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Models of Neutrino Masses and Mixings: a Progress Report

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The exp. situation for masses and mixing is still unclear

- LSND: true or false? -> MiniBooNE soon (??) will tell
- what is the absolute scale of ν masses? θ_{13}? (θ_{23}-45°) ?
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana)......

Different classes of models are still possible:

If LSND true

sterile ν(s)??

CPT violat'n??

- "3-1" or "3-n"

If LSND false → 3 light ν's are OK

We assume this case here

- Degenerate ($m^2 >> \Delta m^2$)

- Inverse hierarchy

- Normal hierarchy

Model building

Quality factors for models:
(higher standards by now!)

- Based on the most general lagrangian compatible with some simple symmetry or dynamical principle
- Should be complete: address at least charged leptons and neutrinos ($U_{P\text{-}NMS} = U^+_e U_\nu$, and the gauge symmetry connects ch. leptons and LH neutrinos)
- As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.
- The necessary vev configuration should be a minimum of the most general potential for a region of parameter space
- The stability under radiative corrections and higher dim operators must be checked
- Simplicity, economy of fields and parameters, predictivity…

Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103,
G.A, R. Franceschini, hep-ph/051220,
G.A., F. Feruglio,Y. Li hep-ph/0610165;
F. Feruglio, C. Hagedorn, Y. Li, L. Merlo, hep-ph/0702194

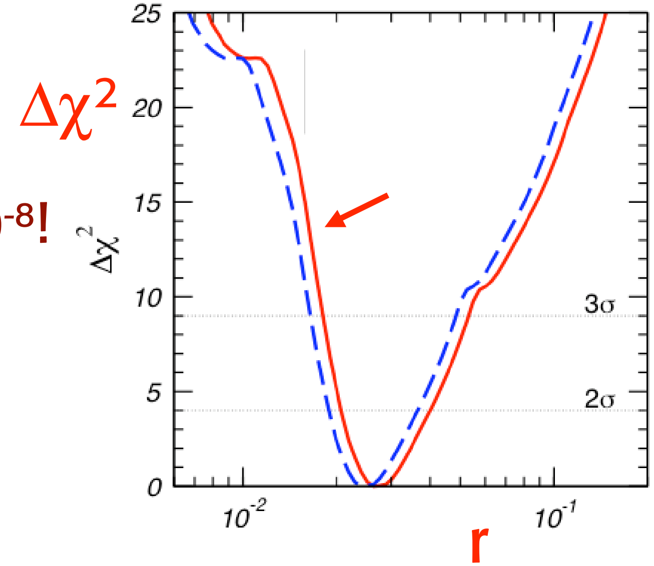
Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048],
G.A., hep-ph/0410101, F. Feruglio, hep-ph/0410131,
G.A, hep-ph/0611117.

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$



Only a few years ago could be as small as 10^{-8}!

Precisely at 2σ: $0.025 < r < 0.049$

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) ⟶ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

2-σ interval $0.32 < \sin^2\theta_{23} < 0.62$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

Very small θ_{13} theoretically hard

"Normal" models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

"Exceptional" models: θ_{23} very close to maximal and/or θ_{13} very small
or: a special value for θ_{12}....

Natural models of the “normal” type are not too difficult
to build up (with normal or inverse or degenerate hierarchy)

Review: G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048],

It is reasonable to attribute hierarchies in masses and
mixings to differences in some flavour quantum number(s).

A simplest flavour (or horizontal) symmetry is $U(1)_F$

For example, simple models based on see-saw and $U(1)_F$
work for all quarks and leptons, explain all small numbers,
are natural and compatible with (SUSY) GUT’s, e.g $SU(5)xU(1)_F$
(accomodation rather than prediction).

Larger flavour symmetry groups have also been studied.
They are more predictive but less flexible.
The problem of the "best" flavour group is still open.

The most ambitious models try to combine (SUSY) SO(10)
GUT's with a suitable flavour group

Here we concentrate on "exceptional" models, in particular on models for "tri-bimaximal" mixing

The most general mass matrix for $\theta_{13} = 0$ and θ_{23} maximal is given by (after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Tri-bimaximal Mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

Fogli et al '05

$\sin^2\theta_{12} = 1/3$: 0.290-0.342

$\sin^2\theta_{23} = 1/2$: 0.39-0.53

$\sin^2\theta_{13} = 0$: < 0.02

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

It is interesting to construct models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) are very interesting (minimal solution)

Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103
GA, Feruglio hep-ph/0610165
....

Alternative models based on $SU(3)_F$ or $SO(3)_F$

Verzielas, G. Ross   King
....

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

Thus A4 transf.s can be written as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

x, x′ in same class if

C_1, C_2, C_3, C_4 are equivalence classes [$x' \sim gxg^{-1}$] g: group element

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1', 1"

In the (S-diag basis) consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

(a_1,a_2,a_3) $\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

while, under S, 1" is inv.

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

Recall:

$S^2 = T^3 = (ST)^3 = 1$

- 1: $S=1,\ T=1$
- 1': $S=1,\ T=\omega$
- 1'': $S=1,\ T=\omega^2$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

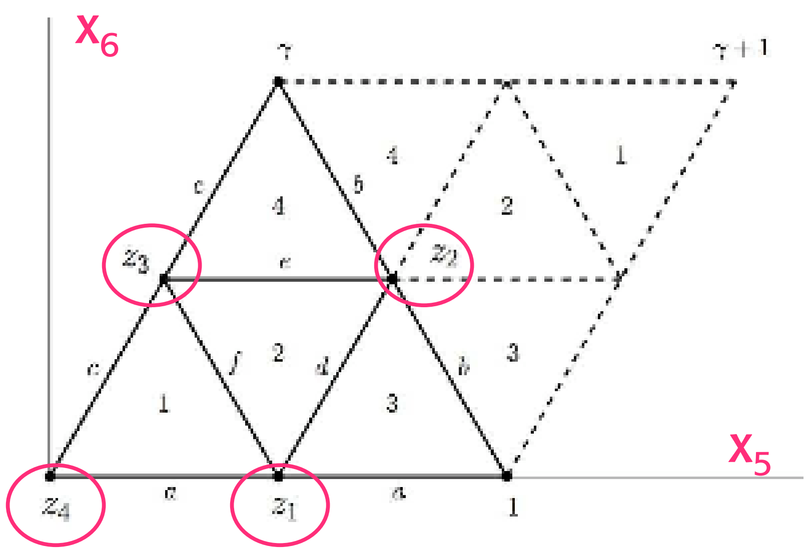
$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

What can be the origin of A4? G.A., F. Feruglio,Y. Li hep-ph/0610165

A4 (or some other discrete group) could arise from extra dimensions (by orbifolding with fixed points) as a remnant of 6-dim spacetime symmetry.



$z=x_5+ix_6$

A torus with identified points:

$z \to z + 1$

$z \to z + \gamma \quad \gamma=\exp(i\pi/3)$

and a parity $z \to -z$

leads to 4 fixed points (equivalent to a tethraedron).

There are 4D branes at the fixed points where the SM fields live (additional gauge singlets are in the bulk)

- A4 interchanges the fixed points

Under A4 the most common classification is:

lepton doublets $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1', 1''$ respectively

gauge singlet flavons $\phi, \phi', \xi, (\xi') \sim 3, 3, 1, (1)$ respectively

driving fields (for SUSY version) $\phi_0, \phi'_0, \xi_0 \sim 3, 3, 1$

Additional symmetries: broken $U(1)_F$ symmetry (ch. lepton masses) with e^c, μ^c, τ^c charges (3 or 4,2,0)

and a discrete symmetry (dep. on versions) : for example

Z: $(e^c, \mu^c, \tau^c) \to -i\,(e^c, \mu^c, \tau^c)$, $l \to i l$, $\phi \to \phi$, $(\xi, \phi') \to -(\xi, \phi')$

Structure of the model

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda,$ $\quad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$

!!!

$$\begin{aligned} \langle\varphi'\rangle &= (v', 0, 0) \\ \langle\varphi\rangle &= (v, v, v) \\ \langle\xi\rangle &= u \end{aligned}$$

$$m_l = v_d \frac{v}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu \omega^2 & y_\mu \omega \\ y_\tau & y_\tau \omega & y_\tau \omega^2 \end{pmatrix}$$

Spectrum free.
Diagonalized by U_e:

the big plus of A4

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

$$l \to \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} l = V l$$

From here it follows that U_{HPS} is the mixing matrix

m_ν in the basis of diagonal charged leptons is:

$$m_\nu\big|_{ldiag} \sim V^* \begin{bmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{bmatrix} V^* = \begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

$$V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

which in turn can be written as:

$$m_\nu\big|_{ldiag} \sim U^T \begin{bmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{bmatrix} U$$

with:

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The crucial issue is to guarantee the strict alignment

$$
\begin{aligned}
\langle \varphi' \rangle &= (v', 0, 0) \\
\langle \varphi \rangle &= (v, v, v) \\
\langle \xi \rangle &= u
\end{aligned}
$$

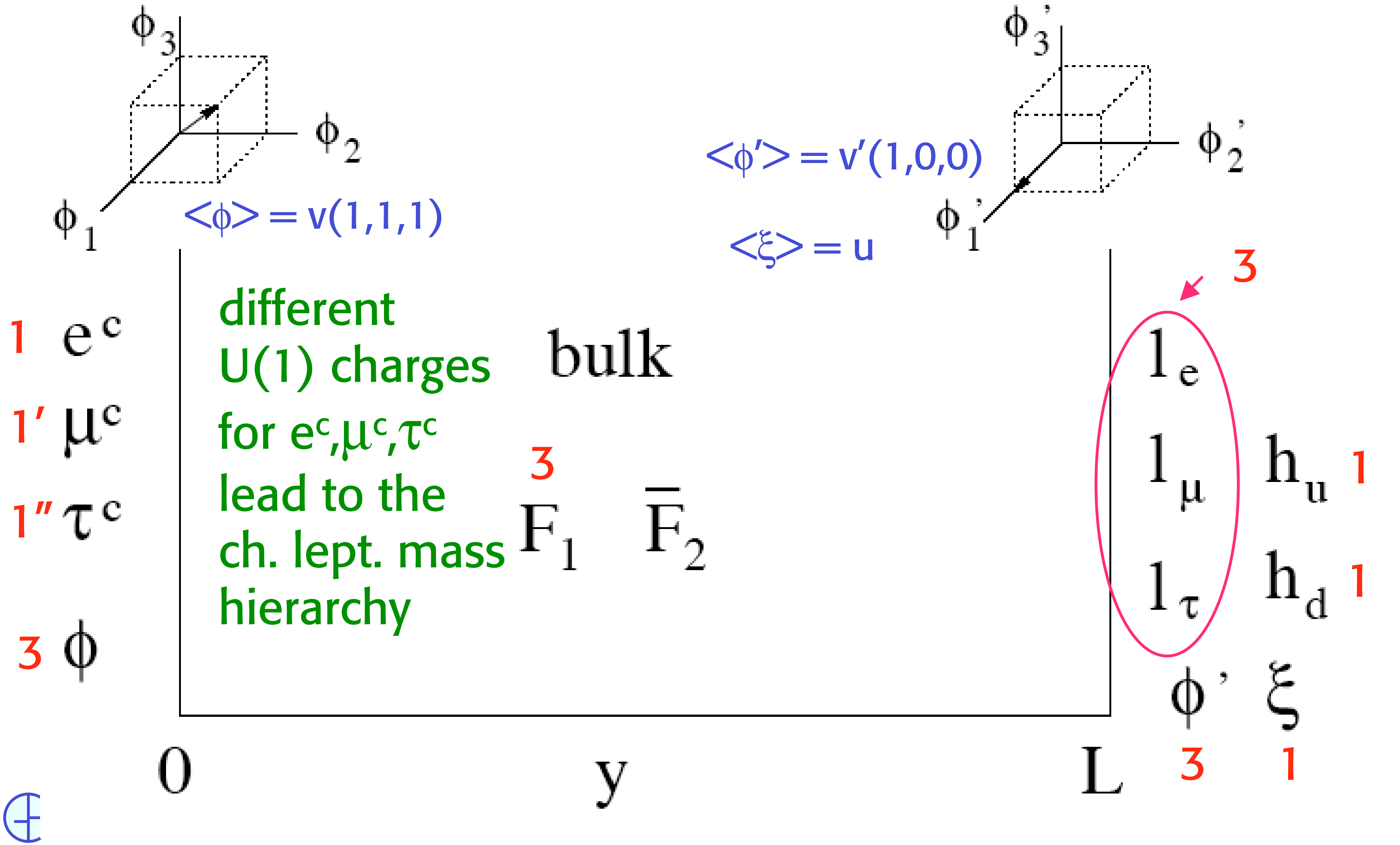
We have constructed a number of completely natural versions of the model, e.g.:

- a version in 5 dimensions (economic in flavon fields)
- a SUSY version in 4-dim (with more fields)

GA, Feruglio hep-ph/0504165

The model has 1 compactified extra dim. and 2 branes

(crucial issue: guarantee and protect the vev alignment)



In lowest approximation the action is:

$$
\begin{aligned}
S &= \int d^4x dy \Big\{ \Big[iF_1\sigma^\mu\partial_\mu\overline{F}_1 + iF_2\sigma^\mu\partial_\mu\overline{F}_2 + \frac{1}{2}(F_2\partial_y F_1 - \partial_y F_2 F_1 + h.c.) \Big] \\
&- M(F_1F_2 + \overline{F}_1\overline{F}_2) \\
&+ V_0(\varphi)\delta(y) + V_L(\varphi', \xi)\delta(y-L) \\
&+ \left[Y_e e^c(\varphi F_1) + Y_\mu \mu^c(\varphi F_1)'' + Y_\tau \tau^c(\varphi F_1)' + h.c.\right]\delta(y) \\
&+ \left[\frac{x_a}{\Lambda^2}\xi(ll)h_u h_u + \frac{x_d}{\Lambda^2}(\varphi' ll)h_u h_u + Y_L(F_2 l)h_d + h.c.\right]\delta(y-L)\Big\} + ...
\end{aligned}
$$

a Z-parity has also been imposed

$$
(f^c, l, F, \varphi, \varphi', \xi) \xrightarrow{Z} (-if^c, il, iF, \varphi, -\varphi', -\xi)
$$

After integrating out of the F fields one obtains the required effective 4-dim action

$$
\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a\xi(ll) + x_d(\varphi' ll) + h.c. + ...
$$

In the flavour basis:

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

$m_\nu = U\, \mathrm{diag}(a+d, a, -a+d) U^T$ (in units of v_u^2/Λ) and $U = U_{HPS}$

In terms of physical param.s (moderate normal hierarchy):

$$|m_1|^2 = \left[-r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_2|^2 = \frac{1}{8\cos^2\Delta(1-2r)}\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_3|^2 = \left[1 - r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.053\ \mathrm{eV})^2$$

⊕ A moderate fine tuning is needed for r

A version with see-saw is also possible

ν_R is a triplet of A4: $\nu^c \sim 3$ No change for ch leptons

Dirac Majorana

$$w_l = ...+y(\nu^c l)+x_A\xi(\nu^c\nu^c)+x_B(\varphi_T\nu^c\nu^c)$$

[Discrete parity Z: $\omega,\omega^2,\omega^2,\omega^2$ for l, ν^c, ϕ_T, ξ respectively]

$m_\nu^D \sim 1$

$$M_{RR} \sim \begin{bmatrix} A & 0 & 0 \\ 0 & A & D \\ 0 & D & A \end{bmatrix}$$

$m_\nu = m_\nu^{DT} M_{RR}^{-1} m_\nu^D \sim M_{RR}^{-1}$

The mass matrix appears just as the inverse of what was before, so that the mixing matrix is the same.

Eigenvalues are the inverse: one can produce inverse hierarchy with realistic θ_{12}, θ_{23} and very small θ_{13}

The model crucially depends on the precise vev alignment

$$\begin{aligned} \langle \varphi' \rangle &= (v', 0, 0) \\ \langle \varphi \rangle &= (v, v, v) \\ \langle \xi \rangle &= u \end{aligned}$$

The extra dimension with 2 branes allows the decoupling of the ϕ and ξ, ϕ' potentials.

A discrete symmetry is also essential: a separate continuous rotation symmetry on the 2 branes would make any disalignment illusory.

An alternative in 4 dimensions is a SUSY model with driving fields and a superpotential where all terms allowed by symmetry are present (with added fields $\xi', \phi_0, \phi'_0, \xi_0$).

In our models

- all terms allowed by symmetry are present
- all correct'ns are under control and can be made negligible

The 4-dim SUSY version (written in the T-diag basis)

In this basis the ch. leptons are diagonal!

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

$$\begin{aligned} w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\ &+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2 \end{aligned}$$

and the potential $V = \sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$

The assumed simmetries are summarised here

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2
$U(1)_F$		2q	q	1								

The driving field have zero vev. So the minimization is:

$$\frac{\partial w}{\partial \varphi_{01}^T} = M\varphi_{T\,1} + \frac{2g}{3}(\varphi_{T\,1}^2 - \varphi_{T\,2}\varphi_{T\,3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{01}^S} = g_2\tilde{\xi}\varphi_{S\,1} + \frac{2g_1}{3}(\varphi_{S\,1}^2 - \varphi_{S\,2}\varphi_{S\,3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{02}^T} = M\varphi_{T\,3} + \frac{2g}{3}(\varphi_{T\,2}^2 - \varphi_{T\,1}\varphi_{T\,3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{02}^S} = g_2\tilde{\xi}\varphi_{S\,3} + \frac{2g_1}{3}(\varphi_{S\,2}^2 - \varphi_{S\,1}\varphi_{S\,3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{03}^T} = M\varphi_{T\,2} + \frac{2g}{3}(\varphi_{T\,3}^2 - \varphi_{T\,1}\varphi_{T\,2}) = 0 \qquad \frac{\partial w}{\partial \varphi_{03}^S} = g_2\tilde{\xi}\varphi_{S\,2} + \frac{2g_1}{3}(\varphi_{S\,3}^2 - \varphi_{S\,1}\varphi_{S\,2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S\,1}^2 + 2\varphi_{S\,2}\varphi_{S\,3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\tilde{\xi} = 0$$

$$\xi = u$$

$$\varphi_S = (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2$$

In the paper
w at NLO is also
studied

NLO corrections studied in detail

to m_l — 1st non trivial correction at $o(1/\Lambda^3)$

LO is $1/\Lambda$

to m_ν

LO is $1/\Lambda^2$

$$\frac{x_c}{\Lambda^3}(\varphi_T\varphi_S)'(ll)''h_uh_u \qquad \frac{x_d}{\Lambda^3}(\varphi_T\varphi_S)''(ll)'h_uh_u \qquad \frac{x_e}{\Lambda^3}\xi(\varphi_T ll)h_uh_u$$

to vevs

LO is 1

$$\begin{array}{lcl} \langle\varphi_T\rangle & \rightarrow & (v_T'+\delta v_T, \delta v_T, \delta v_T) \\ \langle\varphi_S\rangle & \rightarrow & (v_S+\delta v_1, v_S+\delta v_2, v_S+\delta v_3) \\ \langle\xi\rangle & \rightarrow & u \\ \langle\tilde{\xi}\rangle & \rightarrow & \delta u' \end{array}$$

δv_T, δv_S, δv_i, $\delta u' \sim o(1/\Lambda)$

All observables get a correction of order $1/\Lambda$

From exp (eg θ_{12}) must be less than 5%

$$0.0022 < \frac{v_S}{\Lambda} \approx \frac{v_T}{\Lambda} \approx \frac{u}{\Lambda} < 0.05$$

In particular $\theta_{13} < \sim 0.05$,

$|tg^2\theta_{23} - 1| < \sim 0.05$

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices are BOTH diagonalised by

$$U_u, U_d \sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

As a result VCKM is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$). But even those are essentially the same for u and d quarks)

Note: NOT straightforward to embed these models in a GUT: with these assignments A4 does not commute with SU(5)

If $l \sim 3$ then all $5^* \sim 3$, so that $d^c_i \sim 3$
if $e^c, \mu^c, \tau^c \sim 1, 1', 1''$ then all $10_i \sim 1, 1', 1''$

Realistic quark mass matrices are not easy to obtain from these assigments

For example, for u quarks at leading order:

$$m_u \sim 1.1 + 1'.1'' + 1''.1' \sim a\, u_1 u_1 + b\, (u_2 u_3 + u_3 u_2)$$

or

$$m_u \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$$

Which implies $|m_c| = |m_t|$ and maximal U_{23}

Recent directions of research:

- Different (larger) finite groups

Ma;
Kobayashi et al;
Luhn, Nasri, Ramond [$\Delta(3n^2)$];
.....

- Trying to improve the quark mixings

Carr, Frampton
Feruglio et al
.....

- Construct GUT models with approximate tri-bimaximal mixing

Ma, Sawanaka, Tanimoto; Ma;
Morisi, Picarello, Torrente Lujan;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C x SU(2)_L x SU(2)_R$];
.....

Better quarks: use T′ (also called $SL_2(F_3)$) the double covering group of A4 (A4 is not a subgroup of T′)

Aranda, Carone, Lebed
Carr, Frampton
Feruglio et al

24 transformations.
Irreducible representations: 1, 1′, 1″, 2, 2′, 2″, 3

Equivalent to A4 for leptons. For quarks use 1(3rd family) + 2, 2′, 2″ (1st&2nd families)

- t,b masses at renormalizable level (unsuppressed)
- V_{cb}, V_{ts} from doublet flavons (do not couple to leptons)
- 1st generation masses and mixings from subleading effects

Similar to old U(2) models

Barbieri, Dvali, Hall '96
Barbieri, Hall, Raby, Romanino '97
Barbieri, Hall, Romanino '97

GUT-compatible A4-models

All doublets ~ 3 and all singlets ~ 1, 1′, 1″ for quarks and leptons is not compatible with SU(5), SO(10).

It is OK with $SU(4)_C x SU(2)_L x SU(2)_R$ (Pati-Salam)

King, Malinsky

SU(5)-compatible classifications have been tried:

5* ~3, 10 ~ 1, 1′, 1″ — Ma, Sawanaka, Tanimoto

all in 3 — Ma; Morisi, Picarello, Torrente Lujan;

Problem still open

Conclusion

From experiment: a good first approximation for quarks:

$$V_{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Models based on A4 indeed lead to this pattern

All this is highly non trivial but no real illumination has followed!!