Neutrino Oscillations in Venice

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Neutrino mass
and neutrinoless double beta decay

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and MPI Heidelberg
The status of the present knowledge of the neutrino oscillation phenomena is schematically depicted in this slide.

Three quantities are unknown at present:

a) The mass $m_1$

b) The angle $\theta_{13}$

c) Whether the normal or inverted hierarchy is realized.
However, $\nu$ masses are much smaller than the masses of other fermions. Is that a possible "Hint of" a new mass-generating mechanism?
The most popular theory of why neutrinos are so light is the —

**See-Saw Mechanism**

*(Gell-Mann, Ramond, Slansky (1979), Yanagida(1979), Mohapatra, Senjanovic(1980))*

It assumes that the very heavy neutrinos $N_R$ exist. Their mass plays an analogous role as the scale $\Lambda$ of Weinberg, i.e., $m_\nu \sim \nu^2/M_N$. Both the light and heavy neutrinos are Majorana fermions.
Goals: Dirac or Majorana particle?
Or equivalently, is the total lepton number conserved?

Majorana: The neutrino is its own antiparticle

Ettore Majorana
How can we tell whether the total lepton number is conserved?

A partial list of processes where the lepton number would be violated:

Neutrinoless $\beta\beta$ decay: $(Z,A) \rightarrow (Z\pm 2,A) + 2e^{(\pm)}$, $T_{1/2} > \sim 10^{25}$ y

Muon conversion: $\mu^- + (Z,A) \rightarrow e^+ + (Z-2,A)$, $BR < 10^{-12}$

Anomalous kaon decays: $K^+ \rightarrow \pi^- \mu^+\mu^+$, $BR < 10^{-9}$

Flux of $\bar{\nu}_e$ from the Sun: $BR < 10^{-4}$

Flux of $\nu_e$ from a nuclear reactor: $BR < ?$

Observing any of these processes would mean that the lepton number is not conserved, and that neutrinos are massive Majorana particles.

It turns out that the study of the $0\nu\beta\beta$ decay is by far the most sensitive test of the total lepton number conservation, so we restrict further discussion to this process.
Symbolic representation of the two $\beta\beta$ decay modes

2$\nu$ $\beta\beta$ decay: a standard process in nuclear physics

0$\nu$ $\beta\beta$ decay: a hypothetical process

$\rightarrow$ $m_\nu \neq 0$ since helicity has to "flip"

$\rightarrow$ $\bar{\nu} = \nu$

From G. Gratta
### Candidate Nuclei for Double Beta Decay

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>Q (MeV)</th>
<th>Abund. (%)</th>
</tr>
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<tbody>
<tr>
<td>$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$</td>
<td>4.271</td>
<td>0.187</td>
</tr>
<tr>
<td>$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$</td>
<td>2.040</td>
<td>7.8</td>
</tr>
<tr>
<td>$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$</td>
<td>2.995</td>
<td>9.2</td>
</tr>
<tr>
<td>$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$</td>
<td>3.350</td>
<td>2.8</td>
</tr>
<tr>
<td>$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$</td>
<td>3.034</td>
<td>9.6</td>
</tr>
<tr>
<td>$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$</td>
<td>2.013</td>
<td>11.8</td>
</tr>
<tr>
<td>$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$</td>
<td>2.802</td>
<td>7.5</td>
</tr>
<tr>
<td>$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$</td>
<td>2.228</td>
<td>5.64</td>
</tr>
<tr>
<td>$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$</td>
<td>2.533</td>
<td>34.5</td>
</tr>
<tr>
<td>$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$</td>
<td>2.479</td>
<td>8.9</td>
</tr>
<tr>
<td>$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$</td>
<td>3.367</td>
<td>5.6</td>
</tr>
</tbody>
</table>

All candidate nuclei on this list have $Q > 2\text{MeV}$. The nuclei with an arrow are used in the present or planned large mass experiments. For most of the nuclei in this list the $2\nu\beta\beta$ decay has been observed.
Whatever processes cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term: Schechter and Valle,82

By adding only Standard model interactions we obtain

$$(\bar{\nu})_R \rightarrow (\nu)_L$$  Majorana mass term

Hence observing the $0\nu\beta\beta$ decay guaranties that $\nu$ are massive Majorana particles.
What is the nature of the `black box'? In other words, what is the mechanism of the $0\nu\beta\beta$ decay?

All these diagrams can contribute to the $0\nu\beta\beta$ decay amplitude.
Linking LNV to LFV Summary:

\[ B_{\mu \rightarrow e\gamma} = \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu_\mu \nu_e)} \quad B_{\mu \rightarrow e} = \frac{\Gamma(\mu^- + (Z,A) \rightarrow e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \rightarrow \nu_\mu + (Z,A))} \]

- **SM extensions with low (∼ TeV) scale LNV**
  
  Left-right symmetric model, R-parity violating SUSY, etc. possibly \( \Gamma_{0\nu\beta\beta} \) unrelated to \( m_{\beta\beta}^2 \)

\[ \mathcal{R} = \frac{B_{\mu \rightarrow e}}{B_{\mu \rightarrow e\gamma}} \gg 10^{-2} \]

- **SM extensions with high (GUT) scale LNV**
  
  \[ \mathcal{R} \sim O(\alpha/\pi) \sim 10^{-3} - 10^{-2} \]

**In absence of fine-tuning or hierarchies in flavor couplings. Important caveat!**

See: V. Cirigliano et al., PRL93,231802(2004)
\( \mu - e \) conversion in nuclei within the CMSSM seesaw: universality versus non-universality

E. Arganda\(^a\), M. J. Herrero\(^a\) and A. M. Teixeira\(^b\)

Ratio of the branching ratios for \( \mu \) conversion to \( \mu \to e + \gamma \) as a function of the Higgs mass. Note the typical value of \(~1/200\).
What is the relation of the deduced fundamental parameters and the neutrino mixing matrix? Or, in other words, what is the relation between the $0\nu\beta\beta$ decay rate and the absolute neutrino mass?

As long as the mass eigenstates $\nu_i$ that are the components of the flavor neutrinos $\nu_e$, $\nu_\mu$, and $\nu_\tau$ are Majorana neutrinos, the $0\nu\beta\beta$ decay will occur, with the rate

$$
1/T_{1/2} = G(E_{\text{tot}}, Z) (M^{0\nu})^2 <m_{\beta\beta}>^2,
$$

where $G(E_{\text{tot}}, Z)$ is easily calculable phase space factor, $M^{0\nu}$ is the nuclear matrix element, calculable with difficulties (and discussed later), and

$$
<m_{\beta\beta}> = | \sum_i |U_{ei}|^2 \exp(i\alpha_i) m_i |,
$$

where $\alpha_i$ are unknown Majorana phases (only two of them are relevant). Using the formula above we can relate $<m_{\beta\beta}>$ to other observables related to the absolute neutrino mass.
from observational cosmology, $M = m_1 + m_2 + m_3$

from $\beta$ decay

minimum mass, not observable

blue shading: normal hierarchy, $\Delta m_{23}^2 > 0$.
red shading: inverted hierarchy $\Delta m_{23}^2 < 0$

shading: best fit parameters, lines 95% CL errors.

Thanks to A. Piepke
Note as a curiosity:
\(<m_{\beta\beta}>\) may vanish even though all \(m_i\) are nonvanishing and all \(\nu_i\) are Majorana neutrinos.

**What can we do in that case?**
In principle, although probably not in practice, we can look for the lepton number violation involving muons.

Numerical example: take \(\theta_{13} = 0\), and Majorana phase \(\alpha_2 - \alpha_1 = \pi\) (only for this choice of phases can \(<m_{\beta\beta}>\) vanish when \(\theta_{13} = 0\)).

\(<m_{\beta\beta}> = 0 \text{ if } m_1/m_2 = \tan^2\theta_{12}, \text{ with } m_2 = (m_1^2 + \Delta m_{\text{sol}}^2)^{1/2}.\)

That happens for \(m_1 = 4.58\) meV and \(m_2 = 10\) meV (this is, therefore, fine tuning).

But then \(<m_{\mu e}> = \sin2\theta_{12}\cos\theta_{23}/2 \times (m_1 + m_2) = 4.78\) meV,
Which is, at least in principle, observable using \(\mu^- + (Z,A) \rightarrow e^+ + (Z-2,A).\)
The issue of nuclear matrix elements

A provocative question: Do we know at all how large the matrix elements really are? Or, in other words, why there is so much variation among the published calculated matrix elements?

This suggests an uncertainty of as much as a factor of 5. Is it really so bad?
Why it is difficult to calculate the matrix elements accurately?

Contributions of different angular momenta $J$ of the neutron pair that is transformed in the decay into the proton pair with the same $J$.

Note the opposite signs, and thus tendency to cancel, between the $J = 0$ (pairing) and the $J \neq 0$ (ground state correlations) parts.

The same restricted s.p. space is used for QRPA and NSM. There is a reasonable agreement between the two methods.
The radial dependence of $M^{0\nu}$ for the three indicated nuclei. The contributions summed over all components ss shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below. Note that these two parts essentially cancel each other for $r > 2-3$ fm. This is a generic behavior. Hence the treatment of small values of $r$ and large values of $q$ are quite important. The curves are from QRPA. However, essentially identical curves are obtained in NSM.
The radial dependence of $M^{0\nu}$ for the indicated nuclei, evaluated in the nuclear shell model. (Menendes et al, arXiv:0801.3760). Note the similarity to the QRPA evaluation of the same function.
Calculated values of the nuclear matrix elements, QRPA vs. NSM
There is a steady progress in the sensitivity of the searches for $0\nu\beta\beta$ decay. Several experiments that are funded and almost ready to go will reach sensitivity to $\sim 0.1$ eV. There is one (so far unconfirmed) claim that the $0\nu\beta\beta$ decay of $^{76}$Ge was actually observed. The deduced mass $<m_{\beta\beta}>$ would be then 0.3–0.7 eV.
What should happen next?

1) In a number of new experiments (CUORE, EXO, Majorana, MOON, SuperNEMO, GERDA, etc) the amount of source will be increased from the present ~10 kg to ~100 kg, and the sensitivity from the ~$10^{25}$ y to ~$10^{26-27}$ y, covering the `degenerate` mass region.

2) This should open the door for ~ton $0\nu\beta\beta$ decay experiments that will reach into the `inverted hierarchy` region.

3) Next generation of experiments on LFV will extend the sensitivity considerably. In parallel, running of LHC will shed light on the existence of particles with ~TeV masses.

4) Hopefully, progress in the nuclear structure calculation will remove some or most of the uncertainty in the $0\nu\beta\beta$ nuclear matrix elements.
## “Selected” Projects

<table>
<thead>
<tr>
<th>Project</th>
<th>Description</th>
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<tbody>
<tr>
<td>CUORE</td>
<td>TeO$_2$ Crystal bolometers</td>
</tr>
<tr>
<td>EXO</td>
<td>Liquid Xe TPC, daughter tag</td>
</tr>
<tr>
<td>GERDA</td>
<td>Bare Ge detectors in $\Upsilon$ LAr</td>
</tr>
<tr>
<td>Majorana</td>
<td>Ge det. in traditional cryostat</td>
</tr>
<tr>
<td>MOON</td>
<td>Scint. sandwiching Mo foils</td>
</tr>
<tr>
<td>SuperNEMO</td>
<td>Foils, tracking and scint.</td>
</tr>
</tbody>
</table>
$0\nu\beta\beta$ half-lives for $<m_{\beta\beta}> = 100$ meV based on the matrix elements of Simkovic et al. (arXiv:0710.2055). This is a conservative range based on the estimated QRPA uncertainty in the nuclear matrix elements. The estimates are highly correlated, if one of them is indeed near its upper edge, all of them are.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Half-life Range (years)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>$(1-3) \times 10^{26}$</td>
<td>GERDA plans, Phase II, to reach $2 \times 10^{26}$y</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>$(0.5 - 1.2) \times 10^{26}$</td>
<td></td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>$(0.25 - 1) \times 10^{26}$</td>
<td></td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>$(0.25 - 1) \times 10^{26}$</td>
<td>CUORE plans to reach $(2-6) \times 10^{26}$y</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>$(0.5 - 4) \times 10^{26}$</td>
<td>EXO-200 plans to reach $6 \times 10^{25}$y</td>
</tr>
</tbody>
</table>

Note: The sensitivity to $<m_{\beta\beta}>$ scales as $1/(T_{1/2})^{1/2}$
Conclusions

• There is overwhelming evidence for a small but finite neutrino mass: Need drastic ideas to understand it
• Testing the lepton number conservation will tell us whether neutrino are Majorana fermions or not, and thus help in pointing us in the right direction
• Hopefully, we will learn a lot more in the next few years