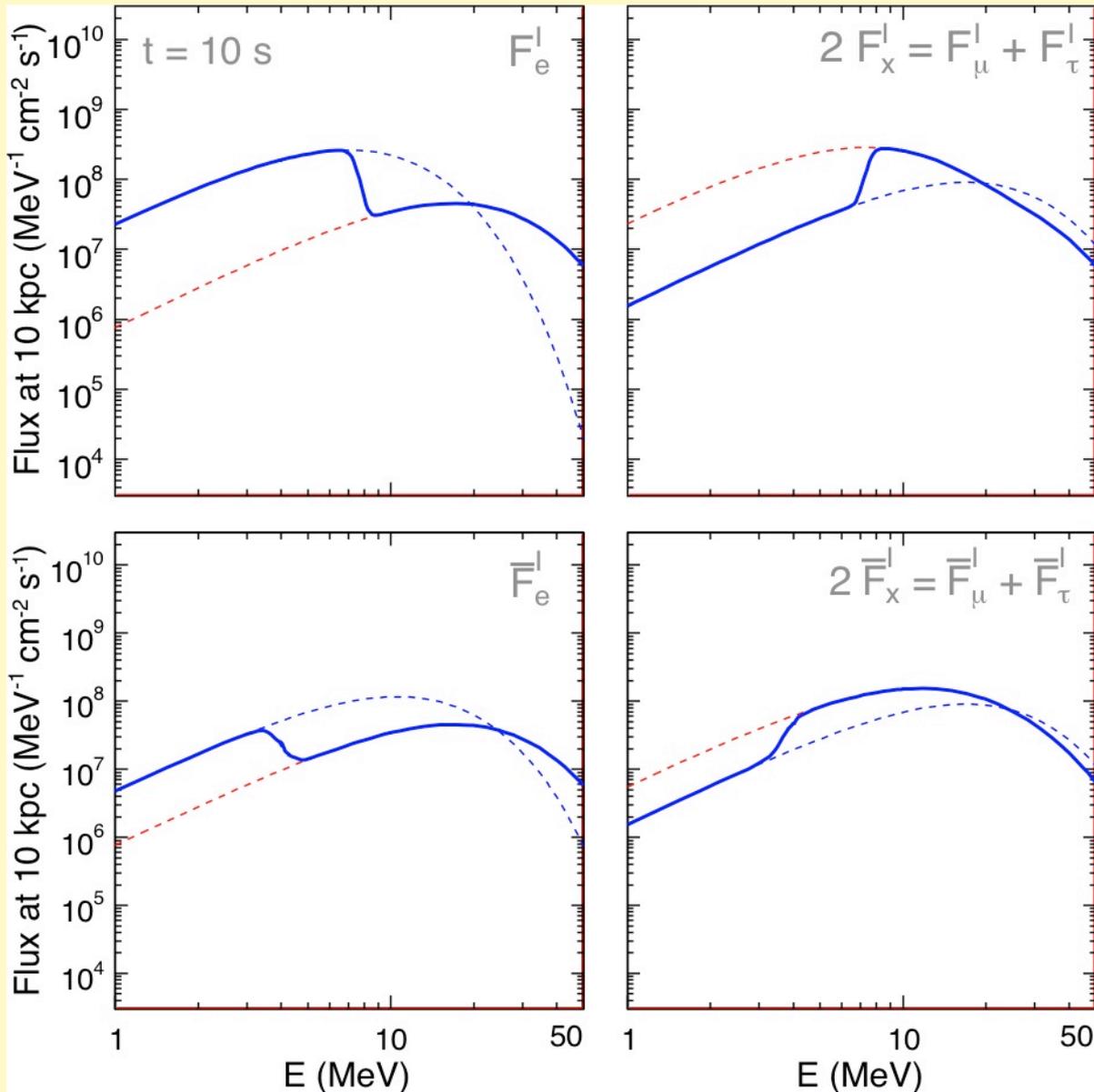
# Unoscillated fluxes at the neutrinosphere (projected at d=10 kpc)



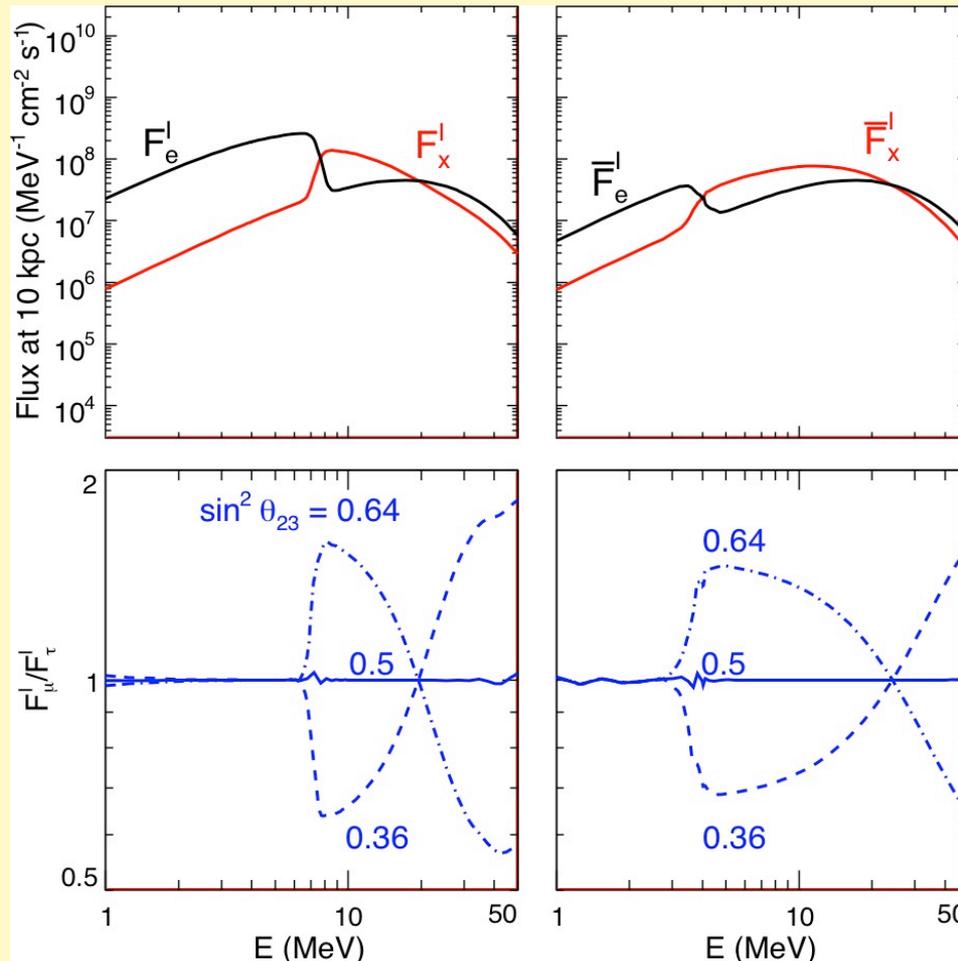
# Intermediate fluxes at the end of collective effects



# Complete swap above some critical energies:

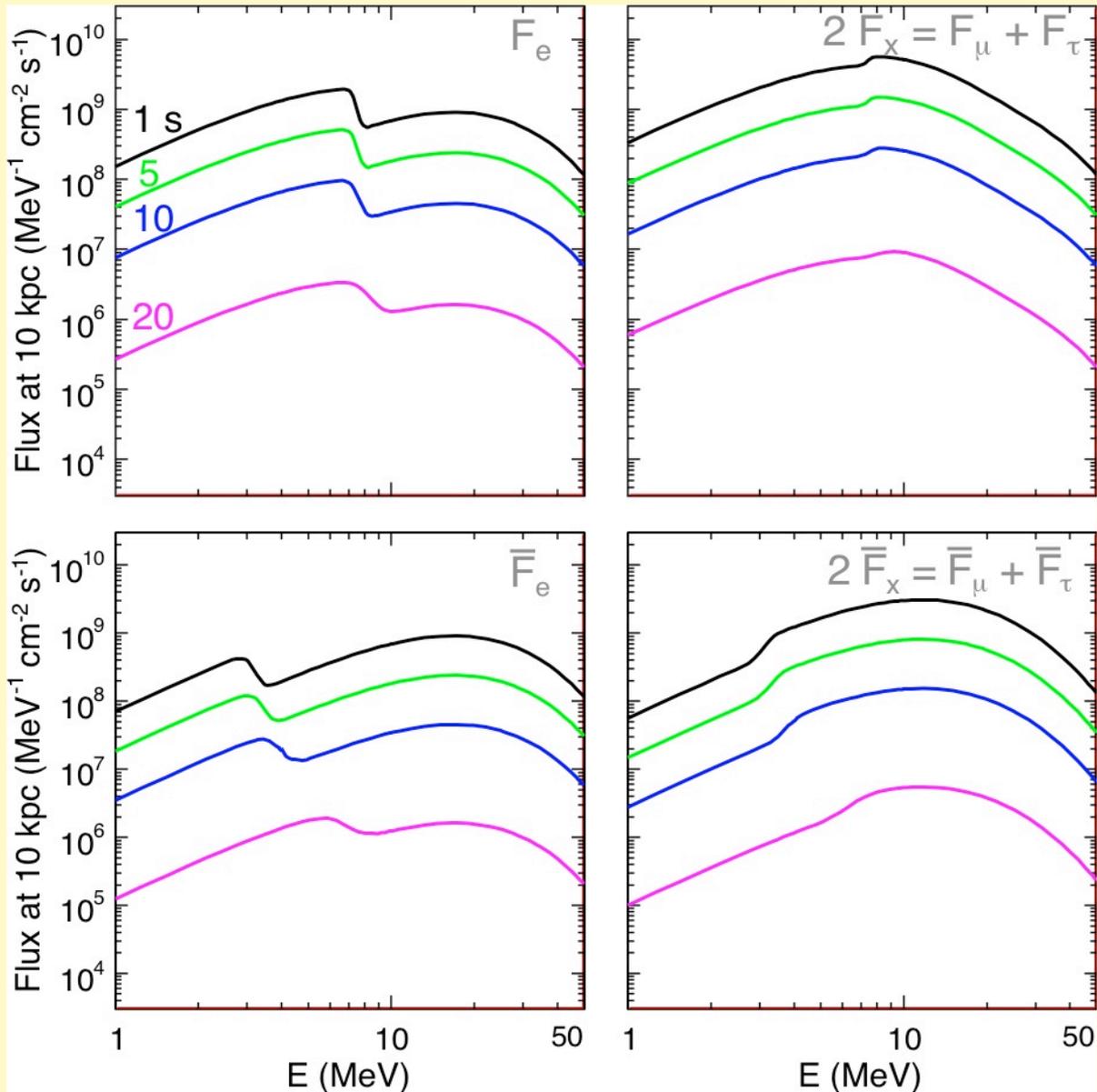


In principle, the mu/tau fraction (unobservable!) could probe  $\theta_{23}$

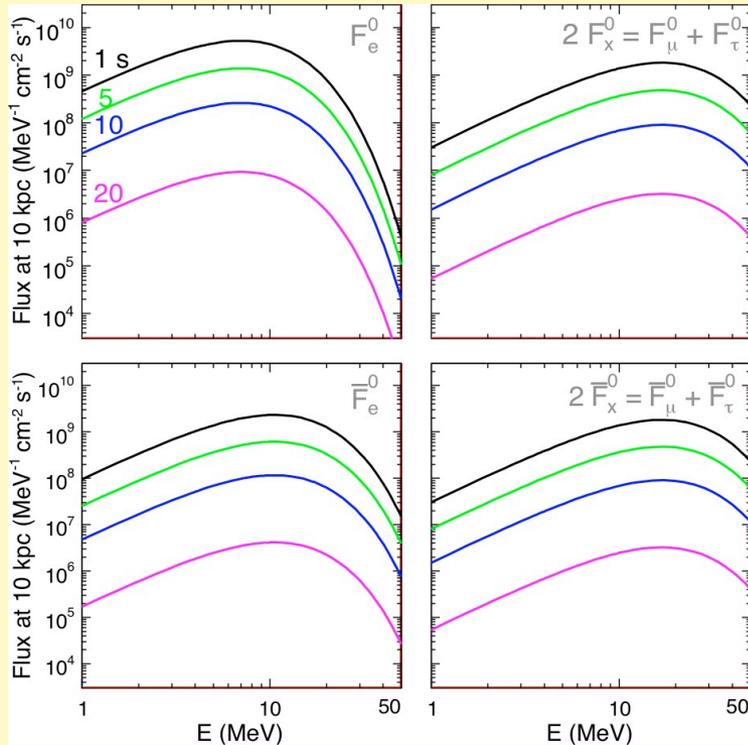


Effects due to solar  $\delta m^2$  terms and to 1-loop mu-tau neutrino potential turn out to be small (at or below percent level)

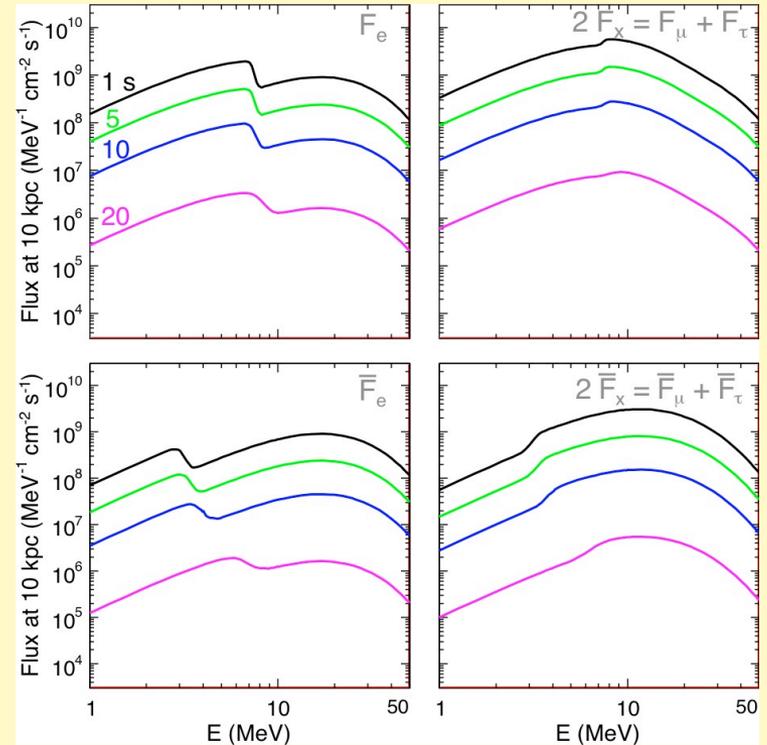
# Final fluxes at the detector



## Unoscillated

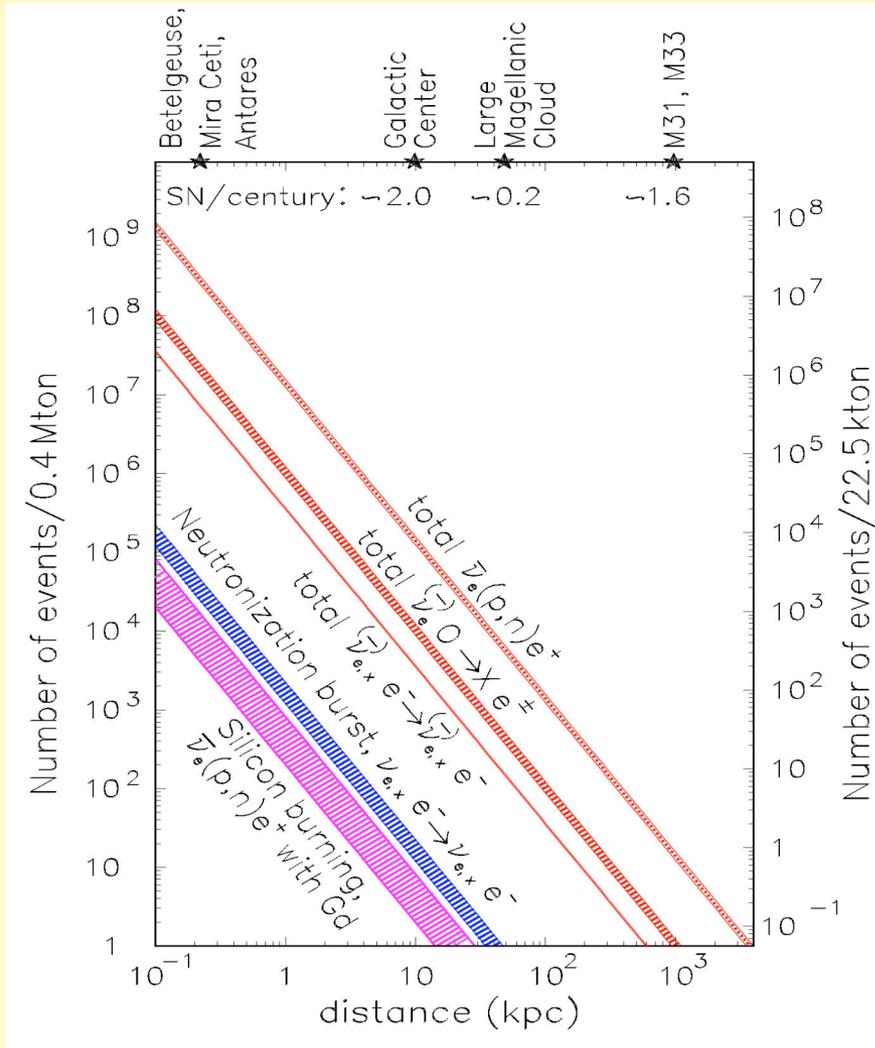


## Oscillated



The spectral split features emerge at any time in the interval  $\sim 1$ -20 s. If realized in nature, they should be observable in time-integrated energy spectra.

Total event rates may be high; but low-energy thresholds for (anti)neutrinos are challenging.



# CONCLUSIONS

In the dense supernova core, neutrinos are a nontrivial background to themselves - perhaps more important than the matter background

As a consequence, collective flavor transformation phenomena occur. Their understanding has significantly improved in the last few years.

Spectral splits seem to provide relevant observable signatures of self-interactions in inverted hierarchy (provided that  $\theta_{13}$  is nonzero)

It remains to understand how robust such collective effects are in non-idealized SNe (e.g., in turbulent and asymmetric explosions)

Furthermore, one may wonder whether collective effects may have a feedback on the explosion dynamics itself (hard to implement!)



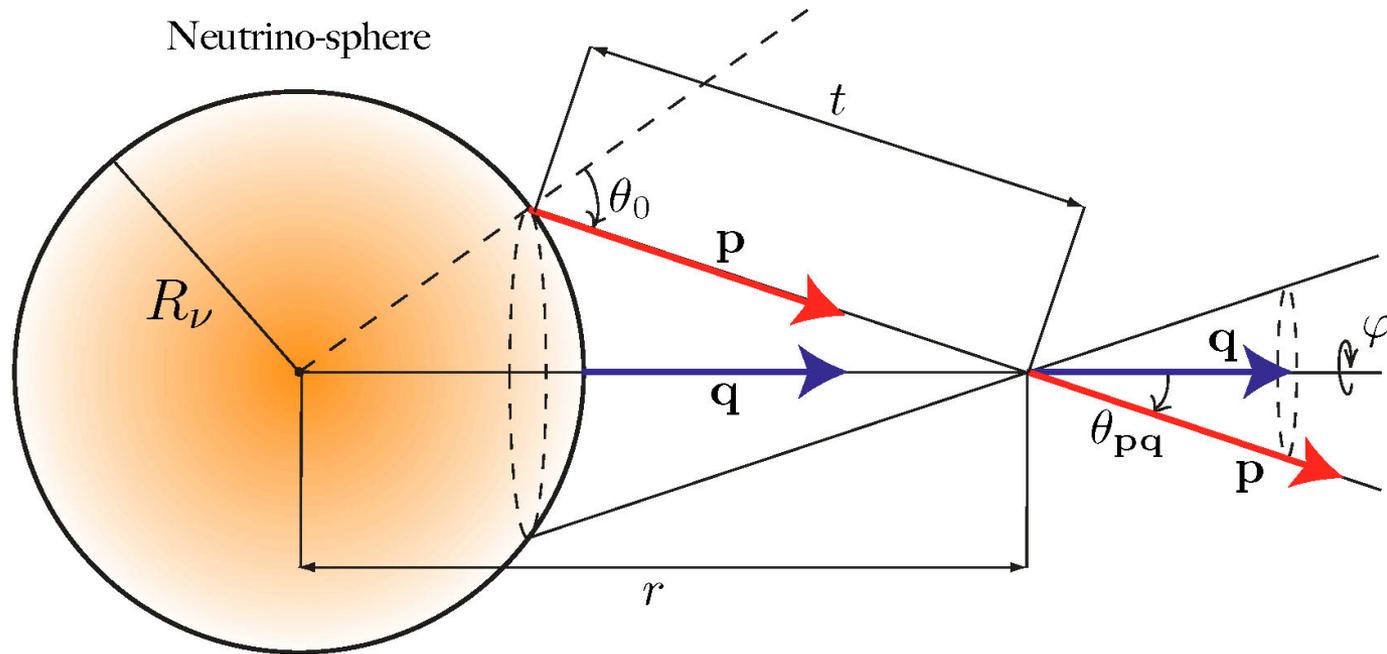
The next galactic supernova might reveal collective effects we never thought we'd see!

Thank you  
for your  
attention.



Backup slides

# Geometry: the neutrino bulb model



$$r \sin \theta_{pq} = R_\nu \sin \theta_0 \quad t = \sqrt{r^2 - R_\nu^2 \sin^2 \theta_0} - R_\nu \cos \theta_0$$

# Single-angle vs Multi-angle (individual components $P_i$ )

