NEUTRINO MASSES AND MIXINGS: A THEORETICAL PERSPECTIVE

GUIDO ALTARELLI
Theoretical Physics Division, CERN, CH - 1211 Geneva 23
and
Università di Roma Tre, Rome, Italy
E-mail: guido.altarelli@cern.ch

and

FERRUCCIO FERUGLIO
Università di Padova and I.N.F.N., Sezione di Padova, Padua, Italy
E-mail: feruglio@pd.infn.it

ABSTRACT

We briefly review the recent activity on neutrino masses and mixings which was prompted by the confirmation of neutrino oscillations by the Superkamiokande experiment.

1. Introduction

Recent data from Superkamiokande [1)] have provided a more solid experimental basis for neutrino oscillations as an explanation of the atmospheric neutrino anomaly. In addition the solar neutrino deficit, observed by several experiments [2)], is also probably an indication of a different sort of neutrino oscillations. Results from the laboratory experiment by the LSND collaboration [3)] can also be considered as a possible indication of yet another type of neutrino oscillation. Neutrino oscillations [a] imply neutrino masses. The extreme smallness of neutrino masses in comparison with quark and charged lepton masses indicates a different nature of neutrino masses, linked to lepton number violation and the Majorana nature of neutrinos. Thus neutrino masses provide a window on the very large energy scale where lepton number is violated and on Grand Unified Theories (GUTs) [5)]. The new experimental evidence on neutrino masses could also give an important feedback on the problem of quark and charged lepton masses, as all these masses are possibly related in GUTs. In particular the observation of a nearly maximal mixing angle for $\nu_\mu \to \nu_\tau$ is particularly interesting.

Perhaps also solar neutrinos may occur with large mixing angle. At present solar neutrino mixings can be either large or very small, depending on which particular solution will eventually be established by the data. Large mixings are very interesting because a first guess was in favour of small mixings in the neutrino sector in analogy to what is observed for quarks. If confirmed, single or double maximal mixings can provide an important hint on the mechanisms that generate neutrino masses. The

[a] Alternative explanations such as neutrino decay and violations of the equivalence principle appear to be disfavoured by the present data [4)].

purpose of this article is to provide a concise review of the implication of neutrino masses and mixings on our picture of particle physics. We will not review in detail the status of the data but rather concentrate on their conceptual impact.

The experimental status of neutrino oscillations is still very preliminary. While the evidence for the existence of neutrino oscillations from solar and atmospheric neutrino data is rather convincing by now, the values of the mass squared differences Δm^2 and mixing angles are not firmly established. For the observed ν_e suppression of solar neutrinos, for example, three possible solutions are still possible [6]. Two are based on the MSW mechanism [7], one with small (MSW-SA: $\sin^2 2\theta_{sun} \sim 5.5\ 10^{-3}$) and one with large mixing angle (MSW-LA: $\sin^2 2\theta_{sun} \gtrsim 0.2$). The third solution is in terms of vacuum oscillations (VO) with large mixing angle (VO: $\sin^2 2\theta_{sun} \sim 0.75$). However, it is important to keep in mind that the Δm^2 values of the above solutions are determined by the experimental result that the suppression is energy dependent. This is obtained by comparing experiments with different thresholds. The Cl experiment shows a suppression larger than by a factor of 2, which is what is shown by Ga and water experiments [2]. If the Cl indication is disregarded, then new energy independent solutions would emerge, with large Δm^2 and maximal mixing. For example, good fit of all data, leaving those on Cl aside, can be obtained with Δm^2 as large as $\Delta m^2 \lesssim 10^{-3}\mathrm{eV}^2$ [8,9]. For atmospheric neutrinos the preferred value of Δm^2, in the range $10^{-3} - 10^{-2}$ eV^2, is still affected by experimental uncertainties and could sizeably drift in one sense or the other, but the fact that the mixing angle is large appears established ($\sin^2 2\theta_{atm} \gtrsim 0.9$ at 90% C.L.) [10,11,12]. Another issue which is still open is the claim by the LSND collaboration of an additional signal of neutrino oscillations in a terrestrial experiment [3]. This claim was not so-far supported by a second recent experiment, Karmen [13], but the issue is far from being closed.

Given the present experimental uncertainties the theorist has to make some assumptions on how the data will finally look like in the future. Here we tentatively assume that the LSND evidence will disappear. If so then we only have two oscillations frequencies, which can be given in terms of the three known species of light neutrinos without additional sterile kinds (i.e. without weak interactions, so that they are not excluded by LEP). We then take for granted that the frequency of atmospheric neutrino oscillations will remain well separated from the solar neutrino frequency, even for the MSW solutions. The present best values are [6,10,11,12,14] $(\Delta m^2)_{atm} \sim 3.5\ 10^{-3}$ eV^2 and $(\Delta m^2)_{MSW-SA} \sim 5\ 10^{-6}$ eV^2 or $(\Delta m^2)_{VO} \sim 10^{-10}$ eV^2. We also assume that the electron neutrino does not participate in the atmospheric oscillations, which (in absence of sterile neutrinos) are interpreted as nearly maximal $\nu_\mu \to \nu_\tau$ oscillations as indicated by the Superkamiokande [1] and Chooz [15] data. However the data do not exclude a non-vanishing U_{e3} element. In the Superkamiokande allowed region the

bound by Chooz [15] amounts to $|U_{e3}| \lesssim 0.2$ [10,11,14].

2. Direct Limits on Neutrino Masses

Neutrino oscillations are due to a misalignment between the flavour basis, $\nu' \equiv (\nu_e, \nu_\mu, \nu_\tau)$, where ν_e is the partner of the mass and flavour eigenstate e^- in a left-handed weak isospin $SU(2)$ doublet (similarly for ν_μ and ν_τ)) and the neutrino mass eigenstates $\nu \equiv (\nu_1, \nu_2, \nu_3)$:

$$|\nu'\rangle = U|\nu\rangle, \tag{1}$$

where U is the 3 by 3 mixing matrix. Thus, in presence of mixing, neutrinos cannot be all massless and actually the presence of two different oscillation frequencies implies at least two different non zero masses. Neutrino oscillations are only sensitive to differences Δm^2 so that the absolute scale of squared masses is not fixed by the observed frequencies. But the existing direct bounds on neutrino masses, together with the observed frequencies, imply that all neutrino masses are by far smaller than any quark or lepton masses. In fact the following direct bounds hold: $m_{\nu_e} \lesssim\sim 5$ eV, $m_{\nu_\mu} \lesssim 170$ KeV and $m_{\nu_\tau} \lesssim 18$ MeV [16]. Since the observed Δm^2 indicate mass splittings much smaller than that, the limit on m_{ν_e} is actually a limit on all neutrino masses. Moreover from cosmology we know [17] that the sum of masses of (practically) stable neutrinos cannot exceed a few eV, say $\sum m_{\nu_i} \lesssim 6$ eV, corresponding to a fraction of the critical density for neutrino hot dark matter $\Omega_\nu h^2 \lesssim 0.06$ (the present value of the reduced Hubble constant h being around 0.7). In conclusion, the heaviest light neutrinos that are allowed are three nearly degenerate neutrinos of mass around or somewhat below 2 eV. In this case neutrinos would be of cosmological relevance as hot dark matter and contribute a relevant fraction of the critical density. But at present there is no compelling experimental evidence for the necessity of hot dark matter [17,18]. As a consequence, neutrino masses can possibly be much smaller than that. In fact, for widely split neutrino masses the heaviest neutrino would have a mass around ~ 0.06 eV as implied by the atmospheric neutrino frequency.

An additional important constraint on neutrino masses, which will be relevant in the following, is obtained from the non observation of neutrino-less double beta decay. This is an upper limit on the ν_e Majorana mass, or equivalently, on $m_\nu^{ee} = \sum_i U_{ei}^2 m_i$, which is at present quoted to be $m_\nu^{ee} \lesssim 0.2$ eV [19].

3. Neutrino Masses and Lepton Number Violation

Neutrino oscillations imply neutrino masses which in turn demand either the existence of right-handed neutrinos (Dirac masses) or lepton number (L) violation (Majorana masses) or both. Given that neutrino masses are extremely small, it is really difficult from the theory point of view to avoid the conclusion that L must be violated. In fact, it is only in terms of lepton number violation that the smallness of neutrino

masses can be explained as inversely proportional to the very large scale where L is violated, of order M_{GUT} or even M_{Planck}.

Recall that an ordinary Dirac mass term is of the form $m_{RL} = \overline{\nu_R} m_D \nu_L$ plus its hermitian conjugate m_{LR}, where $\nu_{R,L} = 1/2(1 \pm \gamma_5)\nu$ and ν is the neutrino field. The field ν_L annihilates a left-handed neutrino and creates a right-handed antineutrino. Correspondingly the field $\overline{\nu_L}$ creates a left-handed neutrino and annihilates a right-handed antineutrino. Now left-handed neutrinos ν_L and right-handed antineutrinos $(\bar{\nu})_R$ are indeed the only observed neutrino states. Thus in principle one could assume that right-handed neutrinos and left-handed antineutrinos do not exist at all. The field ν_L and its conjugate $\overline{\nu_L}$ would then suffice and there would be no ν_R and $\overline{\nu_R}$ fields in the theory. In the SM lagrangian density only the term $i\overline{\nu_L} \not{D} \nu_L$ appears, where D_μ is the $SU(2) \bigotimes U(1)$ gauge covariant derivative. Clearly if no ν_R is allowed there is no possible Dirac mass for a neutrino.

But if L is violated, there is no conserved quantum number that really makes neutrinos and antineutrinos different and a new type of mass term is possible. For a massive neutrino, the positive helicity state that Lorentz invariance demands to be associated with the state ν_L of given momentum can be $(\bar{\nu})_R$ (for a charged particle with mass, say an e_L^-, one can go to the rest frame, rotate the spin by 180^o and boost again to the original momentum to obtain e_R^-). If neutrinos and antineutrinos are not really distinct particles, both Lorentz and TCP invariance are satisfied by just allowing ν_L and $(\bar{\nu})_R$ (TCP changes one into the other at fixed momentum). For a massive charged particle of spin 1/2 one needs four states, while only two are enough for an intrinsically neutral particle. If L is violated we can have a Majorana mass term $m_{LL} = \overline{(\nu^c)_R} m \nu_L = \nu_L^T C m \nu_L$ where $(\nu^c)_R = C\overline{\nu_L}^T$ and C is the 4 by 4 matrix in Dirac space that implements charge conjugation (the field $(\nu^c)_R$ annihilates a $(\bar{\nu})_R$ exactly as the field $\overline{\nu_L}^T$ does, the transposition only indicating that we want it as a column vector). Clearly, the Majorana mass term m_{LL} violates L by two units. Also, since in the SM ν_L is a weak isospin doublet, m_{LL} transforms as a component of an isospin triplet. In the following, as we are only interested in flavour indices and not in Dirac indices, we will simply denote m_{LL} by $m_{LL} = \nu_L^T m \nu_L$, omitting the Dirac matrix C. Note that if L is violated and ν_R also exists, then a second type of Majorana mass is also possible which is $m_{RR} = \nu_R^T m \nu_R$, where we again omitted C. Clearly also m_{RR} violates L by two units, but, since ν_R is a gauge singlet, m_{RR} is invariant under the SM gauge group. In conclusion, if ν_R does not exist, we can only have a Majorana mass m_{LL} if L is violated. If ν_R exists and L is violated, we can have both Dirac m_{LR} and Majorana masses m_{LL} and m_{RR}.

Imagine that one wanted to give masses to neutrinos and, at the same time, avoid the conclusion that lepton number is violated. Then he/she must assume that ν_R exists and that neutrinos acquire Dirac masses through the usual Higgs mechanism as quark and leptons do. Technically this is possible. But there are two arguments against this possibility. The first argument is that neutrino masses are extremely

small so that the corresponding Yukawa couplings would be enormously smaller than those of any other fermion. Note that within each generation the spread of masses is by no more than a factor 10^{1-2}. But the spread between the t quark and the heaviest neutrino would exceed a factor of 10^{11}. A second argument arises from the fact that once we introduce ν_R in the theory, then the L violating term $m_{RR} = \nu_R^T m \nu_R$ is allowed in the lagrangian density by the gauge symmetry. In the minimal SM, i.e. without ν_R, we understand L and B conservation as accidental global symmetries that hold because there is no operator term of dimension ≤ 4 that violates B and L but respects the gauge symmetry. For example, the transition $u + u \to e^+ + \bar{d}$ is allowed by colour, weak isospin and hypercharge gauge symmetries, but corresponds to a four-fermion operator of dimension 6: $O_6 = (\lambda/M^2)(e^T d)(u^T u)$. This term is suppressed by the dimensional factor $1/M^2$. In the assumption that the SM extended by Supersymmetry is an effective low energy theory which is valid up to the GUT scale, as suggested by the compatibility of observed low energy gauge couplings with the notion of unification at $M_{GUT} \sim 10^{16}$ GeV, the large mass M is identified with M_{GUT}. In fact the factor $1/M_{GUT}^2$ can be obtained from the propagator of a superheavy intermediate gauge boson with the right quantum numbers (e.g. those of an SU(5) generator). In supersymmetric models with R invariance the status of B and L conservation as an accidental symmetry is maintained. In the presence of ν_R, the dimension 3 operator corresponding to m_{RR} is gauge symmetric but violates L. By dimensions we expect a mass factor in front of this operator in the lagrangian density, and in the absence of a protective symmetry, we expect it of the order of the cut-off i.e. of order M_{GUT} or larger. Thus, L number violation is naturally induced by the presence of ν_R, unless we enforce it by hand.

4. L Violation Explains the Smallness of Neutrino Masses: the See-Saw Mechanism

Once we accept L violation we gain an elegant explanation for the smallness of neutrino masses as they turn out to be inversely proportional to the large scale where lepton number is violated. If L is not conserved, even in the absence of ν_R, Majorana masses can be generated for neutrinos by dimension five operators of the form

$$O_5 = L_i^T \lambda_{ij} L_j \phi \phi / M \tag{2}$$

with ϕ being the ordinary Higgs doublet, λ a matrix in flavour space and M a large scale of mass, of order M_{GUT} or M_{Planck}. Neutrino masses generated by O_5 are of the order $m_\nu \sim v^2/M$ for $\lambda_{ij} \sim$ O(1), where $v \sim$ O(100 GeV) is the vacuum expectation value of the ordinary Higgs.

We consider that the existence of ν_R is quite plausible because all GUT groups larger than SU(5) require them. In particular the fact that ν_R completes the representation 16 of SO(10): 16=$\bar{5}$+10+1, so that all fermions of each family are contained

in a single representation of the unifying group, is too impressive not to be significant. At least as a classification group SO(10) must be of some relevance. Thus in the following we assume that there are both ν_R and lepton number violation. With these assumptions the see-saw mechanism [20] is possible. In its simplest form it arises as follows. Consider the mass terms in the lagrangian corresponding to Dirac and RR Majorana masses (for the time being we consider LL Majorana mass terms as comparatively negligible):

$$\mathcal{L} = -\overline{R} m_D L' + \frac{1}{2}\overline{R} M \overline{R}^T + \ h.c. \tag{3}$$

For notational simplicity we denoted ν_L and ν_R by L' and R, respectively (the prime defining the ν flavour basis, see eq. (1)). The 3 by 3 matrices m_D and M are the Dirac and Majorana mass matrices in flavour space (M is symmetric, $M = M^T$, while m_D is, in general, non hermitian and non symmetric). We expect the eigenvalues of M to be of order M_{GUT} or more because RR Majorana masses are $SU(3) \times SU(2) \times U(1)$ invariant, hence unprotected and naturally of the order of the cutoff of the low-energy theory. Since all ν_R are very heavy we can integrate them away. For this purpose we write down the equations of motion for $\bar{R}$ in the static limit, *i.e.* neglecting their kinetic terms:

$$-\frac{\partial \mathcal{L}}{\partial \bar{R}} = m_D L' - M \bar{R}^T = 0 \tag{4}$$

From this, by solving for $\bar{R}^T$ and transposing, we obtain:

$$\bar{R} = L'^T m_D^T M^{-1} \tag{5}$$

We now replace in the lagrangian, eq. (3), this expression for $\bar{R}$ and we get:

$$\mathcal{L} = -\frac{1}{2} L'^T m_\nu L' \qquad , \tag{6}$$

where the resulting neutrino mass matrix is:

$$m_\nu = m_D^T M^{-1} m_D \tag{7}$$

This is the well known see-saw mechanism [20] result: the light neutrino masses are quadratic in the Dirac masses and inversely proportional to the large Majorana mass. If some ν_R are massless or light they would not be integrated away but simply added to the light neutrinos. Notice that the above results hold true for any number n of heavy neutral fermions R coupled to the 3 known neutrinos. In this more general case M is an n by n symmetric matrix and the coupling between heavy and light fields is described by the rectangular n by 3 matrix m_D.

Here we assumed that the additional non renormalizable terms from O_5 are comparatively negligible, otherwise they should simply be added. After elimination of the heavy right-handed fields, at the level of the effective low energy theory, the two

types of terms are equivalent. In particular they have identical transformation properties under a chiral change of basis in flavour space. The difference is, however, that in the see-saw mechanism, the Dirac matrix m_D is presumably related to ordinary fermion masses because they are both generated by the Higgs mechanism and both must obey GUT-induced constraints. Thus if we assume the see-saw mechanism more constraints are implied. In particular we are led to the natural hypothesis that m_D has a largely dominant third family eigenvalue in analogy to m_t, m_b and m_τ which are by far the largest masses among u quarks, d quarks and charged leptons. Once we accept that m_D is hierarchical it is very difficult to imagine that the effective light neutrino matrix, generated by the see-saw mechanism, could have eigenvalues very close in absolute value.

5. The Neutrino Mixing Matrix

Given the definition of the mixing matrix U in eq. (1) and the transformation properties of the effective light neutrino mass matrix m_ν:

$$\begin{aligned} L'^T m_\nu L' &= L^T U^T m_\nu U L \\ U^T m_\nu U &= \mathrm{Diag}[e^{i\phi_1} m_1, e^{i\phi_2} m_2, m_3] \equiv m_{diag} \end{aligned} \tag{8}$$

we obtain the general form of m_ν:

$$m_\nu = U m_{diag} U^T \tag{9}$$

The matrix U can be parameterized in terms of three mixing angles and one phase, exactly as for the quark mixing matrix V_{CKM}. In addition we have the two phases ϕ_1 and ϕ_2 that are present because of the Majorana nature of neutrinos. Thus, in general, 9 parameters are added to the SM when non vanishing neutrino masses are included: 3 eigenvalues, 3 mixing angles and 3 CP violating phases.

Maximal atmospheric neutrino mixing and the requirement that the electron neutrino does not participate in the atmospheric oscillations, as indicated by the Superkamiokande [1)] and Chooz [15)] data, lead directly to the following structure of the U_{fi} ($f = e, \mu, \tau$, $i = 1, 2, 3$) mixing matrix, apart from sign convention redefinitions

$$U_{fi} = \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & +1/\sqrt{2} \end{bmatrix} \quad . \tag{10}$$

This result is obtained by a simple generalization of the analysis of ref. [21,22)] to the case of arbitrary solar mixing angle ($s \equiv \sin\theta_{sun}$, $c \equiv \cos\theta_{sun}$): $c = s = 1/\sqrt{2}$ for maximal solar mixing (e.g. for vacuum oscillations $\sin^2 2\theta_{sun} \sim 0.75$) , while $\sin^2 2\theta_{sun} \sim 4s^2 \sim 5.5 \cdot 10^{-3}$ for the small angle MSW [7)] solution. The vanishing of U_{e3} guarantees that ν_e does not participate in the atmospheric oscillations and the relation

$|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$ implies maximal mixing for atmospheric neutrinos. Also, in the limit $U_{e3} = 0$, all CP violating effects vanish and we can neglect the additional phase parameter generally present in U_{fi}: the matrix U is real and orthogonal and equal to the product of a rotation by $\pi/4$ in the 23 plane times a rotation in the 12 plane:

$$U_{fi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad . \tag{11}$$

Note that we are assuming only two frequencies, given by

$$\Delta_{sun} \propto m_2^2 - m_1^2, \qquad \Delta_{atm} \propto m_3^2 - m_{1,2}^2 \tag{12}$$

The numbering 1,2,3 corresponds to our definition of the frequencies and in principle may not coincide with the family index although this will be the case in the models that we favour. The effective light neutrino mass matrix is given by eq. (9). We disregard the phases $\phi_{1,2}$ but in the following $m_{1,2}$ can be of either sign.

For generic s, using eq. (9), one finds

$$m_\nu = \begin{bmatrix} 2\epsilon & \delta & \delta \\ \delta & \frac{m_3}{2} + \epsilon_2 & -\frac{m_3}{2} + \epsilon_2 \\ \delta & -\frac{m_3}{2} + \epsilon_2 & \frac{m_3}{2} + \epsilon_2 \end{bmatrix} \quad , \tag{13}$$

with

$$\epsilon = (m_1 c^2 + m_2 s^2)/2 \ , \quad \delta = (m_1 - m_2)cs/\sqrt{2} \ , \quad \epsilon_2 = (m_1 s^2 + m_2 c^2)/2 \quad . \tag{14}$$

We see that the existence of one maximal mixing and $U_{e3} = 0$ are the most important input that lead to the matrix form in eqs. (13-14). The value of the solar neutrino mixing angle can be left free. While the simple parametrization of the matrix U in eq. (10) is quite useful to guide the search for a realistic pattern of neutrino mass matrices, it should not be taken too literally. In particular the data do not exclude a non-vanishing U_{e3} element. As already mentioned, the bound by Chooz [15] amounts to $|U_{e3}| \lesssim 0.2$. Thus neglecting $|U_{e3}|$ with respect to s in eq. (10) is not completely justified. Also note that in presence of a large hierarchy $|m_3| \gg |m_{1,2}|$ the effect of neglected parameters in eq. (10) can be enhanced by $m_3/m_{1,2}$ and produce seizable corrections. A non vanishing U_{e3} term can lead to different $(m_\nu)_{12}$ and $(m_\nu)_{13}$ terms. Similarly a deviation from maximal mixing $U_{\mu 3} \neq U_{\tau 3}$ distorts the ϵ_2 terms in the 23 sector of m_ν. Therefore, especially for a large hierarchy, there is more freedom in the small terms in order to construct a model that fits the data than it is apparent from eq. (13).

6. Possible Mass Hierarchies and Approximate Textures

Given the observed frequencies and our notation in eq. (12), there are three

possible hierarchies of mass eigenvalues [23]:

$$\begin{aligned} \text{A} &: |m_3| >> |m_{2,1}| \\ \text{B} &: |m_1| \sim |m_2| >> |m_3| \\ \text{C} &: |m_1| \sim |m_2| \sim |m_3| \end{aligned} \tag{15}$$

(in case A there is no prejudice on the m_1, m_2 relation). For B and C different subcases are then generated according to the relative sign assignments for $m_{1,2,3}$.

For each case we can set to zero the small masses and mixing angles and find the effective light neutrino matrices which are obtained both for double and single maximal mixing. Note that here we are working in the basis where the charged lepton masses are diagonal, and approximately given by $m_l = \text{Diag}[0, 0, m_\tau]$. For model building one has to arrange both the charged lepton and the neutrino mass matrices so that the neutrino results coincide with those given here after diagonalization of charged leptons. For example, in case A, $m_{diag} = \text{Diag}[0, 0, m_3]$ and we obtain

$$A: \quad m_{diag} = \text{Diag}[0,0,1]\, m_3 \qquad \rightarrow \qquad m_\nu/m_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad . \tag{16}$$

In this particular case the results are the same for double and single maximal mixing. Note that the signs correspond to the phase convention adopted in eq. (10). If one prefers all signs to be positive it is sufficient to invert the sign of the third row of the matrix U in eq. (10). We can similarly proceed in the other cases and we obtain the results in table I (where the overall mass scale was dropped).

We recall that, once a solution to the solar neutrino problem is chosen and a set of suitable small perturbation terms is introduced, oscillation phenomena are unable to distinguish between the cases A, B and C displayed in table I. However, from the model building point of view, each texture in table I represent an independent possibility in a zeroth order approximation. Moreover, table I could be generalized by allowing general phase differences among the 3 neutrino masses as shown in eq. (8). For instance, for double maximal mixing and $|m_1| = |m_2| = |m_3|$ we find the general texture

$$\frac{m_\nu}{m_3} = \frac{1}{2} \begin{bmatrix} p_{12} & \frac{m_{12}}{\sqrt{2}} & \frac{m_{12}}{\sqrt{2}} \\ \frac{m_{12}}{\sqrt{2}} & 1 + \frac{p_{12}}{2} & -1 + \frac{p_{12}}{2} \\ \frac{m_{12}}{\sqrt{2}} & -1 + \frac{p_{12}}{2} & 1 + \frac{p_{12}}{2} \end{bmatrix} \quad , \tag{17}$$

where $p_{12}(m_{12}) \equiv e^{i\phi_1} + (-)e^{i\phi_2}$. When $p_{12} = 0$, eq. (17) reproduces the cases C1 and C2 of table I, while for $m_{12} = 0$ C0 and C3 are obtained.

7. The Case of Nearly Degenerate Neutrino Masses

The configurations B) and C) imply a very precise near degeneracy of squared

Table 1. Zeroth order form of the neutrino mass matrix for double and single maximal mixing, according to the different possible hierarchies given in eq. (15).

	m_{diag}	double maximal mixing	single maximal mixing
A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
C0	Diag[1,1,1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
C3	Diag[1,1,-1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

masses. For example, the case C) is the only one that could in principle accommodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. We think that it is not at all clear at the moment that a hot dark matter component is really needed [18] but this could be a reason in favour of the fully degenerate solution. Then the common mass should be around 1-3 eV. The solar frequency could be given by a small 1-2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2-3 splitting. A strong constraint arises in this case from the non observation of neutrino-less double beta decay which requires that the ee entry of m_ν must obey $|(m_\nu)_{ee}| \leq 0.2$ eV [19]. Perhaps this bound can somewhat be relaxed if the error from the theoretical ambiguities on nuclear matrix elements is increased. As observed in ref. [24], if this bound is below 1 eV, it can only be satisfied if bimixing is realized (that is double maximal mixing, with solar neutrinos explained by the VO solution or may be by the large angle MSW solution). The reason is that, as seen from eqs. (13,14), $(m_\nu)_{ee} = m_1 c^2 + m_2 s^2$. Now s^2 is either very small or very close to c^2. For the required cancellation one needs opposite signs for m_1 and m_2 and comparable values of c^2 and s^2, hence nearly maximal solar mixing.

A problem with this type of solution is that we would need a relative splitting $|\Delta m/m| \sim \Delta m^2_{atm}/2m^2 \sim 10^{-3} - 10^{-4}$ and a much smaller one for solar neutrinos especially if explained by vacuum oscillations: $|\Delta m/m| \sim 10^{-10} - 10^{-11}$. As mentioned above we consider it unplausible that starting from hierarchical Dirac matrices we end up via the see-saw mechanism into a nearly perfect degeneracy of squared masses. Thus models with degenerate neutrinos could only be natural if the dominant contributions directly arise from non renormalizable operators like O_5 in eq. (2) because they are apriori unrelated to other fermion mass terms. The degeneracy of neutrinos should be guaranteed by some slightly broken symmetry. Notice however that, even if arranged by a symmetry at the GUT scale, it is doubtful that such a precise degeneracy could be stable against renormalization group corrections when running down at low energy [25]. Models based on discrete or continuous symmetries have been proposed. For example in the models of ref. [26] the symmetry is SO(3). In the unbroken limit neutrinos are degenerate and charged leptons are massless. When the symmetry is broken the charged lepton masses are much larger than neutrino splittings because the former are first order while the latter are second order in the electroweak symmetry breaking.

A model which is simple to describe but difficult to derive in a natural way is one [27,53] where up quarks, down quarks and charged leptons have "democratic" mass matrices, with all entries equal (in first approximation):

$$m^u_D,\ m^d_D,\ m^l_D \propto \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

where we have also indicated the diagonal form with two vanishing eigenvalues. In

this limit the CKM matrix is the identity, because u and d quarks are diagonalized by the same matrix (recall that $V_{CKM} = U_u^\dagger U_d$). This matrix U can be chosen with $U_{e3} = 0$ and is given by (note the analogy with the quark model eigenvalues π^0, η and η'):

$$U = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \quad . \tag{19}$$

The mass matrix of eq. (18) is invariant under a discrete $S_{3L} \times S_{3R}$ permutation symmetry. The same requirement leads to the general neutrino mass matrix:

$$m_\nu \propto a \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad , \tag{20}$$

the two independent invariants being allowed by the Majorana nature of the light neutrinos. If b vanishes the neutrinos are degenerate. In the presence of small terms that break the permutation symmetry this degeneracy is removed and the neutrino mixing matrix may remain very close to U in eq. (19). At the same time, the lightest quarks and charged leptons may acquire a non-vanishing mass. Notice that the atmospheric neutrino mixing is nearly maximal: $\sin^2 2\theta = 8/9$, while the solar mixing angle is maximal.

An intermediate possibility between the case of degenerate neutrino masses and the case of hierarchical ones is the situation B in eq. (15). With two almost degenerate heavier states and a nearly massless neutrino we cannot reach a mass range interesting for cosmological purposes. However, since the experimentally accessible quantities are the squared mass differences, case B remains an open possibility. The small terms required to go beyond the zeroth order approximation can be controlled by a spontaneously broken flavour symmetry. For instance, working only with the light neutrinos, the $U(1)_Q$ charge $Q \equiv (L_e - L_\mu - L_\tau)$ allows for a texture of the kind B1, double maximal mixing, at leading order [11]. The non-vanishing entries $(m_\nu)_{12}$ and $(m_\nu)_{13}$ are expected to be of the same order, although not necessarily equal. The vanishing entries can be filled by the ratio of the VEV $\langle\varphi\rangle$ of a scalar field carrying two units of Q and a mass scale M providing the cut-off to the low-energy theory:

$$m_\nu = m \begin{bmatrix} \epsilon & 1 & 1 \\ 1 & \epsilon & \epsilon \\ 1 & \epsilon & \epsilon \end{bmatrix} \quad , \tag{21}$$

where $\epsilon = \langle\varphi\rangle/M$ and only the order-of-magnitudes are indicated. For values of ϵ smaller than 1 one obtains a small perturbation of the zeroth order texture. The mixing matrix has a large, not necessarily maximal, mixing angle in the 23 sector, a nearly maximal mixing angle in the 12 sector and a mixing angle of order ϵ in the 13 sector. The mass parameter m should be close to $10^{-1} - 10^{-2}$ eV to provide the

frequency required by the atmospheric oscillations. Finally, $m_2^2 - m_1^2 \sim m^2\epsilon$. In this model the VO solution to the solar neutrino deficit would require a very tiny breaking term, $\epsilon \sim \mathrm{O}(10^{-7})$.

8. The Case of Hierarchical Neutrino Masses

We now discuss models of type A with large effective light neutrino mass splittings and large mixings. In general large splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2 by 2 matrix context the requirement of large splitting and large mixings leads to a condition of vanishing determinant. For example the matrix

$$m \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \quad . \tag{22}$$

has eigenvalues 0 and $1 + x^2$ and for x of 0(1) the mixing is large. Thus, in the limit of neglecting small mass terms of order $m_{1,2}$, the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the 2 by 2 subdeterminant 23 of the 3 by 3 mixing matrix vanishes. The problem is to show that this approximate vanishing can be arranged in a natural way without fine tuning. We have discussed suitable possible mechanisms in our papers [28,23,29]. We in particular favour a class of models where, in the limit of neglecting terms of order $m_{1,2}$ and, in the basis where charged leptons are diagonal, the Dirac matrix m_D, defined by $\bar{R} m_D L$, takes the approximate form:

$$m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{bmatrix} \quad . \tag{23}$$

This matrix has the property that for a generic Majorana matrix M one finds:

$$m_\nu = m_D^T M^{-1} m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{bmatrix} \quad . \tag{24}$$

The only condition on M^{-1} is that the 33 entry is non zero. It is important for the following discussion to observe that m_D given by eq. (23) under a change of basis transforms as $m_D -> V^\dagger m_D U$ where V and U rotate the right and left fields respectively. It is easy to check that in order to make m_D diagonal we need large left mixings. More precisely m_D is diagonalized by taking $V = 1$ and U given by

$$U = \begin{bmatrix} c & -s & 0 \\ s c_\gamma & c c_\gamma & -s_\gamma \\ s s_\gamma & c s_\gamma & c_\gamma \end{bmatrix} \quad , \tag{25}$$

with

$$s_\gamma = -x/r \quad , \qquad c_\gamma = 1/r \quad , \qquad r = \sqrt{1+x^2} \qquad . \tag{26}$$

The matrix U is directly the neutrino mixing matrix. The mixing angle for atmospheric neutrino oscillations is given by:

$$\sin^2 2\theta = 4 s_\gamma^2 c_\gamma^2 = \frac{4x^2}{(1+x^2)^2} \qquad . \tag{27}$$

Thus the bound $\sin^2 2\theta \gtrsim 0.9$ translates into $0.7 \lesssim |x| \lesssim 1.4$. It is interesting to recall that in refs. [30,31] it was shown that the mixing angle can be amplified by the running from a large mass scale down to low energy.

We have seen that, in order to explain in a natural way widely split light neutrino masses together with large mixings, we need an automatic vanishing of the 23 subdeterminant. This in turn is most simply realized by allowing some large left-handed mixing terms in the Dirac neutrino matrix. By left-handed mixing we mean non diagonal matrix elements that can only be eliminated by a large rotation of the left-handed fields. Thus the question is how to reconcile large left-handed mixings in the leptonic sector with the observed near diagonal form of V_{CKM}, the quark mixing matrix. Strictly speaking, since $V_{CKM} = U_u^\dagger U_d$, the individual matrices U_u and U_d need not be near diagonal, but V_{CKM} does, while the analogue for leptons apparently cannot be near diagonal. However nothing forbids for quarks that, in the basis where m_u is diagonal, the d quark matrix has large non diagonal terms that can be rotated away by a pure right-handed rotation. We suggest that this is so and that in some way right-handed mixings for quarks correspond to left-handed mixings for leptons.

In the context of (Susy) SU(5) [5] there is a very attractive hint of how the present mechanism can be realized. In the $\bar{5}$ of SU(5) the d^c singlet appears together with the lepton doublet (ν, e). The (u, d) doublet and e^c belong to the 10 and ν^c to the 1 and similarly for the other families. As a consequence, in the simplest model with mass terms arising from only Higgs pentaplets, the Dirac matrix of down quarks is the transpose of the charged lepton matrix: $m_D^d = (m_D^l)^T$. Thus, indeed, a large mixing for right-handed down quarks corresponds to a large left-handed mixing for charged leptons. At leading order we may have:

$$m_D^d = (m_D^l)^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} v_d \tag{28}$$

In the same simplest approximation with 5 or $\bar{5}$ Higgs, the up quark mass matrix is symmetric, so that left and right mixing matrices are equal in this case. Then small mixings for up quarks and small left-handed mixings for down quarks are sufficient to guarantee small V_{CKM} mixing angles even for large d quark right-handed mixings.

If these small mixings are neglected, we expect:

$$m_D^u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} v_u \tag{29}$$

When the charged lepton matrix is diagonalized the large left-handed mixing of the charged leptons is transferred to the neutrinos. Note that in SU(5) we can diagonalize the u mass matrix by a rotation of the fields in the 10, the Majorana matrix M by a rotation of the 1 and the effective light neutrino matrix m_ν by a rotation of the $\bar{5}$. In this basis the d quark mass matrix fixes V_{CKM} and the charged lepton mass matrix fixes neutrino mixings. It is well known that a model where the down and the charged lepton matrices are exactly the transpose of one another cannot be exactly true because of the e/d and μ/s mass ratios. It is also known that one remedy to this problem is to add some Higgs component in the 45 representation of SU(5) [32]. A different solution [33] will be described later. But the symmetry under transposition can still be a good guideline if we are only interested in the order of magnitude of the matrix entries and not in their exact values. Similarly, the Dirac neutrino mass matrix m_D is the same as the up quark mass matrix in the very crude model where the Higgs pentaplets come from a pure 10 representation of SO(10): $m_D = m_D^u$. For m_D the dominance of the third family eigenvalue as well as a near diagonal form could be an order of magnitude remnant of this broken symmetry. Thus, neglecting small terms, the neutrino Dirac matrix in the basis where charged leptons are diagonal could be directly obtained in the form of eq. (23).

9. An Example with Horizontal Abelian Charges

We give here an explicit example [29] of the mechanism under discussion in the framework of a unified Susy $SU(5)$ theory with an additional $U(1)_F$ flavour symmetry [34]. This model is to be taken as merely indicative, in that some important problems, like, for example, the cancellation of chiral anomalies are not tackled here. But we find it impressive that the general pattern of all what we know on fermion masses and mixings is correctly reproduced at the level of orders of magnitude. We regard the present model as a low-energy effective theory valid at energies close to $M_{GUT} \ll M_{Pl}$. We can think to obtain it by integrating out the heavy modes from an unknown underlying fundamental theory defined at an energy scale close to M_{Pl}. From this point of view the gauge anomalies generated by the light supermultiplets listed below can be compensated by another set of supermultiplets with masses above M_{GUT}, already eliminated from the low-energy theory. In particular, we assume that these additional supermultiplets are vector-like with respect to $SU(5)$ and chiral with respect to $U(1)_F$. Their masses are then naturally expected to be of the order of the $U(1)_F$ breaking scale, which, in the following discussion, turns out to be near M_{Pl}. It is possible to check explicitly the possibility of canceling the gauge anomalies in this

way but, due to our ignorance about the fundamental theory, it is not particularly instructive to illustrate the details here. In this model the known generations of quarks and leptons are contained in triplets Ψ_{10}^a and $\Psi_{\bar{5}}^a$, $(a = 1, 2, 3)$ transforming as 10 and $\bar{5}$ of $SU(5)$, respectively. Three more $SU(5)$ singlets Ψ_1^a describe the right-handed neutrinos. We assign to these fields the following F-charges:

$$\Psi_{10} \sim (3, 2, 0) \tag{30}$$
$$\Psi_{\bar{5}} \sim (3, 0, 0) \tag{31}$$
$$\Psi_1 \sim (1, -1, 0) \tag{32}$$

We start by discussing the Yukawa coupling allowed by $U(1)_F$-neutral Higgs multiplets φ_5 and $\varphi_{\bar{5}}$ in the 5 and $\bar{5}$ $SU(5)$ representations and by a pair θ and $\bar{\theta}$ of $SU(5)$ singlets with $F = 1$ and $F = -1$, respectively.

In the quark sector we obtain [b]:

$$m_D^u = (m_D^u)^T = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u \ , \qquad m_D^d = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_d \ , \tag{33}$$

from which we get the order-of-magnitude relations:

$$\begin{aligned} m_u : m_c : m_t &= \lambda^6 : \lambda^4 : 1 \\ m_d : m_s : m_b &= \lambda^6 : \lambda^2 : 1 \end{aligned} \tag{34}$$

and

$$V_{us} \sim \lambda \ , \qquad V_{ub} \sim \lambda^3 \ , \qquad V_{cb} \sim \lambda^2 \ . \tag{35}$$

Here $v_u \equiv \langle \varphi_5 \rangle$, $v_d \equiv \langle \varphi_{\bar{5}} \rangle$ and λ denotes the ratio between the vacuum expectation value of $\bar{\theta}$ and an ultraviolet cut-off identified with the Planck mass M_{Pl}: $\lambda \equiv \langle \bar{\theta} \rangle / M_{Pl}$. To correctly reproduce the observed quark mixing angles, we take λ of the order of the Cabibbo angle. For non-negative F-charges, the elements of the quark mixing matrix V_{CKM} depend only on the charge differences of the left-handed quark doublet [34]. Up to a constant shift, this defines the choice in eq. (30). Equal F-charges for $\Psi_{\bar{5}}^{2,3}$ (see eq. (31)) are then required to fit m_b and m_s. We will comment on the lightest quark masses later on.

At this level, the mass matrix for the charged leptons is the transpose of m_D^d:

$$m_D^l = (m_D^d)^T \tag{36}$$

and we find:

$$m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1 \tag{37}$$

[b] In eq. (33) the entries denoted by 1 in m_D^u and m_D^d are not necessarily equal. As usual, such a notation allows for O(1) deviations.

The O(1) off-diagonal entry of m_D^l gives rise to a large left-handed mixing in the 23 block which corresponds to a large right-handed mixing in the d mass matrix. In the neutrino sector, the Dirac and Majorana mass matrices are given by:

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & \lambda' & \lambda' \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \quad , \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix} \bar{M} \quad , \tag{38}$$

where $\lambda' \equiv \langle\theta\rangle/M_{Pl}$ and $\bar{M}$ denotes the large mass scale associated to the right-handed neutrinos: $\bar{M} \gg v_{u,d}$.

After diagonalization of the charged lepton sector and after integrating out the heavy right-handed neutrinos we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\bar{M}} \tag{39}$$

where we have taken $\lambda \sim \lambda'$. The O(1) elements in the 23 block are produced by combining the large left-handed mixing induced by the charged lepton sector and the large left-handed mixing in m_D. A crucial property of m_ν is that, as a result of the sea-saw mechanism and of the specific $U(1)_F$ charge assignment, the determinant of the 23 block is *automatically* of $O(\lambda^2)$ (for this the presence of negative charge values, leading to the presence of both λ and λ' is essential [23]).

It is easy to verify that the eigenvalues of m_ν satisfy the relations:

$$m_1 : m_2 : m_3 = \lambda^4 : \lambda^2 : 1 \quad . \tag{40}$$

The atmospheric neutrino oscillations require $m_3^2 \sim 10^{-3}$ eV^2. From eq. (39), taking $v_u \sim 250$ GeV, the mass scale $\bar{M}$ of the heavy Majorana neutrinos turns out to be close to the unification scale, $\bar{M} \sim 10^{15}$ GeV. The squared mass difference between the lightest states is of $O(\lambda^4)$ m_3^2, appropriate to the MSW solution to the solar neutrino problem. Finally, beyond the large mixing in the 23 sector, corresponding to $s_\gamma \sim c_\gamma$ in eq. (25), m_ν provides a mixing angle $s \sim (\lambda/2)$ in the 12 sector, close to the range preferred by the small angle MSW solution. In general U_{e3} is non-vanishing, of $O(\lambda^3)$.

In general, the charge assignment under $U(1)_F$ allows for non-canonical kinetic terms that represent an additional source of mixing. Such terms are allowed by the underlying flavour symmetry and it would be unnatural to tune them to the canonical form. The results quoted up to now remain unchanged after including the effects related to the most general kinetic terms, via appropriate rotations and rescaling in the flavour space (see also ref. [35]).

Obviously, the order of magnitude description offered by this model is not intended to account for all the details of fermion masses. Even neglecting the parameters

associated with the CP violating observables, some of the relevant observables are somewhat marginally reproduced. For instance we obtain $m_u/m_t \sim \lambda^6$ which is perhaps too large. However we find it remarkable that in such a simple scheme most of the 12 independent fermion masses and the 6 mixing angles turn out to have the correct order of magnitude. Notice also that our model prefers large values of $\tan\beta \equiv v_u/v_d$. This is a consequence of the equality $F(\Psi^3_{10}) = F(\Psi^3_{\bar{5}})$ (see eqs. (30) and (31)). In this case the Yukawa couplings of top and bottom quarks are expected to be of the same order of magnitude, while the large m_t/m_b ratio is attributed to $v_u \gg v_d$ (there may be factors O(1) modifying these considerations, of course). We recall here that in supersymmetric grand unified models large values of $\tan\beta$ are one possible solution to the problem of reconciling the boundary condition $m_b = m_\tau$ at the GUT scale with the low-energy data [36]. Alternatively, to keep $\tan\beta$ small, one could suppress m_b/m_t by adopting different F-charges for the $\Psi^3_{\bar{5}}$ and Ψ^3_{10}.

Additional contributions to flavour changing processes (both in the quark and in the lepton [37] sectors) and to CP violating observables are generally expected in a supersymmetric grand unified theory. However, a reliable estimate of the corresponding effects would require a much more detailed definition of the theory than attempted here. Crucial ingredients such as the mechanism of supersymmetry breaking and its transmission to the observable sector have been ignored in the present note. We are implicitly assuming that the omission of this aspect of the flavour problem does not substantially alter our discussion.

A common problem of all $SU(5)$ unified theories based on a minimal higgs structure is represented by the relation $m^l_D = (m^d_D)^T$ that, while leading to the successful $m_b = m_\tau$ boundary condition at the GUT scale, provides the wrong prediction $m_d/m_s = m_e/m_\mu$ (which, however, is an acceptable order of magnitude equality). We can easily overcome this problem and improve the picture [33] by introducing an additional supermultiplet $\bar{\theta}_{24}$ transforming in the adjoint representation of $SU(5)$ and possessing a negative $U(1)_F$ charge, $-n$ $(n > 0)$. Under these conditions, a positive F-charge f carried by the matrix elements $\Psi^a_{10}\Psi^b_{\bar{5}}$ can be compensated in several different ways by monomials of the kind $(\bar{\theta})^p(\bar{\theta}_{24})^q$, with $p + nq = f$. Each of these possibilities represents an independent contribution to the down quark and charged lepton mass matrices, occurring with an unknown coefficient of O(1). Moreover the product $(\bar{\theta}_{24})^q\varphi_{\bar{5}}$ contains both the $\bar{5}$ and the $\overline{45}$ $SU(5)$ representations, allowing for a differentiation between the down quarks and the charged leptons. The only, welcome, exceptions are given by the O(1) entries that do not require any compensation and, at the leading order, remain the same for charged leptons and down quarks. This preserves the good $m_b = m_\tau$ prediction. Since a perturbation of O(1) in the subleading matrix elements is sufficient to cure the bad $m_d/m_s = m_e/m_\mu$ relation, we can safely assume that $\langle\bar{\theta}_{24}\rangle/M_{Pl} \sim \lambda^n$, to preserve the correct order-of-magnitude predictions in the remaining sectors.

A general problem common to all models dealing with flavour is that of recovering

the correct vacuum structure by minimizing the effective potential of the theory. It may be noticed that the presence of two multiplets θ and $\bar{\theta}$ with opposite F charges could hardly be reconciled, without adding extra structure to the model, with a large common VEV for these fields, due to possible analytic terms of the kind $(\theta\bar{\theta})^n$ in the superpotential. We find therefore instructive to explore the consequences of allowing only the negatively charged $\bar{\theta}$ field in the theory.

It can be immediately recognized that, while the quark mass matrices of eqs. (33) are unchanged, in the neutrino sector the Dirac and Majorana matrices get modified into:

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & 0 & 0 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \quad , \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} \bar{M} \quad . \tag{41}$$

The zeros are due to the analytic property of the superpotential that makes impossible to form the corresponding F invariant by using $\bar{\theta}$ alone. These zeros should not be taken literally, as they will be eventually filled by small terms coming, for instance, from the diagonalization of the charged lepton mass matrix and from the transformation that put the kinetic terms into canonical form. It is however interesting to work out, in first approximation, the case of exactly zero entries in m_D and M, when forbidden by F.

The neutrino mass matrix obtained via see-saw from m_D and M has the same pattern as the one displayed in eq. (39). A closer inspection reveals that the determinant of the 23 block is identically zero, independently from λ. This leads to the following pattern of masses:

$$m_1 : m_2 : m_3 = \lambda^3 : \lambda^3 : 1 \quad , \qquad m_1^2 - m_2^2 = \mathrm{O}(\lambda^9) \quad . \tag{42}$$

Moreover the mixing in the 12 sector is almost maximal:

$$\frac{s}{c} = \frac{\pi}{4} + \mathrm{O}(\lambda^3) \quad . \tag{43}$$

For $\lambda \sim 0.2$, both the squared mass difference $(m_1^2 - m_2^2)/m_3^2$ and $\sin^2 2\theta_{sun}$ are remarkably close to the values required by the vacuum oscillation solution to the solar neutrino problem. This property remains reasonably stable against the perturbations induced by small terms (of order λ^5) replacing the zeros, coming from the diagonalization of the charged lepton sector and by the transformations that render the kinetic terms canonical. We find quite interesting that also the just-so solution, requiring an intriguingly small mass difference and a bimaximal mixing, can be reproduced, at least at the level of order of magnitudes, in the context of a "minimal" model of flavour compatible with supersymmetric SU(5). In this case the role played by supersymmetry is essential, a non-supersymmetric model with $\bar{\theta}$ alone not being distinguishable from the version with both θ and $\bar{\theta}$, as far as low-energy flavour properties

are concerned.

10. Other Similar or Alternative Proposals

Finally, let us compare our model with other recent proposals [38]. Textures for the effective neutrino mass matrix similar to m_ν in eq. (39) were derived in refs. [39,40], also in the context of an $SU(5)$ unified theory with a $U(1)$ flavour symmetry. In these works, however, the O(1) entries of m_ν are uncorrelated, due to the particular choice of U(1) charges. The diagonalization of such matrix, for generic O(1) coefficients, leads to only one light eigenvalue and to two heavy eigenvalues, of O(1), in units of $v_u^2/\bar{M}$. Then the required pattern $m_3 \gg m_2 \sim m_1$ has to be fixed by hand. On the contrary, in our model the desired pattern is automatic, since, as emphasized above, the determinant of the 23 block in m_ν is vanishing at the leading order. Other models in terms of $U(1)$ horizontal charges have been proposed in refs. [41,42,43,30,44]. Clearly a large mixing for the light neutrinos can be provided in part by the diagonalization of the charged lepton sector. As we have seen, in $SU(5)$, the left-handed mixing carried by charged leptons is expected to be, at least in first approximation, directly linked to the right-handed mixing for the d quarks and, as such, perfectly compatible with the available data. This possibility was remarked, for instance, in refs. [45,46,47,31] where the implementation was in terms of asymmetric textures, of the Branco et al. type [48], used as a general parametrization of the existing data consistent with the constraints imposed by the unification program. On the other hand, our model aims to a dynamical explanation of the flavour properties, although in a simplified setting [c].

It is interesting to note that the mechanism discussed sofar can be embedded in an $SO(10)$ grand-unified theory in a rather economic way [46]. The 33 entries of the fermion mass matrices can be obtained through the coupling $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H$ among the fermions in the third generation, $\mathbf{16}_3$, and a Higgs tenplet $\mathbf{10}_H$. The two independent VEVs of the tenplet v_u and v_d give mass, respectively, to t/ν_τ and b/τ. The key-point to obtain an asymmetric texture is the introduction of an operator of the kind $\mathbf{16}_2\mathbf{16}_H\mathbf{16}_3\mathbf{16}'_H$. This operator is thought to arise by integrating out an heavy $\mathbf{10}$ that couples both to $\mathbf{16}_2\mathbf{16}_H$ and to $\mathbf{16}_3\mathbf{16}'_H$. If the $\mathbf{16}_H$ develops a VEV breaking $SO(10)$ down to $SU(5)$ at a large scale, then, in terms of $SU(5)$ representations, we get an effective coupling of the kind $\overline{\mathbf{5}}_2\mathbf{10}_3\overline{\mathbf{5}}_H$, with a coefficient that can be of order one. This coupling contributes to the 23 entry of the down quark mass matrix and to the 32 entry of the charged lepton mass matrix, realizing the desired asymmetry. To distinguish the lepton and quark sectors one can further introduce an operator of the form $\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H\mathbf{45}_H$, $(i,j=2,3)$, with the VEV of the $\mathbf{45}_H$ pointing in the $B-L$ direction. Additional operators, still of the type $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_H\mathbf{16}'_H$ can contribute to the

[c] Also ref. [47] suggests an horizontal $U(2)$ symmetry to justify the assumed textures.

matrix elements of the first generation. The mass matrices look like:

$$m_D^u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} v_u \quad , \qquad m_D^d = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' & -\epsilon/3 & 1 \end{bmatrix} v_d \quad , \tag{44}$$

$$m_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} v_u \quad , \qquad m_D^e = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma + \epsilon & 1 \end{bmatrix} v_d \quad . \tag{45}$$

They provide a good fit of the available data in the quarks and the charged lepton sector in terms of 5 parameters (one of which is complex). In the neutrino sector one obtains a large θ_{23} mixing angle, $\sin^2 2\theta_{12} \sim 6.6 \cdot 10^{-3}$ eV^2 and θ_{13} of the same order of θ_{12}. Mass squared differences are sensitive to the details of the Majorana mass matrix.

Looking at models with three light neutrinos only, i.e. no sterile neutrinos, from a more general point of view, we stress that in the above models the atmospheric neutrino mixing is considered large, in the sense of being of order one in some zeroth order approximation. In other words it corresponds to off diagonal matrix elements of the same order of the diagonal ones, although the mixing is not exactly maximal. The idea that all fermion mixings are small and induced by the observed smallness of the non diagonal V_{CKM} matrix elements is then abandoned. An alternative is to argue that perhaps what appears to be large is not that large after all. The typical small parameter that appears in the mass matrices is $\lambda \sim \sqrt{m_d/m_s} \sim \sqrt{m_\mu/m_\tau} \sim 0.20 - 0.25$. This small parameter is not so small that it cannot become large due to some peculiar accidental enhancement: either a coefficient of about 3, or an exponent of the mass ratio which is less than 1/2 (due for example to a suitable charge assignment), or the addition in phase of an angle from the diagonalization of charged leptons and an angle from neutrino mixing. One may like this strategy of producing a large mixing by stretching small ones if, for example, he/she likes symmetric mass matrices, as from left-right symmetry at the GUT scale. In left-right symmetric models smallness of left mixings implies that also right-handed mixings are small, so that all mixings tend to be small. Clearly this set of models tend to favour moderate hierarchies and a single maximal mixing, so that the SA-MSW solution of solar neutrinos is preferred. For example, consider the 23 submatrix only, for simplicity. Assume that at order zero the neutrino Dirac matrix is given by

$$m_D^\nu \sim \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{46}$$

For a generic Majorana matrix M also the effective light neutrino matrix m_ν has the same structure in the 23 sector. If for charged leptons one has

$$m_l \sim \begin{bmatrix} \lambda^2 & r\lambda \\ r\lambda & 1 \end{bmatrix} \tag{47}$$

with $r \sim \mathrm{O}(1)$ and $\lambda \sim \mathrm{O}(\sqrt{m_\mu/m_\tau})$ (in fact the eigenvalues are of order 1 and λ^2, respectively), then the symmetric charged lepton matrix is diagonalized by a unitary matrix (the same for left and right fields). In the basis where m_l is diagonal, we then have

$$m_\nu \sim \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c & -s \\ s & c \end{bmatrix} = \begin{bmatrix} s^2 & sc \\ sc & c^2 \end{bmatrix} \tag{48}$$

As a result the 23 sub-determinant vanishes and $\sin 2\theta \sim 2sc \sim 2r\lambda$ is large for $r \sim 2-3$. Models of this type have been discussed in refs. [49].

To further discuss how a large neutrino mixing could be generated starting from symmetric matrices and small mixings for quarks and leptons we can go, without loss of generality, to a basis where both the charged lepton Dirac mass matrix and the RR Majorana matrix are diagonal. In fact, after diagonalization of the charged lepton Dirac mass matrix, we still have the freedom of a change of basis for the right-handed neutrino fields, in that the right-handed charged lepton and neutrino fields, as opposed to left-handed fields, are uncorrelated by the $SU(2) \bigotimes U(1)$ gauge symmetry. We can use this freedom to make the Majorana matrix diagonal: $M^{-1} = V^T d_M V$ with $d_M = \mathrm{Diag}[1/M_1, 1/M_2, 1/M_3]$. Then if we parametrize the matrix $V m_D$ by z_{ab} we have:

$$m_{\nu ab} = (m_D^T M^{-1} m_D)_{ab} = \sum_c \frac{z_{ca} z_{cb}}{M_c}. \tag{49}$$

From this expression we see that, while we can always arrange the twelve parameters z_{ab} and M_a to arbitrarily fix the six independent matrix elements of m_ν, case A is special in that it can be approximately reproduced in two particularly simple ways, without relying on precise cancellations among different terms:

i) There are only two large entries in the z matrix, $|z_{c2}| \sim |z_{c3}|$, and the three eigenvalues M_a are of comparable magnitude (or, at least, with a less pronounced hierarchy than for the z matrix elements). Then, the subdeterminant 23 vanishes and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This is the possibility discussed in detail in section 8 and 9.

ii) One of the right-handed neutrinos is particularly light and, in first approximation, it is only coupled to μ and τ. Thus, $M_c \sim \eta$ (small) and $z_{c1} \sim 0$. In this case [11] the 23 subdeterminant vanishes, and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility has been especially emphasized in refs. [50].

In particular mechanism ii) is compatible with symmetric mass matrices. For example, one could want to preserve left-right symmetry at the GUT scale. Then, the observed smallness of left-handed mixings for quarks would also demand small right-handed mixings. So we now assume that m_D is nearly diagonal (always in the basis where charged leptons and M_{RR} are diagonal) with all its off diagonal terms proportional to some small parameter ϵ. Starting from

$$m_D \propto \begin{bmatrix} \epsilon^p & x\epsilon \\ x\epsilon & 1 \end{bmatrix}, \qquad M^{-1} \propto \begin{bmatrix} r_2 & 0 \\ 0 & 1 \end{bmatrix} \qquad . \tag{50}$$

where x is $0(1)$, we obtain:

$$m_\nu = m_D^T M^{-1} m_D \propto \begin{bmatrix} \epsilon^{2p} r_2 + x^2\epsilon^2 & x\epsilon^{p+1} r_2 + x\epsilon \\ x\epsilon^{p+1} r_2 + x\epsilon & x^2\epsilon^2 r_2 + 1 \end{bmatrix} \quad . \tag{51}$$

For sufficiently small M_2 the terms in r_2 are dominant. For $p = 1, 2$, which we consider as typical cases, it is sufficient that $\epsilon^2 r_2 \gg 1$. Assuming that this condition is satisfied, consider first the case with p=2. We have

$$m_\nu = m_D^T M^{-1} m_D \propto x^2\epsilon^2 r_2 \begin{bmatrix} \frac{\epsilon^2}{x^2} & \frac{\epsilon}{x} \\ \frac{\epsilon}{x} & 1 \end{bmatrix} \quad . \tag{52}$$

This case is qualitatively similar to that described by eq. (48). The determinant is naturally vanishing (to the extent that the terms in r_2 are dominant), so that the mass eigenvalues are widely split. However the mixing is nominally small: $\sin 2\theta$ is of order $2\epsilon/x$. It could be numerically large enough if $1/x \sim 2 - 3$ and ϵ is of the order of the Cabibbo angle $\epsilon \sim 0.20 - 0.25$. This is what we call "stretching": the large neutrino mixing is explained in terms of a small parameter which is not so small and can give a perhaps sufficient amount of mixing if enhanced by a possibly large coefficient.

A more interesting case is obtained for $p = 1$, that gives:

$$m_\nu = m_D^T M^{-1} m_D \propto \epsilon^2 r_2 \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix} \quad . \tag{53}$$

In this case the small parameter ϵ is completely factored out and for $x \sim 1$ the mixing is nearly maximal. The see-saw mechanism has created large mixing from almost nothing: all relevant matrices entering the see-saw mechanism are nearly diagonal . Clearly, the crucial factorization of the small parameter ϵ^2 only arises for $p = 1$. It is possible to extend the previous model to the 3 by 3 case and to support the resulting almost diagonal textures by means of suitable flavour symmetries [51]. Models along this line have been also considered in ref. [52].

Notice that we do not need a symmetric matrix m_D as that given in eq. (50) to get the desired effective neutrino mass matrix. The relevant condition is $(m_D)_{22} = (m_D)_{23}$, which can be realized in a asymmetric Dirac mass matrix. For instance we can equally well take

$$m_D \propto \begin{bmatrix} \epsilon & x\epsilon \\ y & 1 \end{bmatrix} \quad , \tag{54}$$

with $y \leq 1$ arbitrary. For $\epsilon^2 r_2 \gg 1$ we end up again with m_ν given in eq. (53).

The previous models are all in trouble if the atmospheric neutrino mixing is exactly maximal or very close to maximal. There are a few cases in the literature, not of compelling elegance, where the mixing is arranged to be exactly maximal. For instance, in ref. [53] a discrete group generated by the elements:

$$R = 1/2 \begin{bmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & -1 & 1 \end{bmatrix} \quad , \qquad T_n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{n}} & 0 \\ 0 & 0 & e^{-\frac{2\pi i}{n}} \end{bmatrix} \quad , \qquad (n \geq 2) \tag{55}$$

has been suggested to constrain the leading order form of the mass matrices. Indeed, if R and T_n act on the neutrinos in their flavour basis and if the required Higgs multiplets are singlets under the discrete symmetry, then, for $n > 2$, the most general invariant mass matrix for the light neutrinos has the form:

$$m_\nu = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad . \tag{56}$$

that reproduces the case $C3$ in table I (first column), that is the degenerate case with a double maximal mixing. By invoking an appropriate transformation property for the relevant Higgs doublet, it is possible to obtain a hierarchical and diagonal mass matrix in the charged lepton sector. A suitable spontaneous breaking of the discrete symmetry is then required to generate the correct pattern of masses and mixings beyond the leading approximation. Another example is found in ref. [26].

11. Outlook and Conclusion

By now there are rather convincing experimental indications for neutrino oscillations. If so, then neutrinos have non zero masses. As a consequence, the phenomenology of neutrino masses and mixings is brought to the forefront. This is a very interesting subject in many respects. It is a window on the physics of GUTs in that the extreme smallness of neutrino masses can only be explained in a natural way if lepton number is violated. Then neutrino masses are inversely proportional to the large scale where lepton number is violated. Also, the pattern of neutrino masses and mixings can provide new clues on the long standing problem of quark and lepton mass matrices. The actual value of neutrino masses is important for cosmology as neutrinos are candidates for hot dark matter: nearly degenerate neutrinos with a common mass around 2 eV would significantly contribute to the matter density in the universe.

While the existence of oscillations appears to be on a solid ground, many important experimental ambiguities remain. The most solid indication is that atmospheric neutrino oscillations occur with nearly maximal mixing and Δm^2 in the $10^{-3} - 10^{-2}$ eV^2 range. It is not clear if ν_μ disappear into mainly ν_τ or into some unknown sterile neutrino kind (disappearance into ν_e is disfavoured by the data). For solar neutrinos it is not yet clear which of the three solutions, MSW-SA, MSW-LA and VO, is true, and the possibility also remains of different solutions if not all of the experimental input is correct (for example, energy independent solutions are resurrected if the Homestake result is modified). Finally a confirmation of the LNSD alleged signal is necessary. In our discussion here we assumed that the LNSD evidence will fade away, so that we could restrict to the 3 known kinds of neutrinos without new sterile species.

The three neutrino mass eigenvalues can either be nearly degenerate or widely split. Solutions with three nearly degenerate neutrinos with a common mass around

2 eV, as for neutrinos of cosmological relevance, are strongly constrained by the bound on neutrinoless double beta decay, and only nearly maximal mixing for solar neutrinos is allowed. In any case, because of the smallness of $\Delta m/m$, solutions of this kind are not easy to theoretically justify in a natural way. So it looks that probably neutrinos are not cosmologically relevant.

We argued in favour of widely split solutions for neutrino masses. Reconciling large splittings with large mixing(s) requires some natural mechanism to implement a vanishing determinant condition. This can be obtained in the see-saw mechanism if one light right handed neutrino is dominant, or a suitable texture of the Dirac matrix is imposed by an underlying symmetry. In a GUT context, the existence of right handed neutrinos indicates SO(10) at least as a classification group generated by physics at M_{Pl}. The symmetry group at M_{GUT} could be either (Susy) SU(5) or SO(10) or a larger group. We have presented a class of natural models where large right handed mixings for quarks are transformed into large left handed mixings for leptons by the approximate transposition relation $m_d = m_e^T$ which is often realized in SU(5) models. We have shown that these models can be naturally implemented by simple assignments of U(1) horizontal charges.

While we favour models based on asymmetric mass matrices, (approximately) symmetric matrices as, for example, produced in left-right symmetric models are not excluded. We have seen that, in the see-saw mechanism, it is even possible to have nearly maximal mixing starting from all Dirac matrices nearly diagonal in the basis where the RR Majorana matrix is diagonal. Alternatively the large neutrino mixing could be generated by an enhancement of formally small terms. This is because the typical small term in quark or charged lepton mass matrices is of the order of the Cabibbo angle $\lambda \sim 0.22$ which is not that small. This enhancement can be the result of a coherent addition of not so small coefficients from diagonalizing the charged leptons and from the neutrino matrix itself. Or it could arise from a small exponent of m_μ/m_τ for example arising from suitable values of abelian charges.

In conclusion the fact that some neutrino mixing angles are large, while surprising at the start, was eventually found to be well compatible, without any major change, with our picture of quark and lepton masses within GUTs. Rather it provides us with new important clues that can become sharper when the experimental picture will be further clarified.

12. Acknowledgements

We are grateful to Milla Baldo-Ceolin for inviting us to this exceptionally interesting Workshop and for her beautiful organization and splendid hospitality.

13. References

1) Y. Fukuda et al., hep-ex/9805006, hep-ex/9805021 and hep-ex/9807003.
2) The data from Homestake, Gallex, Sage, Kamiokande and Superkamiokande on solar neutrinos are summarised and the list of references given in J. N. Bahcall, P. I. Krastev and A. Yu Smirnov, hep-ph/9807216.
3) C. Athanassopoulos et al., Phys. Rev. Lett. 77 (1996) 3082; nucl-ex/9706006, nucl-ex/9709006.
4) P. Lipari and M. Lusignoli, hep-ph/9901350;
   G.L. Fogli, E. Lisi, A. Marrone and G. Scioscia, Phys. Rev. D59 (1999) 117303 hep-ph/9902267;
   G.L. Fogli, E. Lisi, A. Marrone and G. Scioscia, hep-ph/9904248;
   S. Choubey and S. Goswami, hep-ph/9904257;
   P. Lipari and M. Lusignoli, hep-ph/9905229;
   S. Pakvasa, hep-ph/9905426.
5) G.G.Ross, "Grand Unified Theories", Benjamin, 1985;
   R.N.Mohapatra, "Unification and Supersymmetry" Springer-Verlag, 1986.
   C. Kounnas, A.Masiero, D.V.Nanopoulos and K.A.Olive, "Grand Unification with and without Supersymmetry and Cosmolgical Implications", World Sci. 1984.
6) P. Osland and G. Vigdel, hep-ph/9806339;
   J. N. Bahcall, P. I. Krastev and A. Yu Smirnov, hep-ph/9807216;
   V. Berezinsky, G. Fiorentini and M. Lissia, hep-ph/9904225;
   J.N. Bahcall, P.I. Krastev and A.Y. Smirnov, hep-ph/9905220.
7) L. Wolfenstein, Phys. Rev. D17 (1978) 2369;
   S.P. Mikheyev and A. Yu Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913.
8) A.Strumia, hep-ph/9904245.
9) P.F. Harrison, D.H. Perkins and W.G. Scott, hep-ph/9904297.
10) M. C. Gonzalez-Garcia, H. Nunokawa, O. L. G. Peres, J. W. F. Valle, hep-ph/9807305;
    G. L. Fogli, E. Lisi, A. Marrone and G. Scioscia, hep-ph/9808205;
    S. M. Bilenkii, C. Giunti, W. Grimus, hep-ph/9809368.
11) R. Barbieri, L. J. Hall, D. Smith, A. Strumia and N. Weiner, hep/ph 9807235.
12) Y. Suzuki, presented at the WIN99 Conference, Cape Town, January 1999.
13) B. Armbruster et al., Phys. Rev. C57 (1998) 3414 and G. Drexlin, talk at Wein'98.
14) G.L. Fogli, E. Lisi, A. Marrone and G. Scioscia, hep-ph/9904465.
15) M. Apollonio et al., Phys. Lett. B 420 (1998) 397.
16) Particle Data Group, Eur. Phys. J. C3(1998)1.

17) For a recent review, see, for example: M.S.Turner and J.A.Thyson, astro-ph/9901113.
18) See, for example, L. K. Krauss, hep-ph/9807376.
19) L. Baudis et al., hep-ex/9902014.
20) M. Gell-Mann, P. Ramond and R. Slansky in Supergravity, ed. P. van Nieuwenhuizen and D. Z. Freedman, North-Holland, Amsterdam, 1979, p.315; T. Yanagida, in Proceedings of the Workshop on the unified theory and the baryon number in the universe, ed. O. Sawada and A. Sugamoto, KEK report No. 79-18, Tsukuba, Japan, 1979. See also R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
21) V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, hep-ph/9806387;
V. Barger, T.J. Weiler and K. Whisnant, hep-ph/9807319.
22) S.M. Bilenky and C. Giunti, hep-ph/9802201;
A.J. Baltz, A.S. Goldhaber and M. Goldhaber, hep-ph/9806540.
23) G. Altarelli and F. Feruglio, JHEP 11(1998)21, hep-ph/9809596.
24) F. Vissani, hep-ph/9708483;
H. Georgi and S.L. Glashow, hep-ph/9808293.
25) J. Ellis and S. Lola, hep-ph/9904279;
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, hep-ph/9904395 and hep-ph/9905381.
26) R. Barbieri, L. J. Hall, G. L. Kane and G. G. Ross, hep-ph/9901228.
27) A. Ioannisian, J. W. F. Valle, Phys. Lett. B332 (1994) 93, hep-ph/9402333;
M. Fukugita, M. Tanimoto, T. Yanagida, Phys. Rev. D57 (1998) 4429, hep-ph/9709388;
M. Tanimoto, hep-ph/9807283 and hep-ph/9807517;
H. Fritzsch, Z. Xing, hep-ph/9808272;
R. N. Mohapatra, S. Nussinov, hep-ph/9808301 and hep-ph/9809415;
M. Fukugita, M. Tanimoto, T. Yanagida, hep-ph/9809554;
Yue-Liang Wu, hep-ph/9810491;
J. I. Silva-Marcos, hep-ph/9811381;
H. Fritzsch, Z. Xing, hep-ph/9903499.
28) G. Altarelli and F. Feruglio, Phys. Lett. B439(1998)112, hep-ph/9807353.
29) G. Altarelli and F. Feruglio, Phys. Lett. B451(1999) 388, hep-ph/9812475.
30) M. Tanimoto, Phys. Lett. B360 (1995) 41, hep-ph/9508247;
G. K. Leontaris, S. Lola, G. G. Ross, Nucl. Phys. B454 (1995) 25, hep-ph/9505402;
G. K. Leontaris, S. Lola, C. Scheich, J.D. Vergados, Phys. Rev. D53 (1996) 6381, hep-ph/9509351;
J. Ellis, G.K. Leontaris, S. Lola and D.V. Nanopoulos, hep-ph/9808251.
31) K.Hagiwara and N.Okamura, hep-ph/9811495.
32) H. Georgi and C. Jarlskog, Phys. Lett. B86(1979) 297.

33) J. Ellis and M. K. Gaillard, Phys. Lett. B88(1979) 315.
34) C. Froggatt and H. B. Nielsen, Nucl. Phys. B147 (1979) 277.
35) M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B420 (1994) 468, hep-ph/9310320.
36) M. Carena, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B406 (1993) 59;
P. Langacker and N. Polonsky, Phys. Rev. D49 (1194) 1451;
N. Polonsky, Phys. Rev. D54 (1996) 4537.
37) W. Buchmuller, D. Delepine and F. Vissani, hep-ph/9904219.
38) For early references on large neutrino mixing in GUTs, see:
J. Bijnens and C. Wetterich, Nucl.Phys.B292(1987)443;
K.S. Babu and S.M. Barr, Phys. Lett. B381 (1996) 202, hep-ph/9511446;
Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B396 (1997) 150;
G. Costa and E. Lunghi, Nuovo Cim. A110 (1997) 549;
B. Brahmachari and R.N. Mohapatra, Phys. Rev. D58 (1998), hep-ph/9710371;
A. S. Joshipura, A. Yu. Smirnov, hep-ph/9806376;
J. C. Pati, hep-ph/9807315;
I. S. Sogami, H. Tanaka and T. Shinohara, hep-ph/9807449;
Y. Nomura and T. Yanagida, hep-ph/9807325;
K. Oda, E. Takasugi, M. Tanaka and M. Yoshimura, hp-ph/9808241;
J. Sato and T. Yanagida, hep-ph/9809307.
39) J. K. Elwood, N. Irges and P. Ramond, hep-ph/9807228;
P. Ramond, hep-ph/9808489;
P. Ramond, hep-ph/9809401.
40) W. Buchmuller and T. Yanagida, hep-ph/9810308.
41) P. Binetruy, S. Lavignac, S. Petcov and P. Ramond, Nucl. Phys. B496 (1997) 3, hep-ph/9610481;
N. Irges, S. Lavignac, P. Ramond, Phys. Rev. D58 (1998) 5003, hep-ph/9802334;
Y. Grossman, Y. Nir, Y. Shadmi, hep-ph/9808355.
42) C. D. Froggatt, M. Gibson and H. B. Nielsen, Phys. Lett. B446 (1999) 256, hep-ph/9811265.
43) M. Bando and T. Kugo, hep-ph/9902204.
44) E. Ma and D. P. Roy, Phys. Rev. D59 (1999) 097702.
45) Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B409 (1997) 220.
46) C. H. Albright and S. M. Barr, Phys. Rev. D58 (1998) 013002, hep-ph/9712488;
C. H. Albright, K. S. Babu and S. M. Barr, Phys. Rev. Lett. 81 (1998) 1167, hep-ph/9802314;
C. H. Albright and S. M. Barr, hep-ph/9901318.
47) Z. Berezhiani and A. Rossi, hep-ph/9811447;

R. Barbieri, L. Giusti, L. Hall and A. Romanino, hep-ph/9812239.
48) G. C. Branco and J. I. Silva-Marcos, Phys. Lett. B331 (1994) 390, see also G. C. Branco, L. Lavoura and F. Mota, Phys. Rev. D39(1989)3443.
49) S. Lola and G. G. Ross, hep-ph/9902283;
K. Babu, J. Pati and F. Wilczek, hep-ph/9912538.
50) S. F. King, Phys. Lett. B439 (1998) 350, hep-ph/9806440; S. F. King, hep-ph/9904210;
S. Davidson and S. F. King, Phys. Lett. B445 (1998) 191, hep-ph/9808333;
Q. Shafi and Z. Tavartkiladze, Phys. Lett. B451 (1999) 129, hep-ph/9901243.
51) G. Altarelli, F. Feruglio and I. Masina, in preparation.
52) M. Jezabek and Y. Sumino, Phys. Lett. B440 (1998) 327, hep-ph/9807310;
T. Blazek, S. Raby and K. Tobe, hep-ph/9903340;
R. Barbieri, P. Creminelli and A. Romanino, hep-ph/9903460;
M. Jezabek and Y. Sumino, hep-ph/9904382.
53) C. Wetterich, hep-ph/9812426.