Leptonic CP-violation: Zero, Maximal or between the Two Extremes

Yasaman Farzan

yasaman@theory.ipm.ac.ir

IPM

Tehran-Iran

Reference

This talk is based on

“Leptonic CP violation: Zero, maximal or between the two extremes",

Y. F. and A. Smirnov, JHEP 0701 (2007) 059

hep-ph/0610337

Symmetric Neutrino Mass Matrix

Assumption: three light Majorana neutrinos

$$m_\nu = \begin{bmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{bmatrix}.$$

Diagonalizing m_ν

$$U_{PMNS} = U_{23}(\theta_{23})\Gamma_\delta U_{13}(\theta_{13})\Gamma_{-\delta} U_{12}(\theta_{12})\Gamma_M.$$

$$\Gamma_\delta \equiv \mathrm{diag}(1, 1, e^{i\delta}), \quad \Gamma_M \equiv \mathrm{diag}(1, e^{i\phi_2/2}, e^{i\phi_3/2}).$$

What does the value of δ teach us?

Neutrino mass models which have a prediction for the value of δ:

- Zee model: No CP-violation;
- A_4 symmetry: Maximal Dirac phase, $\delta = \pi/2$
  Babu, Ma, Valle, PLB 552 (03) 207.
- $m_{\mu\mu} = m_{e\tau(\mu)} = 0$ or $m_{\tau\tau} = m_{e\tau(\mu)} = 0$: $\delta \simeq \pi/2$
  Matsuda and Nishiura, PRD74 (06) 33014; Kaneko et al., JHEP508 (05) 73; Xing PLB 530 (02) 159.

Goal and Agenda

The final goal of studying the CP-violation in the lepton sector is to unveil the underlying theory by measuring δ and/or Majorana phases.

Here, we try to pave the way for this goal by searching for principles, symmetries as well as phenomenological and empirical relations that lead to particular values of δ and $\phi_{1,2}$

Plan of talk

- Zero Dirac phase
  – No CP-violation $\equiv$ zero Dirac as well as Majorana phases; Rephasing Invariants;
  – $\delta = 0$ but $\phi_1, \phi_2 \neq 0$: Conditions on m_ν;

Plan of talk

- Zero Dirac phase
  – No CP-violation $\equiv$ zero Dirac as well as Majorana phases; Rephasing Invariants;
  – $\delta = 0$ but $\phi_1, \phi_2 \neq 0$: Conditions on m_ν;
- Maximal δ; $\mu - \tau$ reflection symmetry;

Plan of talk

- Zero Dirac phase
  – No CP-violation $\equiv$ zero Dirac as well as Majorana phases; Rephasing Invariants;
  – $\delta = 0$ but $\phi_1, \phi_2 \neq 0$: Conditions on m_ν;
- Maximal δ; $\mu - \tau$ reflection symmetry;
- Arbitrary δ between zero and maximal value: Generalized $\mu - \tau$ reflection symmetry.

Plan of talk

- Zero Dirac phase
  – No CP-violation $\equiv$ zero Dirac as well as Majorana phases; Rephasing Invariants;
  – $\delta = 0$ but $\phi_1, \phi_2 \neq 0$: Conditions on m_ν;
- Maximal δ; $\mu - \tau$ reflection symmetry;
- Arbitrary δ between zero and maximal value: Generalized $\mu - \tau$ reflection symmetry.
- Relations between the phases of the CKM and PMNS matrix

No CP-violation

In the standard basis,

$$m_\nu = U_{PMNS}\mathrm{Diag}(|m_1|,|m_2|,|m_3|)U_{PMNS}^T$$

$$U_{PMNS} = U_{23}(\theta_{23})\Gamma_\delta U_{13}(\theta_{13})\Gamma_{-\delta}U_{12}(\theta_{12})\Gamma_M.$$

$$\Gamma_\delta \equiv \mathrm{diag}(1,1,e^{i\delta}), \quad \Gamma_M \equiv \mathrm{diag}(1,e^{i\phi_2/2},e^{i\phi_3/2})$$

- Real $m_\nu \Rightarrow$ No CP-violation

No CP-violation

In the standard basis,

$$m_\nu = U_{PMNS}\text{Diag}(|m_1|,|m_2|,|m_3|)U^T_{PMNS}$$

$$U_{PMNS} = U_{23}(\theta_{23})\Gamma_\delta U_{13}(\theta_{13})\Gamma_{-\delta}U_{12}(\theta_{12})\Gamma_M.$$

$$\Gamma_\delta \equiv \text{diag}(1,1,e^{i\delta}), \quad \Gamma_M \equiv \text{diag}(1,e^{i\phi_2/2},e^{i\phi_3/2})$$

- Real $m_\nu \Rightarrow$ No CP-violation
- The opposite is not valid: m_ν may appear complex but its CP-invariance may still be maintained. This is because of the possibility of unphysical phases.

Rephasing

As $\nu_e \to e^{i\alpha_e}\nu_e \quad \nu_\mu \to e^{i\alpha_\mu}\nu_\mu \quad \nu_\tau \to e^{i\alpha_\tau}\nu_\tau$:

$$m_\nu = \begin{bmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{bmatrix}$$

changes as follows:

$$\begin{bmatrix} m_{ee}e^{2i\alpha_e} & m_{e\mu}e^{i(\alpha_e+\alpha_\mu)} & m_{e\tau}e^{i(\alpha_e+\alpha_\tau)} \\ m_{e\mu}e^{i(\alpha_e+\alpha_\mu)} & m_{\mu\mu}e^{2i\alpha_\mu} & m_{\mu\tau}e^{i(\alpha_\mu+\alpha_\tau)} \\ m_{e\tau}e^{i(\alpha_e+\alpha_\tau)} & m_{\mu\tau}e^{i(\alpha_\mu+\alpha_\tau)} & m_{\tau\tau}e^{2i\alpha_\tau} \end{bmatrix}$$

Rephasing Invariants

$$I_1 \equiv m_{e\mu}^2 m_{ee}^* m_{\mu\mu}^*$$

$$I_2 \equiv m_{e\tau}^2 m_{ee}^* m_{\tau\tau}^*$$

$$I_3 \equiv m_{\mu\tau}^2 m_{\tau\tau}^* m_{\mu\mu}^*$$

$$I_4 \equiv m_{e\mu}^2 m_{\mu\mu}^* / m_{e\tau}^2 m_{\tau\tau}^* = I_1/I_2$$

$$I_5 \equiv m_{e\tau} m_{\mu\tau} m_{\tau\tau}^* / m_{e\mu} = \sqrt{I_3 I_2 / I_1}$$

Y.F. and A. Smirnov, JHEP 0701 (2007) 059; Sarkar and Singh, hep-ph/0608030.

These invariant are sensitive not only to δ but also on the Majorana phases. As we will see, they can help to derive the Majorana phases

No CP-violation: $\delta, \phi_1, \phi_2 = 0$

CP is conserved if and only if

- All diagonal elements of m_ν are nonzero: Im$I_1 = 0$, Im$I_2 = 0$ and Im$I_5 = 0$;

No CP-violation: $\delta, \phi_1, \phi_2 = 0$

CP is conserved if and only if

- All diagonal elements of m_ν are nonzero:
  Im$I_1 = 0$, Im$I_2 = 0$ and Im$I_5 = 0$;
- $m_{ee} = 0$ but $m_{\mu\mu}, m_{\tau\tau} \neq 0$:
  Im$I_3 = 0$ and Im$I_5 = 0$

No CP-violation: $\delta, \phi_1, \phi_2 = 0$

CP is conserved if and only if

- All diagonal elements of m_ν are nonzero:
  Im$I_1 = 0$, Im$I_2 = 0$ and Im$I_5 = 0$;
- $m_{ee} = 0$ but $m_{\mu\mu}, m_{\tau\tau} \neq 0$:
  Im$I_3 = 0$ and Im$I_5 = 0$
- $m_{ee} = m_{\mu\mu} = 0$ and $m_{\tau\tau} \neq 0$:
  Im$I_5 = 0$;

No CP-violation: $\delta, \phi_1, \phi_2 = 0$

CP is conserved if and only if

- All diagonal elements of m_ν are nonzero:
  Im$I_1 = 0$, Im$I_2 = 0$ and Im$I_5 = 0$;
- $m_{ee} = 0$ but $m_{\mu\mu}, m_{\tau\tau} \neq 0$:
  Im$I_3 = 0$ and Im$I_5 = 0$
- $m_{ee} = m_{\mu\mu} = 0$ and $m_{\tau\tau} \neq 0$:
  Im$I_5 = 0$;
- $m_{ee} = m_{\mu\mu} = m_{\tau\tau} = 0$ (disfavored by the data):
  no CP-violation

Zero δ

$$h \equiv m_\nu \cdot m_\nu^\dagger$$

In the flavor basis

$$\mathrm{Im}[h_{e\mu} h_{\mu\tau} h_{\tau e}] = \Delta m_{12}^2 \Delta m_{32}^2 \Delta m_{13}^2 J_{CP}$$

where

$$J_{CP} = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin\delta$$

If either of $h_{e\mu}$, $h_{\mu\tau}$ or $h_{\tau e}$ vanishes, J_{CP} will also vanish. Let us consider them one by one.

Vanishing $h_{e\mu}$

$$h_{e\mu} \Rightarrow \sin\delta = 0, \quad s_{13} \simeq -0.5 \sin 2\theta_{12} \cot\theta_{23} \Delta m^2_{21} / \Delta m^2_{31}$$

Numerically, $|s_{13}| \sim 0.016$ which is beyond the reach of

Double CHOOZ

F. Ardellier et al., hep-ex/0405032

Daya Bay

Daya Bay Collaboration, hep-ex/0701029

Long baseline experiments

Itow, hep-ex/0106019.

Positive s_{13}-signal $\Rightarrow h_{e\mu} \neq 0$

An example of vanishing $h_{e\mu}$

$$m_\nu = \begin{bmatrix} 0 & k & -k \\ k & A & A \\ -k & A & A(1+\epsilon) \end{bmatrix} \Rightarrow h_{e\mu} = 0$$

To accommodate the neutrino data $k \ll A$ and $\epsilon \ll 1$.
k and A can be made real but in this basis ϕ_ϵ can induce Majorana CP-violating phase.
Using the rephasing invariant I_3:

$$\phi_3 \simeq -\phi_\epsilon.$$

Vanishing $h_{e\tau}$ and $h_{\tau\mu}$

- $h_{\tau\mu} = 0$ is NOT compatible with the neutrino data.

Vanishing $h_{e\tau}$ and $h_{\tau\mu}$

- $h_{\tau\mu} = 0$ is NOT compatible with the neutrino data.
- $h_{e\tau} = 0$, similarly to $h_{e\mu} = 0$, results in

$$\sin\delta = 0, \quad s_{13} \simeq 0.5 \sin 2\theta_{12} \tan\theta_{23} \Delta m^2_{21}/\Delta m^2_{31}$$

Vanishing $h_{e\tau}$ and $h_{\tau\mu}$

- $h_{\tau\mu} = 0$ is NOT compatible with the neutrino data.
- $h_{e\tau} = 0$, similarly to $h_{e\mu} = 0$, results in

$$\sin\delta = 0, \quad s_{13} \simeq 0.5 \sin 2\theta_{12} \tan\theta_{23} \Delta m^2_{21}/\Delta m^2_{31}$$

- $s_{13} \ll 0.1$: No signal of nonzero s_{13} in forthcoming experiments.

General Form of m_ν with $\sin\delta \ll 1$

$$m_\nu = \begin{bmatrix} r-2sx-t & s(1+x\eta-\alpha)+t\eta & -s(1-x\eta-\alpha) \\ \cdots & r+2t\alpha-s\eta & t \\ \cdots & \cdots & r-2t\alpha+s\eta \end{bmatrix}$$

where $\alpha, \eta \ll x \sim 1$.

$$\delta \sim \mathrm{Max}[\eta, \alpha] \ll 1$$

α, η and x are real but r, s and t are in general complex.

For normal hierarchical scheme: $|s| \ll |t| \simeq |r|$ and for quasi-degenerate scheme $|s| \sim |t| \sim |r|$

$$\cos\theta_{23} = \frac{(1-\alpha)}{\sqrt{2}}, \quad \sin\theta_{13} = \frac{\eta}{\sqrt{2}}, \quad \cot 2\theta_{12} = \frac{x}{\sqrt{2}}$$

Maximal CP-violation, $|\sin\delta| = 1$

Maximal CP-violation is a basis-independent concept:

For given θ_{12}, θ_{23} and θ_{13} (to be measured by the reactor experiments), J_{CP} (to be extracted from $P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$) is maximal.

General consideration

We are looking for symmetries that have a prediction for δ.

We naturally expect such a symmetry to involve both neutrino and its CP-conjugate:

$$\nu_\alpha \Rightarrow P_{\alpha,\beta} \nu^c_\beta$$

where $P_{\alpha\beta}$ is a unitary matrix and α and β are flavor indices

$\mu - \tau$ reflection symmetry (which differs from the famous $\mu - \tau$ exchange symmetry) is an example of such a symmetry with

$$P = \begin{bmatrix} \xi_1 & 0 & 0 \\ 0 & 0 & \xi_2 \\ 0 & \xi_2 & 0 \end{bmatrix}$$

$\mu - \tau$ reflection symmetry

$$\nu_e \rightarrow \xi_1 \nu_e^c \quad \nu_\mu \rightarrow \xi_2 \nu_\tau^c \quad \nu_\tau \rightarrow \xi_2 \nu_\mu^c$$

where ξ_i are pure phases. After proper rephasing

$$m_\nu = \begin{bmatrix} f & we^{-i\sigma} & -we^{i\sigma} \\ \cdots & y & z \\ \cdots & \cdots & y \end{bmatrix}$$

With real parameters.

Harrison and Scott, PLB 547 (02) 219; Grimus and Lavoura, PLB 579 (04) 113.

Consequences of the $\mu - \tau$ reflection

$$m_\nu = \begin{bmatrix} f & we^{-i\sigma} & -we^{i\sigma} \\ \cdots & y & z \\ \cdots & \cdots & y \end{bmatrix}$$

- $\cos 2\theta_{23} = 0$

Consequences of the $\mu - \tau$ reflection

$$m_\nu = \begin{bmatrix} f & we^{-i\sigma} & -we^{i\sigma} \\ \cdots & y & z \\ \cdots & \cdots & y \end{bmatrix}$$

- $\cos 2\theta_{23} = 0$
- Zero Majorana phases: $\sin\phi_2 = \sin\phi_3 = 0$

Consequences of the $\mu - \tau$ reflection

$$m_\nu = \begin{bmatrix} f & we^{-i\sigma} & -we^{i\sigma} \\ \cdots & y & z \\ \cdots & \cdots & y \end{bmatrix}$$

- $\cos 2\theta_{23} = 0$
- Zero Majorana phases: $\sin\phi_2 = \sin\phi_3 = 0$
- $\cos\delta s_{13} = 0$

$$\sin\sigma = 0: \quad s_{13} = 0$$

$$\sin\sigma \neq 0: \quad \cos\delta = 0$$

$\sin\sigma = 0$ corresponds to special (real) case of $\mu - \tau$ exchange symmetry.

To establish $\mu - \tau$ reflection

- $\cos 2\theta_{23} = 0$
  T2K; NoνA

To establish $\mu - \tau$ reflection

- $\cos 2\theta_{23} = 0$
  T2K; NoνA
- Zero Majorana phases: $\sin\phi_2 = \sin\phi_3 = 0$

To establish $\mu - \tau$ reflection

- $\cos 2\theta_{23} = 0$
  T2K; NoνA
- Zero Majorana phases: $\sin\phi_2 = \sin\phi_3 = 0$
- $\sin\theta_{13}\cos\delta = 0$

Generalized $\mu - \tau$ reflection

$$\nu_\alpha \Rightarrow \sum_\beta P_{\alpha\beta} \nu^c_\beta \quad \text{where}$$

$$P(\alpha,\phi) = U_{23}(\alpha)\mathrm{Diag}[1,1,e^{i\phi}]U^T_{23}(\alpha)$$

Special case of $e^{i\phi} = -1$ and $\alpha = \pi/4$ yields

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

which corresponds to the $\mu - \tau$ reflection symmetry.

Consequences and prerequisites

$$P(\alpha,\phi)\cdot m_\nu \cdot P(\alpha,\phi) = m_\nu^*$$

is satisfied if and only if

$\theta_{23} = \alpha$

and

$$\delta = -\frac{\phi}{2}, \phi_2 = 0 \text{ or } \pi \quad \text{and } \phi_3 = -\phi$$

Thus, $\phi_3 = 2\delta$. To establish the symmetry, in addition to measuring θ_{23} and δ, one also has to measure ϕ_2 and ϕ_3.

Relating δ_{CKM} and δ

Conditions can be formulated that lead to simple and immediate relations between the phases, δ_{CKM} and δ

In the following, we formulate such conditions in the context of Quark Lepton Complementarity (QLC). Raidal, PRL **93**, 161801 (04); Minakata and Smirnov, PRD **70**, 073009 (04); Ferrandis and Pakvasa, PRD **71**, 033004 (05); Kang, Kim and Lee, PLB **619**, 129 (05); Li and Ma, PRD **71**, 097301 (05); Cheung et al, PRD **72**, 036003 (05); Xing, PLB **618**, 141 (05); Datta, Everett and Ramond, PLB **620**, 42 (05).

Formulating the assumption

We make the following assumptions:

1) Type-I seesaw mechanism; in the flavor basis

$m_\nu = U_L^* m_D^{diag} M_N^{-1} m_D^{diag} U_L^\dagger;$

2) Due to the quark-lepton symmetry or unification

$U_L = V_{CKM}^\dagger;$

3) The matrix $m_D^{diag} M_N^{-1} m_D^{diag}$ is diagonalized by a bi-maximal rotation

$U_{bm} = U_{23}^m U_{12}^m,$

where U_{ij}^m is the maximal $(\pi/4)$ rotation in the $ij-$ plane.

Barger et al, PLB **437** (98) 107.

Consequence of this assumption

Consequence: $U_{PMNS} = V^{\dagger}_{CKM} U_{bm}$

$$|\sin\theta_{13}| = \frac{1}{\sqrt{2}} \left| V^{\dagger}_{td} + V^{\dagger}_{cd} \right| \simeq \frac{\sin\theta_C}{\sqrt{2}}$$

and

$$\sin\delta \approx \frac{|V_{ub}|}{\sin\theta_C} \sin\delta_{CKM} \simeq 0.02$$

Ambiguity: Under $V^{\dagger}_{CKM} \to V^{\dagger}_{CKM} \mathrm{Diag}[e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}]$ δ changes but $\sin\theta_{13}$ remains invariant.

Testing the prediction

The prediction for s_{13} lies in a range that can be probed by forthcoming experiments:

Double CHOOZ

F. Ardellier et al., hep-ex/0405032

Daya Bay

Daya Bay Collaboration, hep-ex/0701029 and J. Cao, Nucl. Phys. Proc. Suppl. **155** (2006) 229

T2K

Y. Itow *et al.*, arXiv:hep-ex/0106019.

NOν A

Ayres *et al.* [NOvA Collaboration], arXiv:hep-ex/0503053.

Summary

We have studied symmetries and principles that yield certain values for δ:

- zero (or a very small) phase;
  Rephasing invariants General m_ν with $|\sin\delta| \ll 1$
- a maximal CP-violating phase, $\delta = \pi/2$;
- an arbitrary phase that depends on the parameter of symmetry transformation:
  Generalized $\mu - \tau$ reflection symmetry: $\phi_2 = 0 \quad \phi_3 = 2\delta$
- We have discussed a possible relation between the phases of the CKM and PMNS matrices: $\sin\delta \simeq 0.02$

$\delta = 0$ or $\delta \ll 1$

Rephasing invariants: I_1, I_2, I_3, I_4, I_5

If Im[I_1]=0, Im[I_2]=0 and Im[I_5]=0, m_ν will be CP-invariant: $\delta = \phi_2 = \phi_3 = 0$. We have formulated the general form of m_ν for $|\sin\delta| \ll 1$.

any of off-diagonal elements of $(m_\nu \cdot m_\nu^\dagger) = 0 \Rightarrow J_{CP} = 0$

- $h_{e\mu} = (m_\nu \cdot m_\nu^\dagger)_{e\mu} = 0$:
  $s_{13} = -0.5 \sin 2\theta_{12} \cot\theta_{23} \Delta m_{21}^2 / \Delta m_{31}^2 \sim 0.02$
- $(m_\nu \cdot m_\nu^\dagger)_{\tau\mu} = 0$: Not compatible with the data
- $h_{\tau e} = (m_\nu \cdot m_\nu^\dagger)_{\tau e} = 0$:
  $s_{13} = 0.5 \sin 2\theta_{12} \tan\theta_{23} \Delta m_{21}^2 / \Delta m_{31}^2 \sim 0.02$

Finding $s_{13} \sim 0.1$ will exclude $h_{e\mu(\tau)} = 0$.

Maximal Dirac Phase

The $\mu - \tau$ reflection symmetry yields $\sin\delta = 1$ but it is not a necessary condition for maximal Dirac phase.

The $\mu - \tau$ reflection symmetry also requires (and implies)

- Maximal 23-mixing: $\cos 2\theta_{23} = 0$;
- Zero Majorana phases

Generalized $\mu - \tau$ reflection

$\nu_\alpha \rightarrow \sum_\beta P_{\alpha\beta}(\alpha,\phi)\nu^c_\beta$ where

$$P(\alpha,\phi) = U_{23}(\alpha)\mathrm{Diag}[1,1,e^{i\phi}]U^T_{23}(\alpha).$$

- $\theta_{23} = \alpha$
- $\delta = -\phi/2, \phi_2 = 0, \phi_3 = -\phi$

$$\delta = \phi_3/2$$