STERILE NEUTRINOS?

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ABSTRACT

The notion of sterile neutrinos is discussed. The schemes of mixing of four massive neutrinos, which imply the existence of sterile neutrinos, are briefly considered. Several model independent methods that allow to reveal possible transitions of solar neutrinos into sterile states are presented.

1. Introduction

The notion of sterile neutrinos was introduced by B. Pontecorvo in 1967 [1]. In the last years the possibility of transitions of flavor neutrinos into noninteracting sterile states has been widely discussed in the literature. This interest in sterile neutrinos is connected mainly with the result of the LSND experiment [2] that together with the results of atmospheric [3,4] and solar [5,6] neutrino experiments imply the existence of sterile neutrinos [7,8,9,10,11].

2. Notion of sterile neutrinos

Let us start with the discussion of the possibilities of sterile neutrinos to appear in neutrino mixing schemes (see, for example, Refs. [12,13,14]). Flavor neutrinos ν_e, ν_μ, ν_τ, are determined by the standard charged-current (CC) and neutral-current (NC) weak interactions

$$\mathcal{L}_I^{\rm CC} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\rm CC}\, W^\alpha + {\rm h.c.}\,, \qquad j_\alpha^{\rm CC} = 2\sum_{\ell=e,\mu,\tau} \overline{\nu_{\ell L}}\,\gamma_\alpha\,\ell_L\,, \tag{1}$$

$$\mathcal{L}_I^{\rm NC} = -\frac{g}{2\cos\theta_W}\, j_\alpha^{\rm NC}\, Z^\alpha\,, \qquad j_\alpha^{\rm NC} = \sum_{\ell=e,\mu,\tau} \overline{\nu_{\ell L}}\,\gamma_\alpha\,\nu_{\ell L}\,, \tag{2}$$

that conserve the electron L_e, muon L_μ and tau L_τ lepton numbers,

$$\sum L_e = {\rm const}\,, \qquad \sum L_\mu = {\rm const}\,, \qquad \sum L_\tau = {\rm const}\,. \tag{3}$$

If there is neutrino mixing, the conservation laws (3) are violated. A *neutrino*

mass term that does not conserve lepton numbers has the general form

$$\mathcal{L}^M = -\overline{n_R}\, M\, n_L + \text{h.c.}\,, \tag{4}$$

where n_L and n_R are N-component columns and M is a $N\times N$ matrix. There are two different possibilities for n_L:

1. Only flavor fields ν_ℓ ($\ell = e, \mu, \tau$) enter into n_L:

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}\,. \tag{5}$$

In this case M is a 3×3 matrix and for the mixing we have

$$\nu_{\ell L} = \sum_i U_{\ell i}\, \nu_{iL} \qquad (\ell = e, \mu, \tau)\,, \tag{6}$$

where U is a unitary matrix and ν_i is the field of neutrinos with mass m_i. The nature of neutrinos ν_i with definite mass depends on n_R:

1-1. If

$$n_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \tau_{\tau R} \end{pmatrix}\,, \tag{7}$$

where $\nu_{\ell R}$ ($\ell = e, \mu, \tau$) are right-handed neutrino fields, the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved and the neutrinos ν_i with definite mass are *Dirac particles*. The corresponding mass term is called "Dirac mass term".

1-2. If

$$n_R = \begin{pmatrix} (\nu_{eL})^c \\ (\nu_{\mu L})^c \\ (\nu_{\tau L})^c \end{pmatrix}\,, \tag{8}$$

where $(\nu_{\ell L})^c = \mathcal{C}\, \overline{\nu_{\ell L}}^T$ is the charge-conjugated (right-handed) component of the left-handed field $\nu_{\ell L}$ ($\ell = e, \mu, \tau$ and $\mathcal{C}$ is the matrix of charge conjugation), there are no conserved lepton numbers and neutrinos with definite masses are *Majorana particles* ($\nu_i^c = \nu_i$). The corresponding mass term is called "Majorana mass term".

From Eq. (6) it follows that in both cases only transitions between active neutrinos $\nu_\ell \leftrightarrows \nu_{\ell'}$ are possible. Notice that with the investigation of neutrino oscillations it is not possible to distinguish the case of Dirac neutrinos from the case of Majorana neutrinos [15].

2. In the column n_L not only the flavor fields $\nu_{\ell L}$ ($\ell = e, \mu, \tau$) but also other fields ν_{sL} ($s = s_1, s_2, \ldots$) enter

$$n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \end{pmatrix} . \tag{9}$$

The fields ν_{sL} do not enter in the standard CC and NC weak interactions (1), (2) and are called "sterile". It is possible that there are three sterile fields ν_{sL} that are the charge-conjugated components of right-handed neutrino fields, $\nu_{sL} = (\nu_{sR})^c$ ($s = e, \mu, \tau$). However, the sterile fields could also be fields of some other particles (SUSY, ...).

In this case, for the neutrino mixing we have

$$\nu_{\alpha L} = \sum_{i=1}^{3+N_s} U_{\alpha i}\, \nu_{iL} \,. \tag{10}$$

where N_s is the number of sterile fields and ν_i ($i = 1, \ldots, 3 + N_s$) is the field of neutrinos with mass m_i. Let us stress that the number of sterile fields depends on the concrete scheme of neutrino mixing.

The nature of neutrinos with definite mass depends on n_R. If $n_R = (n_L)^c$, the ν_i's are Majorana particles. If $\nu_{sL} = (\nu_{sR})^c$ ($s = e, \mu, \tau$), the corresponding mass term is called "Dirac-Majorana mass term".

If all the masses m_i of the fields ν_i in Eq.(10) are small, not only transitions between flavor neutrinos $\nu_\ell \leftrightarrows \nu_{\ell'}$ but also transitions between flavor and sterile neutrinos $\nu_\ell \leftrightarrows \nu_s$ are possible.

3. Possible ways to reveal the existence of the sterile neutrinos

There are two possible ways to reveal the existence of sterile neutrinos:

1. Through the determination of the number of massive neutrinos. If this number is larger than the number of flavor neutrinos (three) sterile neutrinos must exist.

2. Through the measurement of the total transition probability of neutrino with definite flavor (ν_e or ν_μ) into all possible flavor neutrinos, $\sum_{\ell'=e,\mu,\tau} P_{\nu_\ell \to \nu_{\ell'}}$ ($\ell = e, \mu$). This probability can be determined from the investigation of NC induced neutrino processes. If

$$\sum_{\ell'=e,\mu,\tau} P_{\nu_\ell \to \nu_{\ell'}} < 1 \qquad (\ell = e, \mu) \,, \tag{11}$$

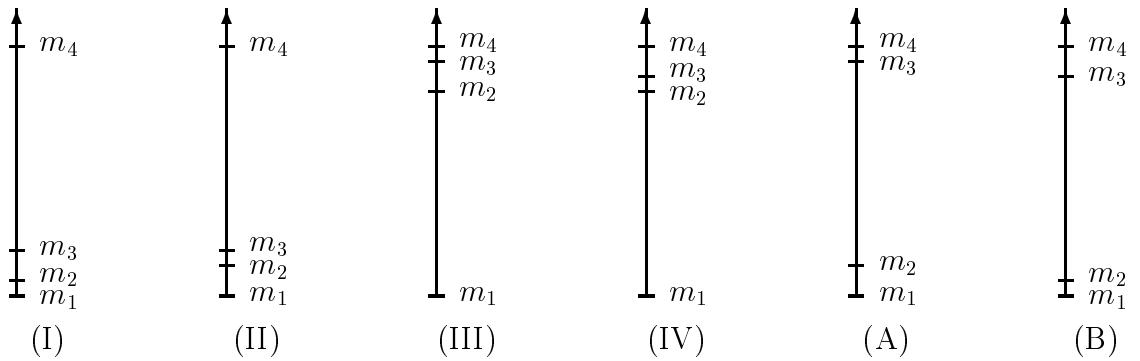

Figure 1: The six types of neutrino mass spectra that can accommodate the solar, atmospheric and LSND scales of Δm^2. The different distances between the masses on the vertical axes symbolize the different scales of Δm^2. Class 1 is constituted by the spectra (I)–(IV), whereas class 2 comprises the spectra (A) and (B).

from the unitarity of the mixing matrix it follows that active flavor neutrinos transform into sterile states.

4. Schemes of mixing of four massive neutrinos

The data of neutrino oscillation experiments indicate the existence of three different scales of neutrino mass-squared difference Δm^2:

1. $\Delta m^2_{\rm atm} \sim 10^{-3}\,{\rm eV}^2$ from atmospheric neutrino experiments [3,4];
2. $\Delta m^2_{\rm sun} \sim 10^{-5}\,{\rm eV}^2$ (or $10^{-10}\,{\rm eV}^2$) from solar neutrino experiments [5,6,16,17];
3. $\Delta m^2_{\rm LSND} \sim 1\,{\rm eV}^2$ from the LSND experiment [2].

Three different scales of Δm^2 can be accommodated only if at least four massive neutrinos exist in nature [7,8,9,10,11]. In other words, the existing data indicate that sterile neutrinos exist.

In the framework of the minimal scheme with four massive neutrinos, from the existing data it follows [9,10] that the dominant transitions of solar neutrinos are $\nu_e \to \nu_s$. Below we will present the corresponding arguments.

Figure 1 shows the six types of spectra of four massive neutrinos that can accommodate the solar, atmospheric and LSND ranges of neutrino mass-squared differences. These spectra are divided in two classes: class 1 constituted by the spectra (I)–(IV) and class 2 comprising the spectra (A) and (B). In the spectra of class 1 a group of three close masses is separated from the fourth mass by the LSND gap of $\sim 1\,{\rm eV}^2$. In the spectra of class 2 two pairs of close masses are separated by the LSND gap.

In the case of the spectra of class 1 the amplitude of $\nu_\mu \to \nu_e$ transitions in short-baseline (SBL) experiments is strongly suppressed and the upper bound of the corresponding oscillation amplitude obtained from the exclusion curves of SBL $\bar{\nu}_e$ and ν_μ disappearance experiments, is smaller than the value of the amplitude of $\nu_\mu \to \nu_e$ oscillations found in the LSND experiment [8]. Thus, the schemes of mixing of four massive neutrinos belonging to class 1 are not compatible with the existing neutrino oscillation data.

Only the schemes of mixing of four massive neutrinos with mass spectra of class 2,

$$\text{(A)} \qquad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{sun}}}_{\text{LSND}} \qquad \text{and} \qquad \text{(B)} \qquad \underbrace{\overbrace{m_1 < m_2}^{\text{sun}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}}, \tag{12}$$

can describe all the existing neutrino oscillation data. In this case, the largest mass squared difference $\Delta m^2_{41} \equiv m^2_4 - m^2_1$ is relevant for the oscillations in SBL reactor and accelerator experiments. The corresponding transition probabilities are [18]

$$P_{\nu_\alpha \to \nu_\beta} = \frac{1}{2}\, A_{\alpha;\beta} \left(1 - \cos \frac{\Delta m^2_{41} L}{2p}\right) , \tag{13}$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \frac{1}{2}\, B_{\alpha;\alpha} \left(1 - \cos \frac{\Delta m^2_{41} L}{2p}\right) . \tag{14}$$

Here L is the distance between neutrino source and neutrino detector, p is neutrino momentum, and $A_{\alpha;\beta}$, $B_{\alpha;\alpha}$ are the oscillation amplitudes given by

$$A_{\alpha;\beta} = 4 \left| \sum_i U_{\beta i}\, U^*_{\alpha i} \right|^2 , \tag{15}$$

$$B_{\alpha;\alpha} = 4 \left(\sum_i |U_{\alpha i}|^2 \right) \left(1 - \sum_i |U_{\alpha i}|^2 \right) , \tag{16}$$

where the index i runs over the values $1, 2$ or $3, 4$.

The quantities $\sum_i |U_{\alpha i}|^2$ for $\alpha = e, \mu$ are constrained by the results of SBL reactor $\bar{\nu}_e \to \bar{\nu}_e$ and accelerator $\nu_\mu \to \nu_\mu$ disappearance experiments in which no indications in favor of neutrino oscillations were found:

$$\sum_i |U_{\alpha i}|^2 \leq a^0_\alpha \qquad \text{or} \qquad 1 - \sum_i |U_{\alpha i}|^2 \leq a^0_\alpha \, . \tag{17}$$

Here

$$a^0_\alpha = \frac{1}{2} \left(1 - \sqrt{1 - B^0_{\alpha;\alpha}} \right) . \tag{18}$$

where $B^0_{\alpha;\alpha}$ is the upper bound of the amplitude of $\nu_\alpha \to \nu_\alpha$ transitions obtained from the exclusion curves of SBL reactor and accelerator disappearance experiments.

Using the exclusion plots obtained in the reactor Bugey [19] experiment and in the accelerator CDHS [20] and CCFR [21] experiments we have [22]

$$a_e^0 \lesssim 4 \times 10^{-2} \qquad \text{for} \qquad \Delta m_{41}^2 \gtrsim 4 \times 10^{-2}\,\text{eV}^2\,, \tag{19}$$

$$a_\mu^0 \lesssim 2 \times 10^{-1} \qquad \text{for} \qquad \Delta m_{41}^2 \gtrsim 3 \times 10^{-1}\,\text{eV}^2\,. \tag{20}$$

Further constraints on the quantities $\sum_i |U_{\alpha i}|^2$ for $\alpha = e, \mu$ can be found by taking into account the results of solar and atmospheric neutrino experiments. The survival probabilities of solar ν_e's and atmospheric ν_μ's are constrained by (see [18,8,14])

$$P^{\text{sun}}_{\nu_e \to \nu_e} \geq \sum_i |U_{\alpha i}|^4\,, \qquad P^{\text{atm}}_{\nu_\mu \to \nu_\mu} \geq \left(1 - \sum_i |U_{\alpha i}|^2\right)^2\,, \tag{21}$$

where the index i runs over 1,2 in scheme A and over 3,4 in scheme B.

Let us introduce the quantities

$$c_\alpha \equiv \sum_i |U_{\alpha i}|^2 \qquad (\alpha = e, \mu, \tau, s)\,, \tag{22}$$

with the index i running over 1,2 in scheme A and over 3,4 in scheme B.

From Eqs. (19), (20) and (21) we conclude [8] that the four-neutrino schemes A and B are compatible with the results of solar and atmospheric neutrino experiments only for

$$c_e \leq a_e^0 \qquad \text{and} \qquad 1 - c_\mu \leq a_\mu^0\,. \tag{23}$$

Constraints on the elements of the mixing matrix U can be also obtained from the limit on the effective number of neutrinos N_ν in Big-Bang Nucleosynthesis (BBN) (see, for example, [23]). The analysis of recent data yields the upper bound $N_\nu \leq 3.2$ at 95% CL [24], which implies that [9,10]

$$c_s \ll 1\,. \tag{24}$$

Taking now into account the unitarity relation

$$\sum_{\alpha = e,\mu,\tau,s} c_\alpha = 2\,, \tag{25}$$

from Eqs. (23) and (24) we come to the conclusion that c_e and c_s are small and c_μ and c_τ are large. In the approximation

$$c_e \ll 1\,, \qquad c_s \ll 1\,, \qquad c_\mu \simeq 1\,, \qquad c_\tau \simeq 1\,, \tag{26}$$

in scheme A we have

$$\begin{aligned} \nu_{eL} &= \cos\vartheta\,\nu_{3L} + \sin\vartheta\,\nu_{4L}\,, \\ \nu_{sL} &= -\sin\vartheta\,\nu_{3L} + \cos\vartheta\,\nu_{4L}\,, \\ \nu_{\mu L} &= \cos\gamma\,\nu_{1L} + \sin\gamma\,\nu_{2L}\,, \\ \nu_{\tau L} &= -\sin\gamma\,\nu_{1L} + \cos\gamma\,\nu_{2L}\,. \end{aligned} \tag{27}$$

where ϑ and γ are mixing angles. The corresponding mixing relations in scheme B can be obtained from Eq. (27) with the change $1,2 \leftrightarrows 3,4$. From Eq. (27) one can see that the dominant transitions of solar neutrinos are $\nu_e \to \nu_s$ and the dominant transitions of atmospheric neutrinos and neutrinos in long-baseline (LBL) experiments are $\nu_\mu \to \nu_\tau$.

5. Possible tests for $\nu_e \to \nu_s$ transitions of solar neutrinos

Here we will consider possible model independent methods of searching for $\nu_e \to \nu_s$ transitions in future solar neutrino experiments [25].

In the SNO experiment [26] solar neutrinos will be detected through the observation of the CC reaction

$$\nu_e + d \to e^- + p + p\,, \tag{28}$$

of the NC reaction

$$\nu + d \to \nu + p + n\,, \tag{29}$$

and of the elastic-scattering (ES) reaction

$$\nu + e^- \to \nu + e^-\,. \tag{30}$$

Because of the large energy thresholds ($\sim 5\,\mathrm{MeV}$ for the CC and ES processes and 2.2 MeV for the NC process), mainly ^{8}B neutrinos will be detected in the SNO experiment. Let us write the initial flux of ^{8}B neutrinos as a function of energy E in the form

$$\phi_{^8\mathrm{B}}(E) = X(E)\,\Phi_{^8\mathrm{B}}\,. \tag{31}$$

Here $\Phi_{^8\mathrm{B}}$ is the total flux and $X(E)$ is a known function $\left(\int X(E)\,\mathrm{d}E = 1\right)$ that characterizes the spectrum of ν_e's in the decay $^8\mathrm{B} \to {}^8\mathrm{Be} + e^+ + \nu_e$. The NC event rate is given by

$$N_{\mathrm{NC}} = \langle \sum_{\ell=e,\mu,\tau} P_{\nu_e\to\nu_\ell}\rangle_{\nu d}\ \langle\sigma^{\mathrm{NC}}_{\nu d}\rangle\ \Phi_{^8\mathrm{B}}\,. \tag{32}$$

Here $\langle\sum_{\ell=e,\mu,\tau} P_{\nu_e\to\nu_\ell}\rangle_{\nu d}$ is the average value of the total transition probability of solar ν_e's into all possible flavor neutrinos and

$$\langle\sigma^{\mathrm{NC}}_{\nu d}\rangle \simeq 4.7\times 10^{-43}\,\mathrm{cm}^2 \tag{33}$$

is the average value of the cross section of the NC process (29).

It is obvious that $\langle\sum_{\ell=e,\mu,\tau} P_{\nu_e\to\nu_\ell}\rangle_{\nu d} < 1$ if $\nu_e \to \nu_s$ transitions take place. We cannot, however, determine the quantity $\langle\sum_{\ell=e,\mu,\tau} P_{\nu_e\to\nu_\ell}\rangle_{\nu d}$ from (32) in a model independent way, because knowledge of the total flux $\Phi_{^8\mathrm{B}}$ is needed.

Information on the total flux $\Phi_{^8\mathrm{B}}$ can be obtained from the data of the Super-Kamiokande experiment. Indeed, we have

$$\Sigma_{\nu e} = \langle \sum_{\ell=e,\mu,\tau} P_{\nu_e\to\nu_\ell}\rangle_{\nu e}\ \langle\sigma_{\nu_\mu e}\rangle\ \Phi_{^8\mathrm{B}}\,, \tag{34}$$

where

$$\langle \sigma_{\nu_\mu e} \rangle \simeq 2 \times 10^{-45}\,\mathrm{cm}^2 \tag{35}$$

is the average cross section of ν_μ-e scattering and

$$\Sigma_{\nu e} = N_{\nu e} - \int_{E_{\rm th}} \left(\sigma_{\nu_e e} - \sigma_{\nu_\mu e}\right) \phi_{\nu_e}(E)\, \mathrm{d}E\,, \tag{36}$$

is the NC contribution to the $\nu_e e \to \nu_e e$ event rate $N_{\nu e}$. In Eq. (36) $\sigma_{\nu_e e}$ ($\sigma_{\nu_\mu e}$) is the cross section of ν_e-e (ν_μ-e) scattering and $\phi_{\nu_e}(E)$ is the flux of solar ν_e's with energy E on the earth, that will be measured in the SNO experiment through the observation of the CC reaction (28). Combining the relations (32) and (34) we obtain the ratio

$$R = \frac{\Sigma_{\nu e}\ \langle \sigma^{\rm NC}_{\nu d} \rangle}{N_{\rm NC}\ \langle \sigma_{\nu_\mu e} \rangle} = \frac{\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} \rangle_{\nu e}}{\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} \rangle_{\nu d}}\,, \tag{37}$$

that is independent from the the total flux $\Phi_{^8\mathrm{B}}$. Only measurable (and known) quantities enter in the ratio R. From Eq. (37) it is obvious that $R \neq 1$ only if $\sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} < 1$, *i.e.* if solar neutrinos transfer into sterile states. Let us notice, however, that the ratio R can be different from one only if the probability $\sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell}$ depends on the neutrino energy.

Another possibility to reveal transitions of solar neutrinos into sterile states could be realized by combining the Super-Kamiokande recoil electron spectrum and the spectrum of ν_e's on the earth that will be determined in the SNO experiment. We have

$$\Sigma_{\nu e}(T) = \langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} \rangle_{\nu e, T}\ \langle \frac{\mathrm{d}\sigma_{\nu_\mu e}}{\mathrm{d}T} \rangle\ \Phi_{^8\mathrm{B}}\,, \tag{38}$$

where T is the recoil electron kinetic energy in the Super-Kamiokande experiment,

$$\Sigma_{\nu e}(T) = N_{\nu e}(T) - \int_{E_{\rm min}(T)} \left(\frac{\mathrm{d}\sigma_{\nu_e e}}{\mathrm{d}T} - \frac{\mathrm{d}\sigma_{\nu_\mu e}}{\mathrm{d}T} \right) \phi_{\nu_e}(E)\, \mathrm{d}E\,, \tag{39}$$

and $E_{\rm min}(T) = \frac{T}{2}\sqrt{1 + \frac{2m_e}{T}}$. From relation (38) it is obvious that if the quantity

$$R_{\rm ES}(T) = \Sigma_{\nu e}(T) \Big/ \langle \frac{\mathrm{d}\sigma_{\nu_\mu e}}{\mathrm{d}T} \rangle \tag{40}$$

depends on T it means that the probability $P_{\nu_e \to \nu_s} = 1 - \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell}$ is different from zero and depends on neutrino energy. The dependence on T of the ratio

$$R(T) = \frac{\Sigma_{\nu e}(T)\ \langle \sigma_{\nu_\mu e} \rangle}{\Sigma_{\nu e}\ \langle \frac{\mathrm{d}\sigma_{\nu_\mu e}}{\mathrm{d}T} \rangle} \tag{41}$$

is presented in Fig. 2. The dashed line corresponds to the small mixing angle MSW solution of the solar neutrino problem in the case of $\nu_e \to \nu_s$ transitions with $\Delta m^2 = 4.5 \times 10^{-6}\,\mathrm{eV}^2$, $\sin^2 2\vartheta = 7.0 \times 10^{-3}$.

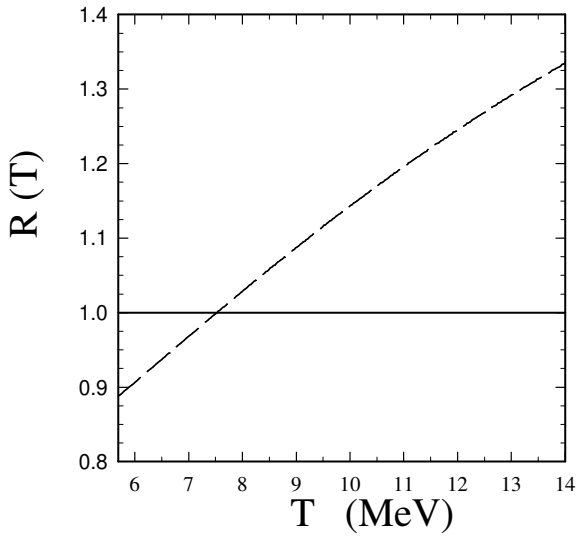

Figure 2: Ratio $R(T)$ (see Eq.(41)) calculated under the assumption of ν_e–$\nu_{\rm s}$ mixing with the values of the mixing parameters $\Delta m^2 = 4.5 \times 10^{-6}\,{\rm eV}^2$, $\sin^2 2\vartheta = 7.0 \times 10^{-3}$ (the small mixing angle MSW solution).

The most direct way to search for $\nu_e \to \nu_s$ transitions of atmospheric neutrinos is to investigate the NC process

$$\nu + N \to \nu + \pi^0 + X\,. \tag{42}$$

This possibility has been discussed in Ref. [27].

6. Conclusions

If we accept all the existing indications in favor of neutrino oscillations we come to the conclusion that sterile neutrinos must exist. Thus, from the phenomenological point of view the problem of sterile neutrinos is connected with the correctness of the existing indications in favor of neutrino oscillations and first of all with the correctness of the indications obtained in the LSND experiment, the only accelerator SBL experiment in which neutrino oscillations have been observed.

Through the investigation of NC processes it is possible to search for transitions of active neutrinos into sterile states in solar, atmospheric SBL and LBL experiments. Several possible tests are based only on the unitarity of the mixing matrix and can be done in a model independent way.

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8. References

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