NEW EFFECT OF TOTAL NEUTRINO CONVERSION IN A MEDIUM OF NONPERIODIC CONSTANT-DENSITY LAYERS AND IN THE EARTH [*]

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ABSTRACT

A new effect of total neutrino conversion is possible when neutrino propagates through multi-layer medium of nonperiodic constant density layers. The effect can take place in the oscillations in the Earth of the Earth-core-crossing solar and atmospheric neutrinos.

1. Introduction

It was pointed out in [1)] that the $\nu_2 \rightarrow \nu_e$, $\nu_\mu \rightarrow \nu_e$ ($\nu_e \rightarrow \nu_{\mu(\tau)}$) oscillations of solar and atmospheric neutrinos in the Earth [a], caused by neutrino mixing in vacuum, can be strongly amplified by a new type of resonance-like effect which differs from the MSW one and takes place when the neutrinos traverse the Earth core on the way to the detector (see also [2,3)]). As numerical calculations have shown, at small mixing angles ($\sin^2 2\theta \lesssim 0.05$), the maxima due to this new enhancing mechanism in the corresponding transition probabilities, $P(\nu_2 \rightarrow \nu_e) \equiv P_{e2}$ and $P(\nu_\mu \rightarrow \nu_e)$ ($P(\nu_e \rightarrow \nu_{\mu(\tau)})$), are absolute maxima and dominate in P_{e2} and $P(\nu_\mu \rightarrow \nu_e)$: the values of the probabilities at these maxima in the simplest case of two-neutrino mixing are considerably larger - by a factor of $\sim (2.5-4.0)$ ($\sim (3.0-7.0)$), than the values of P_{e2} and $P(\nu_\mu \rightarrow \nu_e) = P(\nu_e \rightarrow \nu_{\mu(\tau)})$ at the local maxima associated with the MSW effect taking place in the Earth core (mantle). The effect of the new enhancement is less dramatic at large mixing angles. The enhancement is present and dominates also in the $\nu_2 \rightarrow \nu_e$ transitions in the case of $\nu_e - \nu_s$ mixing and in the $\nu_e \rightarrow \nu_s$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_s$ transitions at small mixing angles [1,2)]. Even at small mixing angles the enhancement is relatively wide [1,2,3)] in the Nadir angle [b], h, and in the neutrino energy - it is somewhat wider than the MSW resonance. It also exhibits rather strong energy dependence. As was shown in [1)] (see also [2,3,4)]), the new mechanism of enhancement of the neutrino

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[a]The $\nu_2 \rightarrow \nu_e$ transition probability accounts, as is well-known, for the Earth effect in the solar neutrino survival probability in the case of the MSW two-neutrino $\nu_e \rightarrow \nu_{\mu(\tau)}$ and $\nu_e \rightarrow \nu_s$ transition solutions of the solar neutrino problem, ν_s being a sterile neutrino.

[b]The Nadir angle determines uniquely the neutrino trajectory through the Earth.

transitions in the Earth can play an important role in the interpretation of the results of the experiments with solar and atmospheric neutrinos [5,6]; it can also be of crucial importance for obtaining information about possible small mixing angle ν_μ ($\bar{\nu}_\mu$) and ν_e ($\bar{\nu}_e$) oscillations. The enhancement of interest was interpreted in [1] as an effect of constructive interference between the various probability amplitudes (notably of the neutrino transitions in the Earth mantle and in the Earth core), entering into the sum representing the probability amplitude of the transition in the Earth.

The exact conditions for the enhancement of the probabilities P_{e2}, $P(\nu_\mu \to \nu_e)$ ($P(\nu_e \to \nu_{\mu(\tau)})$, etc. by the new mechanism can be studied analytically [1] owing to the results of a detailed numerical analysis [7] (see also [8]) which showed that for the calculation of the probabilities of interest the two-layer model of the Earth density distribution provides an excellent approximation to the more complicated density distributions predicted by the existing models of the Earth [9,10]. In the two-layer model, the neutrinos crossing the Earth mantle and the core traverse effectively two layers with constant but different densities, $\bar{\rho}_{man}$ and $\bar{\rho}_c$, and chemical composition or electron fraction numbers, Y_e^{man} and Y_e^c. The neutrino transitions of interest [c] result from two-neutrino oscillations taking place i) first in the mantle over a distance X' with a mixing angle θ'_m and oscillation length L_{man}, ii) then in the core over a distance X'' with mixing angle $\theta''_m \neq \theta'_m$ and oscillation length $L_c \neq L_{man}$, and iii) again in the mantle over a distance X' with θ'_m and L_{man}. Utilizing the above prescription, analytic expressions for the probabilities of neutrino transitions in the Earth P_{e2}, $P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_{\mu(\tau)})$, etc. were derived and were used to study their extrema [1]. Assuming that the neutrino oscillation parameters $\Delta m^2 > 0$ and $\cos 2\theta > 0$ are fixed and treating the neutrino paths X' and X'' as independent variables, it was found in [1] that in addition to the maxima corresponding to the MSW effect in the Earth mantle or in the Earth core, there exists a new maximum in P_{e2} and $P(\nu_\mu \to \nu_e)$. The latter takes place when the relative phases acquired by the states of the two energy-eigenstate neutrinos after the neutrinos have crossed the mantle, $\Delta E' X' = 2\pi X'/L_{man}$, and the core, $\Delta E'' X'' = 2\pi X''/L_c$, obey the constraints,

$$\Delta E' X' = \pi(2k+1), \quad \Delta E'' X'' = \pi(2k'+1), \quad k, k' = 0, 1, 2, ... , \tag{1}$$

and if the inequality [1,3]

$$\cos(2\theta''_m - 4\theta'_m + \theta) \ < 0, \tag{2}$$

is fulfilled. Condition (2) is valid for the probability P_{e2}. When equalities (1) hold, (2) ensures that P_{e2} has a maximum. At the maximum P_{e2} takes the form [1]

$$P_{e2}^{max} = \sin^2(2\theta''_m - 4\theta'_m + \theta). \tag{3}$$

The analogs of eqs. (1) - (3) for the probability $P(\nu_{\mu(e)} \to \nu_{e(\mu;\tau)})$ can be obtained

[c]The densities $\bar{\rho}_{man,c}$ should be considered as mean densities along the neutrino trajectories.

by formally setting [1] $\theta = 0$ while keeping $\theta'_m \neq 0$ and $\theta''_m \neq 0$ in (1) - (3) [d]. The term "neutrino oscillation length resonance" (NOLR) was used in [1] to denote the enhancement of P_{e2}, $P(\nu_\mu \to \nu_e)$, etc. associated with the conditions (1) - (2).

The new mechanism of strong enhancement of the probabilities P_{e2}, $P(\nu_\mu \to \nu_e)$ ($P(\nu_e \to \nu_{\mu(\tau)})$), etc. for the Earth-core-crossing neutrinos, was identified in [1] with the NOLR. This interpretation was suggested by several test cases studied. Doubts about the correctness of such an interpretation remained since i) for the parameters of the Earth [9] (core radius, mantle width, etc.), used in the calculations, the set of conditions (1) - (2), as was shown in [1], cannot be exactly satisfied at small mixing angles, and ii) although the phase $\Delta E'X'$ was close to π at the relevant absolute maxima of P_{e2} and $P(\nu_\mu \to \nu_e)$ in the test cases studied, the values of the phase $\Delta E''X''$ differed quite substantially from those required by eq. (1). Moreover, the same (or similar) enhancement mechanism was found to be operative in the $\nu_2 \to \nu_e$ transitions in the case of $\nu_e - \nu_s$ mixing and in the $\nu_e \to \nu_s$ transitions, at small mixing angles [1]. However, as was noticed in [1], the conditions (1) - (2) are not even approximately fulfilled for the $\nu_2 \to \nu_e \cong \nu_s \to \nu_e$ and $\nu_e \to \nu_s$ transitions at small mixing angles; similar conclusions were reached for the $\bar{\nu}_\mu \to \bar{\nu}_s$ transitions [2].

The indicated results stimulated the systematic study of the extrema of the probabilities P_{e2}, $P(\nu_\mu \to \nu_e)$, $P(\nu_e \to \nu_s)$, etc. of the neutrino transitions in the Earth and more generally, in a medium of non-periodic constant density layers, which was performed in [11]. We have found, in particular, that in addition to the local maxima associated with the MSW effect and the NOLR, the probabilities of interest have absolute maxima caused by a new resonance-like effect of enhancement and corresponding to a total neutrino conversion. The complete set of such absolute maxima was derived. The latter were shown to be absolute maxima in any possible independent variables: the neutrino energy, the widths of one of the layers, etc. The conditions for existence of the new maxima were derived and we have shown that they are fulfilled, in particular, for the transitions of the Earth-core-crossing solar and atmospheric neutrinos. We have shown also that the strong resonance-like enhancement of these transitions is due to the new effect of total neutrino conversion. Further, it was demonstrated in [11] that the NOLR and the newly found resonance-like enhancement effect are caused by a maximal constructive interference between the amplitudes of the neutrino transitions in the different density layers. The NOLR and the new enhancement effect were shown to take place in the transitions of neutrinos crossing two layers of different constant densities and widths as well. Thus, the local and absolute maxima they produce in the neutrino transition probabilities have nothing to do with the parametric resonance, possible in a medium with periodic change of density [12]. Therefore the general physical interpretation of the strong resonance-like enhancement of the transitions in the Earth of the Earth-core-crossing

[d]The conditions for the corresponding maximum in $P(\nu_e \to \nu_s)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_s)$ coincide in form with those for $P(\nu_{\mu(e)} \to \nu_{e(\mu;\tau)})$.

solar and atmospheric neutrinos, given in [1)], was confirmed.

The Earth enhancement of the two-neutrino transitions of interest of the solar and atmospheric neutrinos at relatively small mixing angles has been discussed rather extensively, see, e.g., refs. [13,14,8,15)]. Some of the articles contain plots of the probabilities P_{e2} and/or $P(\nu_\mu \to \nu_e)$ or $P(\nu_e \to \nu_{\mu(\tau)})$ on which one can clearly recognize now the dominating maximum due to the new enhancement effect. However, this maximum was invariably interpreted to be due to the MSW effect in the Earth core (see, e.g., [13,14)]) before the appearance of the study [1)].

In the present article we give a rather concise review of some of the results derived in [11)]. Certain of our results are illustrated by the realistic examples of the transitions of neutrinos passing through the Earth. The latter are obtained using the Stacey model from 1977 [9)] as a reference Earth model. The density distribution in the Stacey model is spherically symmetric and there are two major density structures - the core and the mantle. The core has a radius $R_c = 3485.7$ km, the Earth mantle depth is approximately $R_{man} = 2885.3$ km, and the Earth radius is $R_\oplus = 6371$ km [e].

2. Neutrino Oscillations in Matter with Constant Density

We shall analyze the simple case of mixing of two weak-eigenstate neutrinos ν_α and ν_β, $\alpha \neq \beta = e, \mu, \tau, s$, ν_s being a sterile neutrino, in vacuum. The mixing matrix relating the states of the neutrinos $\nu_{\alpha,\beta}$ and of the neutrinos $\nu_{1,2}$ having definite masses $m_{1,2}$ in vacuum can be chosen in the form:

$$\left(\begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} \right) = \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right) \left(\begin{array}{c} \nu_1 \\ \nu_2 \end{array} \right), \tag{4}$$

where θ is the vacuum mixing angle. For relativistic and stable neutrinos $\nu_{1,2}$, the evolution equation for neutrino propagation in constant-density medium reads

$$i\frac{\mathrm{d}}{\mathrm{d}t} \left(\begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} \right) = \frac{\Delta E}{2} \left(\begin{array}{cc} -\cos(2\theta_m) & \sin(2\theta_m) \\ \sin(2\theta_m) & \cos(2\theta_m) \end{array} \right) \left(\begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} \right) = \mathcal{M} \left(\begin{array}{c} \nu_\alpha \\ \nu_\beta \end{array} \right). \tag{5}$$

Here θ_m is the mixing angle in matter,

$$\cos(2\theta_m) = \frac{1}{\Delta E} \left((\Delta m^2/2E)\cos(2\theta) - V_{\alpha\beta}\right), \tag{6}$$

E being the neutrino energy, ΔE is the difference between the energies of the two energy-eigenstate neutrinos in matter,

$$\Delta E = \frac{\Delta m^2}{2E}\sqrt{\left(\cos(2\theta) - \frac{2EV_{\alpha\beta}}{\Delta m^2}\right)^2 + \sin^2(2\theta)}, \tag{7}$$

[e] The mean values of the matter densities of the core and of the mantle read, respectively: $\bar{\rho}_c \cong 11.5$ g/cm^3 and $\bar{\rho}_{man} \cong 4.5$ g/cm^3. The density distribution in the 1977 Stacey model practically coincides with that in the more recent PREM model [10)]. For Y_e we have used the standard values [9,10,16)] (see also [4)]) $Y_e^{man} = 0.49$ and $Y_e^c = 0.467$.

$\Delta m^2 = m_2^2 - m_1^2$, and $V_{\alpha\beta}$ is the difference between the effective potentials of ν_α and ν_β in the medium. We shall always assume in what follows that

$$\cos(2\theta) > 0, \quad \Delta m^2 > 0. \tag{8}$$

In the case of neutrino propagation in an electrically neutral unpolarized cold medium, like the Earth, one has:

$$V_{e\mu} = \sqrt{2}G_F N_e, \quad V_{es} = \sqrt{2}G_F(N_e - \frac{1}{2}N_n), \quad V_{\mu s} = -\sqrt{2}G_F N_n/2, \tag{9}$$

where N_e and N_n are the electron and the neutron number densities in the medium, respectively. The antineutrino states $\bar{\nu}_\alpha$ and $\bar{\nu}_\beta$ satisfy the same equations (5) with $V_{\alpha\beta}$ replaced by $V_{\bar{\alpha}\bar{\beta}} = -V_{\alpha\beta}$ in eqs. (6) and (7). For the Earth $N_e \cong N_n$ and in addition to $V_{e\mu} > 0$ we have $V_{es} > 0$ and $V_{\bar{\mu}\bar{s}} > 0$. If (8) holds, the neutrino mixing can be enhanced by the Earth matter only in the cases of ν_μ $(\nu_e) \rightarrow \nu_e$ $(\nu_{\mu;\tau})$ $(\nu_2 \rightarrow \nu_e)$, $\nu_e \rightarrow \nu_s$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_s$ transitions, while for, e.g., $\cos(2\theta) > 0$ and $\Delta m^2 < 0$ this will be true for the $\bar{\nu}_\mu$ $(\nu_e) \rightarrow \bar{\nu}_e$ $(\nu_{\mu;\tau})$ $(\bar{\nu}_2 \rightarrow \bar{\nu}_e)$, $\bar{\nu}_e \rightarrow \bar{\nu}_s$ and $\nu_\mu \rightarrow \nu_s$ transitions.

The evolution of the neutrino system is given by the unitary matrix (see, e.g., [17,18])

$$U = \exp(-i\mathcal{M}t) = \cos\phi - i(\boldsymbol{\sigma n})\sin\phi, \tag{10}$$

where

$$\phi = \frac{1}{2}\Delta E\ t \tag{11}$$

and

$$\mathbf{n} = (\sin(2\theta_m), 0, -\cos(2\theta_m)) \tag{12}$$

is a real unit vector. The probabilities of the $\nu_\alpha \rightarrow \nu_\beta$ $(\nu_\beta \rightarrow \nu_\alpha)$ transition, $P_{\alpha\beta(\beta\alpha)}$, and of the ν_α (ν_β) survival, $P_{\alpha\alpha(\beta\beta)}$, have the form:

$$P_{\alpha\beta} = P_{\beta\alpha} = |U_{\beta\alpha}|^2 = (n_1 \sin\phi)^2 + (n_2 \sin\phi)^2 = (\sin\phi\sin(2\theta_m))^2 \tag{13}$$

and

$$P_{\alpha\alpha} = P_{\beta\beta} = |U_{\alpha\alpha}|^2 = \cos^2\phi + (n_3 \sin\phi)^2 = 1 - P_{\alpha\beta}. \tag{14}$$

When neutrinos propagate in a constant-density medium the transition probability $P_{\alpha\beta}$ can be non-negligible even in the case of small vacuum mixing angle θ if the neutrino mixing in matter is maximal, $\theta_m = \pi/4$. This is the well-known MSW effect which takes place when the resonance condition

$$\cos(2\theta_m) = 0 \tag{15}$$

is fulfilled. In order to have total neutrino conversion, $P_{\alpha\beta} = 1$, the additional requirement

$$\cos\phi = 0 \tag{16}$$

has to be satisfied. For a fixed time of propagation t, or a distance $X \cong t$ traveled by the neutrino, there always exists a solution of these system of two equations. The *absolute* minima $P_{\alpha\beta} = 0$ correspond, as it follows from (13), to the curves $\sin\phi = 0$.

In the next Sections we discuss the case of neutrino propagation through three constant-density layers and through the Earth. A new phenomenon of maximal constructive interference between the probability amplitudes of the transitions in the different layers takes place in this case [11)]. It leads to a substantial enhancement of the transition probability. Even when the oscillation parameters have values very different from those required by the resonance conditions (15) for the individual layers, the probability $P_{\alpha\beta}$ can reach its absolute maximum, $P_{\alpha\beta} = 1$, due to this effect.

3. Medium with Thee Constant - Density Layers

Consider transitions of neutrinos traversing three layers of constant density and chemical composition when the first and the third layers have the same density, chemical composition and width which differ, however, from those of the second layer. This corresponds to the physically important case of solar and atmospheric neutrinos crossing the Earth core on the way to detectors on the Earth surface, the first and third layers being the Earth mantle, and the Earth core playing the role of the second layer.

Let us denote the effective potential differences and the lengths of the paths of the neutrinos (antineutrinos) in the first (third) and second layers by $V'_{\alpha\beta}$ ($V'_{\bar{\alpha}\bar{\beta}}$), $V''_{\alpha\beta}$ ($V''_{\bar{\alpha}\bar{\beta}}$) and X', X'', respectively. We shall assume without loss of generality that

$$0 \leq |V'_{\alpha\beta(\bar{\alpha}\bar{\beta})}| < |V''_{\alpha\beta(\bar{\alpha}\bar{\beta})}|. \qquad (17)$$

The transitions of ν_α and ν_β crossing the three layers will be determined by the two sets of two parameters - the mixing angle in matter (6) and the phase difference (11) with $t = X'$ or X'', which characterize the transitions in the first (third) and second layers: θ'_m and ϕ', and θ''_m and ϕ''. An expression for the probability $P_{\alpha\beta}$ in terms of ϕ', θ'_m, ϕ'', θ''_m, etc. can be found in [1)] and we are not going to reproduce it here.

For fixed $\sin^2(2\theta)$ and $\Delta m^2/E$ and given density and chemical composition in the two layers, the mixing angles θ'_m and θ''_m are also fixed for each layer. The phases ϕ' and ϕ'', which depend on the widths of the layers X' and X'', can be independent variables of the system if the two widths X' and X'' can be varied independently. The extrema of $P_{\alpha\beta}$ are determined by the system of two equations $\frac{dP_{\alpha\beta}}{d\phi'} = \frac{dP_{\alpha\beta}}{d\phi''} = 0$,

$$min\ P_{\alpha\beta}, max\ P_{\alpha\beta} : \ \left\{ \begin{array}{l} \sin(2\theta'_m)\ F(2\theta''_m - 2\theta'_m, \pi/2; \phi'', 2\phi' + \pi/2) = 0, \\ F(2\theta''_m - 2\theta'_m, 2\theta'_m; \phi'' + \pi/2, 2\phi') = 0, \end{array} \right. \qquad (18)$$

where

$$F(Y, Z; \varphi, \psi) = \sin Y \cos Z \sin\varphi + \sin Z \left[\sin\varphi\cos\psi\cos Y + \cos\varphi\sin\psi\right], \qquad (19)$$

and by two known supplementary conditions on the values of the second derivatives of $P_{\alpha\beta}$ at the points where eq. (18) hold. This is an interesting case with a clear physical meaning and we are going to investigate it in the present Section. In this way the NOLR resonance in the $\nu_2 \to \nu_e$, ν_μ $(\nu_e) \to \nu_e$ $(\nu_{\mu;\tau})$ and $\nu_e \to \nu_s$ transitions was discovered [1]. However, let us note that it does not correspond to the real case of the Earth, which will be considered in the next Section.

Using the explicit expression for $P_{\alpha\beta}$ and varying the phases ϕ' and ϕ'', we find [11] the following system of extrema of $P_{\alpha\beta}$ in the case $V'_{\alpha\beta} > 0$ (i.e., for the ν_μ $(\nu_e) \to \nu_e$ $(\nu_{\mu;\tau})$, $\nu_e \to \nu_s$ and $\bar\nu_\mu \to \bar\nu_s$ transitions in the Earth). There exist *absolute* maxima of $P_{\alpha\beta}$, corresponding to a *total* neutrino conversion,

$$\textit{case } A: \ \textit{max } P_{\alpha\beta} = 1, \tag{20}$$

for ϕ' and ϕ'' satisfying

$$\textit{solution } A: \ \begin{cases} \tan\phi' = \pm\sqrt{\dfrac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}}, \\ \tan\phi'' = \pm\dfrac{\cos 2\theta'_m}{\sqrt{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}}, \end{cases} \tag{21}$$

where the signs are correlated. The *absolute* maxima are realized in the region A

$$\textit{region } A: \ \begin{cases} \cos(2\theta''_m) \le 0, \\ \cos(2\theta''_m - 4\theta'_m) \ge 0. \end{cases} \tag{22}$$

On the line belonging to region A,

$$\textit{region } B: \ \cos 2\theta'_m = 0, \tag{23}$$

we have

$$\textit{case } B: \ \textit{max } P_{\alpha\beta} = \sin^2 2\theta'_m = 1, \tag{24}$$

provided

$$\textit{solution } B: \ \begin{cases} \cos 2\phi' = 0, \ \textit{or } 2\phi' = \frac{\pi}{2}(2k'+1), \ k' = 0, 1, ..., \\ \sin\phi'' = 0, \ \textit{or } 2\phi'' = 2\pi k'', \ k'' = 0, 1, ... \ . \end{cases} \tag{25}$$

Besides these *absolute* maxima, there exist two regions,

$$\textit{region } C: \ \cos(2\theta''_m) \ge 0, \tag{26}$$

$$\textit{region } D: \ \cos(2\theta''_m - 4\theta'_m) \le 0, \tag{27}$$

with maxima

$$\textit{case } C: \ \textit{max } P_{\alpha\beta} = \sin^2 2\theta''_m, \tag{28}$$

$$\textit{case } D: \ \textit{max } P_{\alpha\beta} = \sin^2(2\theta''_m - 4\theta'_m), \tag{29}$$

which correspond to the solutions

$$solution\ C: \quad \begin{cases} \sin\phi' = 0, \ \ or \ 2\phi' = 2\pi k', \ \ k' = 0, 1, ..., \\ \cos\phi'' = 0, \ \ or \ 2\phi'' = \pi(2k''+1), \ \ k'' = 0, 1, ..., \end{cases} \tag{30}$$

$$solution\ D: \quad \begin{cases} \cos\phi' = 0, \ \ or \ 2\phi' = \pi(2k'+1), \ \ k' = 0, 1, ..., \\ \cos\phi'' = 0, \ \ or \ 2\phi'' = \pi(2k''+1), \ \ k'' = 0, 1, ..., \end{cases} \tag{31}$$

respectively.

Obviously, the case B and solution C correspond to the MSW effect taking place in the first (third) and in the second layer, while solution D is just the NOLR solution considered in [1)] (compare eqs. (27), (29) and (31) with eqs. (1), (2) and (3)). As can be shown [11)], solutions A and D correspond to a new resonance-like effect of constructive interference between the transition amplitudes in the first (and third) and the second layers. Due to this effect the transition probability can reach its maximal value, $P_{\alpha\beta} = 1$, if the oscillation parameters $\sin^2(2\theta)$ and $\Delta m^2/E$ belong to the region A, eq. (22). Correspondingly, one can have a total neutrino conversion, $P_{\alpha\beta} = 1$, even if the MSW resonance does not take place in any of the three layers. Thus, conditions (21) and (22) represent new conditions for a total neutrino conversion in the medium with three constant-density layers considered (or in the Earth).

As was shown in [11)], we get the same system of maxima also in the $\nu_2 \to \nu_e$ transition probability, $P(\nu_2 \to \nu_e) \equiv P_{e2}$. The corresponding results for the probability P_{e2} can formally be obtained from the results eqs. (21) - (31) for the probability $P_{\alpha\beta}$ by replacing $2\theta''_m$ and $2\theta'_m$ with $(2\theta''_m - \theta)$ and $(2\theta'_m - \theta)$, respectively [f].

Similar analysis can be performed for the transitions of neutrinos traversing a medium of two layers of different constant densities and chemical compositions. The probability $P_{\alpha\beta}$ exhibits a system of of maxima which is completely analogous to that obtained above for the three-layer medium [11)]. In particular, analogs of solutions A, corresponding to a total neutrino conversion exist in the two layer case as well: i.e., one can have a *total* neutrino conversion, $P_{\alpha\beta} = 1$, even if the MSW resonance does not take place in any of the two layers. These results imply, in particular, that in order to have a large transition probability, $P_{\alpha\beta} \cong 1$, in a medium with constant-density layers, a periodic density profile is not required even when the MSW resonance is not realized in any of the layers. Thus, the strong enhancement of $P_{\alpha\beta}$, corresponding to the solution A (and to the solution D), has nothing to do with the parametric resonances in $P_{\alpha\beta}$, possible in a medium with periodically varying density and discussed in refs. [12)].

4. Transitions of Neutrinos Traversing the Earth Core

In the case of transitions in the Earth of (solar and atmospheric) neutrinos which

[f]In [11)] the set of *absolute* minima of $P_{\alpha\beta}$ and P_{e2} has also been derived.

pass through the Earth mantle, the core and the mantle again, the two lengths X' and X'' are not independent due to the spherical symmetry of the Earth. We can choose as independent variables, for example, $\cos h$, h being the Nadir angle, and $\Delta m^2/E$. The corresponding extrema conditions for the probability $P_{\alpha\beta}$ read [11]:

$$\Delta E'' \left(\frac{\mathrm{d}F}{\mathrm{d}\phi''} R_\oplus \cos h - \frac{\mathrm{d}F}{\mathrm{d}(2\phi')} X' \right) = 0, \tag{35}$$

$$\begin{aligned} &\frac{\mathrm{d}F}{\mathrm{d}(2\phi')} 2X' \frac{y - 2V'_{\alpha\beta}\cos(2\theta)}{4\Delta E'^2} + \frac{\mathrm{d}F}{\mathrm{d}\phi''} X'' \frac{y - 2V''_{\alpha\beta}\cos(2\theta)}{4\Delta E''^2} \\ &+ 2V'_{\alpha\beta}(\Delta E')^{-2} \sin(2\theta)\sin\phi' \left[\cos(4\theta'_m - 2\theta''_m)\sin\phi'\sin\phi'' - \cos(2\theta'_m)\cos\phi'\cos\phi''\right] \\ &- V''_{\alpha\beta}(\Delta E'')^{-2}\sin(2\theta)\sin\phi'' \left[\cos(4\theta'_m - 2\theta''_m)\sin^2\phi' + \cos(2\theta''_m)\cos^2\phi'\right] = 0, \end{aligned} \tag{36}$$

where $F \equiv F(2\theta''_m - 2\theta'_m, 2\theta'_m; \phi'', 2\phi')$ is defined in eq. (19), and $y \equiv \Delta m^2/E$. In general, the extrema of $P_{\alpha\beta}$ in the variables ϕ' and ϕ'' do not correspond to extrema in the variables $\cos h$ and $\Delta m^2/E$. It is not difficult to check, however, that the solutions (21) for the absolute maxima, corresponding to a total neutrino conversion, $P_{\alpha\beta} = 1$, are solutions of the system of equations (35) - (36) as well. They give the absolute maxima of the probability $P_{\alpha\beta}$ in any variable [11]. Equation (21) gives also the complete set of such solutions. The solutions (30) and (31) for the local maxima with $P_{\alpha\beta} < 1$ no longer correspond to extrema in the variables $\cos h$ and $\Delta m^2/E$. New solutions for the local maxima of $P_{\alpha\beta}$, which do not coincide with the solutions associated with phases equal to multiples of π, are possible. The same conclusions are valid for the absolute and local maxima of the probability P_{e2}.

The solutions (21), providing a total neutrino conversion, $P_{\alpha\beta} = 1$ ($P_{e2} = 1$), exist for the Earth density profile and for all neutrino transitions of interest, $\nu_2 \to \nu_e$, $\nu_\mu \to \nu_e$, $\nu_e \to \nu_{\mu(\tau)}$, $\nu_e \to \nu_s$ and $\bar{\nu}_\mu \to \bar{\nu}_s$, and at small, intermediate and large mixing angles. They are realized for very different sets of values of the phases $2\phi'$ and $2\phi''$, which are not multiples of π. In Table 1 we give a rather complete list of these solutions for the transitions $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_{\mu(\tau)}$) of the Earth-core-crossing neutrinos; similar results for the $\nu_2 \to \nu_e$, $\nu_e \to \nu_s$ and $\bar{\nu}_\mu \to \bar{\nu}_s$ transitions can be found in [11]. The maximal neutrino conversion solutions A, eq. (21), are responsible for the strong resonance-like enhancement of the $\nu_\mu \to \nu_e$, $\nu_e \to \nu_{\mu(\tau)}$, $\nu_e \to \nu_s$, etc. transitions in the Earth of the Earth-core crossing solar and atmospheric neutrinos, discussed in [1,2,3].

5. Instead of Conclusions: a Brief Summary of the Implications

The implications of the new resonance-like enhancement of the probability P_{e2} for the tests of the MSW $\nu_e \to \nu_{\mu(\tau)}$ and $\nu_e \to \nu_s$ transition solutions of the solar neutrino problem have been discussed in [1,3,11] and in much greater detail in [4] (see also [15,19]). For values of Δm^2 from the small mixing angle (SMA) MSW solution region and the geographical latitudes at which the Super-Kamiokande, SNO and ICARUS detectors are located, the enhancement takes place in the $\nu_e \to \nu_{\mu(\tau)}$ case for values of the ^{8}B neutrino energy lying in the interval $\sim (5 - 12)$ MeV to which these detectors are sensitive. Accordingly, at small mixing angles the new resonance is predicted [4)] to produce a much bigger - by a factor of up to ~ 6, day-night (D-N) asymmetry in the Super-Kamiokande sample of solar neutrino events, whose night fraction is due to the core-crossing neutrinos, in comparison with the asymmetry determined by using the *whole night* event sample. As a consequence, it can be possible to test a substantial part of the MSW $\nu_e \to \nu_{\mu(\tau)}$ SMA solution region in the $\Delta m^2 - \sin^2 2\theta$ plane by performing *core* D-N asymmetry measurements [4,15)]. The Super-Kamiokande collaboration has already successfully applied this approach to the analysis of their solar neutrino data [5)]: the limit the collaboration has obtained on the D-N asymmetry utilizing only the *core* event sample permitted to exclude a small part of the MSW SMA solution region. In contrast, the current Super-Kamiokande upper limit on the *whole night* D-N asymmetry [5)] does not allow to probe the SMA solution region: the predicted asymmetry is too small (see, e.g., [4)]).

The new effect of total neutrino conversion can and should be operative also [1,11)] in the $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_{\mu(\tau)}$) transitions of the atmospheric neutrinos crossing the Earth core if the atmospheric ν_μ and $\bar{\nu}_\mu$ indeed take part in large mixing vacuum $\nu_\mu(\bar{\nu}_\mu) \leftrightarrow \nu_\tau(\bar{\nu}_\tau)$, oscillations with $\Delta m^2 \sim (10^{-3} - 8 \times 10^{-3})$ eV^2, as is strongly suggested by the Super-Kamiokande data [6)], and if all three flavour neutrinos are mixed in vacuum. The new resonance-like enhancement can take place practically for all neutrino trajectories through the core (e.g., , for the trajectories with $h = (0^\circ - 23^\circ)$). It can produce an excess of e-like events at $-1 \leq \cos\theta_z \leq -0.8$, θ_z being the Zenith angle, in the Super-Kamiokande multi-GeV atmospheric neutrino data and can be responsible for at least part of the strong Zenith angle dependence of the Super-Kamiokande multi-GeV and sub-GeV $\mu-$like data [1,2,3,11)].

As the results obtained in refs. [1,2,3,4,11)] indicate, it is not excluded that some of the current or future high statistics solar and/or atmospheric neutrino experiments will be able to observe directly the resonance-like enhancement of the neutrino transitions due to the new effect of total neutrino conversion. Solar neutrino experiments located at lower geographical latitudes than the existing ones or those under construction (see the article by J.M. Gelb et al. quoted in [14)]) would be better suited for this purpose. An atmospheric neutrino detector capable of reconstructing with relatively good precision, e.g.,, the energy and the direction of the momentum of the incident neutrino in each neutrino-induced event, would allow to study in detail the new effect in the oscillations of the atmospheric neutrinos crossing the Earth.

6. References

1) S.T. Petcov, Phys. Lett. **B434** (1998) 321, (E) **B444** (1998) 584.

2) M. Chizhov, M. Maris and S.T. Petcov, Report SISSA 53/98/EP, 31 July 1998 (hep-ph/9810501).
3) S.T. Petcov, Report SISSA 103/98/EP (hep-ph/9809587); in *New Era in Neutrino Physics*, eds. H. Minakata and O. Yasuda (Universal Academy Press, Inc. Tokyo, Japan, 1998), p. 219 (hep-ph/9811205).
4) M. Maris and S.T. Petcov, Phys. Rev. D **56** (1997) 7444.
5) Y. Fukuda et al., Super-Kamiokande Coll., Phys. Rev. Lett. **82** (1999) 1810; M. Smy, Super-Kamiokande Coll., hep-ex/9903034.
6) Y. Fukuda et al., Phys. Rev. Lett. **81** (1998) 1562; K. Scholberg, Super-Kamiokande Coll., these Proceedings.
7) M. Maris and S.T. Petcov, to be published; M. Maris, Q.Y. Liu and S.T. Petcov, study performed in December of 1996 (unpublished).
8) P.I. Krastev and S.T. Petcov, Phys. Lett. **B205** (1988) 84.
9) F.D. Stacey, *Physics of the Earth, 2^{nd} edition*, John Wiley and Sons, London, New York, 1977.
10) A.D. Dziewonski and D.L. Anderson, Physics of the Earth and Planetary Interiors **25**, 297 (1981).
11) M. Chizhov and S.T. Petcov, Reports SISSA 28/99/EP (hep-ph/9903424) and 29/99/EP (hep-ph/9903399), March 1999.
12) V.K. Ermilova et al., Short Notices of the Lebedev Institute **5** (1986) 26; E.Kh. Akhmedov, Yad. Fiz. **47** (1988) 475; P.I. Krastev and A.Yu. Smirnov, Phys. Lett. **B226** (1989) 341.
13) S.P. Mikheyev and A.Yu. Smirnov, in *Massive Neutrinos in Astrophysics and in Particle Physics*, eds. O. Fackler and J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1986), p. 355; E.D. Carlson, Phys. Rev. **D34** (1986) 1454; A. Dar et al., Phys. Rev. **D35** (1987) 3607; G. Auriemma et al., **D37** (1988) 665; A. Nicolaidis, Phys. Lett. **B200** (1988) 553.
14) S.P. Mikheyev and A.Yu. Smirnov, quoted in [13]; M. Cribier et al., Phys. Lett. **B182** (1986) 89; A.J. Baltz and J. Weneser, Phys. Rev. **D50** (1994) 5971; J.M. Gelb, W. Kwong and S.P. Rosen, Phys. Rev. Lett. **78** (1997) 2296.
15) Q.Y. Liu, M. Maris and S.T. Petcov, Phys. Rev. D **56** (1997) 5991.
16) R. Jeanloz, Annu. Rev. Earth Planet. Sci. **18**, 356 (1990); C.J. Allègre et al., Earth Planet. Sci. Lett. **134**, 515 (1995).
17) J.J. Sakurai, *Modern Quantum Mechanics* (The Benjamin/Cummings Publishing Company, Menlo Park, California, 1985).
18) C.W. Kim and A. Pevsner, *Neutrinos in Physics and Astrophysics* (Harwood Academic Press, Chur, Switzerland, 1993).
19) M. Maris and S.T. Petcov, Phys. Rev. D**58** (1998) 113008.

Table 1: The values of $\Delta m^2/E \leq 20 \times 10^{-7}$ eV^2/MeV and of $\sin^2 2\theta$, at which the absolute maxima of $P_{e\mu} = P_{\mu e}$, corresponding to a total neutrino conversion, $P_{e\mu} = 1$ (solution A, eq. (21)), take place for neutrinos crossing the Earth along trajectories passing through the Earth core: $h = 0^\circ$; 13°; 23°; 30°. The values of $2\phi'$, $2\phi''$, $\sin^2 2\theta'_m$, $\sin^2 2\theta''_m$ and $\sin^2(2\theta''_m - 4\theta'_m)$ at the absolute maxima are also given.

$P(\nu_e \rightarrow \nu_{\mu(\tau)}) = P(\nu_\mu \rightarrow \nu_e)$							
h°	$\frac{\Delta m^2}{E}$ $[10^{-7}\ \frac{eV^2}{MeV}]$	$\sin^2 2\theta$	$2\phi'/\pi$	$2\phi''/\pi$	$\sin^2 2\theta'_m$	$\sin^2 2\theta''_m$	$\sin^2(2\theta''_m - 4\theta'_m)$
0.	7.201	.034	.923	.949	.111	.614	1.000
0.	4.821	.154	.509	2.334	.749	.208	.296
0.	11.256	.547	2.174	4.693	.795	.996	.285
0.	3.407	.568	.65	3.612	.846	.160	.881
0.	18.113	.803	3.933	9.129	.923	1.000	.710
0.	11.292	.841	2.425	6.167	.987	.891	.716
0.	19.307	.903	4.314	10.382	.979	.987	.845
0.	4.587	.948	1.167	4.705	.792	.285	.996
0.	19.862	.994	4.628	11.738	.992	.900	.981
0.	11.373	.994	2.702	7.577	.954	.708	.981
13.	7.024	.039	.931	.950	.131	.553	1.000
13.	4.537	.169	.496	2.258	.845	.179	.133
13.	10.071	.379	1.887	3.180	.645	.999	.068
13.	10.954	.626	2.272	4.503	.869	.973	.383
13.	3.267	.654	.732	3.456	.777	.153	.966
13.	6.736	.802	1.477	4.059	.997	.580	.688
13.	15.802	.874	3.653	7.680	.977	.972	.791
13.	18.249	.843	4.205	8.603	.949	.997	.762
13.	10.741	.927	2.529	5.936	.998	.797	.859
13.	18.770	.979	4.542	9.914	.999	.922	.954
23.	6.460	.051	.910	.922	.195	.387	1.000
23.	3.812	.224	.499	2.008	1.000	.125	.111
23.	10.101	.449	2.194	2.675	.726	.991	.134
23.	15.419	.744	3.875	5.236	.899	.999	.612
23.	3.417	.848	1.031	2.948	.711	.177	1.000
23.	9.655	.875	2.495	4.011	1.000	.786	.772
23.	16.565	.897	4.368	6.275	.984	.969	.828
23.	20.010	.979	5.470	8.061	.999	.935	.955
30.	5.376	.065	.765	.738	.352	.180	.982
30.	9.038	.199	2.062	.920	.418	1.000	.030
30.	2.908	.437	.742	1.427	.735	.095	.969
30.	14.060	.707	4.159	2.702	.885	.998	.548
30.	8.290	.825	2.491	2.021	1.000	.723	.691
30.	19.839	.913	6.326	4.357	.982	.986	.860
30.	17.123	.952	5.528	3.941	.999	.936	.909
30.	13.989	.997	4.694	3.597	.969	.785	.984
30.	8.909	1.000	3.139	2.741	.882	.551	.999