A PARAMETRIC RESONANCE IN OSCILLATIONS OF SOLAR AND ATMOSPHERIC NEUTRINOS IN THE EEARTH

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ABSTRACT

Neutrino oscillations in matter can exhibit a specific resonance enhancement – parametric resonance, which is different from the MSW resonance. Recently it has been shown that the oscillations of atmospheric and solar neutrinos inside the earth can undergo parametric enhancement when neutrino trajectories cross the core of the earth. In this talk I discuss stability of the parametric resonance with respect to the variations of the zenith angle of the neutrino source, higher-order resonances and prospects of the experimental observation of the parametric resonance in neutrino oscillations.

1. Introduction

It is well known that neutrino oscillations in matter can differ significantly from oscillations in vacuum, the best studied example being the Mikheyev-Smirnov-Wolfenstein (MSW) effect [1,2]. The MSW effect is an enhancement of the neutrino flavor transition probability due to an amplification of the oscillation *amplitude* in matter. However, possible enhancement effects of matter on neutrino oscillations are not limited to the MSW effect. It has been pointed out in [3,4] that the probability of neutrino flavor transition can also be strongly enhanced if the oscillation *phase* undergoes certain modification in matter. This can happen if the change of the matter density along the neutrino path is correlated in a certain way with the change of the oscillation phase. This amplification of the neutrino oscillation probability in matter due to specific phase relationships has an interesting property that it can accumulate if the matter density profile along the neutrino path repeats itself, *i.e.* is periodic. The phenomenon is analogous to the resonance in dynamical systems whose parameters periodically vary with time – parametric resonance. It was therefore called parametric resonance of neutrino oscillations [3,4]. In particular, in [4] the parametric resonance was considered for neutrino oscillations in a matter with a periodic step function ("castle wall") density profile, which allows an exact analytic solution.

The parametric resonance can lead to large probabilities of neutrino flavor transition in matter even if the mixing angles both in vacuum and in matter are small. This happens because each half-wave oscillation of the transition probability is placed on the top of the previous one, *i.e.* the transition probability builds up. If mixing angle in matter is very small (matter density is far from the MSW resonance density), the

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parametric resonance enhancement of neutrino oscillations can manifest itself only if the neutrinos pass through a large number of periods of density modulation, *i.e.* travel a sufficiently long distance. However, if matter density is not very far from the MSW resonance one, an interesting interplay between the MSW effect and parametric effects can occur. In particular, a strong parametric enhancement of neutrino oscillations can take place even if the neutrinos pass only through 1 - 2 periods of density modulation [5)].

Although the parametric resonance of neutrino oscillations is certainly an interesting physical phenomenon, it requires that very special conditions be satisfied. Unfortunately, these conditions cannot be created in the laboratory because this would require either too long a baseline or neutrino propagation in a matter of too high a density (see Sec. 5 below). Until recently it was also unclear whether a natural object exists where these conditions can be satisfied for any known source of neutrinos. This situation has changed with a very important observation by Liu and Smirnov [6)] (see also [7)]), who have shown that the parametric resonance conditions can be approximately satisfied for the oscillations of atmospheric ν_μ into sterile neutrinos ν_s inside the earth.

It is known that the earth consists of two main structures – the mantle and the core. Within the mantle and within the core the matter density changes rather slowly (the density variation scale is large compared to the typical oscillation lengths of atmospheric and solar neutrinos), but at their border it jumps sharply by about a factor of two. Therefore to a good approximation one can consider the mantle and the core as structures of constant densities equal to the corresponding average densities (two-layer model). Neutrinos coming to the detector from the lower hemisphere at zenith angles Θ in the range defined by $\cos\Theta = (-1) \div (-0.837)$ traverse the earth's mantle, core and then again mantle. Therefore such neutrinos experience a periodic "castle wall" potential, and their oscillations can be parametrically enhanced. Even though the neutrinos pass only through "1.5 periods" of density modulations (this would be exactly one period and a half if the distances neutrinos travel in the mantle and in the core were equal), the parametric effects on neutrino oscillations in the earth can be quite strong. Subsequently, it has been pointed out by Petcov [8)] that the parametric resonance conditions can also be satisfied (and to even a better accuracy) for the $\nu_2 \leftrightarrow \nu_e$ oscillations in the earth in the case of the ν_e - $\nu_{\mu(\tau)}$ mixing [a]. This, in particular, may have important implications for the solar neutrino problem. The parametric resonance in the oscillations of solar and atmospheric neutrinos in the earth was further explored in a number of papers [9,10,11)]. In this talk I discuss the stability of the parametric resonance with respect to the variation of the zenith angle of neutrino trajectory, parametric resonances of higher order, and the prospects of

[a]In [8)] a different name for this phenomenon was suggested – neutrino oscillation length resonance. We prefer to follow the original terminology of [3,4)].

experimental observation of the parametric resonance in neutrino oscillations.

2. Parametric resonance in neutrino oscillations in the earth

We shall briefly summarize here some of the formulas describing the parametric resonance in neutrino oscillations in the earth. More detailed discussion can be found in [9,12].

Consider the oscillations in a 2-flavor neutrino system in a matter with periodic step function density profile. We will be assuming that one period of density modulation consists of two parts of the lengths R_m and R_c, with the corresponding effective matter densities N_m and N_c ("castle wall" density profile). For the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations the effective matter density coincides with the electron number density, whereas for the $\nu_{e,\mu,\tau} \leftrightarrow \nu_s$ oscillations in an isotopically symmetric matter it is a factor of two smaller. The parametric resonance in such a system occurs when the oscillations phases $2\phi_m$ and $2\phi_c$ acquired over the intervals R_m and R_c are odd integer multiples of π [4,6,7]. Let us denote

$$\delta = \frac{\Delta m^2}{4E}\,, \qquad V_i = \frac{G_F}{\sqrt{2}} N_i\,, \qquad \omega_i = \sqrt{(\cos 2\theta_0\, \delta - V_i)^2 + (\sin 2\theta_0\, \delta)^2} \qquad (i = m,\ c)\,. \tag{1}$$

Here E, Δm^2 and θ_0 are the neutrino energy, mass squared difference and vacuum mixing angle, respectively. The difference of the neutrino eigenenergies in a matter of density N_i is $2\omega_i$, so that the oscillations phases acquired over the intervals R_m and R_c are

$$2\phi_m = 2\omega_m R_m\,, \qquad 2\phi_c = 2\omega_c R_c\,. \tag{2}$$

The parametric resonance conditions can be written as [3,4,5,6,7,8,9]

$$\phi_1 = \frac{\pi}{2} + k\pi\,, \qquad \phi_2 = \frac{\pi}{2} + k'\pi\,, \qquad k, k' = 0, 1, 2, ... \tag{3}$$

The trajectories of neutrinos traversing the earth are determined by their nadir angle Θ_n. The distances R_m and R_c that neutrinos travel in the mantle (each layer) and in the core are given by

$$R_m = R\left(\cos\Theta_n - \sqrt{r^2/R^2 - \sin^2\Theta_n}\right)\,, \qquad R_c = 2R\sqrt{r^2/R^2 - \sin^2\Theta_n}\,. \tag{4}$$

Here $R = 6371$ km is the earth's radius and $r = 3486$ km is the radius of the core. The matter density in the mantle of the earth ranges from 2.7 g/cm^3 at the surface to 5.5 g/cm^3 at the bottom, and that in the core ranges from 9.9 to 12.5 g/cm^3 (see, *e.g.*, [16]). The electron number fraction Y_e is close to 1/2 both in the mantle and in the core. Taking the average matter densities in the mantle and core to be 4.5 and 11.5 g/cm^2 respectively, one finds for the $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations involving only active neutrinos the following values of V_m and V_c: $V_m = 8.58 \times 10^{-14}$ eV, $V_c = 2.19 \times 10^{-13}$

eV. For transitions involving sterile neutrinos $\nu_e \leftrightarrow \nu_s$ and $\nu_{\mu,\tau} \leftrightarrow \nu_s$, these parameters are a factor of two smaller.

3. Stability of the parametric resonance conditions

If the parametric resonance conditions (3) are satisfied, strong parametric enhancement of the oscillations of core crossing neutrinos in the earth can occur [6,7,8,9,10,11]. We shall now discuss these conditions. The phases ϕ_m and ϕ_c depend on the neutrino parameters Δm^2, θ_0 and E and also on the distances R_m and R_c that the neutrinos travel in the mantle and in the core. The path lengths R_m and R_c vary with the nadir angle; however, as can be seen from (4), their changes are correlated and they cannot take arbitrary values. Therefore if for some values of the neutrino parameters a value of the nadir angle Θ_n exists for which, for example, the first condition in Eq. (3) is satisfied, it is not obvious if at the same value of Θ_n the second condition will be satisfied as well. In other words, it is not clear if the parametric resonance conditions can be fulfilled for neutrino oscillations in the earth for at least one set of the neutrino parameters Δm^2, θ_0 and E. However, as was shown in [8,9], not only the parametric resonance conditions are satisfied (or approximately satisfied) for a rather wide range of the nadir angles covering the earth's core, they are fulfilled for the ranges of neutrino parameters which are of interest for the neutrino oscillations solutions of the solar and atmospheric neutrino problems. In particular, the conditions for the principal resonance ($k = k' = 0$) are satisfied to a good accuracy for $\sin^2 2\theta_0 \lesssim 0.1$, $\delta \simeq (1.1 \div 1.9) \times 10^{-13}$ eV^2, which includes the ranges relevant for the small mixing angle MSW solution of the solar neutrino problem and for the subdominant $\nu_\mu \leftrightarrow \nu_e$ and $\nu_e \leftrightarrow \nu_\tau$ oscillations of atmospheric neutrinos.

The fact that the parametric resonance conditions can be satisfied so well for neutrino oscillations in the earth is rather surprising. It is a consequence of a number of remarkable numerical coincidences. It has been known for some time [17,6,18] that the potentials V_m and V_c corresponding to the matter densities in the mantle and core, the inverse radius of the earth R^{-1}, and typical values of $\delta \equiv \Delta m^2/4E$ of interest for solar and atmospheric neutrinos, are all of the same order of magnitude – $(3 \times 10^{-14}$ – $3 \times 10^{-13})$ eV. It is this surprising coincidence that makes appreciable earth effects on the oscillations of solar and atmospheric neutrinos possible. However, for the parametric resonance to take place, a coincidence by an order of magnitude is not sufficient: the conditions (3) have to be satisfied at least within a 50% accuracy [9]. This is exactly what takes place. In addition, in a wide range of the nadir angles Θ_n, with changing Θ_n the value of the parameter δ at which the resonance conditions (3) are satisfied slightly changes, but the fulfillment of these conditions is not destroyed. We shall show now that in this row of mysterious coincidences, at least the last one – the stability of the parametric resonance conditions with respect to variations of the nadir angle – has a simple explanation.

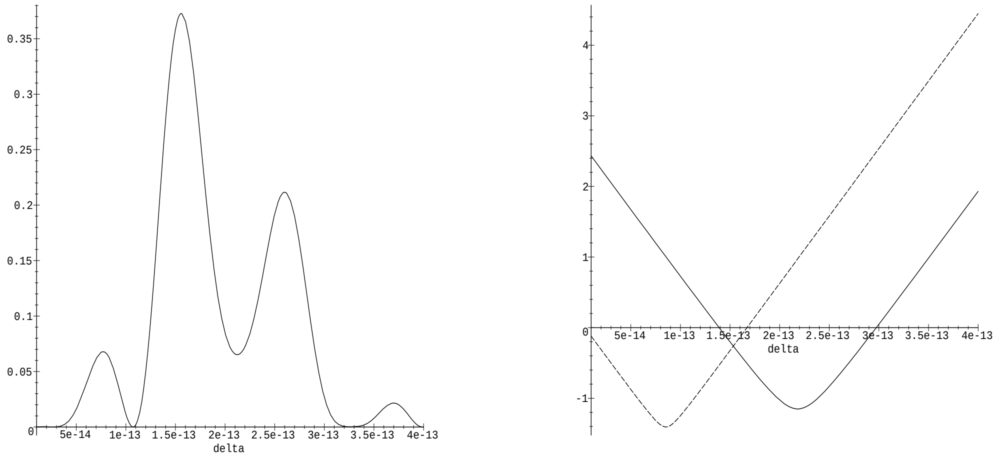

Fig. 1. Transition probability P for $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations of core-crossing neutrinos in the earth, left panel, and the corresponding phases $\phi_m - \pi/2$ (dashed curve) and $\phi_c - \pi/2$ (solid curve), right panel. $\sin^2 2\theta_0 = 0.01$, $\Theta_n = 27.3°$.

We begin with simple qualitative arguments. Consider the principal parametric resonance for which $k = k' = 0$. In fig. 1 the flavor transition probability P and the parameters $\phi_m - \pi/2$ and $\phi_c - \pi/2$ are plotted as functions of δ for $\sin^2 2\theta_0 = 0.01$ and $\Theta_n = 27.3°$. The highest peak of P corresponds to $\delta \simeq 1.55 \times 10^{-13}$ eV at which, as can be seen in the figure, both $\phi_m - \pi/2$ and $\phi_c - \pi/2$ nearly cross the abscissa, *i.e.* the conditions (3) are approximately satisfied. This is the parametric resonance peak; the smaller peaks on its left and right are due to the MSW resonances in the mantle and in the core, respectively. The minima of the curves $\phi_m - \pi/2$ and $\phi_c - \pi/2$ correspond to the minima of ω_m and ω_c. The values of R_m and R_c determine the steepness of the curves. With increasing nadir angle, R_m increases and R_c decreases, which means that the branches of the $\phi_m - \pi/2$ curve become steeper and those of the $\phi_c - \pi/2$ curve become less steep. Increasing the steepness of $\phi_m - \pi/2$ would push the intersection point of the $\phi_m - \pi/2$ and $\phi_c - \pi/2$ curves upwards; however, decreasing the steepness of $\phi_c - \pi/2$ would push this intersection point in the opposite direction. As a result, in a wide range of the nadir angles Θ_n, with changing Θ_n the intersection point shifts along the abscissa but remains close to it, *i.e.* the parametric resonance conditions are not destroyed. It is easy to see that this only happens when the intersection takes place between the minima of $\phi_m - \pi/2$ and $\phi_c - \pi/2$, *i.e.* for δ in the range

$$V_m/\cos 2\theta_0 < \delta < V_m/\cos 2\theta_0\,. \tag{5}$$

If the intersection takes place outside this interval, the changes of the steepness of the $\phi_m - \pi/2$ and $\phi_c - \pi/2$ curves due to changing Θ_n will push their intersection point in the same direction, and this will quickly destroy the parametric resonance conditions. Therefore outside the range (5) the parametric resonance can only occur in rather narrow intervals of Θ_n. Interestingly, the interval (5) is not only favored due to the wide range of allowed values of Θ_n, but also corresponds to the region where the strongest parametric enhancement of neutrino oscillations takes place [8,9].

We shall now quantify the above arguments. Each of the two parametric resonanceconditions in (3) can be solved with respect to the value δ_0 of the parameter δ at the resonance:

$$\delta_0 = \cos 2\theta_0\, V_m \pm \sqrt{[(2k+1)\pi]^2/4R_m^2 - \sin^2 2\theta_0\, V_m^2} \tag{6}$$

$$\delta_0 = \cos 2\theta_0\, V_c \pm \sqrt{[(2k'+1)\pi]^2/4R_c^2 - \sin^2 2\theta_0\, V_c^2} \tag{7}$$

The parametric resonance can occur if the values of δ_0 given by these two expressions coincide or nearly coincide. Notice that the existence of the radicals in (6) and (7) in general puts upper limits on the vacuum mixing angle θ_0. In particular, for the principal resonance, assuming that the resonance condition can be satisfied for all neutrino trajectories that cross the core (including the vertical ones), from (7) one finds [9]

$$\sin^2 2\theta_0 \le \frac{\pi^2}{4(R_c)^2_{max}\, V_c^2} \simeq 0.04\,. \tag{8}$$

If one excludes small nadir angles, the constraint becomes less stringent. For example, for $\sin^2\Theta_n \ge 0.12$ one obtains $\sin^2 2\theta_0 \le 0.07$. The analogous upper bound following from (6) are less restrictive except for the trajectories close to the edge of the core of the earth.

The $(\pm)$ signs in Eqs. (6) and (7) correspond to different regimes of the parametric resonance. We shall now choose the plus sign in (6) and the minus sign in (7), which for small θ_0 corresponds to the interval (5). The other cases can be considered similarly [b].

The consistency of Eqs. (6) and (7) requires

$$\cos 2\theta_0\,(V_c - V_m) = \sqrt{\frac{[(2k+1)\pi]^2}{4R_m^2} - \sin^2 2\theta_0\, V_m^2} + \sqrt{\frac{[(2k'+1)\pi]^2}{4R_c^2} - \sin^2 2\theta_0\, V_c^2}\,. \tag{9}$$

For small vacuum mixing angles θ_0 this simplifies to [c]

$$(V_c - V_m) = \frac{\pi}{2}\left(\frac{2k+1}{R_m} + \frac{2k'+1}{R_c}\right)\,. \tag{10}$$

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[b] In the cases that correspond to the two plus signs, the parametric resonance does not lead to a maximum of the transition probability; for two minus signs there exist parametric maxima, but they are typically smaller than in the region of δ that we are considering.

[c] For the particular case $k = k' = 0$ this condition was obtained in [8].

Eq. (9) was obtained from the requirement that both conditions in (3) are satisfied at the same value of δ. If it is fulfilled, with changing Θ_n the resonance value of δ changes, but the parametric resonance conditions are still satisfied. Thus, it is the condition (9) that ensures the stability of the parametric resonance conditions with respect to the variations of the nadir angle. It remains now to check if or when this condition is indeed satisfied. We shall do that in the limit of the small θ_0 when the simplified form (10) applies. Similar analysis can be performed with the use of Eq. (9).

The l.h.s. of Eq. (10) is Θ_n independent, so must be the r.h.s. As we have pointed out above, with increasing Θ_n the path length R_m increases whereas R_c decreases, which is just a consequence of the spherical geometry of the earth. One can therefore expect at least a partial cancellation of the changes of the $(2k+1)/R_m$ and $(2k'+1)/R_c$ terms in their sum on the r.h.s. of Eq. (10). For the principal parametric resonance with $k = k' = 0$ this cancellation is indeed quite good in a rather wide range of Θ_n. Using Eq. (4) and it is easy to make sure that while R_m^{-1} and R_c^{-1} change noticeably with Θ_n, their sum remains almost constant in the interval $\Theta_n = 0 \div 23°$. For larger values of Θ_n this sum increases, but the increase does not exceed 25% for $\Theta_n \lesssim 31°$. Thus, if the parametric resonance conditions are satisfied at some value of Θ_n in the interval $0 \leq \Theta_n \lesssim 31°$, they will be approximately satisfied in the whole interval. This explains the stability of the parametric resonance conditions. The fact that they *are* satisfied to a good accuracy for at least one value in the above-mentioned interval is a consequence of the numerical coincidences that we discussed earlier.

4. Higher-order parametric resonances

In addition to the principal parametric resonance with $k = k' = 0$ there exist two higher order resonances of neutrino oscillations in the earth, with $k = 0$, $k' = 1$ and $k = 0$, $k' = 2$. However, these resonances are less interesting than the principal one: the domain of the neutrino parameters in which they can occur for the oscillations of solar neutrino in the earth is currently disfavored by the solar neutrino data (though not completely ruled out) [12]. These resonances are of no interest for the atmospheric neutrino oscillations.

5. Can the parametric resonance of neutrino oscillations be observed?

Besides being an interesting physical phenomenon, the parametric resonance of neutrino oscillations can provide us with an important additional information about neutrino properties. Therefore experimental observation of this effect would be of considerable interest. We shall now discuss the prospects for experimental observation of the parametric resonance of neutrino oscillations in the earth, having in mind mainly the principal resonance. There are two main sources of neutrinos for which the parametric resonance can be important – atmospheric neutrinos and solar neutrinos. Both sources have their advantages and disadvantages from the point of view of the possibility of observation of the parametric resonance. We shall now briefly discuss them.

We start with atmospheric neutrinos. The parametric resonance can occur in the $\nu_\mu \leftrightarrow \nu_s$ [6,7] and also in the subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ channels of oscillations [10]. It can affect the distributions of μ-like events and also (in the case of the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations) lead to interesting peculiarities in the zenith angle distributions of the multi-GeV e-like events.

The observation of the parametric effects is hampered by the loose correlation between the directions of the momenta of atmospheric neutrinos and of the charged leptons which they produce and which are actually detected. Because of this the trajectories of neutrinos coming to the detector are not known very precisely. In addition, the data are presented for certain samples of events (sub-GeV, multi-GeV, upward through-going, upward stopping) which includes collecting data over rather wide energy intervals. The contributions of the parametric peaks may therefore be integrated over together with other possible enhancement peaks – due to the MSW resonances in the mantle and in the core, making the distinction between these effects difficult. Also, strong resonance enhancement effects (both parametric and MSW) can only occur either for neutrinos or for antineutrinos, depending on the sign of Δm^2.[d] The present atmospheric neutrino experiments do not distinguish between neutrinos and antineutrinos, therefore possible matter effects are "diluted" in the sum of the ν- and $\bar{\nu}$-induced events. At certain values of the ratio of the muon and electron neutrino fluxes $r(E, \Theta_n)$ depending on the value of mixing angle θ_{23} the parametric effects on e-like events are suppressed [10].

Atmospheric neutrinos have some advantages for observation of the parametric resonance of neutrino oscillations in the earth. Neutrinos come to the detectors from all directions, which means that practically the whole solid angle covering the earth's core will contribute to the effect. There are no additional suppression factors due to a specific composition of the incoming neutrino flux which may quench the earth's matter effect on the oscillations of solar neutrinos (see below). Parametric effects may provide a unique probe of the neutrino mixing angle θ_{13} with sensitivity going far beyond that of the long-baseline accelerator and reactor experiments [10,23]. Possible ways of improving the prospects of the experimental observation of the parametric effects in the atmospheric neutrino oscillations include using various energy cuts, finer zenith angle binning and detectors capable of detecting the recoil nucleon, which would enable one to reconstruct the direction of an incoming neutrino [10,23].

Solar neutrinos can experience a strong parametric enhancement of their oscillations in the earth if the small mixing angle MSW effect is the correct explanation of

[d] In general, the parametric enhancement can take place even for "wrong sign" Δm^2, but in the case of neutrino oscillations in the earth these effects are small.

the the solar neutrino deficit [8,9]. The parametric enhancement can occur in a wide range of values of $\sin^2 2\theta_0$ and for the nadir angles Θ_n almost completely covering the core of the earth. The trajectory of each detected neutrino is exactly known. For boron neutrinos the resonance occurs at the values of Δm^2 which correspond to the central part of the allowed interval for the small mixing angle MSW solution of the solar neutrino problem.

However, there are some disadvantages, too. Unfortunately, due to their geographical location, the existing solar neutrino detectors have a relatively low time during which solar neutrinos pass through the core of the earth to reach the detector every calendar year. The Super-Kamiokande detector has a largest fractional core coverage time equal to 7%. In [17] it was suggested to build a new detector close to the equator in order to increase the sensitivity to the earth regeneration effect; this would also maximize the parametric resonance effects in oscillations of solar neutrinos in the earth. In the case of the MSW solutions of the solar neutrino problem the probability P_{SE} of finding a solar ν_e after it traverses the earth depends sensitively on the average ν_e survival probability in the sun $\bar{P}_S$ [13,14]:

$$P_{SE} = \bar{P}_S + \frac{1-2\bar{P}_S}{\cos 2\theta_0}\,(P_{2e} - \sin^2\theta_0)\,. \tag{11}$$

The probability P_{2e} can experience a strong parametric enhancement, but in the case of small mixing angle MSW solution of the solar neutrino problem the probability $\bar{P}_S$ for the Super-Kamiokande and SNO experiments turns out to be rather close to 1/2. This means that the effects of passage through the earth on solar neutrinos should be strongly suppressed. The current best fit of the solar neutrino data is not far from the line in the parameter space where $\bar{P}_S$ is exactly equal to 1/2 and $P_{SE} = \bar{P}_S$ (*i.e.* the earth matter effects are absent). Whether or not it will be possible to observe the parametric resonance of the oscillations of solar neutrinos in the earth depends on how close to this line the true values of $\sin^2 2\theta_0$ and Δm^2 are. By now the Super-Kamiokande experiment has not observed, within its experimental accuracy, any enhancement of neutrino signal for earth core crossing neutrinos [24]. This can be because the parametric enhancement of the neutrino oscillations in the earth does not occur (*e.g.* if the true solution of the solar neutrino problem is vacuum oscillations or large mixing angle MSW effect), or because the values of Δm^2 and $\sin^2 2\theta_0$ are too close to those at which $\bar{P}_S = 1/2$. Hopefully, with accumulated statistics of the Super-Kamiokande and forthcoming data from the SNO experiment the situation will soon be clarified.

It is interesting to note that the Super-Kamiokande data on the zenith angle dependence of the solar neutrino events seems to indicate some deficiency of the events due to the core-crossing neutrinos rather than an excess [24], although it is not statistically significant. Should this deficiency be confirmed by future data with better statistics, it could have a natural explanation in terms of the parametric resonance of neutrino oscillations. As follows from (11), the parametric enhancement of P_{2e} for core crossing neutrinos can lead to a deficiency of the events if the neutrino parameters are in the small-$\sin^2 2\theta_0$ part of the allowed region which corresponds to $P_S > 1/2$ (see, *e.g.*, fig. 10 in ref. [25]). In this case one should also have an "opposite sign" overall day-night effect (fewer events during the night than during the day). In any case, given the current experimental constraints on the neutrino parameters, if the small mixing angle MSW effect is the true solution of the solar neutrino problem, the only hope to observe earth matter (day-night) effects on solar neutrinos seems to be through the parametric resonance of oscillations of core crossing neutrinos.

As we have seen, observing the parametric resonance in oscillations of solar and atmospheric neutrinos in the earth is not an easy task. Can one create the necessary matter density profile and observe the parametric resonance in neutrino oscillations in the laboratory (*i.e.* short-baseline) experiments? Unfortunately, the answer to this question seems to be negative. The parametric resonance can occur when the mean oscillation length in matter approximately coincides with the matter density modulation length [3,4,5]: $l_m \simeq L$. In a matter of density N_i the oscillation length is given by $l_m = \pi/\omega_i$ where ω_i was defined in (1). Let us require $l_m \lesssim 1$ km. Assume first that $V_i \gtrsim \delta$, *i.e.* ω_i are dominated by matter density terms. Than for $l_m \lesssim 1$ km one would need a matter of mass density $\rho_i \geq 3.3 \times 10^4$ g/cm^3, clearly not a feasible value. Conversely, for $\rho_i \leq 10$ g/cm^3, one finds $l_m \gtrsim 3300$ km, a distance comparable with the earth's radius. Consider now the opposite case, $\delta \gg V_i$. Then the oscillation length in matter essentially coincides with the vacuum oscillations length which in principle can be rather short provided that the vacuum mixing angle θ_0 is small (otherwise this would contradict reactor and accelerator data). However, in this case there is another problem. Requiring $l_m \lesssim 1$ km one finds $\delta \gtrsim 2.5 \times 10^{-10}$ eV. For $\rho_i \lesssim 10$ g/cm^3 one therefore has $V_i/\delta \lesssim 10^{-3}$. This means that the mixing angles in matter are very close to the vacuum one, $\theta_i \simeq \theta_0(1 + V_i/\delta)$, and so their difference is very small: $\Delta\theta = \theta_2 - \theta_1 \simeq (\Delta V/\delta)\,\theta_0 \lesssim 10^{-3}\theta_0$. When the difference of mixing angles in matter is small, the parametric effects can manifest themselves only if neutrinos travel over a large number of periods, $n \simeq \pi/4\Delta\theta$ [3,4,5,9]. Therefore in this case the necessary baseline is $\sim \pi^2/(4\theta_0\Delta V) \gtrsim 3 \times 10^3$ km, again too large. One can conclude that the sole presently known object where the parametric resonance of neutrino oscillations can take place is the earth, as was first pointed out in [6,7].

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