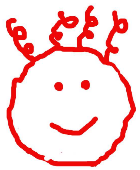
Flavour matters in Leptogenesis

claim: single-flavour approximation (usual Boltzmann Eqns for total lepton number) is unreliable



A Abada, SD, A Ibarra, F-X Josse-Michaux, M Losada, A Riotto

Barbieri etal
Endoh etal
Pilaftsis Underwood
Vives
Abada etal
Nardi etal
...
Pascoli etal
Blanchet etal
Antusch etal
...

1. review:
   - the bayon asymmetry, two neutrino mass differences and the seesaw
   - thermal leptogenesis from N_1 decay, $M_1 \ll M_{2,3}$
2. why flavour matters
   - N_1 production *and* decay involve light leptons
   - lepton flavours are distinguishable (sum probabilities not amplitudes...)
3. how flavour matters
   - phenomenology: "no" leptogenesis bound on neutrino mass scale; mild changes in $\{M_1, \Gamma\}$ where leptogenesis works
   - models: baryon asym can change by orders of magnitude, in specific models

Introduction:why make the BAU in the seesaw?

the Universe contains a matter excess: 6 ± 1 baryons for every 10^{10} photons (WMAP).
$\Rightarrow$ must generate BAU after inflation, before nucleosynthesis ($\tau_U \sim$ sec)
and maintain proton lifetime $\gtrsim 10^{33}$ years

Required ingredients: baryon number violation
C and CP violation
non-equilibrium dynamics

Sakharov

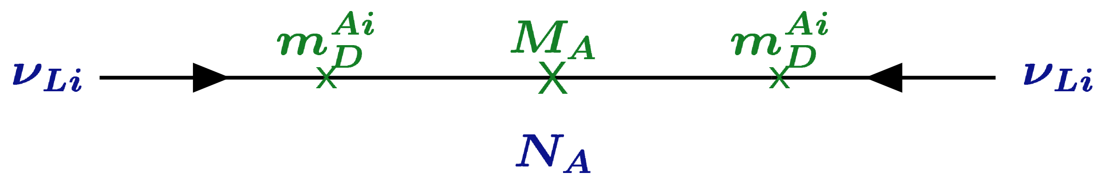
present in SM ...but... not make BAU ... $\Rightarrow$ need ("motivated") New Physics.

$\star$ the seesaw, which naturally explains small neutrino masses, also gives attractive mechanism to make baryon asym: leptogenesis...
$\star$ non-perturbative $\not{B} + \not{L}$ (SM)+ perturbative $\not{L}$ (seesaw) = $\not{B}$ for baryogenesis but not proton decay

The See-Saw in three generations

- in the charged lepton ("flavour") and $N(=\nu_R)$ mass bases, at large energy scale $\gg M_i$:

$$\mathcal{L}_{leptons} = \mathbf{h}_\alpha \, \overline{e}_{R\alpha} \ell_\alpha \cdot \phi + \lambda_{J\alpha} \overline{N}_J \ell_\alpha \cdot \phi - \frac{1}{2} \overline{N_J} M_J N_J^c$$



- at the weak scale, get effective light neutrino mass matrix

$$\frac{1}{2}[m_\nu] = \lambda^T M^{-1} \lambda \langle \phi^0 \rangle^2$$

$$\sim \frac{m_t^2}{10^{14} GeV} \sim .1 eV$$

unflavoured: Fukugita Yanagida
Buchmuller et al
Covi et al
Branco et al
Giudice et al
...

he Baryon Asymmetry by Thermal Leptogenesis (N_1 decay)

Suppose that $M_1 \sim 10^9$ GeV $\ll M_2, M_3$ (simple kinematics $\leftrightarrow$ algebra...)

After inflation, vaccuum energy density is transfered to a hot thermal soup at T_{reheat} (made of particles with gauge interactions). Then...

- $\not{T}\!\!\not{E}$ dynamics: from U expansion $\leftrightarrow$ rates $\lesssim H$. The lightest N_1 produced in the thermal soup after inflation by scatterings ($qt^c \to N\ell_\alpha$) and inverse decays ($\phi\ell_\alpha \to N$). At $T \lesssim M_1$, N_1 decays.

- $\not{C}\!\!\not{P}$: Lepton asym. in flavour α due to $\not{C}\!\!\not{P}$ in N_1 interactions ($\to \lambda$ complex):

$$\epsilon^{\alpha\alpha} = \frac{\Gamma(N_1 \to \phi\ell_\alpha) - \Gamma(\bar{N}_1 \to \bar{\phi}\bar{\ell}_\alpha)}{\Gamma(N \to \phi\ell) + \Gamma(\bar{N}_1 \to \bar{\phi}\bar{\ell})} \qquad (\text{recall } N_1 = \bar{N}_1)$$

- $\not{L}$, $\not{B} + \not{L}$: $\not{L}$ due to M, λ. Non-perturbative SM processes (sphalerons): lepton asym. $\to$ baryon asym.

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim \frac{1}{3}\frac{n_\ell - n_{\bar{\ell}}}{n_\gamma}$$

(caveat: there are many other leptogenesis scenarios!)

non-equilibrium, washout and η

produce N_1 by same $(\not{C}\not{P}, \not{L})$ $\lambda_{1\alpha} N \ell_\alpha \phi$ vertex as they decay through: $\Gamma_{prod} \simeq \Gamma_{decay}$.

Can show: $\Gamma_{dec} \frac{8\pi \langle \phi_0 \rangle^2}{M_1^2} \equiv \tilde{m} > m_1$. "Expect" $\tilde{m} \gtrsim m_{sol}$, $\Rightarrow \Gamma_{decay} \gg H(T = M)$

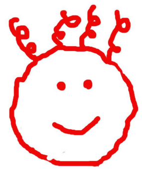
1. for $\Gamma_{prod}, \Gamma_{decay} < H$, make $n_N < n_\gamma$ and a lepton anti-asymmetry $-Y_L$. When n_N decay, they make $+Y_L$ asymmetry. Incomplete cancellation because anti-asymmetry partially washed out while waiting around.
2. $\Gamma_{prod}, \Gamma_{decay} > H$, $\Rightarrow$ make (maximal) thermal density $n_N \simeq n_\gamma$, but, when T drops below M_1, decay " in equilibrium"

- The lepton asym produced in N decay does not survive until *after* inverse decays are "out of equilibrium"

$$\Gamma_{ID}(\phi\ell \to N) \simeq \Gamma_{decay} e^{-M_1/T} \lesssim H$$

- At temperature where $\Gamma_{ID} = H$, have $n_N/n_\gamma \simeq e^{-M/T} \simeq H/\Gamma_{dec}$, so

$$Y_B \sim \frac{1}{3} \frac{n_\ell - n_{\bar{\ell}}}{n_N} \frac{n_N}{n_\gamma} \sim \frac{1}{3} \epsilon \, \eta \frac{1}{g_*} \qquad \eta \sim H/\Gamma \;\; \sim .1 \div .01 \qquad \text{(no flavour)}$$



just a yoctosecond—what about flavour?

with flavour: Barbieri etal
Endoh etal
Pilaftsis Underwood
Vives
Abada etal
Nardi etal

1. produce the (maximal) thermal density $n_N \simeq n_\gamma$ if ($M_1 \lesssim T$, and)
   production rate, $e.g.$ $\sum_\alpha \Gamma(q_L t_R \to \phi \to \ell_\alpha N)$ is fast enough ($\tau_{prod} < \tau_U$):

$$\Gamma_{prod} \sim \sum_\alpha \frac{h_t^2 |\lambda_{\alpha 1}|^2}{4\pi} T > H, \quad \Rightarrow \quad \frac{[\lambda^\dagger \lambda]_{11}}{4\pi} > \frac{10T}{m_{pl}}\Big|_{T=M_1}$$

Suppose that, as "expected", this condition is satisfied.

2. The lepton asym in flavour α (produced from N decay) can survive after Inverse Decays from flavour α turn off ($\tau_{ID} > \tau_U$)

$$\Gamma_{\alpha\alpha} \equiv \Gamma(\ell_\alpha \phi \to N_1) \simeq \frac{|\lambda_{\alpha 1}|^2 M_1 e^{-M_1/T}}{8\pi} < \frac{10T^2}{m_{pl}}$$

At temperature T_α when Inverse Decays from flavour α turn off,

$$\frac{n_N}{n_\gamma}(T_\alpha) \simeq e^{-M_1/T_\alpha} \simeq \frac{H}{\Gamma_{\alpha\alpha}} \equiv \eta_\alpha$$

so

$$Y_B \sim \frac{1}{3} \sum_\alpha \frac{n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}}{n_N} \frac{n_N(T_\alpha)}{n_\gamma} \sim \frac{H}{3g_*} \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\Gamma_{\alpha\alpha}} \qquad \neq \frac{H}{3g_*} \frac{\sum_\alpha \epsilon_{\alpha\alpha}}{\sum_\beta \Gamma_{\beta\beta}}$$

Flavour is irrelevant—its obvious, no?

1. Production, decay of N_1 are controlled by λ; charged lepton Yukawas h_e, h_μ, h_τ are $\ll 1$, a small correction in perturbation theory?
   - Small correction to ϵ. But *not* to dynamics: timescale for leptogenesis is H^{-1} (Sahkarov). All faster interactions should be resummed, eg, into thermal masses.
     Compare rates for h_τ, h_μ to H

     $$\Gamma_\tau \simeq 10^{-2} h_\tau^2 T > H \text{ for } T < 10^{12}\ GeV, \quad \Gamma_\mu > H \text{ for } T < 10^9\ GeV$$

     If the h_τ, h_μ are in equilibrium, flavour defines distinguishable mass eigenstates.
     (If $\Gamma(N \to \phi\ell) > \Gamma_\tau$ should also be included in determination of mass eigenstates) Blanchet diBari Raffelt
   - leptons enter in N_1 production and decay, distinguishable in between $\leftrightarrow$ sum probabilities not amplitudes.

2. lepton asym is a Trace in flavour space, basis-independent, can calculate the asym in the flavour combination into which N_1 decays

   Sure, in field theory, can work in any basis. BUT— in any basis *other* than the flavour basis, h_α put additional fast terms in the Boltzmann Eqns.

outline (again)

1. review:

   - the bayon asymmetry, the seesaw and two neutrino mass differences
   - thermal leptogenesis from N_1 decay, $M_1 \ll M_{2,3}$

2. why flavour matters

   - distinguishable leptons appear in production/decay of N_1 $\Rightarrow$ should evolve flavoured asymmetries.
   - and physics *is* basis independent ??

3. how it matters

   - phenomenology: no leptogenesis bound on neutrino mass scale; mild changes in allowed $\{M_1 \Gamma\}$ parameter space
   - models: baryon asym can change by orders of magnitude, in specific models

pheno consequences 1 - single flavour "old" vs flavoured

"single flavour" approx, successful thermal leptogenesis $\Rightarrow$ light ν mass scale $\lesssim .1$ eV.

"flavoured": more $\not{CP}$, so no bound. Models can be tuned to work for $m_\nu \lesssim$ few eV (cosmo)

"single flavour": no model-indep connection between $\not{CP}$ for leptogenesis and MNS phases.

"flavoured": still no sensitivity of baryon asym. to MNS phases (but can say things in classes of models:Petcov)

There is an envelope, in space of parameters leptogenesis depends on $(M_1,\ \Gamma,\ \epsilon...)$ where leptogenesis can work.

Including flavour gives envelope more dimensions $(M_1, \epsilon_{\alpha\alpha}, \Gamma_{\alpha\alpha})$, little changes to " interesting" regions of the envelope projected onto M_1, Γ space (not move lower bound on T_{reheat})

Antusch+..
Blanchet+..
Josse-Michaux+...

consequences for models...to estimate Y_B

single flavour analytic approximation : $\quad Y_B \simeq \frac{\epsilon\eta}{9g_*}$

$$\eta \simeq \frac{1}{\Gamma/H + H/6\Gamma}\Big|_{T=M_1} \simeq \begin{cases} H/\Gamma & \Gamma \gg H \\ \Gamma/H & \Gamma \ll H \end{cases} \qquad \Rightarrow \text{ calculate } \quad \Gamma, \quad \epsilon = \frac{\Gamma(N_1 \to H\ell_) - \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})}{\Gamma(N \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})}$$

model predictions with flavour : $\epsilon^{ee}, \epsilon^{\mu\mu}, \epsilon^{\tau\tau}, \Gamma^{ee}, \Gamma^{\mu\mu}, \Gamma^{\tau\tau}$

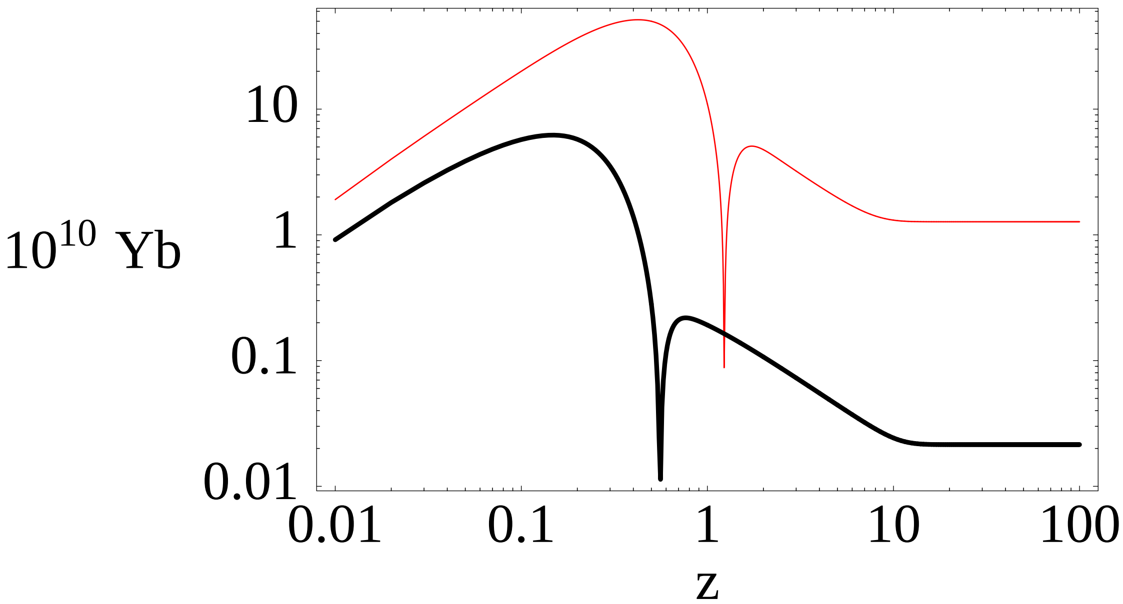
1. "strong washout" of at least one flavour: $\Gamma_\tau \gg \Gamma \gg H$

$$Y_B \simeq \frac{1}{9g_*}\sum_\alpha \epsilon^{\alpha\alpha}\eta^\alpha \qquad \eta^\alpha \simeq \frac{1}{\Gamma^{\alpha\alpha}/H + H/6\Gamma^{\alpha\alpha}}$$

2. weak washout all flavours: $\widetilde{m}^{\alpha\alpha} < m_*$ (assume no initial asymmetry)

$$Y_B \simeq 5 \times 10^{-3}\frac{\tilde{m}}{m_*}\sum_\alpha \epsilon^{\alpha\alpha}\frac{\tilde{m}^{\alpha\alpha}}{m_*} \qquad (Y_N \propto \tilde{m}; \quad C\!\!\!/P \text{ in scatt.} \to \epsilon^{\alpha\alpha}\widetilde{m}^{\alpha\alpha})$$

useful enhancement factor...



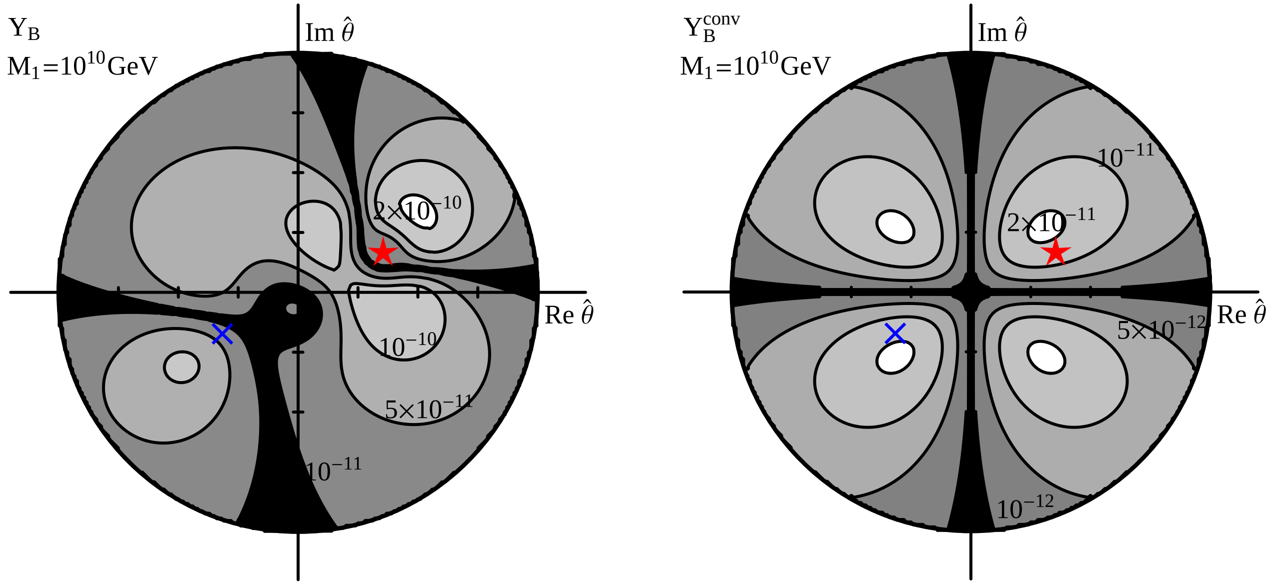
The baryon to entropy ratio, as a function of "time", in flavoured and unflavoured calculation.

$\epsilon_{ee} = 2.5 \times 10^{-6}$, $\epsilon_{\mu\mu} = -2 \times 10^{-6}$, $\epsilon_{\tau\tau} = 10^{-7}$

$M_1 = 10^{10}$ GeV

$\frac{\Gamma_{ee}}{H} \simeq 10$, $\frac{\Gamma_{\mu\mu}}{H} \simeq 30$, $\frac{\Gamma_{\tau\tau}}{H} \simeq 30$

useful factors of 10...



Y_B in the 2RHN model, hierarchical ν_L.

LHS flavoured leptogenesis, RHS "single flavour approx".

(1,2) texture zero (1,3) texture zero

(in plane of complex angle of R matrix)

Summary

thermal leptogenesis is an attractive, minimal mechanism to make the Baryon asymmetry of the Universe.

When the interaction rates of the charged lepton Yukawas are faster than the leptogenesis rates, lepton flavours are distinguishable and asymmetry production should be studied flavour by flavour (or put the charged lepton Yukawas in the Boltzmann Equations).

Resulting "flavoured" Boltzmann Equations are different, and the solutions are different:

- allowed phenomenological parameter space mildly affected
  1) leptogenesis works for degenerate light neutrinos

- baryon asymmetry Y_B obtained in specific models can be (?is?) enhanced
  1) cancellations reduced: $Y_B \propto \sum_\alpha \epsilon^{\alpha\alpha} f(\Gamma^{\alpha\alpha})$
  2) flavoured $\epsilon^{\alpha\alpha}$ can be bigger than ϵ

Eqns for $Y_L^{\alpha\beta}$ in field theory?

Should be able to do Field Theory in any basis we like. Consider eqns of motion for

$$Y_L^{\alpha\beta} \propto \frac{\int d^3k \langle a^\dagger_{\ell_\alpha} a_{\ell_\beta} - a^\dagger_{\bar{\ell}_\beta} a_{\bar{\ell}_\alpha} \rangle}{s}$$

in flavour basis, diagonal elements are $(n_{\ell_\alpha} - n_{\bar{\ell}_\alpha})/s$, off-diagonals encode quantum correlations, and are driven rapidly to zero by charged lepton yukawas when these are in equilibrium.
So at $T \lesssim 10^9$ GeV, in flavour basis, and *only* in flavour basis, the Boltzmann Eqns are:

$$\frac{sH(M_1)}{z}\frac{d}{dz}\left[\begin{array}{ccc} Y_L^{ee} & \cdot & \cdot \\ \cdot & Y_L^{\mu\mu} & \cdot \\ \cdot & \cdot & Y_L^{\tau\tau} \end{array}\right] = \gamma\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\left[\begin{array}{ccc} \epsilon^{ee} & \cdot & \cdot \\ \cdot & \epsilon^{\mu\mu} & \cdot \\ \cdot & \cdot & \epsilon^{\tau\tau} \end{array}\right]$$
$$-\frac{1}{Y_L^{eq}}\left[\begin{array}{ccc} \gamma^{ee} & \cdot & \cdot \\ \cdot & \gamma^{\mu\mu} & \cdot \\ \cdot & \cdot & \gamma^{\tau\tau} \end{array}\right]\left[\begin{array}{ccc} Y_L^{ee} & \cdot & \cdot \\ \cdot & Y_L^{\mu\mu} & \cdot \\ \cdot & \cdot & Y_L^{\tau\tau} \end{array}\right]$$

Same as previous flavoured. Take Trace to get eqn for total Y_L—take it in any basis? (*eg* direction into which N_1 decays? This give the usual BEs) **NO**. Dropped from BE $\sim -|h_\alpha|^2 T Y_L^{\alpha\beta}$ terms that drive off-diagonals to zero.

Boltzmann Equations

- the lightest N_1 is produced in the thermal soup after inflation by scatterings ($qt^c \to N\ell_\alpha$) and inverse decays ($H\ell_\alpha \to N$). At $T \lesssim M_1$ N_1 decays.
- CP violation in N_1 interactions : $\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \to H\ell_\alpha) - \Gamma(N_1 \to \bar{H}\bar{\ell_\alpha})}{\Gamma(N \to H\ell) + \Gamma(N_1 \to \bar{H}\bar{\ell})} \to$ asym. in $n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}$
- non-perturbative SM processes: lepton asymmetry $\to$ baryon asymmetry

described by Boltzmann Eqns for N_1 and lepton asymmetry number densities ($z = \frac{M_1}{T}$ is time var.):

$$\frac{dY_N}{dz} = -\frac{z}{sH}\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\gamma$$

$Y_N = n_N/s$ ($s =$ entropy density) pushed to equilibrium by $\gamma(z)$, which describes thermally averaged N decays (D), scatterings (S), inverse decays(ID). summed on flavour α

$$\frac{d}{dz}\frac{n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}}{s} \equiv \frac{dY_L^{\alpha\alpha}}{dz} = \frac{z}{sH(M)}\left[\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\epsilon^{\alpha\alpha}\gamma - \gamma^{\alpha\alpha}\frac{Y_L^{\alpha\alpha}}{Y_\ell^{eq}}\right]$$

asym $Y_L^{\alpha\alpha}$ produced by out-of-equilibrium Ns, washed out by same processes D, ID, S.
NB tension: $\gamma \gg Hs$ to ensure $Y_N \sim Y_N^{EQ}$ at $T \gtrsim M_1$. But $\gamma \ll Hs$ to minimise washout of $Y_L^{\alpha\alpha}$.

(for afictionados: calculate matrix elements in $T = 0$ field theory. include $\not{C}\not{P}$ in N scattering.)

So Boltzmann Eqns for total lepton number are...

Recall that for $Y_L^{\alpha\alpha} = (n_{\ell_\alpha} - n_{\overline{\ell}_\alpha})/s$

$$\frac{d}{dz} Y_L^{ee} = \frac{z}{sH}\left[\gamma\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon^{ee} - \frac{Y_L^{ee}}{Y_\ell^{eq}}\gamma^{ee}\right], \quad \text{with } \epsilon^{\alpha\alpha} = \frac{\Gamma_{N\to H\ell_\alpha} - \Gamma_{N\to\overline{H\ell}_\alpha}}{\Gamma_{N\to H\ell} + \Gamma_{N\to\overline{H\ell}}}$$

Sum up the flavoured BEs:

$$\sum_\alpha \frac{d}{dz} Y_L^{\alpha\alpha} = \frac{z}{sH}\left[\gamma\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\underbrace{(\epsilon^{ee} + \epsilon^{\mu\mu} + \epsilon^{\tau\tau})}_{\epsilon} - \frac{1}{Y_\ell^{eq}}\underbrace{(\gamma^{ee}Y_L^{ee} + \gamma^{\mu\mu}Y_L^{\mu\mu} + \gamma^{\tau\tau}Y_L^{\tau\tau})}_{\neq\gamma Y_L}\right]$$

$$= \frac{z}{sH}\left[\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon\gamma - \frac{\gamma^{ee}Y_L^{ee} + \gamma^{\mu\mu}Y_L^{\mu\mu} + \gamma^{\tau\tau}Y_L^{\tau\tau}}{Y_\ell^{eq}}\right]$$

Compare to the usual “single flavour” approx= consider lepton number, neglect flavour

$$\frac{dY_L}{dz} = \frac{z}{sH(M)}\left[\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\epsilon\gamma - (\gamma^{ee} + \gamma^{\mu\mu} + \gamma^{\tau\tau})\frac{Y_L^{ee} + Y_L^{\mu\mu} + Y_L^{\tau\tau}}{Y_\ell^{eq}}\right]$$

NOT the same.

Abada etal
Nardi etal

where $z = \frac{M_1}{T}$ is a time var, $\gamma^{\alpha\alpha} = \gamma_D^{\alpha\alpha} + \gamma_{\Delta L=1}^{\alpha\alpha}$ is $N \leftrightarrow H\ell_\alpha$ and $qt^c \leftrightarrow N\ell_\alpha$ etc rates

pheno consequences 1 - no bound on m_ν

In "single flavour" approx, successful thermal leptogenesis $\Rightarrow m_\nu \leq .1$. Recall:

$$Y_B \simeq 10^{-3}\epsilon\eta \qquad\qquad (\eta = H/\Gamma)$$

For degenerate ν_L ($m_\nu > m_{atm}$): Γ increases ($\Gamma \geq C m_\nu \Rightarrow \eta$ decreases), ϵ decreases:

$$\sum_\alpha \epsilon^{\alpha\alpha} \leq \frac{3M_1 \Delta m^2_{atm}}{8\pi v^2 m_\nu}$$

$\Rightarrow$ there is a $m_\nu \simeq 0.1$ eV above which cannot get big enough Y_B.

With flavour, more $C\not{P}$: Individual flavour asyms can grow with light ν mass scale m_ν:

$$\epsilon^{\alpha\alpha} \leq \frac{3M_1 m_\nu}{8\pi v^2}\sqrt{\frac{\Gamma_\alpha}{\Gamma}}$$

Partial decay rates $\Gamma(N \to \phi\ell_\alpha)$ still increase, but

$$Y_B \simeq 10^{-3}\sum_\alpha \epsilon_{\alpha\alpha}\eta_\alpha \qquad\qquad \left(\eta_\alpha = \frac{H}{\Gamma(N \to \phi\ell_\alpha)}\right)$$

can stay same. $\Rightarrow$ no leptogenesis bound $m_\nu \lesssim .1$ eV

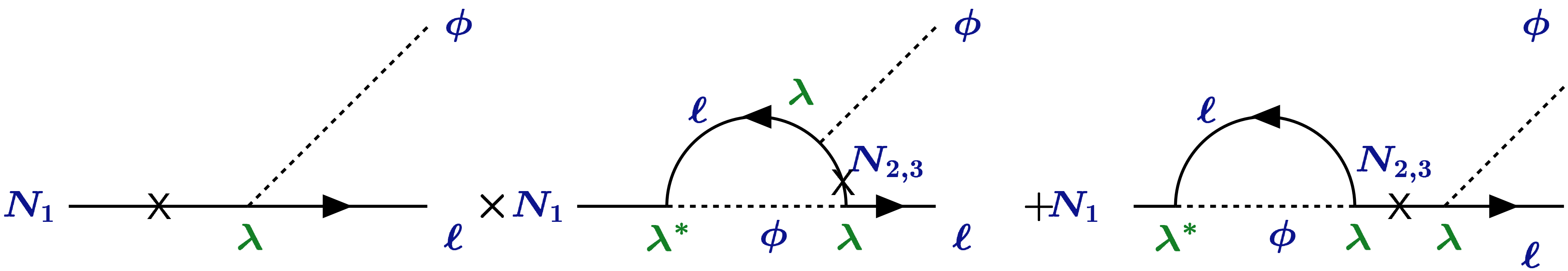
(caveat when $\Gamma(N \to \phi\ell) \gg \Gamma_\tau$, Blanchet diBari Raffelt get $m_\nu < 2$ eV)

unflavoured: Fukugita Yanagida
Buchmuller et al
Covi et al
Branco et al
Giudice et al
...

Thermal Leptogenesis with $M_1 \ll M_{2,3}$: $\not{CP}$ and $\epsilon^{\alpha\alpha}$

- $\not{CP}$, $\not{L}$: generate lepton asymmetry in flavour α due to $\not{CP}$ in N_1 interactions:

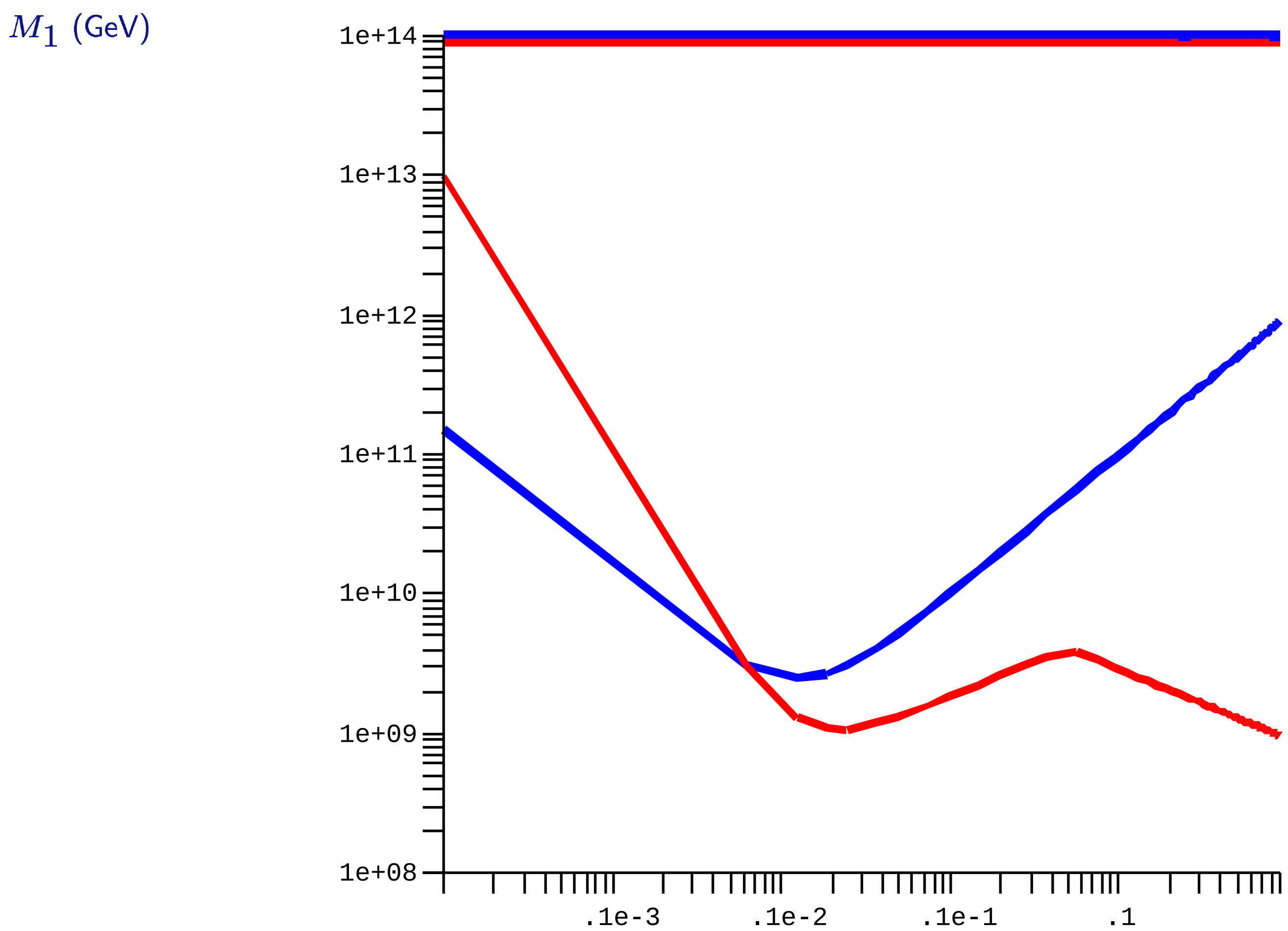
$$\epsilon^{\alpha\alpha} = \frac{\Gamma(N_1 \to H\ell_\alpha) - \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell}_\alpha)}{\Gamma(N \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})} \qquad (\text{recall } N_1 = \bar{N}_1)$$



$$\text{Can show :} \quad \sum_\alpha \epsilon^{\alpha\alpha} < \frac{3}{8\pi} \frac{(m_3 - m_1)M_1}{< H_u >^2} \sim 10^{-6} \frac{M_1}{10^9 \text{GeV}} \quad \Rightarrow M_1 \gtrsim 10^9 \text{ GeV}$$

phenomenological consequences

There is an envelope, in space of parameters leptogenesis depends on : M_1, $\tilde{m}$, ϵ, where leptogenesis *can* work.



$\tilde{m} \propto \Gamma(N \to HL)$

$\log \frac{\tilde{m}}{\text{eV}}$