SUPERHEAVY DARK MATTER

RIOTTO ANTONIO
CERN, Theory Division
CH-1211, Geneva 23, Switzerland
E-mail: riotto@nxth04.cern.ch

ABSTRACT

There are many reasons to believe the present mass density of the universe is dominated by a weakly interacting massive particle (WIMP), a fossil relic of the early universe. Theoretical ideas and experimental efforts have focused mostly on production and detection of *thermal* relics, with mass typically in the range a few GeV to a hundred GeV. Here, I will review scenarios for production of *nonthermal* dark matter. Since the masses of the nonthermal WIMPS are in the range 10^{12} to 10^{16} GeV, much larger than the mass of thermal wimpy WIMPS, they may be referred to as WIMPZILLAS. In searches for dark matter it may be well to remember that "size does matter."

1. Introduction

There is conclusive evidence that the dominant component of the matter density in the universe is dark. The most striking indication of the existence of dark matter is the dynamical motions of astronomical objects. Observations of flat rotation curves for spiral galaxies [1] indicates that the dark component of galactic halos is about ten times the luminous component. Dynamical evidence for DM in galaxy clusters from the velocity dispersion of individual galaxies, as well as from the large x-ray temperatures of clusters, is also compelling [2]. Bulk flows, as well as the peculiar motion of our own local group, also implies a universe dominated by dark matter [3].

The mass of galaxy clusters inferred by their gravitational lensing of background images is consistent with the large dark-to-visible mass ratios determined by dynamical methods [4].

There is also compelling evidence that the bulk of the dark component must be nonbaryonic. The present baryonic density is restricted by big-bang nucleosynthesis to be less than that inferred by the methods discussed above [5]. The theory of structure formation from the gravitational instability of small initial seed inhomogeneities requires a significant nonbaryonic component to the mass density [6].

In terms of the critical density, $\rho_C = 3H_0^2 M_{\rm Pl}^2/8\pi = 1.88 \times 10^{-29}{\rm g\ cm}^{-3}$ with Hubble constant $H_0 \equiv 100h$ km sec^{-1}Mpc^{-1} and Planck mass $M_{\rm Pl}$, the dark-matter density inferred from dynamics is $\Omega_{\rm DM} \equiv \rho_{\rm DM}/\rho_C \gtrsim 0.3$. In addition, the most natural inflation models predict a flat universe, *i.e.,* $\Omega_0 = 1$, while standard big-bang nucleosynthesis implies that ordinary baryonic matter can contribute at most 10% to

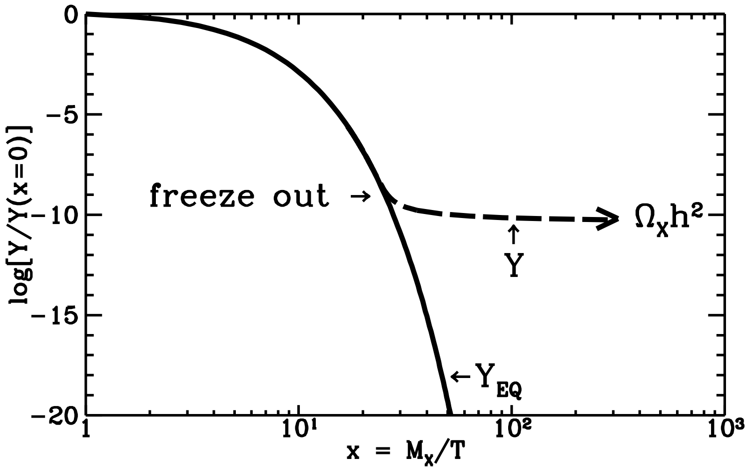

Figure 1: A thermal relic starts in LTE at $T \gg M_X$. When the rates keeping the relic in chemical equilibrium become smaller than the expansion rate, the density of the relic relative to the entropy density freezes out.

Ω_0. This means that about 90% of the matter in our universe may be dark.

2. Thermal Relics—Wimpy WIMPS

It is usually assumed that the dark matter consists of a species of a new, yet undiscovered, massive particle, traditionally denoted by X. It is also often assumed that the dark matter is a thermal relic, *i.e.,* it was in chemical equilibrium in the early universe.

A thermal relic is assumed to be in local thermodynamic equilibrium (LTE) at early times. The *equilibrium* abundance of a particle, say relative to the entropy density, depends upon the ratio of the mass of the particle to the temperature. Define the variable $Y \equiv n_X/s$, where n_X is the number density of WIMP X with mass M_X, and $s \sim T^3$ is the entropy density. The equilibrium value of Y, Y_{EQ}, is proportional to $\exp(-x)$ for $x \gg 1$, while $Y_{EQ} \sim$ constant for $x \ll 1$, where $x = M_X/T$.

A particle will track its equilibrium abundance as long as reactions which keep the particle in chemical equilibrium can proceed rapidly enough. Here, rapidly enough means on a timescale more rapid than the expansion rate of the universe, H. When the reaction rate becomes smaller than the expansion rate, then the particle can no longer track its equilibrium value, and thereafter Y is constant. When this occurs the particle is said to be "frozen out." A schematic illustration of this is given in Fig. 1.

The more strongly interacting the particle, the longer it stays in LTE, and the smaller its eventual freeze-out abundance. Conversely, the more weakly interacting the particle, the larger its present abundance. The freeze-out value of Y is related to the mass of the particle and its annihilation cross section (here characterized by σ_0) by [7)]

$$Y \propto \frac{1}{M_X m_{Pl} \sigma_0} . \tag{1}$$

Since the contribution to Ω is proportional to $M_X n_X$, which in turn is proportional to $M_X Y$, the present contribution to Ω from a thermal relic roughly is *independent* of its mass and depends only upon the annihilation cross section. The cross section that results in $\Omega_X h^2 \sim 1$ is of order 10^{-37}cm^2, of the order the weak scale. This is one of the attractions of thermal relics. The scale of the annihilation cross section is related to a known mass scale.

The simple assumption that dark matter is a thermal relic is surprisingly restrictive. The largest the annihilation cross section can be is roughly M_X^{-2}. This implies that large-mass WIMPS would have such a small annihilation cross section that their present abundance would be too large. Thus one expects a maximum mass for a thermal WIMP, which turns out to be a few hundred TeV [8)].

The standard lore is that the hunt for dark matter should concentrate on particles with mass of the order of the weak scale and with interaction with ordinary matter on the scale of the weak force. This has been the driving force behind the vast effort in dark matter direct detection described in this meeting by Cabrera [9)], Liubarsky [10)], Bernabei [11)], Ramachers [12)], and Baudis [13)].

In view of the unitarity argument, in order to consider *thermal* WIMPZILLAS, one must invoke, for example, late-time entropy production to dilute the abundance of these supermassive particles [14)], rendering the scenario unattractive.

3. Nonthermal Relics—WIMPZILLAS

There are two necessary conditions for the WIMPZILLA scenario. First, the WIMPZILLA must be stable, or at least have a lifetime much greater than the age of the universe. This may result from, for instance, supersymmetric theories where the breaking of supersymmetry is communicated to ordinary sparticles via the usual gauge forces [15)]. In particular, the secluded and the messenger sectors often have accidental symmetries analogous to baryon number. This means that the lightest particle in those sectors might be stable and very massive if supersymmetry is broken at a large scale [16)]. Other natural candidates arise in theories with discrete gauge symmetries [17)] and in string theory and M theory [18,19)].

It is useful here to note that WIMPZILLA decay might be able to account for ultra-high energy cosmic rays above the Greisen–Zatzepin–Kuzmin cutoff [20,21)]. A wimpy little thermal relic would be too light to do the job, a WIMPZILLA is needed.

The second condition for a WIMPZILLA is that it must not have been in equilibrium when it froze out (*i.e.,* it is not a thermal relic), otherwise $\Omega_X h^2$ would be much larger than one. A sufficient condition for nonequilibrium is that the annihilation rate (per particle) must be smaller than the expansion rate: $n_X\sigma|v| < H$, where $\sigma|v|$ is the annihilation rate times the Møller flux factor, and H is the expansion rate. Conversely, if the dark matter was created at some temperature T_* *and* $\Omega_X h^2 < 1$, then it is easy to show that it could not have attained equilibrium. To see this, assume X's were created in a radiation-dominated universe at temperature T_*. Then $\Omega_X h^2$ is given by

$$\Omega_X h^2 = \Omega_\gamma h^2 (T_*/T_0) m_X n_X(T_*)/\rho_\gamma(T_*) \ , \tag{2}$$

where T_0 is the present temperature. Using the fact that $\rho_\gamma(T_*) = H(T_*) M_{Pl} T_*^2$, $n_X(T_*)/H(T_*) = (\Omega_X/\Omega_\gamma) T_0 M_{Pl} T_*/M_X$. One may safely take the limit $\sigma|v| < M_X^{-2}$, so $n_X(T_*)\sigma|v|/H(T_*)$ must be less than $(\Omega_X/\Omega_\gamma) T_0 M_{Pl} T_*/M_X^3$. Thus, the requirement for nonequilibrium is

$$\left(\frac{200\,\mathrm{TeV}}{M_X}\right)^2 \left(\frac{T_*}{M_X}\right) < 1 \ . \tag{3}$$

This implies that if a nonrelativistic particle with $M_X \gtrsim 200$ TeV was created at $T_* < M_X$ with a density low enough to result in $\Omega_X \lesssim 1$, then its abundance must have been so small that it never attained equilibrium. Therefore, if there is some way to create WIMPZILLAS in the correct abundance to give $\Omega_X \sim 1$, nonequilibrium is automatic.

Any WIMPZILLA production scenario must meet these two criteria. Before turning to several WIMPZILLA production scenarios, it is useful to estimate the fraction of the total energy density of the universe in WIMPZILLAS at the time of their production that will eventually result in $\Omega \sim 1$ today.

The most likely time for WIMPZILLA production is just after inflation. The first step in estimating the fraction of the energy density in WIMPZILLAS is to estimate the total energy density when the universe is "reheated" after inflation.

Consider the calculation of the reheat temperature, denoted as T_{RH}. The reheat temperature is calculated by assuming an instantaneous conversion of the energy density in the inflaton field into radiation when the decay width of the inflaton energy, Γ_ϕ, is equal to H, the expansion rate of the universe.

The reheat temperature is calculated quite easily [7]. After inflation the inflaton field executes coherent oscillations about the minimum of the potential. Averaged over several oscillations, the coherent oscillation energy density redshifts as matter: $\rho_\phi \propto a^{-3}$, where a is the Robertson–Walker scale factor. If ρ_I and a_I denotes the total inflaton energy density and the scale factor at the initiation of coherent oscillations, then the Hubble expansion rate as a function of a is

$$H(a) = \sqrt{\frac{8\pi}{3}\frac{\rho_I}{M_{Pl}^2}\left(\frac{a_I}{a}\right)^3} \ . \tag{4}$$

Equating $H(a)$ and Γ_ϕ leads to an expression for a_I/a. Now if all available coherent energy density is instantaneously converted into radiation at this value of a_I/a, one can define the reheat temperature by setting the coherent energy density, $\rho_\phi = \rho_I(a_I/a)^3$, equal to the radiation energy density, $\rho_R = (\pi^2/30)g_* T_{RH}^4$, where g_* is the effective number of relativistic degrees of freedom at temperature T_{RH}. The result is

$$T_{RH} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4} \sqrt{\Gamma_\phi M_{Pl}} = 0.2 \left(\frac{200}{g_*}\right)^{1/4} \sqrt{\Gamma_\phi M_{Pl}} \; . \tag{5}$$

The limit from gravitino overproduction is $T_{RH} \lesssim 10^9$ to 10^{10} GeV.

Now consider the WIMPZILLA density at reheating. Suppose the WIMPZILLA never attained LTE and was nonrelativistic at the time of production. The usual quantity $\Omega_X h^2$ associated with the dark matter density today can be related to the dark matter density when it was produced. First write

$$\frac{\rho_X(t_0)}{\rho_R(t_0)} = \frac{\rho_X(t_{RH})}{\rho_R(t_{RH})} \left(\frac{T_{RH}}{T_0}\right), \tag{6}$$

where ρ_R denotes the energy density in radiation, ρ_X denotes the energy density in the dark matter, T_{RH} is the reheat temperature, T_0 is the temperature today, t_0 denotes the time today, and t_{RH} denotes the approximate time of reheating. More specifically, this is approximately the time at which the universe becomes radiation dominated after inflation. To obtain $\rho_X(t_{RH})/\rho_R(t_{RH})$, one must determine when X particles are produced with respect to the completion of reheating and the effective equation of state between X production and the completion of reheating.

At the end of inflation the universe may have a brief period of matter domination resulting either from the coherent oscillations phase of the inflaton condensate or from the preheating phase [22]. If the X particles are produced at time $t = t_e$ when the de Sitter phase ends and the coherent oscillation period just begins, then both the X particle energy density and the inflaton energy density will redshift at approximately the same rate until reheating is completed and radiation domination begins. Hence, the ratio of energy densities preserved in this way until the time of radiation domination is

$$\frac{\rho_X(t_{RH})}{\rho_R(t_{RH})} \approx \frac{8\pi}{3} \frac{\rho_X(t_e)}{M_{Pl}^2 H^2(t_e)}, \tag{7}$$

where $M_{Pl} \approx 10^{19}$ GeV is the Planck mass and most of the energy density in the universe just before time t_{RH} is presumed to turn into radiation. Thus, using Eq. 6, one may obtain an expression for the quantity $\Omega_X \equiv \rho_X(t_0)/\rho_C(t_0)$, where $\rho_C(t_0) = 3H_0^2 M_{Pl}^2/8\pi$ and $H_0 = 100\, h$ km sec^{-1} Mpc^{-1}:

$$\Omega_X h^2 \approx \Omega_R h^2 \left(\frac{T_{RH}}{T_0}\right) \frac{8\pi}{3} \left(\frac{M_X}{M_{Pl}}\right) \frac{n_X(t_e)}{M_{Pl} H^2(t_e)}. \tag{8}$$

Here $\Omega_R h^2 \approx 4.31 \times 10^{-5}$ is the fraction of critical energy density in radiation today and n_X is the density of X particles at the time when they were produced.

Note that because the reheating temperature must be much greater than the temperature today ($T_{RH}/T_0 \gtrsim 4.2 \times 10^{14}$), in order to satisfy the cosmological bound $\Omega_X h^2 \lesssim 1$, the fraction of total WIMPZILLA energy density at the time when they were produced must be extremely small. One sees from Eq. 8 that

$$\Omega_X h^2 \sim 10^{17}(T_{RH}/10^9\mathrm{GeV})(\rho_X(t_e)/\rho(t_e)). \tag{9}$$

It is indeed a very small fraction of the total energy density extracted in WIMPZILLAS.

This means that if the WIMPZILLA is extremely massive, the challenge lies in creating very few of them. Gravitational production discussed in Section naturally gives the needed suppression. Note that if reheating occurs abruptly at the end of inflation, then the matter domination phase may be negligibly short and the radiation domination phase may follow immediately after the end of inflation. However, this does not change Eq. 8.

4. WIMPZILLA PRODUCTION

4.1. Gravitational Production

First consider the possibility that WIMPZILLAS are produced in the transition between an inflationary and a matter-dominated (or radiation-dominated) universe due to the "nonadiabatic" expansion of the background spacetime acting on the vacuum quantum fluctuations [23)].

The distinguishing feature of this mechanism is the capability of generating particles with mass of the order of the inflaton mass (usually much larger than the reheating temperature) even when the particles only interact extremely weakly (or not at all) with other particles and do not couple to the inflaton. They may still be produced in sufficient abundance to achieve critical density today due to the classical gravitational effect on the vacuum state at the end of inflation. More specifically, if $0.04 \lesssim M_X/H_I \lesssim 2$, where $H_I \sim m_\phi \sim 10^{13}$GeV is the Hubble constant at the end of inflation (m_ϕ is the mass of the inflaton), WIMPZILLAS produced gravitationally can have a density today of the order of the critical density. This result is quite robust with respect to the "fine" details of the transition between the inflationary phase and the matter-dominated phase, and independent of the coupling of the WIMPZILLA to any other particle.

Conceptually, gravitational WIMPZILLA production is similar to the inflationary generation of gravitational perturbations that seed the formation of large scale structures. In the usual scenarios, however, the quantum generation of energy density fluctuations from inflation is associated with the inflaton field that dominated the mass density of the universe, and not a generic, sub-dominant scalar field. Another difference is that the usual density fluctuations become larger than the Hubble radius, while most of the WIMPZILLA perturbations remain smaller than the Hubble radius.

There are various inequivalent ways of calculating the particle production due to interaction of a classical gravitational field with the vacuum (see for example [24], [25], and [26]). Here, I use the method of finding the Bogoliubov coefficient for the transformation between positive frequency modes defined at two different times. For $M_X/H_I \lesssim 1$ the results are quite insensitive to the differentiability or the fine details of the time dependence of the scale factor. For $0.04 \lesssim M_X/H_I \lesssim 2$, all the dark matter needed for closure of the universe can be made gravitationally, quite independently of the details of the transition between the inflationary phase and the matter dominated phase.

Start with the canonical quantization of the X field in an action of the form (with metric $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)\left[d\eta^2 - d\mathbf{x}^2\right]$ where η is conformal time)

$$S = \int dt \int d^3x \, \frac{a^3}{2}\left(\dot{X}^2 - \frac{(\nabla X)^2}{a^2} - M_X^2 X^2 - \xi R X^2\right) \tag{10}$$

where R is the Ricci scalar. After transforming to conformal time coordinate, use the mode expansion

$$X(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2} a(\eta)}\left[a_k h_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger h_k^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}}\right], \tag{11}$$

where because the creation and annihilation operators obey the commutator $[a_{k_1}, a_{k_2}^\dagger] = \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)$, the h_ks obey a normalization condition $h_k h_k^{\prime *} - h_k^\prime h_k^* = i$ to satisfy the canonical field commutators (henceforth, all primes on functions of η refer to derivatives with respect to η). The resulting mode equation is

$$h_k''(\eta) + w_k^2(\eta) h_k(\eta) = 0, \tag{12}$$

where

$$w_k^2 = k^2 + M_X^2 a^2 + (6\xi - 1)a''/a \; . \tag{13}$$

The parameter ξ is $1/6$ for conformal coupling and 0 for minimal coupling. From now on, $\xi = 1/6$ for simplicity but without much loss of generality. By a change in variable $\eta \to k/a$, one can rewrite the differential equation such that it depends only on $H(\eta)$, $H'(\eta)/k$, $k/a(\eta)$, and M_X. Hence, the parameters H_I and a_I correspond to the Hubble parameter and the scale factor evaluated at an arbitrary conformal time η_I, which can be taken to be the approximate time at which Xs are produced (*i.e.*, η_I is the conformal time at the end of inflation).

One may then rewrite Eq. 12 as

$$h_{\tilde{k}}''(\tilde{\eta}) + \left(\tilde{k}^2 + \frac{M_X^2}{H_I^2}\tilde{a}^2\right) h_{\tilde{k}}(\tilde{\eta}) = 0 \; , \tag{14}$$

where $\tilde{\eta} = \eta a_I H_I$, $\tilde{a} = a/a_I$, and $\tilde{k} = k/(a_I H_I)$. For simplicity of notation, drop all the tildes. This differential equation can be solved once the boundary conditions are supplied.

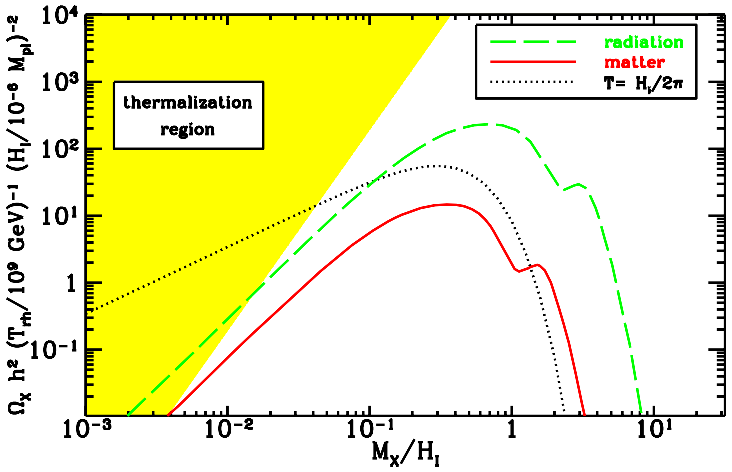

Figure 2: The contribution of gravitationally produced WIMPZILLAS to $\Omega_X h^2$ as a function of M_X/H_I. The shaded area is where thermalization *may* occur if the annihilation cross section is its maximum value. Also shown is the contribution assuming that the WIMPZILLA is present at the end of inflation with a temperature $T = H_I/2\pi$.

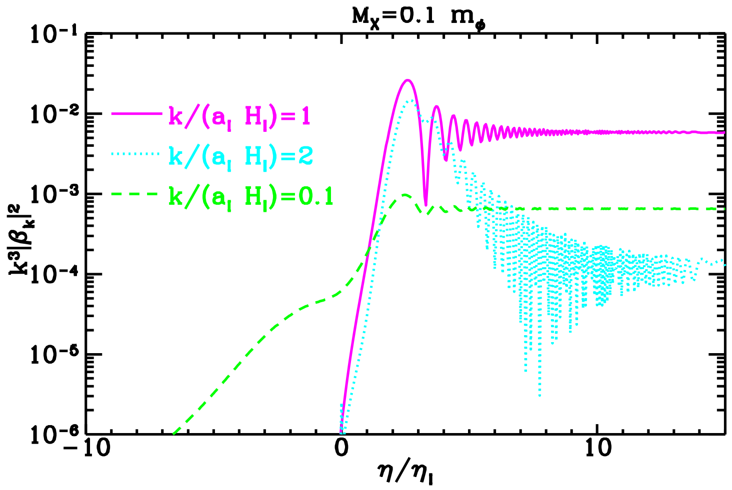

Figure 3: The evolution of the Bogoliubov coefficient with conformal time for several wavenumbers. $\eta = \eta_I$ corresponds to the end of the inflationary era.

The number density of the WIMPZILLAS is found by a Bogoliubov transformation from the vacuum mode solution with the boundary condition at $\eta = \eta_0$ (the initial time at which the vacuum of the universe is determined) into the one with the boundary condition at $\eta = \eta_1$ (any later time at which the particles are no longer being created). η_0 will be taken to be $-\infty$ while η_1 will be taken to be at $+\infty$. Defining the Bogoliubov transformation as $h_k^{\eta_1}(\eta) = \alpha_k h_k^{\eta_0}(\eta) + \beta_k h_k^{*\eta_0}(\eta)$ (the superscripts denote where the boundary condition is set), the energy density of produced particles is

$$\rho_X(\eta_1) = M_X n_X(\eta_1) = M_X H_I^3 \left(\frac{1}{\tilde{a}(\eta_1)}\right)^3 \int_0^\infty \frac{d\tilde{k}}{2\pi^2} \tilde{k}^2 |\beta_{\tilde{k}}|^2, \qquad (15)$$

where one should note that the number operator is defined at η_1 while the quantum state (approximated to be the vacuum state) defined at η_0 does not change in time in the Heisenberg representation.

As one can see from Eq. 14, the input parameter is M_X/H_I. One must also specify the behavior of $a(\eta)$ near the end of inflation. In Fig. 2 (from [23]), I show the resulting values of $\Omega_X h^2$ as a function of M_X/H_I assuming the evolution of the scale factor smoothly interpolates between exponential expansion during inflation and either a matter-dominated universe or radiation-dominated universe. The peak at $M_X/H_I \sim 1$ is similar to the case presented in Ref. [27]. As expected, for large M_X/H_I, the number density falls off faster than any inverse power of M_X/H_I.

Now most of the action occurs around the transition from inflation to the matter-dominated or radiation-dominated universe. This is shown in Fig. 3. Also from Fig. 3 one can see that most of the particles are created with wavenumber of order H_I.

To conclude, there is a significant mass range ($0.1H_I$ to H_I, where $H_I \sim 10^{13}$GeV) for which WIMPZILLAS will have critical density today regardless of the fine details of the transition out of inflation. Because this production mechanism is inherent in the dynamics between the classical gravitational field and a quantum field, it needs no fine tuning of field couplings or any coupling to the inflaton field. However, only if the particles are stable (or sufficiently long lived) will these particles give contribution of the order of critical density.

4.2. Production during Reheating

Another attractive origin for WIMPZILLAS is during the defrosting phase after inflation. It is important to recall that it is not necessary to convert a significant fraction of the available energy into massive particles; in fact, it must be an infinitesimal amount. I now will discuss how particles of mass much greater than T_{RH} may be created in the correct amount after inflation in reheating [28].

In one extreme is the assumption that the vacuum energy of inflation is immediately converted to radiation resulting in a reheat temperature T_{RH}. In this case Ω_X can be calculated by integrating the Boltzmann equation with initial condition $N_X = 0$ at $T = T_{RH}$. One expects the X density to be suppressed by

$\exp(-2M_X/T_{RH})$; indeed, one finds $\Omega_X \sim 1$ for $M_X/T_{RH} \sim 25 + 0.5\ln(m_X^2\langle\sigma|v|\rangle)$, in agreement with previous estimates [20] that for $T_{RH} \sim 10^9$GeV, the WIMPZILLA mass would be about 2.5×10^{10}GeV.

A second (and more plausible) scenario is that reheating is not instantaneous, but is the result of the slow decay of the inflaton field. The simplest way to envision this process is if the comoving energy density in the zero mode of the inflaton decays into normal particles, which then scatter and thermalize to form a thermal background. It is usually assumed that the decay width of this process is the same as the decay width of a free inflaton field.

There are two reasons to suspect that the inflaton decay width might be small. The requisite flatness of the inflaton potential suggests a weak coupling of the inflaton field to other fields since the potential is renormalized by the inflaton coupling to other fields [29]. However, this restriction may be evaded in supersymmetric theories where the nonrenormalization theorem ensures a cancelation between fields and their superpartners. A second reason to suspect weak coupling is that in local supersymmetric theories gravitinos are produced during reheating. Unless reheating is delayed, gravitinos will be overproduced, leading to a large undesired entropy production when they decay after big-bang nucleosynthesis [30].

It is simple to calculate the WIMPZILLA abundance in the slow reheating scenario. It will be important to keep in mind that what is commonly called the reheat temperature, T_{RH}, is not the maximum temperature obtained after inflation. The maximum temperature is, in fact, much larger than T_{RH}. The reheat temperature is best regarded as the temperature below which the universe expands as a radiation-dominated universe, with the scale factor decreasing as $g_*^{-1/3}T^{-1}$. In this regard it has a limited meaning [7,31]. One implication of this is that it is incorrect to assume that the maximum abundance of a massive particle species produced after inflation is suppressed by a factor of $\exp(-M/T_{RH})$.

To estimate WIMPZILLA production in reheating, consider a model universe with three components: inflaton field energy, ρ_ϕ, radiation energy density, ρ_R, and WIMPZILLA energy density, ρ_X. Assume that the decay rate of the inflaton field energy density is Γ_ϕ. Also assume the WIMPZILLA lifetime is longer than any timescale in the problem (in fact it must be longer than the present age of the universe). Finally, assume that the light degrees of freedom are in local thermodynamic equilibrium.

With the above assumptions, the Boltzmann equations describing the redshift and interchange in the energy density among the different components is

$$\begin{aligned}
&\dot\rho_\phi + 3H\rho_\phi + \Gamma_\phi\rho_\phi = 0\\
&\dot\rho_R + 4H\rho_R - \Gamma_\phi\rho_\phi - \frac{\langle\sigma|v|\rangle}{m_X}\left[\rho_X^2 - \left(\rho_X^{EQ}\right)^2\right] = 0\\
&\dot\rho_X + 3H\rho_X + \frac{\langle\sigma|v|\rangle}{m_X}\left[\rho_X^2 - \left(\rho_X^{EQ}\right)^2\right] = 0\ , \qquad (16)
\end{aligned}$$

where dot denotes time derivative. As already mentioned, $\langle\sigma|v|\rangle$ is the thermal average of the X annihilation cross section times the Møller flux factor. The equilibrium energy density for the X particles, ρ_X^{EQ}, is determined by the radiation temperature, $T = (30\rho_R/\pi^2 g_*)^{1/4}$.

It is useful to introduce two dimensionless constants, α_ϕ and α_X, defined in terms of Γ_ϕ and $\langle\sigma|v|\rangle$ as

$$\Gamma_\phi = \alpha_\phi M_\phi \qquad \langle\sigma|v|\rangle = \alpha_X M_X^{-2} \ . \tag{17}$$

For a reheat temperature much smaller than M_ϕ, Γ_ϕ must be small. From Eq. (5), the reheat temperature in terms of α_X and M_X is $T_{RH} \simeq \alpha_\phi^{1/2}\sqrt{M_\phi M_{Pl}}$. For $M_\phi = 10^{13}$GeV, α_ϕ must be smaller than of order 10^{-13}. On the other hand, α_X may be as large as of order unity, or it may be small also.

It is also convenient to work with dimensionless quantities that can absorb the effect of expansion of the universe. This may be accomplished with the definitions

$$\Phi \equiv \rho_\phi M_\phi^{-1} a^3 \ ; \quad R \equiv \rho_R a^4 \ ; \quad X \equiv \rho_X M_X^{-1} a^3 \ . \tag{18}$$

It is also convenient to use the scale factor, rather than time, for the independent variable, so one may define a variable $x = aM_\phi$. With this choice the system of equations can be written as (prime denotes d/dx)

$$\begin{aligned}
\Phi' &= -c_1 \, \frac{x}{\sqrt{\Phi x + R}} \, \Phi \\
R' &= c_1 \, \frac{x^2}{\sqrt{\Phi x + R}} \, \Phi \, + c_2 \, \frac{x^{-1}}{\sqrt{\Phi x + R}} \, \left(X^2 - X_{EQ}^2\right) \\
X' &= -c_3 \, \frac{x^{-2}}{\sqrt{\Phi x + R}} \, \left(X^2 - X_{EQ}^2\right) \ .
\end{aligned} \tag{19}$$

The constants c_1, c_2, and c_3 are given by

$$c_1 = \sqrt{\frac{3}{8\pi}\frac{M_{Pl}}{M_\phi}}\alpha_\phi \qquad c_2 = c_1 \frac{M_\phi}{M_X}\frac{\alpha_X}{\alpha_\phi} \qquad c_3 = c_2 \frac{M_\phi}{M_X} \ . \tag{20}$$

X_{EQ} is the equilibrium value of X, given in terms of the temperature T as (assuming a single degree of freedom for the X species)

$$X_{EQ} = \frac{M_X^3}{M_\phi^3}\left(\frac{1}{2\pi}\right)^{3/2} x^3 \left(\frac{T}{M_X}\right)^{3/2} \exp(-M_X/T) \ . \tag{21}$$

The temperature depends upon R and g_*, the effective number of degrees of freedom in the radiation:

$$\frac{T}{M_X} = \left(\frac{30}{g_*\pi^2}\right)^{1/4} \frac{M_\phi}{M_X}\frac{R^{1/4}}{x} \ . \tag{22}$$

It is straightforward to solve the system of equations in Eq. (19) with initial conditions at $x = x_I$ of $R(x_I) = X(x_I) = 0$ and $\Phi(x_I) = \Phi_I$. It is convenient to express $\rho_\phi(x = x_I)$ in terms of the expansion rate at x_I, which leads to

$$\Phi_I = \frac{3}{8\pi}\frac{M_{Pl}^2}{M_\phi^2}\frac{H_I^2}{M_\phi^2}\, x_I^3 \, . \tag{23}$$

The numerical value of x_I is irrelevant.

Before numerically solving the system of equations, it is useful to consider the early-time solution for R. Here, early times means $H \gg \Gamma_\phi$, *i.e.,* before a significant fraction of the comoving coherent energy density is converted to radiation. At early times $\Phi \simeq \Phi_I$, and $R \simeq X \simeq 0$, so the equation for R' becomes $R' = c_1 x^{3/2}\Phi_I^{1/2}$. Thus, the early time solution for R is simple to obtain:

$$R \simeq \frac{2}{5}c_1\left(x^{5/2} - x_I^{5/2}\right)\Phi_I^{1/2} \qquad (H \gg \Gamma_\phi) \, . \tag{24}$$

Now express T in terms of R to yield the early-time solution for T:

$$\begin{aligned}\frac{T}{M_\phi} \quad &\simeq \quad \left(\frac{12}{\pi^2 g_*}\right)^{1/4} c_1^{1/4}\left(\frac{\Phi_I}{x_I^3}\right)^{1/8} \\ &\quad \times\left[\left(\frac{x}{x_I}\right)^{-3/2} - \left(\frac{x}{x_I}\right)^{-4}\right]^{1/4} \qquad (H \gg \Gamma_\phi) \, .\end{aligned} \tag{25}$$

Thus, T has a maximum value of

$$\begin{aligned}\frac{T_{MAX}}{M_\phi} \quad &= \quad 0.77\left(\frac{12}{\pi^2 g_*}\right)^{1/4} c_1^{1/4}\left(\frac{\Phi_I}{x_I^3}\right)^{1/8} \\ &= \quad 0.77\alpha_\phi^{1/4}\left(\frac{9}{2\pi^3 g_*}\right)^{1/4}\left(\frac{M_{Pl}^2 H_I}{M_\phi^3}\right)^{1/4} \quad ,\end{aligned} \tag{26}$$

which is obtained at $x/x_I = (8/3)^{2/5} = 1.48$. It is also possible to express α_ϕ in terms of T_{RH} and obtain

$$\frac{T_{MAX}}{T_{RH}} = 0.77\left(\frac{9}{5\pi^3 g_*}\right)^{1/8}\left(\frac{H_I M_{Pl}}{T_{RH}^2}\right)^{1/4} \quad . \tag{27}$$

For an illustration, in the simplest model of chaotic inflation $H_I \sim M_\phi$ with $M_\phi \simeq 10^{13}$GeV, which leads to $T_{MAX}/T_{RH} \sim 10^3(200/g_*)^{1/8}$ for $T_{RH} = 10^9$GeV.

We can see from Eq. (24) that for $x/x_I > 1$, in the early-time regime T scales as $a^{-3/8}$, which implies that entropy is created in the early-time regime [31)]. So if one is producing a massive particle during reheating it is necessary to take into account the fact that the maximum temperature is greater than T_{RH}, and that during the early-time evolution, $T \propto a^{-3/8}$.

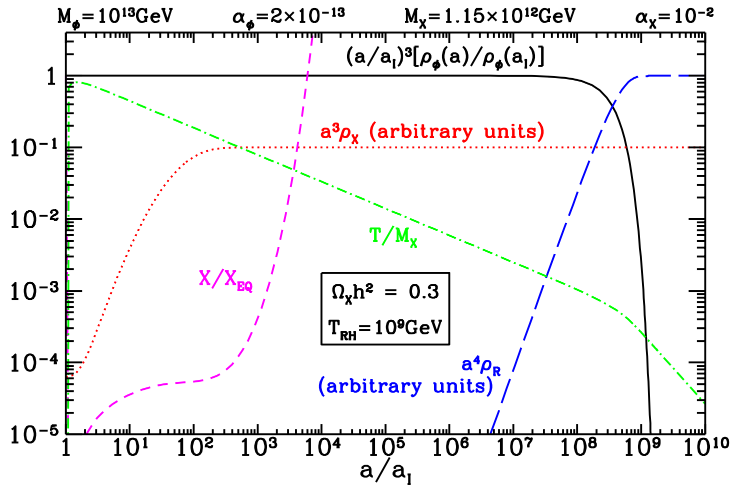

Figure 4: The evolution of energy densities and T/M_X as a function of the scale factor. Also shown is X/X_{EQ}.

An example of a numerical evaluation of the complete system in Eq. (19) is shown in Fig. 4 (from [28)]). The model parameters chosen were $M_\phi = 10^{13}$GeV, $\alpha_\phi = 2 \times 10^{-13}$, $M_X = 1.15 \times 10^{12}$GeV, $\alpha_X = 10^{-2}$, and $g_* = 200$. The expansion rate at the beginning of the coherent oscillation period was chosen to be $H_I = M_\phi$. These parameters result in $T_{RH} = 10^9$GeV and $\Omega_X h^2 = 0.3$.

Figure 4 serves to illustrate several aspects of the problem. Just as expected, the comoving energy density of ϕ (*i.e.*, $a^3\rho_\phi$) remains roughly constant until $\Gamma_\phi \simeq H$, which for the chosen model parameters occurs around $a/a_I \simeq 5 \times 10^8$. But of course, that does not mean that the temperature is zero. Notice that the temperature peaks well before "reheating." The maximum temperature, $T_{MAX} = 10^{12}$GeV, is reached at a/a_I slightly larger than unity (in fact at $a/a_I = 1.48$ as expected), while the reheat temperature, $T_{RH} = 10^9$GeV, occurs much later, around $a/a_I \sim 10^8$. Note that $T_{MAX} \simeq 10^3 T_{RH}$ in agreement with Eq. (27).

From the figure it is clear that $X \ll X_{EQ}$ at the epoch of freeze out of the comoving X number density, which occurs around $a/a_I \simeq 10^2$. The rapid rise of the ratio after freeze out is simply a reflection of the fact that X is constant while X_{EQ} decreases exponentially.

A close examination of the behavior of T shows that after the sharp initial rise of the temperature, the temperature decreases as $a^{-3/8}$ [as follows from Eq. (25)] until $H \simeq \Gamma_\phi$, and thereafter $T \propto a^{-1}$ as expected for the radiation-dominated era.

For the choices of M_ϕ, α_ϕ, g_*, and α_X used for the model illustrated in Fig. 4, $\Omega_X h^2 = 0.3$ for $M_X = 1.15 \times 10^{12}$GeV, in excellent agreement with the mass predicted by using an analytic estimate for the result [28]

$$\Omega_X h^2 = M_X^2 \langle\sigma|v|\rangle \left(\frac{g_*}{200}\right)^{-3/2} \left(\frac{2000 T_{RH}}{M_X}\right)^7 . \tag{28}$$

Here again, the results have also important implications for the conjecture that ultra-high cosmic rays, above the Greisen-Zatsepin-Kuzmin cut-off of the cosmic ray spectrum, may be produced in decays of superheavy long-living particles [19,20,21,32]. In order to produce cosmic rays of energies larger than about 10^{13} GeV, the mass of the X-particles must be very large, $M_X \gtrsim 10^{13}$ GeV and their lifetime τ_X cannot be much smaller than the age of the Universe, $\tau_X \gtrsim 10^{10}$ yr. With the smallest value of the lifetime, the observed flux of ultra-high energy cosmic rays will be reproduced with a rather low density of X-particles, $\Omega_X \sim 10^{-12}$. It has been suggested that X-particles can be produced in the right amount by usual collisions and decay processes taking place during the reheating stage after inflation if the reheat temperature never exceeded M_X [32]. Again, assuming naively that that the maximum number density of a massive particle species X produced after inflation is suppressed by a factor of $(M_X/T_{RH})^{3/2}\exp(-M_X/T_{RH})$ with respect to the photon number density, one concludes that the reheat temperature T_{RH} should be in the range 10^{11} to 10^{15}GeV [20]. This is a rather high value and leads to the gravitino problem in generic supersymmetric models. This is one reason alternative production mechanisms of these superheavy X-particles have been proposed [23,33,34]. However, our analysis show that the situation is much more promising. Making use of Eq. (28), the right amount of X-particles to explain the observed ultra-high energy cosmic rays is produced for

$$\left(\frac{T_{RH}}{10^{10}\ \mathrm{GeV}}\right) \simeq \left(\frac{g_*}{200}\right)^{3/14} \left(\frac{M_X}{10^{15}\ \mathrm{GeV}}\right), \tag{29}$$

where it has been assumed that $\langle\sigma|v|\rangle \sim M_X^{-2}$. Therefore, particles as massive as 10^{15} GeV may be generated during the reheating stage in abundances large enough to explain the ultra-high energy cosmic rays even if the reheat temperature satisfies the gravitino bound.

4.3. Production During Preheating

Another way to produce WIMPZILLAS after inflation is in a preliminary stage of reheating called "preheating" [22], where nonlinear quantum effects may lead to an extremely effective dissipational dynamics and explosive particle production.

Particles can be created in a broad parametric resonance with a fraction of the energy stored in the form of coherent inflaton oscillations at the end of inflation released after only a dozen oscillation periods. A crucial observation for our discussion is that particles with mass up to 10^{15} GeV may be created during preheating [33,35,36], and that their distribution is nonthermal. If these particles are stable, they may be good candidates for WIMPZILLAS [37].

The main ingredient of the preheating scenario introduced in the early 1990s is the nonperturbative resonant transfer of energy to particles induced by the coherently oscillating inflaton fields. It was realized that this nonperturbative mechanism can be much more efficient than the usual perturbative mechanism for certain parameter ranges of the theory [22)]. The basic picture can be seen as follows. Suppose there is a scalar field X with a coupling $g^2\phi^2X^2$ where ϕ is a homogeneous classical inflaton field. The mode equation for X field then can be written in terms of a redefined variable $\chi_k \equiv X_k a^{3/2}$ as

$$\ddot{\chi}_k(t) + [A + 2q\cos(2t)]\chi_k(t) = 0 \tag{30}$$

where A depends on the energy of the particle and q depends on the inflaton field oscillation amplitude. When A and q are constants, this equation is usually referred to as the Mathieu equation which exhibits resonant mode instability for certain values of A and q. In an expanding universe, A and q will vary in time, but if they vary slowly compared to the frequency of oscillations, the effects of resonance will remain. If the mode occupation number for the X particles is large, the number density per mode of the X particles will be proportional to $|\chi_k|^2$. If A and q have the appropriate values for resonance, χ_k will grow exponentially in time, and hence the number density will attain an exponential enhancement above the usual perturbative decay. This period of enhanced rate of energy transfer has been called preheating primarily because the particles that are produced during this period have yet to achieve thermal equilibrium.

This resonant amplification leads to an efficient transfer of energy from the inflaton to other particles which may have stronger coupling to other particles than the inflaton, thereby speeding up the reheating process and leading to a higher reheating temperature than in the usual scenario. Another interesting feature is that particles of mass larger than the inflaton mass can be produced through this coherent resonant effect. This has been exploited to construct a baryogenesis scenario [35)] in which the baryon number violating bosons with masses larger than the inflaton mass are created through the resonance mechanism. A natural variation on this idea is to produce WIMPZILLAS by the same resonance mechanism.

Interestingly enough, what was found [37)] is that in the context of a slow-roll inflation with the potential $V(\phi) = m_\phi^2\phi^2/2$ with the inflaton coupling of $g^2\phi^2X^2/2$, the resonance phenomenon is mostly irrelevant to WIMPZILLA production because too many particles would be produced if the resonance is effective. For the tiny amount of energy conversion needed for WIMPZILLA production, the coupling g^2 must be small enough (for a fixed M_X) such that the motion of the inflaton field at the transition out of the inflationary phase generates just enough nonadiabaticity in the mode frequency to produce WIMPZILLAS . The rest of the oscillations, damped by the expansion of the universe, will not contribute significantly to WIMPZILLA production as in the resonant case. In other words, the quasi-periodicity necessary for a true resonance phenomenon is not present in the case when only an extremely tiny fraction of the energy density is converted into WIMPZILLAS. Of course, if the energy scales are lowered such that a fair fraction of the energy density can be converted to WIMPZILLAS without overclosing the universe, this argument may not apply.

The main finding of a detailed treatment [37)] is that WIMPZILLAS with a mass as large as $10^3 H_I$, where H_I is the value of the Hubble expansion rate at the end of inflation, can be produced in sufficient abundance to be cosmologically significant today.

If the WIMPZILLA is coupled to the inflaton ϕ by a term $g^2\phi^2X^2/2$, then the mode equation in Eq. 13 is now changed to

$$\omega_k^2 + k^2 + \left(M_X^2 + g^2\phi^2\right) a^2 \ , \tag{31}$$

again taking $\xi = 1/6$.

The procedure to calculate the WIMPZILLA density is the same as in Section . Now, in addition to the parameter M_X/H_I, there is another parameter gM_{Pl}/H_I. Now in large-field models $H_I \sim 10^{13}$GeV, so M_{Pl}/H_I might be as large as 10^6. The choice of $g = 10^{-3}$ would yield $gM_{Pl}/H_I = 10^3$.

Fig. 5 (from [37)]) shows the dependence of the WIMPZILLA density upon M_X/H_I for the particular choice $gM_{Pl}/H_I = 10^6$. This would correspond to $g \sim 1$ in large-field inflation models where $M_{Pl}/H_I = 10^6$, about the largest possible value. Note that $\Omega_X \sim 1$ obtains for $M_X/H_I \approx 10^3$. The dashed and dotted curves are two analytic approximations discussed in [37)], while the solid curve is the numerical result. The approximations are in very good agreement with the numerical results.

Fig. 6 (also from [37)]) shows the dependence of the WIMPZILLA density upon gM_{Pl}/H_I. For this graph M_X/H_I was chosen to be unity. This figure illustrates the fact that the dependence of $\Omega_X h^2$ on gM_{Pl}/H_I is not monotonic. For details, see [37)].

4.4. Production in Bubble Collisions

WIMPZILLAS may also be produced [34)] in theories where inflation is completed by a first-order phase transition [38)], in which the universe exits from a false-vacuum state by bubble nucleation [39)]. When bubbles of true vacuum form, the energy of the false vacuum is entirely transformed into potential energy in the bubble walls. As the bubbles expand, more and more of their energy becomes kinetic as the walls become highly relativistic.

In bubble collisions the walls oscillate through each other [40)] and their kinetic energy is dispersed into low-energy scalar waves [40,41)]. We are interested in the potential energy of the walls, $M_P = 4\pi\eta R^2$, where η is the energy per unit area of a bubble wall of radius R. The bubble walls can be visualized as a coherent state of inflaton particles, so the typical energy E of the products of their decays is simply the inverse thickness of the wall, $E \sim \Delta^{-1}$. If the bubble walls are highly relativistic when they

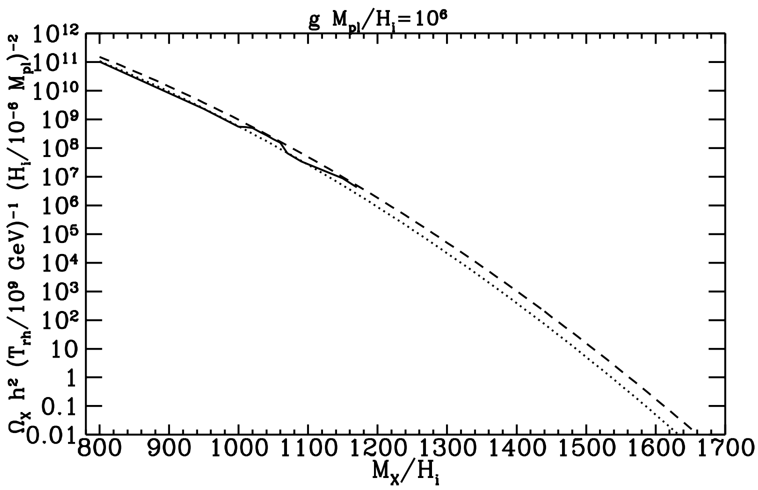

Figure 5: A graph of $\Omega_X h^2$ versus M_X/H_I for $gM_{Pl}/H_I = 10^6$. The solid curve is a numerical result, while the dashed and dotted curves are analytic approximations.

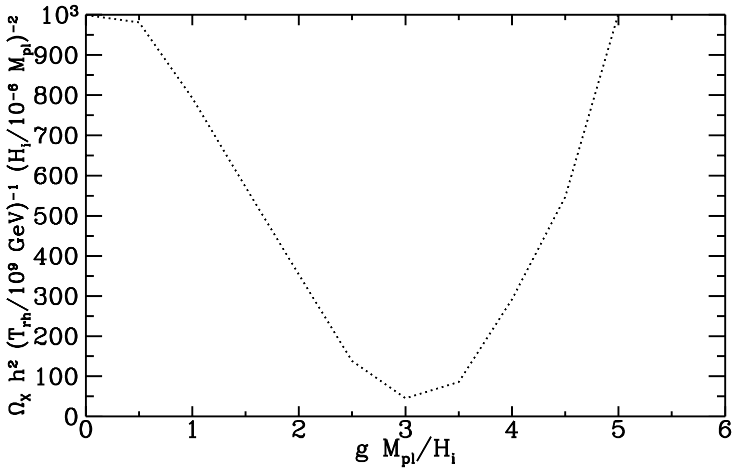

Figure 6: An illustration of the nonmonotonic behavior of the particle density produced with the variation of the coupling constant. The value of M_X/H_I is set to unity.

collide, there is the possibility of quantum production of nonthermal particles with mass well above the mass of the inflaton field, up to energy $\Delta^{-1} = \gamma M_\phi$, with γ the relativistic Lorentz factor.

Suppose for illustration that the WIMPZILLA is a fermion coupled to the inflaton field by a Yukawa coupling $g\phi\overline{X}X$. One can treat ϕ (the bubbles or walls) as a classical, external field and the WIMPZILLA as a quantum field in the presence of this source. The number of WIMPZILLAS created in the collisions from the wall potential energy is $N_X \sim f_X M_P/M_X$, where f_X parametrizes the fraction of the primary decay products in WIMPZILLAS. The fraction f_X will depend in general on the masses and the couplings of a particular theory in question. For the Yukawa coupling g, it is $f_X \simeq g^2 \ln\left(\gamma M_\phi/2M_X\right)$ [41,42]. WIMPZILLAS may be produced in bubble collisions out of equilibrium and never attain chemical equilibrium. Even with T_{RH} as low as 100 GeV, the present WIMPZILLA abundance would be $\Omega_X \sim 1$ if $g \sim 10^{-5}\alpha^{1/2}$. Here $\alpha^{-1} \ll 1$ is the fraction of the bubble energy at nucleation in the form of potential energy at the time of collision. This simple analysis indicates that the correct magnitude for the abundance of WIMPZILLAS may be naturally obtained in the process of reheating in theories where inflation is terminated by bubble nucleation.

5. Conclusions

In this talk I have pointed out several ways to generate nonthermal dark matter. All of the methods can result in dark matter much more massive than the feeble little weak-scale mass thermal relics. The nonthermal dark matter may be as massive as the GUT scale, truly in the WIMPZILLA range.

The mass scale of the WIMPZILLAS is determined by the mass scale of inflation, more exactly, the expansion rate of the universe at the end of inflation. For large-field inflation models, that mass scale is of order 10^{13}GeV. For small-field inflation models, it may be less, perhaps much less.

The mass scale of inflation may one day be measured! In addition to scalar density perturbations, tensor perturbations are produced in inflation. The tensor perturbations are directly proportional to the expansion rate during inflation, so determination of a tensor contribution to cosmic background radiation temperature fluctuations would give the value of the expansion rate of the universe during inflation and set the scale for the mass of the WIMPZILLA.

Undoubtedly, other methods for WIMPZILLA production will be developed. But perhaps even with the present scenarios one should start to investigate methods for WIMPZILLA detection. While wimpy WIMPS must be color singlets and electrically neutral, WIMPZILLAS may be endowed with color and electric charge. This should open new avenues for detection and exclusion of WIMPZILLAS.

The lesson of the talk is illustrated in Fig. 7. WIMPZILLAS may surprise and be the dark matter, and we may learn that size does matter!

Figure 7: Dark matter may be much more massive than usually assumed, much more massive than wimpy WIMPS, perhaps in the WIMPZILLA class.

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7. References

1 B. Fuchs, "DARK98 Proceedings of the Second International Conference on Dark Matter in Astro and Particle Physics," eds. H V Klapdor-Kleingrothaus and L. Baudis, (Institute of Physics Publishing, Bristol and Philadelphia, 1999).

2 G. Evrard, "DARK98 Proceedings."

3 See, *e.g.,* A. Dekel, in Proceedings of the 3rd ESO-VLT Workshop on "Galaxy Scaling Relations: Origins, Evolution and Applications," ed. L. da Costa (Springer).

4 See, *e.g.,* J. A. Tyson, G. P. Kochanski, and I. P.. Dell'Antonio, astro-ph/9801193.

5 S. Sarkar, "DARK98 Proceedings."

6 G. Böerner, "DARK98 Proceedings."

7 E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley, Menlo Park, Ca., 1990).

8 K. Griest and M. Kamionkowski, Phys. Rev. Lett. **64**, 615 (1990).

9 B. Cabrera, "DARK98 Proceedings."

10 I. Liubarsky, "DARK98 Proceedings."
11 R. Bernabei, "DARK98 Proceedings."
12 Y. Ramachers, "DARK98 Proceedings."
13 L. Baudis, "DARK98 Proceedings."
14 J. Ellis, J.L. Lopez and D.V. Nanopoulos, Phys. Lett. **B247**, 257 (1990).
15 See, for instance, G. F. Giudice and R. Rattazzi, hep-ph/9801271.
16 S. Raby, Phys. Rev. D **56**, (1997).
17 K. Hamaguchi, Y. Nomura and T. Yanagida, hep-ph/9805346.
18 K. Benakli, J. Ellis and D.V. Nanopoulos, hep-ph/9803333.
19 D. V. Nanopoulos, "DARK98 Proceedings."
20 V. A. Kuzmin and V. A. Rubakov, Phys. Atom. Nucl. **61**, 1028 (1998).
21 M. Birkel and S. Sarkar, hep-ph/9804285.
22 L. A. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); S. Yu. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **77**, 219 (1996); Phys. Lett. **B390**, 80 (1997); Phys. Rev. Lett. **79**, 1607 (1997); Phys. Rev. D **56**, 653 (1997); G. W. Anderson, A. Linde and A. Riotto, Phys. Rev. Lett. **77**, 3716 (1996); see L. Kofman, *The origin of matter in the Universe: reheating after inflation*, astro-ph/9605155, UH-IFA-96-28 preprint, 16pp., to appear in *Relativistic Astrophysics: A Conference in Honor of Igor Novikov's 60th Birthday*, eds. B. Jones and D. Markovic for a more recent review and a collection of references; see also L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56**, 3258 (1997); J. Traschen and R. Brandenberger, Phys. Rev. D **42**, 2491 (1990); Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D **51**, 5438 (1995).
23 D. J. H. Chung, Edward W. Kolb, and A. Riotto, hep-ph/9802238.
24 S. Fulling, Gen. Rel. and Grav. **10**, 807 (1979).
25 N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982).
26 D. M. Chitre and J. B. Hartle, Phys. Rev. D **16**, 251 (1977); D. J. Raine and C. P. Winlove, Phys. Rev. D **12**, 946 (1975); G. Schaefer and H. Dehnen, Astron. Astrophys. **54**, 823 (1977).
27 N. D. Birrell and P. C. W. Davies, J. Phys. A:Math. Gen. **13**, 2109 (1980).
28 D. J. H. Chung, E. W. Kolb, and A. Riotto, hep-ph/9809453.
29 D. H. Lyth and A. Riotto, hep-ph/9807278.
30 J. Ellis, J. Kim and D. V. Nanopoulos, Phys. Lett. **B145**, 181 (1984); L. M. Krauss, Nucl. Phys. **B227**, 556 (1983); M. Yu. Khlopov and A. D. Linde, Phys. Lett. **138B**, 265 (1984); J. Ellis, D. V. Nanopoulos, and S. Sarkar, Nucl. Phys. **B461**, 597 (1996).
31 R. J. Scherrer and M. S. Turner, Phys. Rev. **D31**, 681 (1985).
32 V. Berezinsky, M. Kachelriess and A. Vilenkin, Phys. Rev. Lett. **79**, 4302 (1997).

33 V. Kuzmin and I. I. Tkachev, hep-ph/9802304.
34 D. J. H. Chung, E. W. Kolb and A. Riotto, hep-ph/9805473.
35 E. W. Kolb, A. D. Linde and A. Riotto, Phys. Rev. Lett. **77**, 4290 (1996).
36 E. W. Kolb, A. Riotto and I. I. Tkachev, *Phys. Lett.* **B423**, 348 (1998).
37 D. J. H. Chung, hep-ph/9809489.
38 D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989).
39 A. H. Guth, Phys. Rev. D **23**, 347 (1981).
40 S. W. Hawking, I. G. Moss and J. M. Stewart, Phys. Rev. D **26**, 2681 (1982).
41 R. Watkins and L. Widrow, Nucl. Phys. **B374**, 446 (1992).
42 A. Masiero and A. Riotto, Phys. Lett. **B289**, 73 (1992).