Coherence and Oscillations of Cosmic neutrinos

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Reference

This talk is based on

Y. F. and A. Yu. Smirnov, "*Coherence and oscillations of cosmic neutrinos*", arXiv:0803.0495 [hep-ph]

Neutrino Telescopes

ICECUBE

In south pole

KM3NET

In Mediterranean

Neutrino Astrophysics

- Learning about sources of neutrinos
- Neutrino properties

Deriving neutrino parameters

Deriving $|U_{\mu 1}|^2$ as a part of the program of reconstructing the unitarity triangle

Y. F. and A. Yu. Smirnov, PRD65 (02) 113001

Deriving θ_{13} and δ from cosmic neutrino data:

Beacom et al, PRL 90 (03) 181301; PRD 69 (04) 17303; Serpico and

Kachelriess, PRL 94 (05) 211102;

Serpico, PRD 73 (06) 047301; Winter, PRD 74 (06) 33015; Blum, Nir and Waxman, arXiv:0706.2070

Separation of wavepackets

Source

Detector

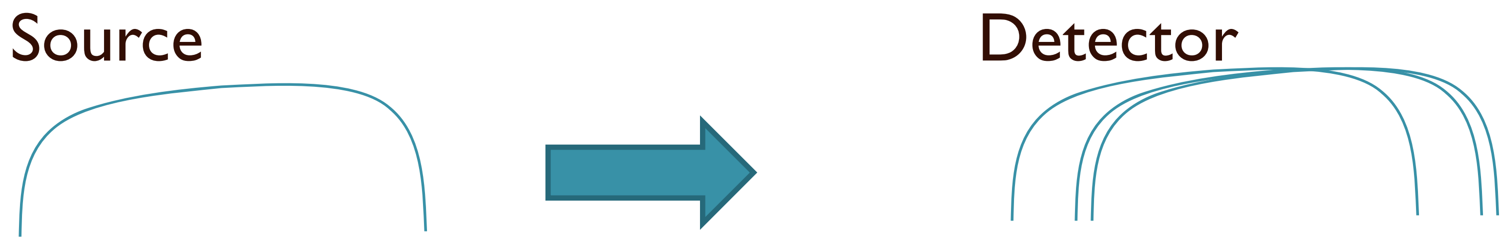
Probability of detecting $|\nu_\beta\rangle$

$\sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$ ← No interference

This happens when

$$\Delta v_g \frac{L}{\bar{v}_g} > \sigma_x = \text{wavepacket size} \qquad \Delta v_g = \frac{\Delta m_{ij}^2}{2E^2}$$

Coherent oscillation



$$\Delta v_g \frac{L}{\bar{v}_g} < \sigma_x = \text{wavepacket size}$$

$$P_{\bar{\alpha}\bar{\beta}} \simeq P_{\alpha\beta} \simeq |U_{\alpha 1}|^2 |U_{\beta 1}|^2 + |U_{\alpha 2}|^2 |U_{\beta 2}|^2 + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 +$$

$$2\mathrm{Re}[U^*_{\beta 1} U_{\alpha 1} U_{\beta 2} U^*_{\alpha 2}] \cos \Delta_{12} + 2\mathrm{Re}[U^*_{\beta 1} U_{\alpha 1} U_{\beta 3} U^*_{\alpha 3}] \cos \Delta_{13}$$

$$+ 2\mathrm{Re}[U^*_{\beta 2} U_{\alpha 2} U_{\beta 3} U^*_{\alpha 3}] \cos \Delta_{32},$$

where $\Delta_{ij} = \Delta m^2_{ij} L/(2E_\nu)$.

For cosmic neutrinos: $\Delta_{ij} \gg 1$

Averaging out the oscillatory term

$$L \gg \frac{2E_\nu}{\Delta m^2_{ij}}$$

- Summing over energy (finite energy resolution)
- Distance (Accumulating data from different sources)

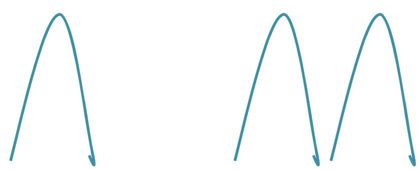
$$\langle \cos \Delta_{ij} \rangle = 0 \Rightarrow \langle P_{\alpha\beta} \rangle = \langle P_{\bar{\alpha}\bar{\beta}} \rangle = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

It will not be possible to discriminate between coherent and incoherent cases.

Discrimination with excellent energy resolution!?

Suppose we are lucky enough to encounter a very powerful source at relatively close distance to yield enough statistics (SN at 3 Mpc ⟶ 300 events ICECUBE;

Ando and Beacom, PRL 95 (05) 61103)



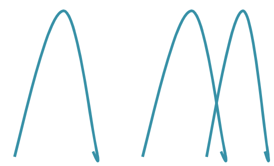
versus

Can we discriminate between the two with high energy resolution by arbitrarily fine energy resolution ?

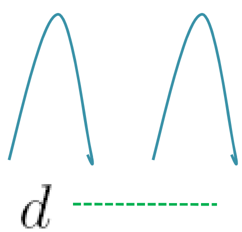
$\sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$ versus $\sum_{ij} \mathrm{Re}[U^*_{\alpha i} U_{\alpha j} U_{\beta j} U^*_{\alpha j}] \cos \frac{(m_i^2 - m_j^2)L}{2E_\nu}$

Restoring coherence at detector

If we measure the energy with precision ΔE_ν better than $\frac{E_\nu^2}{\Delta m^2 L}$, we can resolve the effects of the oscillatory terms (in the case ).

How about the case  ?

Kiers, Nussinov and Weiss, PRD53 (96) 537.



$d = \frac{\Delta m^2}{E^2} L$ Measurement of energy with

Precision ΔE_ν takes time $\Delta t \sim \frac{1}{\Delta E_\nu} \sim d$ during which the second will arrive and start interfering.

What matters is energy spectrum.

By measuring neutrino events at the detector, it would not be possible to determine whether the neutrino flux is composed of short wavepackets or long wavepackets. Only the shape of the spectrum matters.

Stodolosky, PRD 58 (98) 36006; Kiers, Nussinov and Weiss, PRD53 (96) 537.

Coherent broadening of the spectrum:

Consider the two-body decay of a particle at rest.

Kinematics ➡ The spectrum is monochromatic.
Coherent broadening ➡ A narrow spectrum but with finite width given by width of wavepacket $\sigma_E \sim \tau^{-1}$

Coherent broadening

Suppose without coherent broadening the flux is $F(E_\nu)$

Coherent broadening deforms the flux to

$$F + \frac{\sigma_E}{2}\frac{dF}{dE}$$

If $\left(\frac{1}{F}\frac{dF}{dE}\right)^{-1} \sim \sigma_E$, the coherent broadening is significant.

Power law spectrum: $F \propto E^{-n}$ with $n \simeq 2$

Broadening will be important only if $\sigma_E/E > 0.1$

If $\sigma_E/E > 0.1$ and moreover if $\frac{\sigma_E}{E}$ varies with energy the shape of the spectrum will change.

Sharp features will be smeared.

The goal

We are going to estimate the wavepacket width of neutrinos produced, under various circumstances perceivable for cosmic neutrinos.

- Neutrinos produced by free pion and muon.

- Pions and muon scatter off the particles in the medium before decay.

- Pion and muons move spirally in the magnetic field.

The results may have a wider application.

A general remarks

- If $\frac{\sigma_{P_\perp}}{E_\nu} \gtrsim \frac{1\ \text{km}}{100\ \ Mpc} \sim 10^{-21}$ the wavepacket will spread so widely before reaching the detector that cannot be detected.

$\sigma_{p_\perp} \lll\lll\lll \sigma_{p_{||}}$ from now on $\sigma_p \equiv \sigma_{p_{||}}$

- Neutrinos are ultra-relativistic: $\sigma_p = \sigma_E$

- $\frac{\sigma_E}{E}$ is Lorentz invariant.

$$E' = \gamma E(1 + \tfrac{v}{c}\sin\xi)$$

$$\sigma'_E = \gamma \sigma_E(1 + \tfrac{v}{c}\sin\xi)$$

Free decay

$$\pi^+ \rightarrow \nu_\mu + \mu^+ \,,$$
$$\mu^+ \rightarrow \nu_e + \bar{\nu}_\mu + e^+$$

In the rest frame of the parent particle: $\sigma_E|_{\rm rest} \sim \tau_0^{-1}$

In a general frame: $\sigma_E = \frac{1}{\tau_0}\frac{E_\nu}{E_\nu^0}$

where E_ν^0 is the energy of the neutrino in the rest frame of the parent particle.

For the pion decay:

$$\tau_0 = 3 \times 10^{-8}\ \rm sec \qquad E_\nu^0 \sim 30\ \rm MeV \qquad \sigma_E \sim 10^{-15} E_\nu \lll 0.5 E_\nu$$

For the muon decay:

$$\tau_0 = 2 \times 10^{-6}\ \rm sec \qquad E_\nu^0 \sim 30\ \rm MeV \qquad \sigma_E \sim 10^{-17} E_\nu \lll 0.5 E_\nu$$

Do wavepacket separate?

Coherent broadening does not affect the spectrum.

Even after traveling cosmological distances, the wavepackets hardly separate:

$$\frac{d_L}{\sigma_x} \sim 0.1 \left(\frac{\Delta m^2}{8 \times 10^{-5}\ \mathrm{eV}^2} \right) \left(\frac{L}{100\ \mathrm{Mpc}} \right) \left(\frac{10\ \mathrm{TeV}}{E_\nu} \right) \left(\frac{3 \times 10^{-8}\ \mathrm{sec}}{\tau_0} \right)$$

For pion:

For muon:

Magnetic field and/or scattering

Interaction of the parent particles with the magnetic field and/or scattering off the particles in the medium can drastically decrease σ_x .

Magnetic field at source: σ_x ↓ σ_E ↑

Scattering in the thermal bath

Neutrinos are produced by the decay of charged pions and muons in the thermal bath of photons that have a thermal distribution with temperature T_γ in a jet moving with a boost factor of Γ_{jet} relative to us.

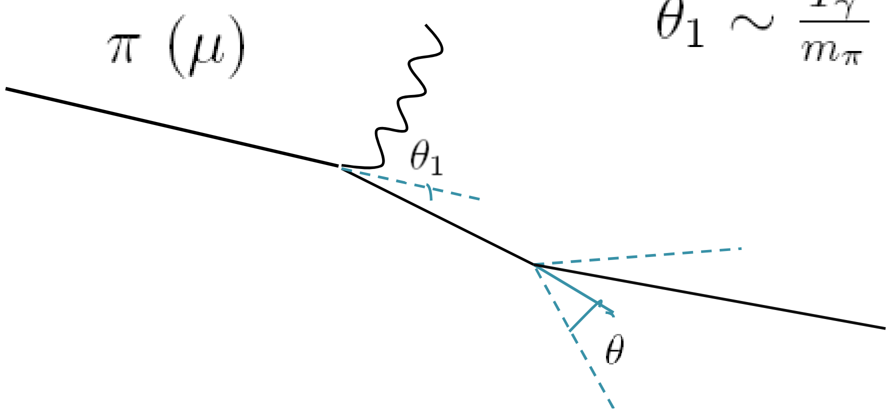
Meszaros and Waxman, PRL 87 (01) 1 71102; Ando and Beacom, PRL 95 (05) 61103

emission cone

$$\theta \sim \frac{m_\pi}{E^j}$$

$$\theta_1 \sim \frac{T_\gamma}{m_\pi}$$

$$\theta_1 \ll \theta$$



Scattering of pions

After the collision, the momentum of the pion is changed by $|\Delta\vec{p}_\pi^j| \sim T_\gamma E_\pi^j/m_\pi$ in a direction transverse to the momentum $(\Delta\vec{p}_\pi^j \cdot \vec{p}_\pi^j \ll |\Delta\vec{p}_\pi^j||\vec{p}_\pi^j|)$

The difference between the energy of neutrinos emitted in our direction before and after collision:

$$\Delta E_\nu \sim T_\gamma (E_\pi^j/m_\pi)^2$$

The length of the wavepacket emitted during the mean free time

$$\sigma_{x,\pi}^{jet} \sim \ell_{col} \frac{m_\pi}{E_\pi^j} \frac{E_\nu^0}{E_\nu^j} = \ell_{col} \frac{m_\pi}{E_\pi^j} \frac{m_\pi^2 - m_\mu^2}{2m_\pi E_\nu^j}$$

$$\ell_{\rm col} \sim \frac{1}{\sigma n_\gamma}$$

$$(\sigma_{x,\pi}^{jet})^{-1} \ll \Delta E_\nu$$

Revisiting the same problem for the muons

Unlike the pion case the spectrum of the neutrinos emitted by muon is continuous. This means after collision still,

$$\Delta E_\nu = 0$$

The wavepackets emitted before and after collision can be coherent.

In each collision the momentum of the muon is rotated by

$$\theta_1 \sim \frac{T_\gamma}{m_\mu}$$

The length of wavepacket is given by the time it takes for the line of sight to exit the emission cone.

Scattering of muons

After N collision the muon momentum will be rotated by an angle of size $\sqrt{N}\theta_1 \sim \sqrt{N} T_\gamma / m_\mu$

Thus, after $N \sim (m_\mu^2 / E_\mu T_\gamma)^2$ collisions, the momentum rotates by $\sim m_\mu / E_\mu^j$ and the line of sight exits the emission cone. N successive collision takes $\Delta t \sim N \ell_{col} \sim \frac{m_\mu^2}{E_\nu T_\gamma} \ell_{col}$. The length of the emitted wavepacket:

$$\sigma_{x,\mu}^{jet} \sim c\Delta t \frac{m_\mu}{E_\mu^j} \frac{E^0}{E_\nu^j} \sim \frac{\ell_{col}}{3} \frac{m_\mu^6}{T_\gamma^2 (E_\nu^j)^2 E_\mu^2},$$

where $E^0 \sim m_\mu / 3$ is the energy of the neutrino in the rest frame of the muon.

Mean free path

$$\ell_{\rm col} \sim \frac{1}{\sigma n_\gamma}$$ with $$n_\gamma = [2\zeta(3)/\pi^2]T^3$$

Meszaros and Waxman, PRL 87 (01) 1 71102: $T_\gamma \sim \text{few keV}$

For $\frac{E_\nu}{\Gamma_{jet}} T_\gamma \lesssim m_\pi^2 \simeq m_\mu^2 \simeq (100 \text{ MeV})^2$, the scattering cross-section is given by the Thompson formula

$$\sigma = \frac{8\pi\alpha^2}{3m_\pi^2}.$$

Wavepacket sizes

In the observer frame, ($E\sigma_x$ is Lorentz invariant)

$$\sigma_{x,\pi} = \sigma_{x,\pi}^{jet} \frac{E_\nu^{jet}}{E_\nu} \sim 3 \times 10^{-5}\ \mathrm{cm} \left(\frac{10\ \mathrm{TeV}}{E_\nu} \right)^2 \left(\frac{4\ \mathrm{keV}}{T_\gamma} \right)^3 \frac{\Gamma_{jet}}{100}$$

$$\sigma_{x,\mu} = \sigma_{x,\mu}^{jet} \frac{E_\nu^{jet}}{E_\nu} \sim 5 \times 10^{-3}\ \mathrm{cm} \left(\frac{4\ \mathrm{keV}}{T_\gamma} \right)^5 \left(\frac{10\ \mathrm{TeV}}{E_\nu} \right)^4 \left(\frac{\Gamma_{jet}}{100} \right)^3$$

$$d_L = 1.2 \cdot 10^{-4}\mathrm{cm} \left(\frac{L}{100\ \mathrm{Mpc}} \right) \left(\frac{\Delta m^2}{8 \cdot 10^{-5}\mathrm{eV}^2} \right) \left(\frac{10\ \mathrm{TeV}}{E} \right)^2$$

Spectrum broadening because of scattering

Pion decay

$$\sigma_E \sim 10^{-13} E_\nu \left(\frac{E_\nu}{10\ \text{TeV}} \frac{100}{\Gamma_{jet}} \right) \left(\frac{T_\gamma}{4\ \text{keV}} \right)^2$$

Muon decay

$$\sigma_E \sim 4 \times 10^{-16} E_\nu \left(\frac{E_\nu}{10\ \text{TeV}} \frac{100}{\Gamma_{jet}} \right)^3 \left(\frac{T_\gamma}{4\ \text{keV}} \right)^5$$

Effect of scattering on spectrum broadening is negligible:

$$\left(\frac{1}{F} \frac{dF}{dE} \right)^{-1} \sim 0.5 E_\nu \gg \sigma_E$$

Effects of B on wavepacket length

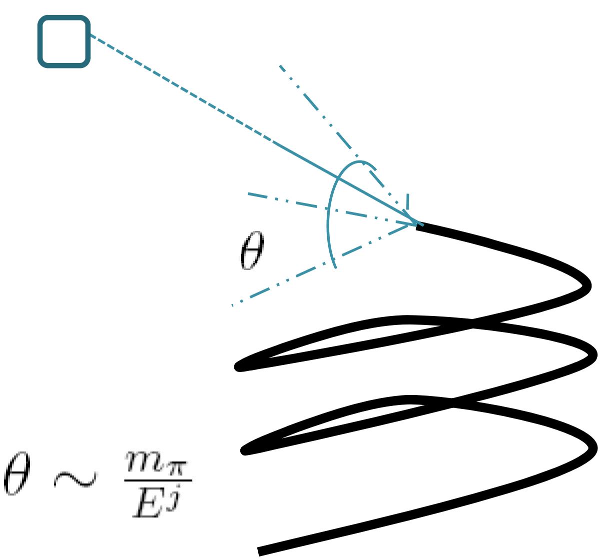
The magnetic field in the jets at the cosmic neutrino sources can be as large as $\sim 10^9$ Gauss. Ando and Beacom, PRL 95 (05)

Classical regime: $eB \ll E^2$

Pion in magnetic field — Spiral trajectory

The energy of the neutrino emitted at t is different from that emitted at $t + \Delta t$:

$$\Delta E_\nu \equiv E_\nu(t + \Delta t) - E_\nu(t).$$



Emission cone with opening angle: $\theta \sim \frac{m_\pi}{E^j}$

Length of the wavepacket

Width of the wavepacket emitted during Δt (Lorentz contraction) $\sigma_E \sim (\Delta t)^{-1} \frac{E_\pi}{m_\pi} \frac{E_\nu}{E_\nu^0}$

By solving the following equation (For $(eB\Delta t/E_\mu) \lesssim m_\mu/E_\mu$

$$\sigma_E = \Delta E_\nu = \frac{1}{\Delta t}\left(\frac{E_\pi}{m_\pi}\right)\left(\frac{E_\nu}{E_\nu^0}\right) = \frac{1}{\Delta t}\left(\frac{2E_\pi E_\nu}{m_\pi^2 - m_\mu^2}\right)$$

we can estimate σ_E . First, we should calculate ΔE_ν

$$\Delta E_\nu \equiv E_\nu(t + \Delta t) - E_\nu(t).$$

The four-momentum of the pion:

$$(E_\pi, \quad p_\pi \sin\theta_\pi \cos\Phi(t), \quad p_\pi \sin\theta_\pi \sin\Phi(t), \quad p_\pi \cos\theta_\pi),$$

Rotation phase: $\Phi(t) = \frac{eB(t-t_0)}{E_\pi}$

$$E_\nu(t)(1, \sin\theta_\nu, 0, \cos\theta_\nu),$$

$$E_\nu(t) = \frac{m_\pi^2 - m_\mu^2}{2[E_\pi - p_\pi \cos(\theta_\nu - \theta_\pi) + 2p_\pi \sin\theta_\pi \sin\theta_\nu \sin^2\frac{\Phi(t)}{2}]}.$$

$$\Delta E_\nu = \frac{2eBE_\nu^2 \sin\theta_\nu \sin\theta_\pi \sin\Phi}{m_\pi^2 - m_\mu^2}\Delta t.$$

To have the line of sight within the emission cone

$$(\theta_\nu - \theta_\pi) \sim \sin\Phi(t) \lesssim m_\pi/p_\pi \ll 1$$

Wavepacket of neutrinos from pion decay

$$\sigma_E \sim \left[\frac{4eBE_\nu^3 m_\pi \sin\theta_\pi \sin\theta_\nu}{(m_\pi^2 - m_\mu^2)^2}\right]^{1/2}$$

Wavepacket size

(for typical values in Meszaros and Waxman, PRL 87 (01) 171102)

$$\sigma_x \sim \sigma_E^{-1} \sim 2\times 10^{-14}\ \mathrm{cm}\left(\frac{\Gamma_{jet}}{100}\frac{10^7\ \mathrm{Gauss}}{B}\right)^{1/2}\left(\frac{10\ \mathrm{TeV}}{E_\nu}\right)^{3/2}$$

Can coherent broadening due to B be significant?

$$\frac{\sigma_E}{E_\nu} = \frac{\sigma_E^{jet}}{E^{jet}} \sim \sqrt{\frac{eBE_\nu m_\pi}{\Gamma_{jet}(m_\pi^2 - m_\mu^2)^2}} \sim 10^{-4} \left(\frac{100}{\Gamma_{jet}} \frac{E_\nu}{10 \text{ TeV}} \frac{B}{10^7 \text{ Gauss}} \right)^{1/2}$$

To have significant broadening

(i.e., to have $\sigma_E \sim \left(\frac{1}{F}\frac{dF}{dE}\right)^{-1} \sim 0.5E_\nu$)

we should go to higher and higher values of energy

Coherent broadening

$$\frac{\sigma_E}{E_\nu} = \frac{\sigma_E^{jet}}{E^{jet}} \sim \sqrt{\frac{eBE_\nu m_\pi}{\Gamma_{jet}(m_\pi^2 - m_\mu^2)^2}} \sim 10^{-4}\left(\frac{100}{\Gamma_{jet}}\ \frac{E_\nu}{10\ \mathrm{TeV}}\ \frac{B}{10^7\ \mathrm{Gauss}}\right)^{1/2}$$

Taking $B \sim 10^7$ Gauss and $\Gamma_{jet} \sim 100$

(the typical values found In Meszaros and Waxman, PRL 87 (01) 1 71102)

$\frac{\sigma_E}{E_\nu} \sim 0.1$ for $E_\nu > 10\ \mathrm{EeV}$

Taking $B \sim 10^9$ Gauss and $\Gamma_{jet} \sim 3$

(the typical values found in Ando and Beacom, PRL 95 (05) 61103)

$\frac{\sigma_E}{E_\nu} \sim 0.1$ for $E_\nu > 30\ \mathrm{PeV}$

Wavepacket of neutrinos from muon decay

The spectrum of neutrinos from muon decay is continuous.

Δt =Time interval during which the angle between line of sight and momentum of muon is smaller than m_μ / E_μ:

$(eB\Delta t / E_\mu) \lesssim m_\mu / E_\mu.$

The width of wavepacket emitted during Δt, at the jet frame, is

$$\sigma_E^{jet} = (\Delta t)^{-1} \left(\frac{E_\mu^{jet}}{m_\mu} \right)^2 = \frac{eB}{m_\mu} \left(\frac{3E_\nu^{jet}}{m_\mu} \right)^2,$$

Wavepackets of different mass eigenstates will be separated *en route*

$$\sigma_x \sim \sigma_E^{-1} \sim 5 \times 10^{-13}\ \mathrm{cm} \left(\frac{10\ \mathrm{TeV}}{E_\nu} \right)^2 \frac{10^9\ \mathrm{Gauss}}{B} \frac{\Gamma_{jet}}{100}$$

Coherent broadening

$$\frac{\sigma_E}{E_\nu} = \frac{\sigma_E^{jet}}{E_\nu^{jet}} = 5 \times 10^{-8} \frac{B}{10^7 \text{ Gauss}} \frac{E_\nu}{10 \text{ TeV}} \frac{100}{\Gamma_{jet}}$$

Taking $B \sim 10^7$ Gauss and $\Gamma_{jet} \sim 100$

(the typical values found In Meszaros and Waxman, PRL 87 (01) 1 71102)

$\frac{\sigma_E}{E_\nu} \sim 0.1$ for $E_\nu > 100 \text{ EeV}$

Taking $B \sim 10^9$ Gauss and $\Gamma_{jet} \sim 3$

(the typical values found in Ando and Beacom, PRL 95 (05) 61103)

$\frac{\sigma_E}{E_\nu} \sim 0.1$ for $E > 10 \text{ PeV}$

Energy Loss in magnetic field

Are such high energies at such high magnetic field possible at all? The answer is NOT no!

- Synchrotron radiation reduces energy

- Acceleration by internal shocks (the same mechanism that accelerated protons in the first place).

Koers and Wijers, arXiv:0711.4791

Conclusion and remarks

We have estimated the size of the neutrino wavepacket produced by decay under various circumstances. Although our main motivation was to study the cosmic neutrinos but our results are general. We have found that the magnetic field at the source can dramatically shorten the wavepacket size.

Neutrinos from pion decay: $\sigma_x \sim \sigma_E^{-1} \sim 2 \times 10^{-14}\ \mathrm{cm} \left(\frac{\Gamma_{jet}}{100}\frac{10^7\ \mathrm{Gauss}}{B}\right)^{1/2} \left(\frac{10\ \mathrm{TeV}}{E_\nu}\right)^{3/2}.$

Neutrinos from muon decay: $\sigma_x \sim \sigma_E^{-1} \sim 5 \times 10^{-11}\ \mathrm{cm} \left(\frac{10\ \mathrm{TeV}}{E_\nu}\right)^{2} \frac{10^7\ \mathrm{Gauss}}{B}\frac{\Gamma_{jet}}{100}$

σ_x is not sensitive to the lifetime of the parent particle but depends on whether the decay is two body or three body. (General application)

Although, observation cannot determine the wavepacket size but shortening of wavepacket can give rise to the "coherent broadening" of the spectrum: $F \longrightarrow F + \frac{\sigma_E}{2}\frac{dF}{dE}$

If $\sigma_E \sim \left(\frac{1}{F}\frac{dF}{dE}\right)^{-1}$ the coherent broadening can be significant

(for cosmic neutrino $\left(\frac{1}{F}\frac{dF}{dE}\right)^{-1} \sim 0.5E$)

$$\frac{\sigma_E}{E_\nu} \sim 0.1 \frac{B}{10^9 \text{ Gauss}} \frac{E_\nu}{10 \text{ PeV}} \frac{3}{\Gamma_{jet}}$$

Coherent broadening can change the shape of the spectrum from power law.