NEUTRINO INDUCED CHARGED LEPTON OSCILLATIONS AND NEUTRINOS FROM THE CORE OF THE EARTH

A. WIDOM[1], E. SASSAROLI[2] and Y. SRIVASTAVA[1,3]

1. Physics Department, Northeastern University, Boston, Mass USA

2. Laboratory of Nuclear Science, Massachusetts Institute of Technology Cambridge, Mass USA.

3. Physics Department-University of Perugia and INFN-Perugia via Pascoli, I-06100 Perugia, Italy.

ABSTRACT

Flavour mixed neutrinos cause oscillations in the decays of muons. The KARMEN anomaly is interpreted here as an experimental observation of this phenomenon. Also, the $(g - 2)$ measurements of the muon should inherit some neutrino induced oscillations. We further discuss the notion that the Earth's core may be a substantial source of (anti-) neutrinos. Observations of terrestrial neutrinos is suggested as a uniqe tool to probe the topography of the core of the Earth.

1. Introduction

This talk is a concise review of several results about oscillations induced by mass mixed states [1),2)], neutrinos [3)] and effects caused by neutrino mixings [4)−7)]. In sec 2, we discuss the basic notion of induced oscillations to be found in our ref [1)−7)] through the example of muon momentum distribution produced from pion decays. In sec. 3, we use the above to understand the "anomalous" oscillations found in the KARMEN data. In sec. 4, we discuss the $(g - 2)$ experiment on the muon and show how its decay probability is modified by neutrino mixings. In sec. 5, we observe that recent findings in geophysics and chemistry have quashed the long held view that the core of the Earth has very little radioactivity. Our proposal is that a measurement of the (anti-) neutrino flux from the core of the Earth may be a unique probe of its composition and the nuclear reactions occurring there.

2. Induced Oscillations

The notion of induced oscillations following from exact $4-$ momentum conservation has been extensively developed in several of our papers listed below[1)−7)]. The principal result to be later used in sec. 3 and 4, may be stated for pion decay very simply. Let a π^+ with $4-$momentum P^λ $(P^2 = M_\pi^2)$ decay into a μ^+ and ν_μ. If ν_μ is a linear superposition of 3 -different mass eigenstates, then $4-$momentum conservation for each "channel" reads

$$P^\lambda(pion) = p_1^\lambda(muon) + k_1^\lambda(\nu\ of\ mass\ m_1);\ (k_1^2 = m_1^2), \tag{2.1a}$$

$$P^\lambda(pion) = p_2^\lambda(muon) + k_2^\lambda(\nu\ of\ mass\ m_2);\ (k_2^2\ =\ m_2^2), \qquad (2.1b)$$

$$P^\lambda(pion) = p_3^\lambda(muon) + k_3^\lambda(\nu\ of\ mass\ m_3);\ (k_3^2\ =\ m_3^2), \qquad (2.1c)$$

so that $p_i^\lambda\ \neq\ p_2^\lambda\ \neq\ p_3^\lambda$, even though $p_1^2\ =\ p_2^2\ =\ p_3^2\ =\ M_\mu^2$. The muon is therefore put in a linear superposition of different momenta (albeit with the same mass M_μ). This leads to an interference pattern in the decay distribution of the muon.

Writing the mixing matrix for the neutrino states as

$$|\nu_\mu >= \sum_a \mathcal{R}_{\mu a}|\nu_a >, \qquad (2.2)$$

the muon survival probabilty (at the secondary space-time vertex x) becomes

$$P(x) \approx e^{-\Gamma\sqrt{x^2}} \sum_{a,b} |\mathcal{R}_{\mu a}|^2 |\mathcal{R}_{\mu b}|^2 cos[(p_1 - p_2)\cdot x]. \qquad (2.3)$$

Graphs showing the resulting interference pattern in the muon decay distribution may be found in ref.(7) for various values of neutrino mixing angles and mass differences.

3. KARMEN Anomaly

The KARMEN group [8),9)] has measured the spectrum of electron neutrinos from μ^+ decaying at rest

$$\mu^+ \to e^+\ +\ \bar{\nu}_\mu\ +\ \nu_e,$$

and found an (anomalous) oscillatory counting rate. We shall apply the interference results derived above to understand this KARMEN anomaly.

As is usual in the standard model, let us view the μ^+ decay as a two step process

$$1.\ \mu^+ \to W_{eff}^+\ +\ \bar{\nu}_\mu, \quad and\ \ 2.\ \ W_{eff}^+\ \to e^+\ +\ \nu_e.$$

The virtual effective W_{eff}^+ (far off the W mass shell) wave function propagates as

$$W_{eff,\alpha}^+(x) = \int \mathcal{D}_{\alpha\beta}(M_W; x-y) J^{+,\beta}(y)(d^4y), \qquad (3.1)$$

where $\mathcal{D}_{\alpha\beta}(M_W; x-y)$ is the W- propagator and $J^{+,\beta}(y)$ is the charged current. For the energies and momenta involved in muon decays, large M_W limit provides an excellent limit, so that

$$W_{eff,\alpha}^+(x) \approx g_W(\frac{1}{M_W^2})\bar{\Psi}(x)\gamma_\alpha(1-\gamma_5)N_\mu(x), \qquad (3.2)$$

where the muon wave function $\bar{\Psi}$ is the one-particle to vacuum matrix element of the muon field operator $\bar{\psi}$

$$\bar{\Psi} =< 0|\bar{\psi}|\mu^+ > . \qquad (3.3)$$

Similarly, N_μ is the vacuum to one-particle matrix element of the muon neutrino field operator

$$N_\mu(x) =< \bar{\nu}_\mu|\nu_\mu(x)|0> . \tag{3.4}$$

For simplicity, let us consider the case of only two flavours. Then, the state

$$< \bar{\nu}_\mu| = cos\phi < \nu_1| + sin\phi < \nu_2|, \tag{3.5}$$

and the muon neutrino field operator ν_μ reads

$$\nu_\mu(x) = cos\phi\ \nu_1(x) + sin\phi\ \nu_2(x), \tag{3.6}$$

so that

$$N_\mu(x) = cos^2\phi\ N_1(x) + sin^2\phi\ N_2(x), \tag{3.7}$$

the muon neutrino wave function has an oscillatory form. As in the previous section, this throws the "recoiling particle" (here W^+_{eff}) into an oscillatory superposition of amplitudes

$$W^+_{eff}(x) = cos^2\phi\ W^+_1(x) + sin^2\phi\ W^+_2(x), \tag{3.8}$$

with

$$W^+_{i\alpha}(x) \approx g_W(\frac{1}{M^2_W})\bar{\Psi}(x)\gamma_\alpha(1-\gamma_5)N_\imath(x) \quad (i=1,2). \tag{3.9}$$

Let p_λ be the initial 4−momentum of the muon and $k_{i\lambda}$ the neutrino 4−momenta with $k_i^2 = m_i^2$ $(i = 1,2)$. There are then two different 4−momenta of the W^+

$$P^\lambda_{eff,i} = (p-k_i)^\lambda \quad (i=1,2). \tag{3.10}$$

For a fixed effective mass M_{effW}, since

$$M^2_{effW} = P^2_{effW_1} = P^2_{effW_2}, \tag{3.11}$$

we have

$$2(k_2-k_1)\cdot p = (m_1^2 - m_2^2). \tag{3.12}$$

In the muon rest frame, the two possible energies ϵ_1 and ϵ_2 have a Bohr transition frequency

$$\omega = |\epsilon_2 \ - \ \epsilon_1| = \frac{(m_2^2-m_1^2)}{2M_\mu}. \tag{3.13}$$

The virtual W^+_{eff} wave function is a product of a neutrino wave function $N_\mu(x)$, which has the usual neutrino oscillation amplitude $cos^2\phi e^{-i\epsilon_1 t} \ + \ sin^2\phi e^{-i\epsilon_2 t}$ and, a muon wave function $\bar{\Psi}(x)$ which has the muon decay probabilty amplitude $e^{-\Gamma t/2}$. Thus, W^+ survival probabilty inherits both and satisfies the differential equation

$$-(\frac{dP}{dt}) = Ze^{-\Gamma t}[1 - sin^2(2\phi)sin^2(\omega t/2)], \tag{3.14}$$

where the constant Z is found by normalization:

$$-\int_o^\infty (\frac{dP}{dt})dt = P(0) - P(\infty) = P(0) = 1. \tag{3.15}$$

In ref.(5), we performed a detailed likelihood analysis of the KARMEN data on ν_e, measured in time bins of $\Delta t_i = 0.5\ sec.$ The mean number of detected ν_e from muon decay (in each bin) is given by

$$\mu_i(\omega, \phi) = -\mathcal{N}[\frac{dP(t_i;\omega,\phi)}{dt}](\Delta t_i), \tag{3.16}$$

where $\mathcal{N} = \Sigma_i \mu_i$ is the total number of events. If events in each bin obey Poisson statistics, then the probability distribution for a counting sequence of n_i events in the $i\ th$ bin is given by

$$\mathcal{P}[n;\omega,\phi] = \Pi_i(\frac{e^{-\mu_i(\omega,\phi)}\mu_i(\omega,\phi)^n}{\Gamma(n_i+1)}). \tag{3.17}$$

The likelihood function for the data is then defined as

$$\Lambda(\omega;\phi) = K\mathcal{P}[n_{data};\omega;\phi], \tag{3.18}$$

where K is an arbitrary constant.

Under the best of circumstances, Λ should exhibit a single maximum in the theoretical parameter space $(\omega;\phi)$. Our likelihood function, shown in Fig. (1), has a nice gaussian peak in both ω as well as ϕ.

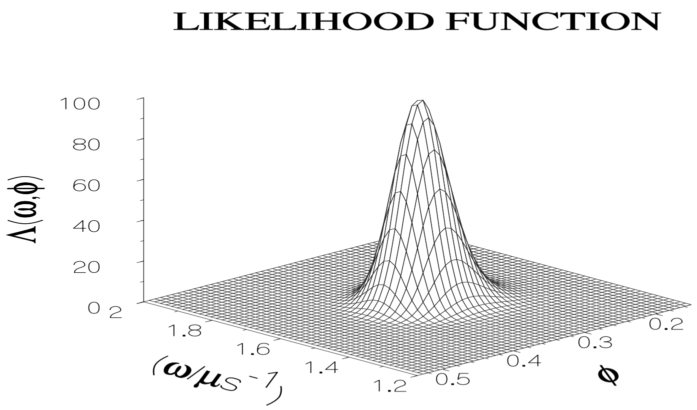

Fig. 1. The likelihood function Λ using the KARMEN data as a function of ω and ϕ

In Fig.(2), we show how well the event rate versus time is reproduced for these parameters.

The best values for the mass difference and the mixing angle are

$$\Delta m^2 = (0.22\ \pm\ .02)\ eV^2\ and\ sin^2(2\phi) = (0.34\ \pm\ 0.10). \tag{3.19}$$

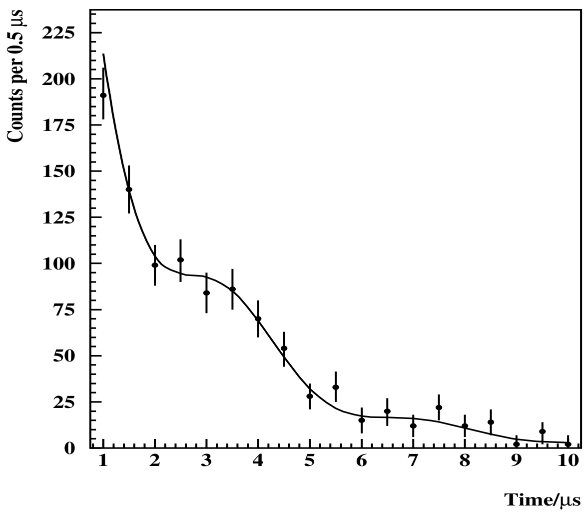

Fig. 2. Theoretical ν_e counting rate as a function of real time and its comparison with KARMEN data

4. (g - 2) of the Muon

Long time ago, Schwinger[10)] showed that in a homogeneous EM field $F_{\mu\nu}$ the mass of a charged lepton is split

$$\mathcal{M} = M - \kappa(\frac{e}{2M})(\frac{1}{2}F_{\lambda\sigma}F^{\lambda\sigma}), \tag{4.1}$$

where κ is the anomalous magnetic moment of the lepton. Thus, in an external magnetic field, the muon mass becomes

$$\mathcal{M} = M \mp \Delta M; \qquad \Delta M = \kappa(e/2M)B \tag{4.2}$$

The survival probability of the muon therefore oscillates so that the number of produced positrons oscillates in real time as

$$P(muon; t) = e^{-(\Gamma t M/E)}[\frac{1 + Acos(\Omega t)}{1 + A}]; \quad (\Omega = \Delta M). \tag{4.3}$$

For the muons derived from pion decay, there should also be a neutrino mixing induced superposition of different muon energies (as discussed in sec. 2). For 2 different neutrino masses, we expect the muon energy to be split symmetrically about its mean value E_o

$$E_{\pm} = E_o \pm \Omega_{\nu}. \tag{4.4}$$

Thus folding the anomalous magnetic moment (κ) oscillations with the neutrino induced oscillations, we have

$$\frac{dW_{tot}(E)}{dE} = \int (d\epsilon)[\frac{dW_1(E+\epsilon)}{dE}][\frac{dW_2(\epsilon)}{d\epsilon}]. \tag{4.5}$$

Hence, the total muon survival amplitude is a product of the respective survival amplitudes

$$\mathcal{S}(t) = \mathcal{S}_1(t)\mathcal{S}_2(t). \tag{4.6}$$

The muon total survival probablity is therefore given by

$$P_{TOT}(muon;t) = [1 - e^{-(t/t_1)^2} sin^2(2\phi) sin^2(\Omega_\nu t)] P(muon;t), \tag{4.7}$$

where through the parameter t_1 we have introduced a gaussian wave packet energy distribution of the muon arising from the pions and $P(muon;t)$ is the distribution in Eq.(4.3). For fast forward muons,

$$\Omega_\nu \approx E_\pi (\frac{\Delta m}{M_\pi})^2. \tag{4.8}$$

Depending upon how sharp the initial muon energy distribution is, that is on the strength of t_1 and, on the neutrino mass difference Δm, we may expect high frequency distortions in the oscillation pattern of the muon during the first few life times. This effect may already have been seen in the CERN muon ($g - 2$) experiments of the '70s by Picasso et al[11]. In these experiments, it was called "detector paralysis" and data from the first life times were simply thrown away.

5. (Anti) Neutrino's from the Earth's Core [3]

It is ironic that we have a better knowledge about nuclear reactions within the Sun than the nature and extent of nuclear reactions within the interior of the Earth. Just as solar neutrinos constitute the only *direct evidence* about nuclear reactions within the Sun, we advocate that an observation of geophysical neutrino flux would not only constitute a *direct evidence* of nuclear activity within the deep interior of the Earth but its measurement would tell us the chemical composition of radioactive materials within the core.

Before discussing the specifics of the proposal, let us summarize what is known about the composition of the Earth and examine a wrong view (of ancient vintage) based on chemistry according to which there should be very little concentration of active materials in the core. The fallacy in the chemical argument has only recently been exposed through direct experimentation. Once we couple this with the discovery of actual nuclear reactors near the surface of the Earth active for hundreds of millions of years, we may safely discard previous pessimism about the lack of much radioactivity in the Earth's core.

Our knowledge about the structure of the Earth has been summarized in the classic work of Jefferies[12]. Measurements of sound wave propagation due to seismic crunching and crackling of Earth quakes have led to the standard geophysical model of the Earth. Briefly, the Earth is pictured as having a shell structure as shown in Fig. 3 (approximately to scale).

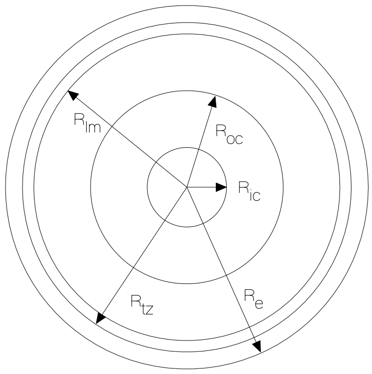

Fig. 3. The shell structure of the Earth includes an inner core with radius R_{ic}, an outer core with radius R_{oc}, a lower mantle with radius R_{lm}, a transition zone with radius R_{tz} and a thin crust to the surface at radius R_e

The interior shell structure of the Earth and radioactive processes within it are poorly understood. Early on, G. Darwin[13] and Rutherford[14] had first invoked Earth's radioactivity to correct Kelvin's big blunder according to which the age of the Earth was much smaller than what the geologists had (more correctly) estimated [15]. Later Jeffries[12] states, ".....it would be interesting to consider what would happen if the present radioactive elements, with their quantities adapted to 4×10^9 years ago, were uniformly distributed...,it is quite possible that radioactive heating could produce fusion in a fraction of the age of the Earth." These lines were prophetic. Naturally occurring nuclear reactors near the surface of the Earth and operating over a long span of time have indeed been discovered[16]. Thus, proving that *nature* and not *nuclear physicists* built the first nuclear reactors on the Earth.

For the longest time it was believed that the bulk of nuclear reactions would occur in the mantle and/ above the mantle. The presumed lack of radioactive materials in the core was based on the argument that Uranium and other radioactive nuclei were not miscible. Relatively recently[17] it has been demonstrated that Uranium is totally miscible at the elevated temperatures of the Earth's core, thereby removing the obstacle to a substantial concentration of radioactive materials in the Earth's core. For a further discussion see [18].

Soon experimental geophysical data will exist in laboratories located on or somewhat below the Earth's surface with large fiducial volume neutrino detectors. The idea is to measure the differential neutrino flux $d^2\Phi$ (per unit time per unit area) within a neutrino energy interval dE and incident from a solid angle direction within $d\Omega$. Let θ be the angle between a line drawn from the laboratory to the center of the Earth and the direction of the solid angle $d\Omega$, and let us assume spherically symmetric neutrino sources[19)]. Then, the differential flux would have the functional form

$$(\frac{d^2\Phi}{dEd\Omega}) = \mathcal{F}(E, cos\theta). \tag{5.1}$$

Let $\eta_e(r, E)d^3\mathbf{r}$ denote the number of neutrino's produced within an earth volume $d^3\mathbf{r}$ per unit time with an energy less than E. Then the flux of geophysical neutrinos seen in a laboratory located on the Earth's surface is described (in an angular range $0 < \theta < \pi/2$) by

$$\mathcal{F}(E, cos\theta) =$$

$$R_e \int_o^{2cos\theta} (\frac{d\eta_e(r = R_e\sqrt{1 + x^2 - 2xcos\theta}, E)}{dE})(dx). \tag{5.2}$$

Under the natural assumption that $\eta_e(r, E)$ is proportional to the mass density $\rho_e(r)$, it is possible to numerically compute the angular distribution of geophysical neutrinos $(d\Phi_e/d\Omega)$. We have carried out such a claculation which is depicted in Fig. 4 below. The mass density was taken from tables provided by Birch[20)]. For the purpose of conversion into neutrino flux, we have assumed one part per million of the nuclei β-decay with a mean life time of 4×10^9 years. These nuclear physics values are in reasonable agreement with those by Jefferies.

Based on the above estimates, we find that in an Earth bound laboratory the total geophysical electron (anti) neutrino flux is atleast comparable to the total solar neutrino flux. The theoretical angular distribution of the geophysical neutrino flux is (of course) broader. Nevertheless, the magnitudes are sufficient to imply some periodic modulations in what may previously have been regarded as a purely solar neutrino flux. Modulations will occur whenever the neutrino beam from the Sun is parallel (or anti parallel) to the neutrino beam from the Earth's core.

The peak in $(d\Phi_e/d\Omega)$ as $cos\theta \to 1$ is due to the fact that the Earth's core contains roughly half of the Earth's mass and thereby half of the Earth's neutrino sources in the model under consideration. The discontinuity of the mass density at the core radius R_{oc} is clearly visible from the structure of the peak.

It appears safe to conclude from our analysis that careful observations of the electron neutrinos from within the Earth would be most helpful in mapping the core topography of the Earth. It may indeed be the only way[21)]. In view of the already discovered nuclear reactors near the surface of the Earth, it is perhaps not unreasonable to enlarge the Jefferies conjecture and entertain the possibility of nuclear fusion occurring in the core of the Earth.

In passing, we would like to comment about a commonly held view which attributes all presently observed ν_e and ν_μ beyond a certain energy threshold to be

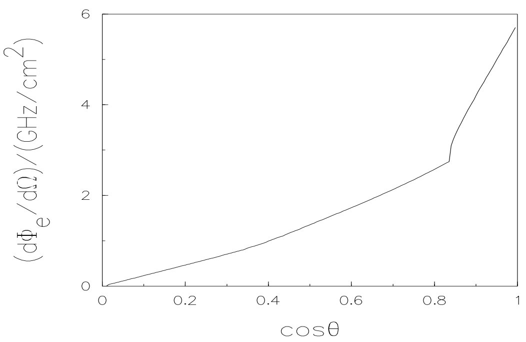

Fig. 4. The angular distribution of geophysical neutrinos in a model where the source density of neutrino production is proportional to the mass density. The geophysical mass density used is due to Birch[20)]

"atmospheric". The argument that only energetic cosmic rays upon hitting the atmosphere could produce such high energy neutrinos from pion decays appears quite plausible. However, since cosmic rays are "everywhere", whenever they hit anything (made up of hadrons) they would produce pions and thus neutrinos in the observed energy range. The nearest big object outside the Earth is the Moon. Cosmic rays hitting the Moon would also produce neutrinos in a similar energy interval and some (those falling within the Earth Moon solid angle) would reach the Earth. If these were sufficient in number, the absolute flux of neutrinos measured in a laboratory on the Earth would not agree with the flux of neutrinos computed using the cosmic rays hitting the Earth.

6. Footnotes and References

1) Y. Srivastava, E. Sassaroli and A. Widom, *Phys Lett* **B344** (1995) 436.
2) Y. Srivastava, E. Sassaroli and A. Widom, *Zeit Fis* **C66** (1995) 601.
3) A. Widom, E. Sassaroli and Y. Srivastava, **hep-ph/ 9812284**; **hep-ph/9707268**.
4) Y. Srivastava, E. Sassaroli and A. Widom, *Euro Phys J* **C2** (1998) 769.
5) S. Palit, Y. Srivastava, A. Widom and E. Sassaroli: **hep-ph/ 9807543**
6) Y. Srivastava, E. Sassaroli and A. Widom, *Proceedings of the DAΦNE Workshop*, Laboratori Nazionali di Frascati, Roma, Italy (1995-96).
7) Y. Srivastava, E. Sassaroli and A. Widom, *Le Rencontres de Physique de la Vale'e d' Aoste*, La Thuile, Aosta Valley, LNF Frascati Physics Series, Ed M. Greco (1996).
8) G. Drexlin *et al Nuc. Inst. and Meth. in Phys.* **A289** (1990)490.

9) B. Armbruster *et al Phys. Rev.* **C57** (1998)3414; **hep-ex/ 9801007**.
10) J. Schwinger, *Phys. Rev* **73** (1948)416; **75** (1949) 898; **82** (1951) 664 (Appendix B).
11)
12) H. Jefferies, *The Earth*, 6th edition, Cambridge University Press, Cambridge (1976).
13) G. Darwin, *Scientific Papers*, Volume 4, 547, Cambridge University Press, Cambridge (1905).
14) Lord Rutherford and B. Boltwood, *Am. J. Sci.* **22** (1906) 1.
15) It is a sobering thought that Kelvin's wrong calculation about the age of the Earth, was the most serious (scientific) hurdle against the theory of evolution. Charles Darvin wrote "Kelvin's ghost stalks me everywhere" and he modified his evolutionary theory *wrongly* for which later other naturalists would criticize Charles Darwin severly. A naturally modest Darwin could hardly imagine that certain physicists (vedi present day cosmologists) are often wrong but never in doubt.
16) G. Boubin, C. Blain, R. Hagemann, M. Cremmer, M. Lucas, L. Merlivat, R. Molina, G. Nieff, F. Prost Marechal, F. Regnaud and E. Roth, *C.R. Acad. Sci.* **275** (1973) 2291.
17) R. Feber, T. Wallace and L. Libby, *Eos* **65** (1984) 785.
18) E. Durrance, *Radioactivity and Geology, Principles and Applications*, John Wiley and Sons, New York (1986).
19) The word neutrino is used here in a generic sense: it obviously includes both neutrinos and antineutrinos.
20) F. Birch, *J. Geo. Res.* **69** (1964) 4377.
21) The conventional elementary tools such as electrons, photons and protons have such small mean free paths that they are useless for probing the inside of the Earth. There could be "monsters" active in the core. So far we have no way of knowing.