HELIOSEISMOLOGY, SOLAR MODELS AND SOLAR NEUTRINOS

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ABSTRACT

We review recent advances concerning helioseismology, solar models and solar neutrinos. Particularly we shall address the following points: i) helioseismic tests of recent SSMs; ii)the accuracy of the helioseismic determination of the sound speed near the solar center; iii)predictions of neutrino fluxes based on helioseismology, (almost) independent of SSMs; iv)helioseismic tests of exotic solar models.

1. Introduction

Without any doubt, in the last few years helioseismology has changed the perspective of standard solar models (SSM).

Before the advent of helioseismic data a solar model had essentially three free parameters (initial helium and metal abundances, Y_{in} and Z_{in}, and the mixing length coefficient α) and produced three numbers that could be directly measured: the present radius, luminosity and heavy element content of the photosphere. In itself this was not a big accomplishment and confidence in the SSMs actually relied on the success of the stellar evoulution theory in describing many and more complex evolutionary phases in good agreement with observational data.

Helioseismology has added important data on the solar structure which provide severe constraint and tests of SSM calculations. For instance, helioseismology accurately determines the depth of the convective zone R_b, the sound speed at the transition radius between the convective and radiative transfer c_b, as well as the photospheric helium abundance Y_{ph}. With these additional constraints there are essentially no free parameters for SSM builders.

In this paper we review recent advances concerning helioseismology, solar models and solar neutrinos. Particularly we shall address the following points: i) helioseismic tests of recent SSMs; ii)the accuracy of the helioseismic determination of the sound speed near the solar center; iii)predictions of neutrino fluxes based on helioseismology, (almost) independent of SSMs; iv)helioseismic tests of exotic solar models.

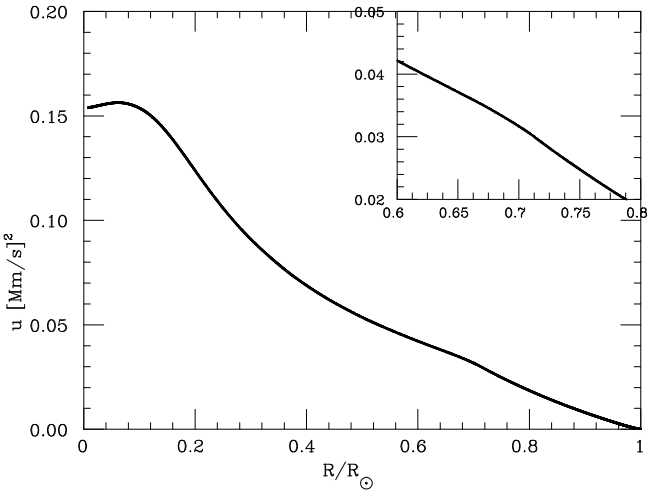

Fig. 1. The isothermal sound speed profile, $u = P/\rho$, as derived from helioseismic observations, from [2].

2. A summary of helioseismic determinations of solar properties

While we refer to e.g. [2] for a review of the method and to [7] for the data, we recall that by measurements of thousands of solar frequencies (p-modes)with a typical accuracy of $\Delta\nu/\nu \simeq 10^{-4}$, one derives:

- a) properties of the present convective envelope, such as depth, density and helium abundance:

$$R_b = 0.711(1 \pm 0.4\%)R_\odot \quad (1)$$
$$\rho_b = 0.192(1 \pm 0.4\%)g/cm^3 \quad (2)$$
$$Y_{ph} = x0.249(1 \pm 4\%) \quad (3)$$

The quoted errors, mostly resulting from systematic uncertainties in the inversion technique, have been estimated conservatively by adding linearly all known individual uncertainties, see [2]. If uncertainties are added in quadrature, the global error is about one third of that indicated in eqs. (1,2,3), see again [2]. This latter procedure was also used by Bahcall et al. [3] with similar results. This yields the so called "1σ" errors. We shall refer to the conservative estimate as the "3σ" determination. We remark however that this terminology is part of a slang, and it does not correspond to well defined confidence level, as one has to combine several essentially systematic errors.

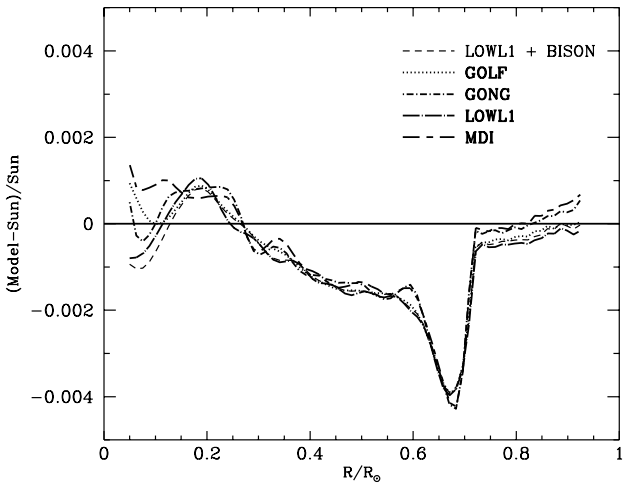

Fig. 2. The predicted BP98 sound speeds compared with five different helioseismological measurements[7], from [1]

- b)sound speed profile. By inversion of helioseismic data one can determine the sound speed in the solar interior. This analysis can be performed in terms of either the isothermal sound speed squared, $u = P/\rho$, or in terms of the adiabatic sound speed squared $c^2 = \partial P/\partial\rho|_{ad} = \gamma P/\rho$, as the coefficient $\gamma = \partial log P/\partial log\rho|_{adiab}$ is extremely well determined by means of the equation of state of the stellar plasma.

  In fig. 1 we show the helioseismic value of u as a function of the radial coordinate $R/R_\odot$. The typical 3σ errors are of order $\pm 0.4\%$ in the intermediate solar region, $R/R_\odot \simeq 0.4$, and increase up to $\pm 2\%$ near the solar center.

3. Helioseismic tests of recent Standard Solar Models

Fig. 2 compares the results of five different observational determinations of the sound speed in the sun with the results of the best solar model of ref. [1], hereafter BP98. This figure suggests several comments:

i)Different measurements yield quite consistent value of the sound speed, to the level 0.1%;

ii)*The solar model of BP98 is in agreeement with helioseismic data to better than 0.5% at any depth in the sun.* We remark that also the properties of the convective envelope predicted by BP98 ($R_b/R_\odot = 0.714$, $\rho_b = 0.186 g/cm^3$, $Y_{ph} = 0.243$) are in good agreement with helioseismic determinations, see eqs. (1,2,3).

iii)On the other hand, the predicted sound speed differs from the helioseismic determination at the level of 0.3-0.4% *just below the convective envelope.*

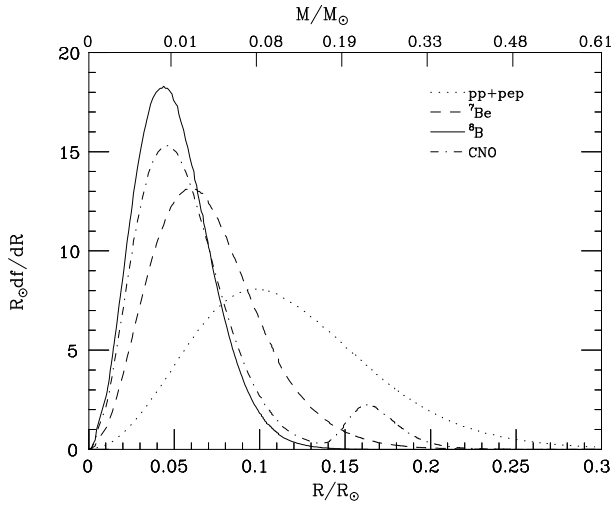

Fig. 3. For the indicated components, *df* is the fraction of neutrinos produced inside the sun within dR. On the bottom (top) scale the radial (mass) coordinate is indicated.

Concerning this last point, we remark that the difference is however within the "3σ" uncertainty of the helioseismic determination. Nevertheless it can be taken as an indication of some imperfection of the SSM. In fact this feature is common to any helioseismic data set, see again Fig. 2. As remarked in [2)] this feature is common to all recent SSM, which include elemental diffusion and use updated opacities, see fig. 4 in [2)]. It is well known [2,1)] that by using older opacities the problem disappears or is reduced, so that one can suspect of the accuracy of the calculated opacity. Furthermore, the well known Lithium deficit in the photosphere –a factor one hundred below the meteoric abundance– in not yet understood. In addition the "mixing length theory" is a rough description of the convective transport which maybe a not too good approximation in the transition between the radiative and the convective region.

In summary all this means that SSM predictions are accurate to the level of one per cent or better, although there are indications of some deficiencies at the level of per mille.

4. The solar sound speed in the neutrino production region

As well known, Boron and Beryllium neutrinos are produced very near to the solar center, see Fig. 3, with maximal production rates respectively at $R_B = 0.04 R_\odot$ and $R_{Be} = 0.06 R_\odot$. Since the p-modes which are observed do not propagate (actually are exponentially dumped) so deeply in the sun the question often arises if present helioseismic data can determine the sound speed in region of Beryllium and Boron production.

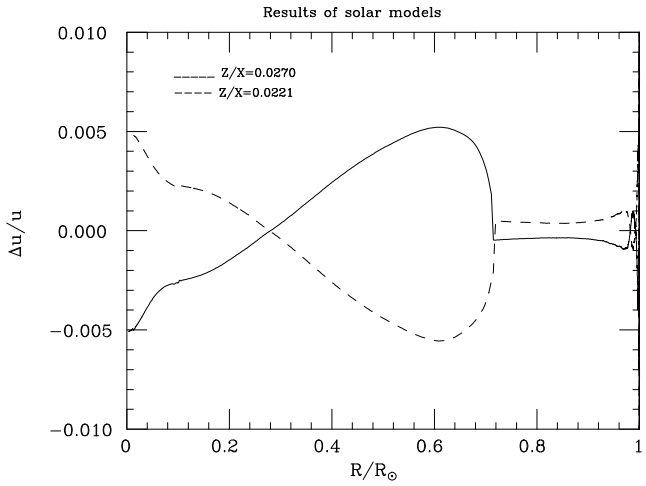

Fig. 4. Difference of the predicted sound speeds of metal rich (poor) models, compared with respect to the SSM prediction, full (dashed) line.

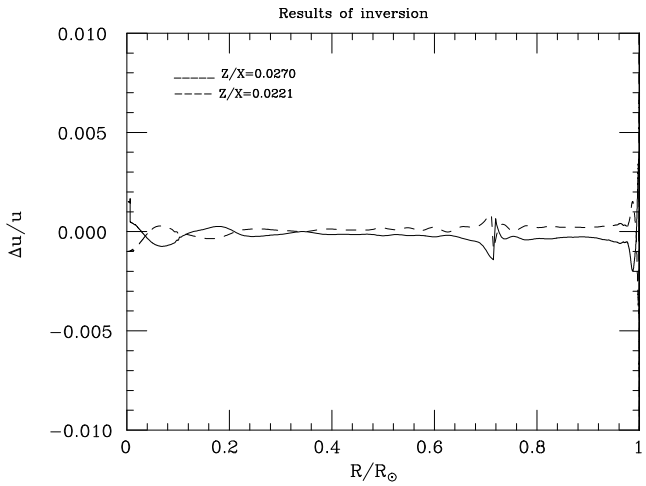

Fig. 5. Difference among the helioseismic sound speeds, obtained by using different starting models.

¿From an extensive analysis of the inversion method and of data available at that time we already concluded in [2] that $u(R \simeq 0)$ is determined with a "1σ" accuracy of 1% In this section we present a simplyfied analysis in order to produce convincing evidence that helioseismology fixes the sound speed near the solar center with such an accuracy.

Essentially there are two questions.
a) *Convergence of the inversion method*: for a given helioseismic data set, how does the reconstructed sound speed depend on the input solar model?
b) *Consistency of helioseimic data*: for a given inversion procedure, how does the result depend on the helioseismic data set?

In order to address the first question we have used as starting model for the inversion procedure the SSM of ref. [4] (Z/X=0.0245), as well as two non-standard models: a metal rich model (Z/X=0.027) and a metal poor one (Z/X=0.022). The predicted sound speed differences are shown in Fig. 4. We remark that the relative difference of u between the metal rich and the metal poor model is 1% at $R \simeq 0$. The helioseismic sound speeds, derived starting from these models and by using the BBSO86 [5]+BISON[6] data set are shown in Fig. 5. The relative difference between the reconstructed sound speeds are anywhere less or of the order of one per mille.

In particular *the helioseismic sound speeds at* $R \simeq 0$ *differ by two per mille although the difference between the models was a factor 5 larger*. This means that the helioseimic sound speed near the center is really determined by data.

The answer to the second question is clearly derived from Fig. 2. For a given starting solar model, inversion of different helioseismic data sets gives reconstructed sound speed which differ as much as two per mille near the solar center.

In conclusion we confirm our, possibly conservative, "1σ" accuracy

$$\Delta u/u(R \simeq 0) = 1\% \tag{4}$$

We remind that what is important for the production of Boron and Beryllium neutrinos is the solar temperature near the center. The knowledge of the sound speed does not give direct information about temperature, since the chemical composition and the equation of state have to be known.

In the energy production zone, the equation of state (EOS) for the solar interior can be approximated, with an accuracy better than 1%, by the EOS of a fully ionized classical perfect gas:

$$KT = u\mu\,, \tag{5}$$

where the "mean molecular weight" is:

$$\mu = m_p/(3/2X + 1/4Y + 1/2)\,. \tag{6}$$

For a given value of u without any assumption on the chemical composition one immediately gets a direct helioseismic constraint on the solar temperature:

$$1/2u < KT/m_p < 4/3u\,, \tag{7}$$

which will be useful for the discussion in the next sections. We remark however that much more strict bounds on the central temperature can be obtained by studying the so called Helioseismically Constrained solar Models HCSM, see [8].

5. Predictions on neutrino fluxes based on helioseismology

The basic idea is to use helioseismology in place of SSM calculations. Neutrino production rates are generally given as:

$$dN/dt = \int dv n_i n_j < \sigma v >_{ij} \propto S_{ij} \int dv n_i n_j T^{\alpha_{ij}} \tag{8}$$

The astrophysical S-factors S_{ij} are given by nuclear physics calculations and/or experiments and the power law coefficients α_{ij} are calculated by using the Gamow formula, see. e.g. [15]. In the usual approach, the nuclear densities $n_i(R)$ and the temperature profile $T(R)$ are given by SSM calculations.

Alternatively one can use helioseismology to constrain or determine the above integrals. This can be accomplished in the following way.

Since helioseismology determines $u = P/\rho$, by using the hydrostatic equilibrium equation, $dP/dR = -GM_R\rho/r^2$, one can determine $\rho = \rho(u)$, i.e. the density profile is also given by means of helioseismology. Furthermore, by using the classical perfect gas law one has $KT = u\mu$, see the previous section, so that also the temperature is given by helioseismology, *except for the mean molecular weight* μ. As previously remarked, one can determine constraints on μ, which translate into constraints on the neutrino production rates.

This approach has been applied to the study of *hep*-neutrinos in [9]. As well known, the excess of highest energy solar-neutrino events observed by Superkamiokande can be in principle explained by an anomalously high *hep*-neutrino flux $\Phi_\nu(hep)$. Without using SSM calculations, from the solar luminosity constraint it was found that $\Phi_\nu(hep)/S_{13}$ cannot exceed the SSM estimate by more than a factor three. If one makes the additional hypothesis that *hep* neutrino production occurs where the ^{3}He concentration is at equilibrium, helioseismology gives an upper bound which is (less then) two times the SSM prediction. We argue that the anomalous *hep*-neutrino flux of order of that observed by Superkamiokande cannot be explained by astrophysics, but rather by a large production cross-section.

In ref. [10] a lower limit on the Beryllium neutrino flux on earth was found, $\Phi(Be)_{min} = 1 \cdot 10^9$ cm^{-2} s^{-1}, in the absence of oscillations, by using helioseismic data, the B-neutrino flux measured by Superkamiokande and the hydrogen abundance at the solar center X_c predicted by Standard Solar Model (SSM) calculations. We remark that this abundance is the only result of SSMs needed for getting $\Phi(Be)_{min}$. Lower bounds for the Gallium signal, $G_{min} = (91 \pm 3)$ SNU, and for the Chlorine signal, $C_{min} = (3.24 \pm 0.14)$ SNU, have also been derived. They are about 3σ above the

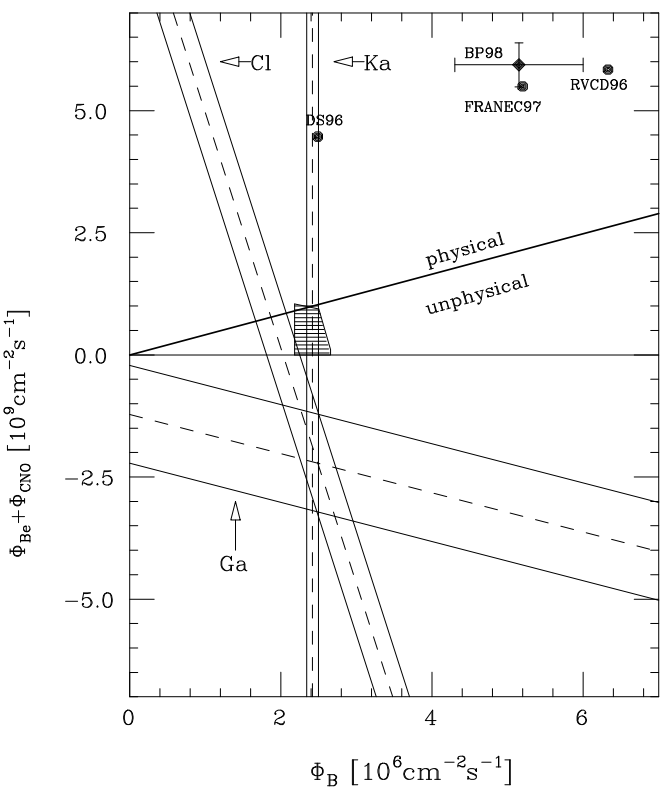

Fig. 6. The ^8B and $^7Be + CNO$ neutrino fluxes, consistent with the luminosity constraint and experimental results for standard neutrinos. The dashed (solid) lines correpond to the central ($\pm 1\sigma$) experimental values for Chlorine, Gallium and $\nu - e$ scattering experiments. The dashed area corresponds to the region within 3σ from each experimental result. The predictions of solar models including element diffusion (full circles) [18,1,17,20], are also shown. The thick diagonal line corresponds to the helioseismic lower limit on $\Phi(Be)$, see text.

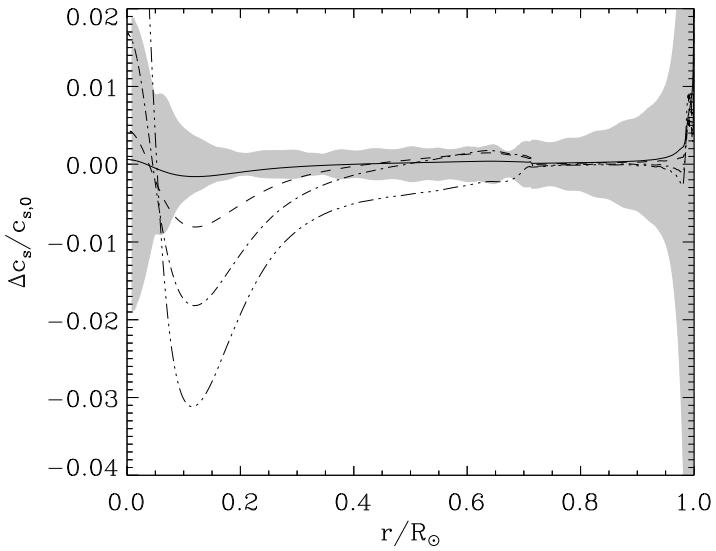

Fig. 7. Difference in sound-speed profiles of present-day solar models with axion losses compared to the reference model in the sense (Reference−Model)/Reference. Different line types correspond to different values of the axion-photon coupling constant: g_{10}=4.5 (solid line), 10 (short-dashed), 15 (dash-dotted), 20 (dash-dot-dot-dotted. The shaded area reflects the "3σ" uncertainties in the infered sound speed of the seismic model. From [16)]

corresponding experimental values, $G_{exp} = (72 \pm 6)$ SNU [11,12)] and $C_{exp} = (2.56 \pm 0.22)$ SNU [13)].

We remark that predictions for X_c are very stable among different (standard and non standard) solar models, see [10)]. In fact X_c is essentially an indicator of how much hydrogen has been burnt so far. The stability of X_c corresponds to the fact that any solar model has to account for the same present and time integrated solar luminosity.

In Fig. 6 we summarize the present situation concerning solar neutrino experiments. Helioseismology, when supplemented with the hydrogen abundance at the solar center X_c given by SSM, provides the lower bound $\Phi(Be) \geq 4 \cdot 10^2 \Phi(B)$ (thick diagonal line). One sees that the region within three sigmas from each experiment is almost completely out of the physical domain.

Along similar lines, a more extensive analysis has been presented in [14)] where the stronger bound on Beryllium neutrinos, $\Phi(Be) \geq 1.6 \cdot 10^9$ cm-2 s^{-1} was found.

6. Helioseismic tests of exotic solar models:axions from the sun?

Helioseismology severely constrains possible deviations from standard solar models, allowing e.g. the derivation of new limits on anomalous solar energy losses. In ref. [16)] as an example of nonstandard energy loss channel, the Primakoff conversion of photons in the Coulomb fields of charged particles, $\gamma + Ze \rightarrow Ze + a$ has been

considered.

Axion emission from the sun alters the hydrogen burning now and in the past. More hydrogen is being burnt and consequently the central solar temperature increases with respect to the SSM prediction. In addition more hydrogen has been burnt into helium in the past, so that the helium abundance in the solar center differs from the SSM value. All this affects the sound speed profile, see Fig. 7, and the photospheric helium abundance.

For an axion-photon coupling $g_{a\gamma} \lesssim 5 \times 10^{-10}$ GeV^{-1} the solar model is almost indistinguishable from the standard case, while $g_{a\gamma} \gtrsim 10 \times 10^{-10}$ GeV^{-1} is probably excluded, corresponding to an axion luminosity of about $0.20\, L_{\odot}$. This constraint on $g_{a\gamma}$ is much weaker than the well-known globular-cluster limit, but about a factor of 3 more restrictive than previous solar limits [16].

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