Broken μ–τ Symmetry and CP Violation or: Aspects of μ–τ Symmetry Breaking

Werner Rodejohann
(MPI Heidelberg)
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Max-Planck-Institut für Kernphysik

- What is μ–τ symmetry? $\longleftrightarrow$ $U_{e3} = 0$ and $\theta_{23} = \pi/4$
- How can we break μ–τ symmetry?
  - Soft breaking $*$
  - $U = U_\ell^\dagger U_\nu$ $*$
  - Planck-scale effects $*$
  - Type II see-saw
  - Renormalization
  - …

Neutrino Mixing

1) "Best-fit matrix":

$$|U| = |U_\ell^\dagger\, U_\nu| = \begin{pmatrix} 0.83 & 0.56 & 0 \\ 0.39 & 0.59 & 1/\sqrt{2} \\ 0.39 & 0.59 & 1/\sqrt{2} \end{pmatrix}$$

diagonalizes the mass matrix

$$m_\nu = \begin{pmatrix} A & B & D \\ B & E & F \\ D & F & G \end{pmatrix} \Rightarrow \text{ one eigenvector is (close to)} \begin{pmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

2) In general, CP violation described by

$$J_{CP} = \mathrm{Im}\{U_{e1}\, U_{\mu 2}\, U_{e2}^*\, U_{\mu 1}^*\} = \frac{1}{8} \sin 2\theta_{12}\, \sin 2\theta_{23}\, \sin 2\theta_{13}\, \cos\theta_{13}\, \sin\delta$$

What is μ–τ symmetry?

simple exchange (Z_2 or S_2) symmetry

$$P_{\mu\tau} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ gives } P_{\mu\tau}\,\nu = P_{\mu\tau} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} = \begin{pmatrix} e \\ \tau \\ \mu \end{pmatrix}$$

apply also to mass term $\overline{\nu}\, m_\nu\, \nu^c$

$$P_{\mu\tau}\, m_\nu\, (P^*_{\mu\tau})^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} A & B & D \\ B & E & F \\ D & F & G \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} A & D & B \\ D & G & F \\ B & F & E \end{pmatrix} \Rightarrow \text{is symmetry if } \begin{matrix} D = B \\ G = E \end{matrix} \Rightarrow m_\nu = \begin{pmatrix} A & B & B \\ B & D & E \\ B & E & D \end{pmatrix}$$

What is μ–τ symmetry?

$$m_\nu = \begin{pmatrix} A & B & B \\ B & D & E \\ B & E & D \end{pmatrix} \text{ has eigenvector } \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \text{ to eigenvalue } D - E$$

$\Rightarrow U_{e3} = 0$ and $\theta_{23} = \pi/4$!!!!

- 6 free parameters: $m_{1,2,3}$, θ_{12} and Majorana phases
  no Dirac phase δ
- Why not charged leptons? $U = U_\ell^\dagger \, U_\nu$
  - if in symmetry basis charged leptons diagonal: $m_\mu = m_\tau$
  - if in symmetry basis charged leptons NOT diagonal:
    $\theta_{13} = \theta_{23} = 0$, $\theta_{12} \neq 0$

∗ more than one Higgs doublet (`Mohapatra; Grimus, Lavoura`)

∗ or (tuned!) quasi-degenerate neutrinos with equal CP parities and softly broken μ–τ symmetry (`Joshipura; Haba, W.R.`)

μ–τ Symmetry bottom-up

insert $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ in mass matrix

$$U = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \Leftrightarrow m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(D+E) & \frac{1}{2}(E-D) \\ \cdot & \cdot & \frac{1}{2}(D+E) \end{pmatrix}$$

$$A = m_1\, \cos^2\theta_{12} + e^{2i\alpha}\, m_2\, \sin^2\theta_{12}$$

$$B = \frac{\sin\theta_{12}\, \cos\theta_{12}}{\sqrt{2}} \left(e^{2i\alpha}\, m_2 - m_1\right)$$

$$D = \left(m_1\, \sin^2\theta_{12} + e^{2i\alpha}\, m_2 \cos^2\theta_{12}\right)$$

$$E = e^{2i\beta}\, m_3$$

Fukuyama and Nishiura; Mohapatra and Nussinov; Ma and Raidal; Lam; Harrison and Scott; Kitabayashi and Yasue; Grimus and Lavoura; Koide; Ghosal; Grimus, Joshipura, Kaneko, Lavoura, Sawanaka, Tanimoto; Mohapatra; de Gouvea; Choubey and W.R.; Mohapatra and W.R.; Mohapatra, Nasri and Yu; Ahn, Kang, Kim and Lee;...

TWO POPULAR CASES

- $\sin^2\theta_{12} = 1/3$: "Tri–bimaximal Mixing"

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \Leftrightarrow m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

- $\sin^2\theta_{12} = 1/2$: "Bimaximal Mixing"

$$U = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\sqrt{\frac{1}{2}} \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix} \Leftrightarrow m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+D) & \frac{1}{2}(A-D) \\ \cdot & \cdot & \frac{1}{2}(A+D) \end{pmatrix}$$

Note: mixing angles independent of mass matrix entries

Usually symmetries based on finite groups $A_4, D_5, S_3, \ldots$, typically with Z_2

normal hierarchy: $m_3 \simeq \sqrt{\Delta m^2_{\rm A}} \gg m_{2,1}$:

$$m_\nu \simeq \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \left(\begin{array}{ccc} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{array} \right) \text{ conserves } L_e$$

inverted hierarchy: $m_2 \simeq m_1 \simeq \sqrt{\Delta m^2_{\rm A}} \gg m_3$, $\theta_{12} = \pi/4$, $\alpha = \pi/2$:

$$m_\nu \simeq \sqrt{\frac{\Delta m^2_{\rm A}}{2}} \left(\begin{array}{ccc} 0 & 1 & 1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{array} \right) \text{ conserves } L_e - L_\mu - L_\tau$$

quasi-degeneracy: $m_3 \simeq m_2 \simeq m_1 \equiv m_0$, $\alpha = 0$, $\beta = \pi/2$:

$$m_\nu \simeq m_0 \left(\begin{array}{ccc} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{array} \right) \text{ conserves } L_\mu - L_\tau$$

A simple $U(1)$ for m_ν?

With L_e, L_μ and L_τ we have only 3 allowed possibilities

L_e Normal Hierarchy Barbieri; Vissani, Buchmüller, Yanagida	$\begin{pmatrix} 0 & 0 & 0 \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	$R = \frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}} \simeq \lvert U_{e3}\rvert^2$ $\tan^2\theta_{23} \simeq 1 + \lvert U_{e3}\rvert$ $\lvert m_{ee}\rvert \simeq \sqrt{\Delta m^2_\odot}$
$L_e - L_\mu - L_\tau$ Inverted Hierarchy Petcov;...	$\begin{pmatrix} 0 & a & b \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	requires U_ℓ: ideal for QLC $\tan^2\theta_{12} \simeq 1 - 4\,\lvert U_{e3}\rvert$ $\lvert m_{ee}\rvert \simeq \sqrt{\Delta m^2_{\rm A}}$
$L_\mu - L_\tau$ quasi–degenerate νs Choubey, W.R.	$\begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$	$\mu - \tau$ symmetric!! $\Rightarrow U_{e3} = 0$ and $\theta_{23} = \pi/4$ $\lvert m_{ee}\rvert \simeq m_0$

S. Choubey, W.R., *Phys. Rev.* D **72**, 033016 (2005)

($L_e + L_\mu + L_\tau$ means Dirac neutrinos...)

See-saw and μ–τ Symmetry

Mohapatra, Nasri, Yu

$$m_D = \begin{pmatrix} a & b & b \\ d & e & f \\ d & f & e \end{pmatrix} \text{ and } M_R = \begin{pmatrix} A & B & B \\ B & D & E \\ B & E & D \end{pmatrix}$$

- gives μ–τ symmetric low energy mass matrix $m_\nu = m_D \, M_R^{-1} \, m_D^T$
- successful thermal leptogenesis possible
- $\mathrm{BR}(\tau \to e\gamma) = \mathrm{BR}(\mu \to e\gamma)$

Breaking of μ–τ symmetry: What will it mean?

$$\theta_{23} \neq \pi/4$$

$$\theta_{13} \neq 0$$

$$\delta \neq 0$$

Possibly correlation between observables

$\Rightarrow$ Long-Baseline Experiments, νfacs, β-beams,...

Alternative: Neutrino Telescopes!

Breaking of μ–τ symmetry: Neutrino Telescopes

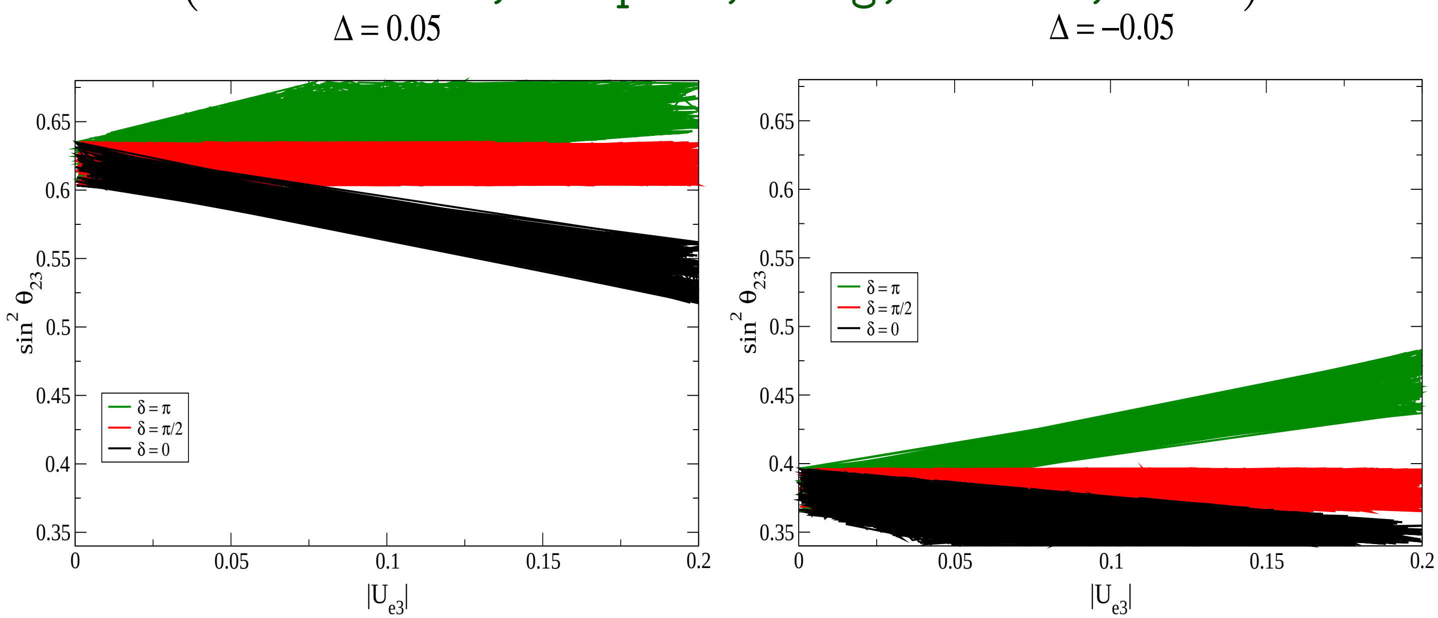
High Energy Neutrinos from $pp \to \pi^\pm$:

$$(\Phi_e : \Phi_\mu : \Phi_\tau)^0 = (1:2:0) \to (1 + 2\,\Delta : 1 - \Delta : 1 - \Delta) \neq (1:1:1)$$

$$\Delta \equiv \frac{1}{4}\,\cos\delta\,\sin 4\theta_{12}\,|U_{e3}| + 2\,s_{12}^2\,c_{12}^2\,\left(\frac{1}{2} - \sin^2\theta_{23}\right)$$

measure breaking of μ–τ symmetry with flux ratios

(Kachelriess, Serpico; Xing; Winter; W.R.)



W.R., JCAP **0701**, 029 (2007)

How to break μ–τ symmetry: soft breaking

Mohapatra, JHEP **0410**, 027 (2004); Grimus *et al.*, NPB **713**, 151 (2005)

Normal hierarchy (L_e)

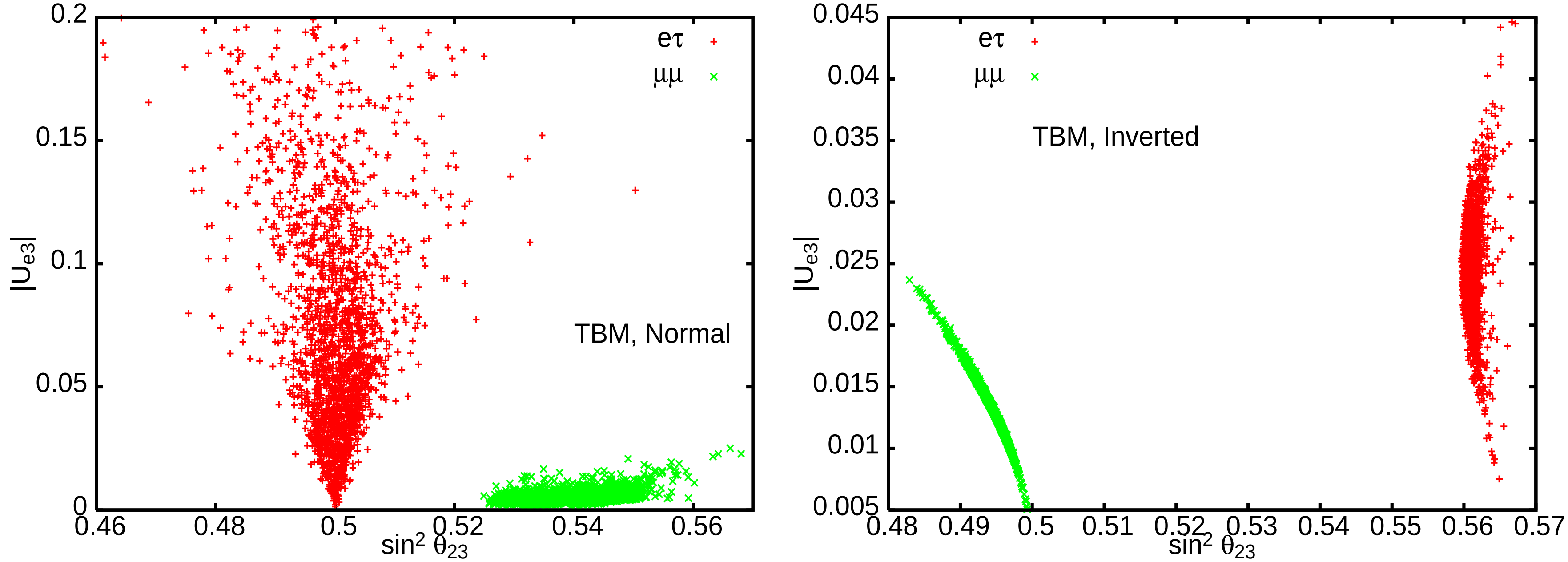
$$m_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & d\,\epsilon \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$ can be broken in e-row or $\mu\tau$ block

$$\begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & b\,\epsilon \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$ gives $$\theta_{13} \simeq \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}} \gg \theta_{23} - \pi/4 \simeq \frac{\Delta m_\odot^2}{\Delta m_A^2}$$

$$\begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & d\,\epsilon \\ \cdot & 1+a\,\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$ gives $$\theta_{13} \simeq \frac{\Delta m_\odot^2}{\Delta m_A^2} \ll \theta_{23} - \pi/4 \simeq \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}}$$

Example tri-bimaximal mixing:

Plentinger, W.R., Phys. Lett. B **625**, 264 (2005)



breaking in $e\tau$ element vs. breaking in $\mu\mu$ element

correlation weaker in inverted hierarchy

Breaking μ–τ Symmetry by a phase

Mohapatra, W.R., Phys. Rev. D **72**, 053001 (2005)

Normal hierarchy

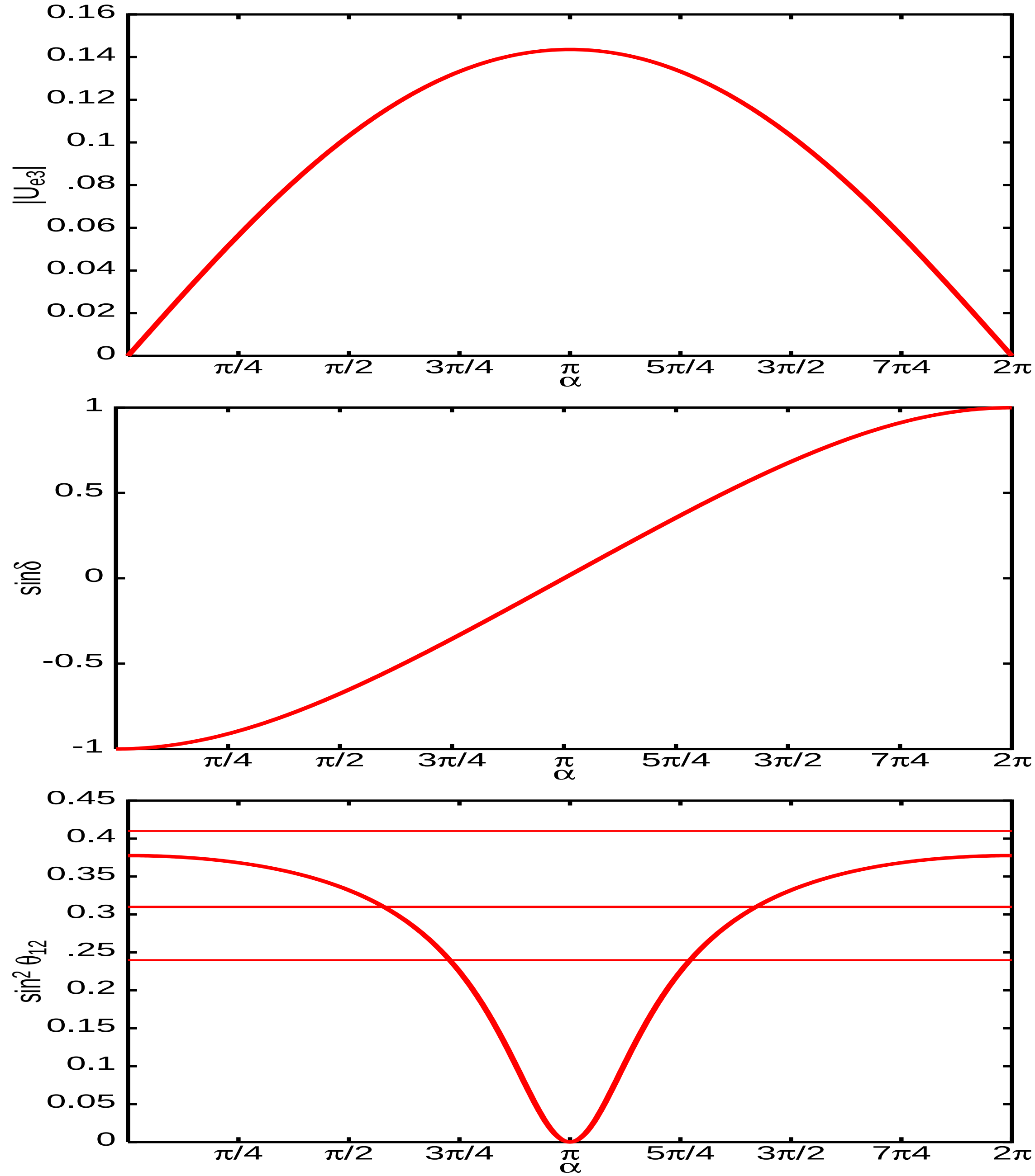
$$m_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} d\,\epsilon^2 & a\,\epsilon & a\,\epsilon \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix} \longrightarrow \begin{pmatrix} d\,\epsilon^2 & a\,\epsilon & a\,\epsilon\, e^{i\alpha} \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$

is only possibility for breaking with phase

Results:

$$\left.\begin{aligned} |U_{e3}| &\simeq \frac{a}{2}\,\epsilon\,\sqrt{1-\cos\alpha} \\ \sin\delta &\simeq -\cos\alpha/2 \\ \tan 2\theta_{12} &\simeq 2\,a\sqrt{1+\cos\alpha} \\ \theta_{23}-\pi/4 &\simeq -\frac{a}{2}\,\epsilon^2\cos\alpha/2 \end{aligned}\right\} \Rightarrow \frac{|U_{e3}|}{\tan 2\theta_{12}} \simeq \frac{\epsilon}{4}\tan\alpha/2$$

maximal $|U_{e3}|$ for $\theta_{12} = 0$



0.16
0.14
0.12
0.1
.08
0.06
0.04
0.02
0
$|U_{e3}|$
π/4
π/2
3π/4
π
5π/4
3π/2
7π4
2π
α
1
0.5
0
-0.5
-1
sinδ
π/4
π/2
3π/4
π
5π/4
3π/2
7π4
2π
α
0.45
0.4
0.35
0.3
.25
0.2
0.15
0.1
0.05
0
$\sin^2\theta_{12}$
π/4
π/2
3π/4
π
5π/4
3π/2
7π4
2π
α

A neat special case

$$m_\nu = \frac{\sqrt{\Delta m_{\rm A}^2}}{2} \begin{pmatrix} d\,\epsilon^2 & a\,\epsilon\,e^{-i\alpha} & a\,\epsilon\,e^{i\alpha} \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$

gives maximal CP violation $(\delta = \pi/2)$!!

$$|U_{e3}| \simeq \frac{a}{\sqrt{2}}\,\epsilon\,\sin\alpha$$

$$\tan 2\theta_{12} \simeq 2\sqrt{2}\,a\cos\alpha$$

Harrison, Scott; Ma; Aizawa *et al.*; **Mohapatra, W.R.; Baba, Yasue; Farzan, Smirnov**

"μ–τ reflection symmetry"

(μ–τ symmetry and CP transformation)

Breaking of μ–τ symmetry: $U = U_\ell^\dagger\, U_\nu$

Always possible to write

Frampton, Petcov, W.R., Nucl. Phys. B **687**, 31 (2004)

$$U_\nu = P \begin{pmatrix} c^\nu_{12}\, c^\nu_{13} & s^\nu_{12}\, c^\nu_{13} & s^\nu_{13} \\ -s^\nu_{12}\, c^\nu_{23} - c^\nu_{12}\, s^\nu_{23}\, s^\nu_{13}\, e^{i\xi} & c^\nu_{12}\, c^\nu_{23} - s^\nu_{12}\, s^\nu_{23}\, s^\nu_{13}\, e^{i\xi} & s^\nu_{23}\, c^\nu_{13}\, e^{i\xi} \\ s^\nu_{12}\, s^\nu_{23} - c^\nu_{12}\, c^\nu_{23}\, s^\nu_{13}\, e^{i\xi} & -c^\nu_{12}\, s^\nu_{23} - s^\nu_{12}\, c^\nu_{23}\, s^\nu_{13}\, e^{i\xi} & c^\nu_{23}\, c^\nu_{13}\, e^{i\xi} \end{pmatrix} Q$$

with $P = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})$ and $Q = \mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$

$$U_\ell = \begin{pmatrix} c^\ell_{12}\, c^\ell_{13} & s^\ell_{12}\, c^\ell_{13} & s^\ell_{13} \\ -s^\ell_{12}\, c^\ell_{23} - c^\ell_{12}\, s^\ell_{23}\, s^\ell_{13}\, e^{i\psi} & c^\ell_{12}\, c^\ell_{23} - s^\ell_{12}\, s^\ell_{23}\, s^\ell_{13}\, e^{i\psi} & s^\ell_{23}\, c^\ell_{13}\, e^{i\psi} \\ s^\ell_{12}\, s^\ell_{23} - c^\ell_{12}\, c^\ell_{23}\, s^\ell_{13}\, e^{i\psi} & -c^\ell_{12}\, s^\ell_{23} - s^\ell_{12}\, c^\ell_{23}\, s^\ell_{13}\, e^{i\psi} & c^\ell_{23}\, c^\ell_{13}\, e^{i\psi} \end{pmatrix}$$

Two possibilities:

(A) μ–τ symmetric U_ν or (B) μ–τ symmetric U_ℓ

(A) Corrections from U_ℓ to μ–τ symmetric U_ν

Assume that U_ℓ is "CKM-like": $\sin\theta_{12}^\ell \equiv \lambda$

$$\sin^2\theta_{12} \simeq \sin^2\theta_{12}^\nu - \cos\phi \, \sin 2\theta_{12}^\nu \, |U_{e3}|$$

$$J_{CP} \simeq \frac{\lambda}{4\sqrt{2}} \sin 2\theta_{12}^\nu \sin\phi \quad \text{which means} \quad |U_{e3}| \simeq \frac{\lambda}{\sqrt{2}}$$

$$\sin^2\theta_{23} \simeq \tfrac{1}{2} + \mathcal{O}(\lambda^2)$$

Mohapatra, W.R., Phys. Rev. D **72**, 053001 (2005)

at leading order

$$\sin^2\theta_{12} = \sin^2\theta_{12}^\nu \pm \sqrt{|U_{e3}|^2 \, \sin^2 2\theta_{12}^\nu - 16\, J_{CP}^2}$$

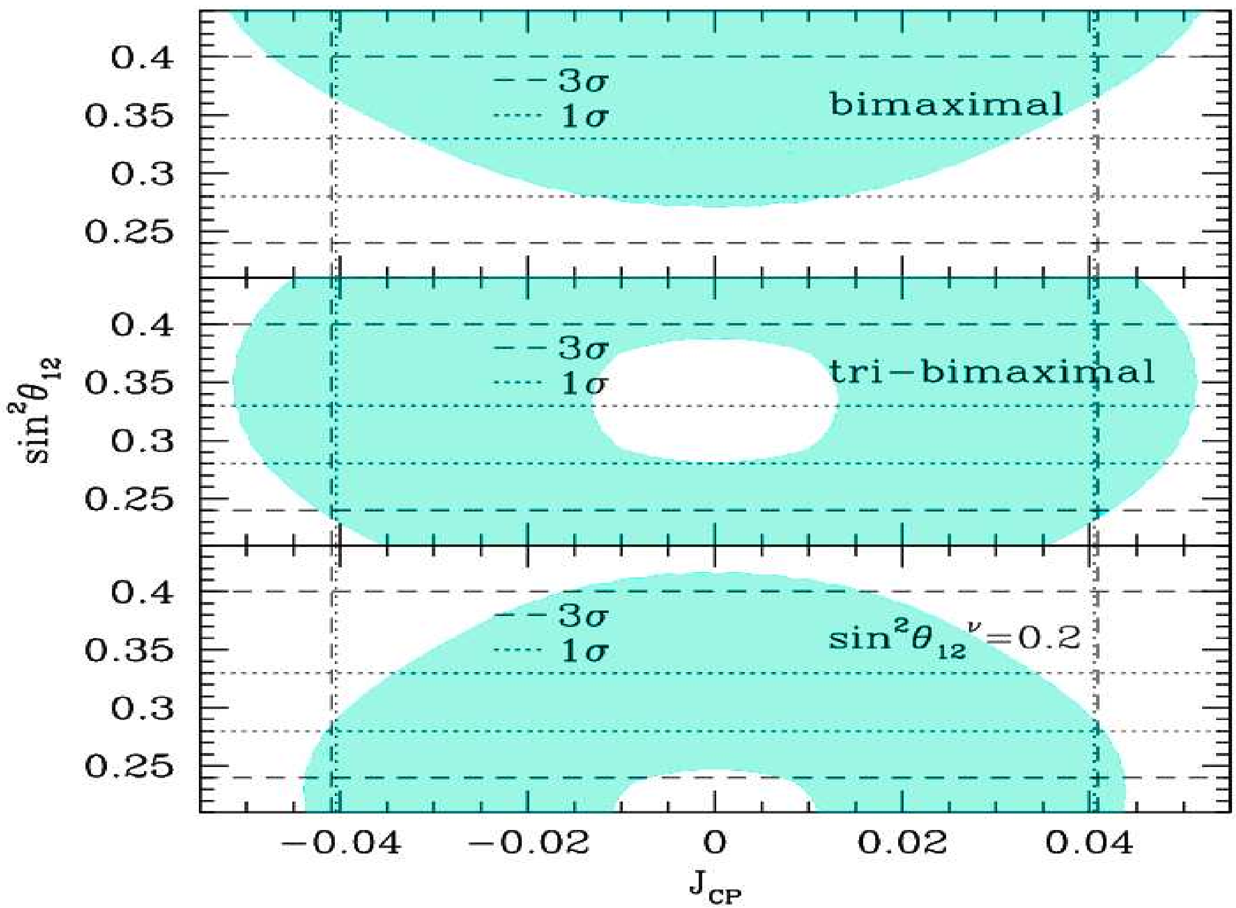
correlation between θ_{12}, $|U_{e3}|$ and leptonic CP violation

if initial tri-bimaximal mixing: $\cos\phi \ll 1 \Rightarrow$ large CP violation

if initial bimaximal mixing: $\cos\phi \simeq 1 \Rightarrow$ suppressed CP violation

Plentinger, W.R., Phys. Lett. B **625**, 264 (2005)

(A) Corrections from U_ℓ to μ–τ symmetric U_ν



Hochmuth, Petcov, W.R., in preparation

(B) Corrections from U_ν to μ–τ symmetric U_ℓ

Assume that U_ν is "CKM-like": $\sin\theta_{23}^{\nu} \equiv \lambda^2$

$$\sin^2\theta_{12} \simeq \sin^2\theta_{12}^{\ell} - \cos\sigma\, \sin 2\theta_{12}^{\ell}\, \lambda_{12}$$

$$|U_{e3}| = \sin\theta_{12}^{\ell}\, \lambda^2 \;,$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} + \cot\theta_{12}^{\ell} \cos(\xi - \sigma + \tau)\, |U_{e3}|$$

$$J_{CP} \simeq -\frac{\lambda^2}{4}\, \sin 2\theta_{12}^{\ell}\, \sin\theta_{12}^{\ell} \sin(\xi - \sigma + \tau)$$

at leading order

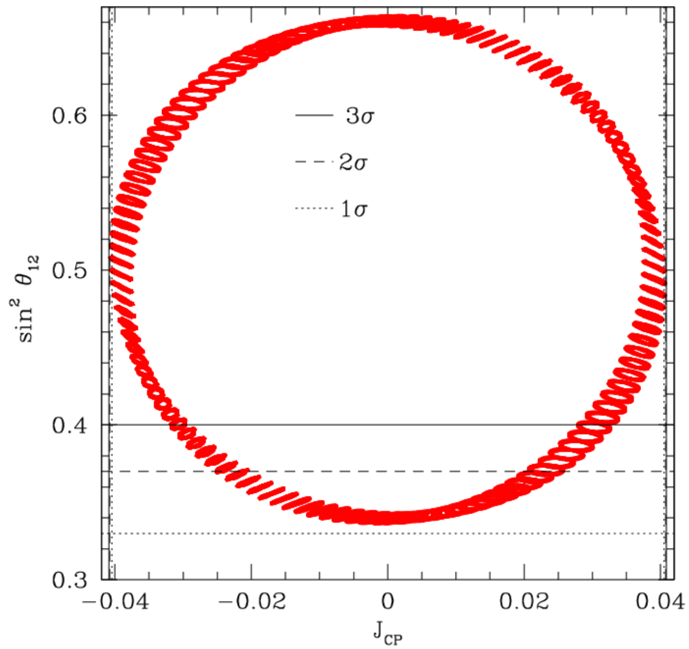
$$\sin^2\theta_{23} \simeq \frac{1}{2} \pm \frac{1}{\sin^2\theta_{12}^{\ell}} \sqrt{|U_{e3}|^2\, \cos^2\theta_{12}^{\ell}\, \sin^2\theta_{12}^{\ell} - 4\, J_{CP}^2}$$

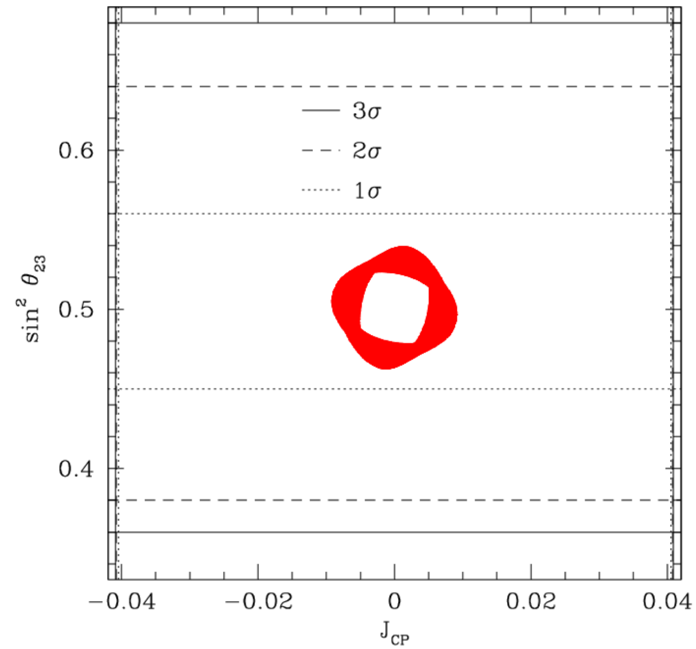
correlation between θ_{23}, $|U_{e3}|$ and leptonic CP violation

Hochmuth, Petcov, W.R., in preparation

Comparison

Example bimaximal mixing and CKM matrix
(Quark-Lepton Complementarity)





U_ν has $U_{e3} = 0$ and $\theta_{23} = \pi/4$

$\frac{1}{2} - \sin^2\theta_{12} \propto \lambda \cos\phi$

$J_{CP}^{\rm lep} \propto \lambda \sin\phi$

U_ℓ has $U_{e3} = 0$ and $\theta_{23} = \pi/4$

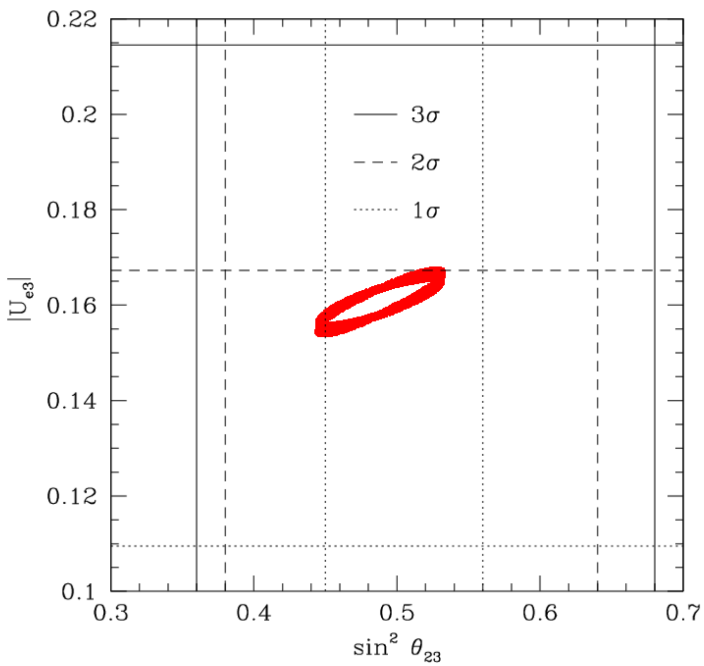
$\frac{1}{2} - \sin^2\theta_{23} \propto \lambda^2 \cos(\xi - \sigma + \tau)$

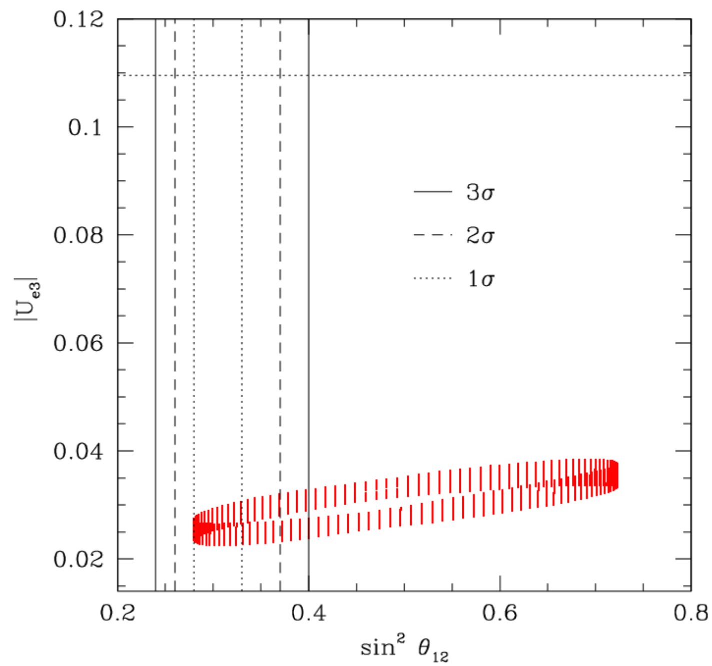
$J_{CP}^{\rm lep} \propto \lambda^2 \sin(\xi - \sigma + \tau)$

Hochmuth, W.R., hep-ph/0607103

Comparison

Example bimaximal mixing and CKM matrix
(Quark-Lepton Complementarity)





U_ν has $U_{e3} = 0$ and $\theta_{23} = \pi/4$
large $|U_{e3}| \simeq \lambda/\sqrt{2}$

U_ℓ has $U_{e3} = 0$ and $\theta_{23} = \pi/4$
small $|U_{e3}| \simeq A\,\lambda^2/\sqrt{2}$

How to break μ–τ symmetry: Planck scale effects

$$\mathcal{L}_{\rm Gr} = \frac{\lambda_{\alpha\beta}}{M_{\rm Pl}} \, (L_\alpha \, \Phi) \, (L_\beta \, \Phi) \stackrel{\rm EWSB}{\longrightarrow} \delta m_\nu = \mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \equiv \mu \, \Delta$$

$$\text{with } \mu \simeq \frac{\langle \Phi \rangle^2}{M_{\rm Pl}} \simeq 2.5 \cdot 10^{-6} \text{ eV}$$

`Barbieri, Ellis, Gaillard; Akhmedov, Berezhiani, Senjanovic; Joshipura; de Gouvea, Valle; Vissani, Narayan, Berezinsky; Koranga, Narayan, Sankar`

- $M_{\rm Pl}$ high energy scale necessarily present
- flavor democratic
- can only be sub-leading effect (past: vacuum oscillations)
- if low scale gravity: sizable effect

How to break μ–τ symmetry: Planck scale effects

$$U = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$$

"unphysical" phases ϕ_1, ϕ_2, ϕ_3 (redefine charged leptons)

phases become not unphysical if additional term added!!

$$m_\nu = \begin{pmatrix} A\, e^{2i\phi_1} & B\, e^{i(\phi_1+\phi_2)} & B\, e^{i(\phi_1+\phi_3)} \\ \cdot & D\, e^{2i\phi_2} & E\, e^{i(\phi_2+\phi_3)} \\ \cdot & \cdot & D\, e^{2i\phi_3} \end{pmatrix} + \mu \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}$$

Planck-corrections to $U_{e3} = 0$ and $\theta_{23} = \pi/4$ only if unphysical phases are non-trivial!!

Dighe, Goswami, W.R., hep-ph/0612328

How to break μ–τ symmetry: Planck scale effects

Special case tri-bimaximal mixing:

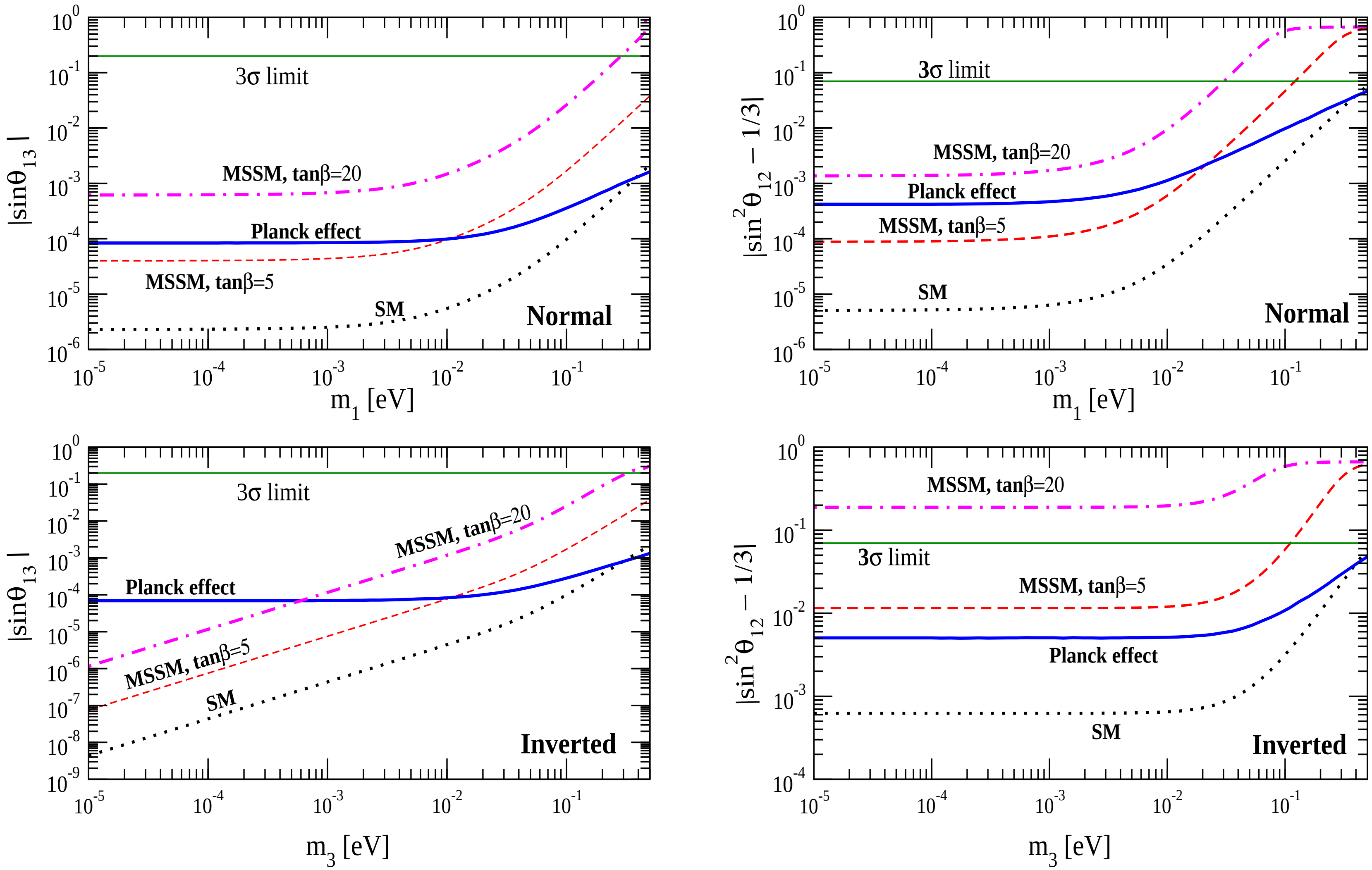
$$m_\nu = \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_2 e^{2i\alpha}}{3}\begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_3 e^{2i\beta}}{2}\begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}$$

term proportional to m_2 is flavor democratic, just as Planck-term:

$\Rightarrow m_2\, e^{2i\alpha} \longrightarrow m_2\, e^{2i\alpha}\, \left(1 + \epsilon_\mu\, e^{-2i\alpha}\right)$ where $\epsilon_\mu = 3\,\mu/m_2 \lesssim 8.9 \cdot 10^{-4}$

No change for any mixing angle if unphysical phases are zero!

Dighe, Goswami, W.R., hep-ph/0612328



- <u>Planck effects can exceed RG effects</u>
- direction of RG fixed by theory (SM or MSSM) and sgn($\Delta m^2_{\rm A}$)
- Planck works in either direction: $\Rightarrow$ relax RG constraints by $(1\ldots 2)^0$ in QD!!

Summary

μ–τ symmetry is simplest way to force $U_{e3} = 0$ and $\theta_{23} = \pi/4$

$$m_\nu = \begin{pmatrix} A & B & B \\ \cdot & D & E \\ \cdot & \cdot & D \end{pmatrix}$$

Even if not realized by Nature, zeroth order mass matrix looks like this!!

- breaking μ–τ symmetry can give testable correlations between $|U_{e3}|$ and $\theta_{23} - \pi/4$ and J_{CP}:
  - soft breaking: possibility to have $|U_{e3}| \ll$ or $\gg |\theta_{23} - \pi/4|$
  - phased breaking: interplay with J_{CP}
  - $U = U_\ell^\dagger \, U_\nu$: magnitude of $|U_{e3}|$ sensitive to the 2 possibilities; correlation of J_{CP} with solar or atmospheric mixing
  - Planck effects: all phases crucial
- if $|U_{e3}| \simeq \theta_{23} - \pi/4 \simeq 0$: typical example for necessity of a discrete symmetry

How to break μ–τ symmetry: Type II see-saw

Include Higgs triplet(s) with

$$m_L = v_L\, f \text{ and } M_R = v_R\, g \text{ where } v_L\, v_R = \gamma\, v^2$$

gives type II see-saw formula

$$m_\nu = v_L \left(f - m_D^T \frac{g^{-1}}{\gamma\, v^2}\, m_D \right) \xrightarrow{\text{LR}} v_L \left(f - m_D^T \frac{f^{-1}}{\gamma\, v^2}\, m_D \right)$$

if $v_R \to \infty$: $m_\nu \to 0$ and pure $V - A$

$\Rightarrow$ smallness of neutrino masses and maximal parity violation!!

- Assume triplet term $m_L = v_L\, f$ conserves μ–τ symmetry
- m_D (or M_R) does not conserve μ–τ symmetry

`M. Lindner, W.R., hep-ph/0703XXX`

Leading Structure of Mass Matrix in Type II See-Saw

$U_{e3} = 0$ and $\theta_{23} = \pi/4$ ("μ–τ symmetry")

$\Rightarrow$ 3 matrices with eigenvector $(0,\, -1/\sqrt{2},\, 1/\sqrt{2})^T$:

(A)	(B)	(C)
$\sqrt{\frac{\Delta m^2_\mathrm{A}}{4}} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}$	$\sqrt{\frac{\Delta m^2_\mathrm{A}}{2}} \begin{pmatrix} 0 & 1 & 1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	$m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix}$
$\sqrt{\Delta m^2_\mathrm{A}} \Rightarrow$ NH	$0 \Rightarrow$ IH	$-m_0 \Rightarrow$ QD
L_e (Barbieri; Vissani; (Buchmüller, Yanagida)	$L_e - L_\mu - L_\tau$ (Petcov)	$L_\mu - L_\tau$ (Choubey, W.R.)
$\Delta m^2_\odot = \theta_{12} = 0$	$\Delta m^2_\odot = 0,\ \theta_{12} = \pi/4$	$\Delta m^2_\mathrm{A} = \theta_{12} = 0$
singular	singular	non-singular

$\Rightarrow$ this term given by $m_L \leftrightarrow$ correction from conventional see-saw term

Corrections from conventional see-saw to leading term

$m_\nu^I \simeq \epsilon \begin{pmatrix} a & b & c \\ \cdot & d & e \\ \cdot & \cdot & f \end{pmatrix}$	$m_\nu^I \simeq \epsilon \begin{pmatrix} a & b & b \\ \cdot & d & e \\ \cdot & \cdot & d \end{pmatrix}$	$m_\nu^I \simeq \epsilon \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}$
anarchical	μ–τ symmetric	democratic
m_D and M_R anarchical	m_D and M_R μ–τ symmetric	m_D democratic and $M_R \propto \mathbb{1}$

Normal Hierarchy:

$$m_L = \sqrt{\frac{\Delta m_{\rm A}^2}{4}} \left(\begin{array}{ccc} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{array} \right) \Rightarrow v_L = \sqrt{\Delta m_{\rm A}^2}/2$$

- singular $\Rightarrow m_L \propto M_R$ not possible ("singular see-saw")
- perturbation matrix has to generate $\Delta m_\odot^2 \Rightarrow \epsilon \sim \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$
- if perturbation anarchical: $|U_{e3}| \sim \theta_{23} - \pi/4 \sim \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$
- if perturbation μ–τ symmetric: $|U_{e3}| = \theta_{23} - \pi/4 = 0$
- easily possible to have $|U_{e3}| \simeq 0$ and $\theta_{23} - \pi/4 \sim \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$ or $|U_{e3}| \sim \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$ and $\theta_{23} \simeq \pi/4$
- if perturbation democratic: tri-bimaximal mixing! ($\sin^2\theta_{12} = 1/3$)

Inverted Hierarchy:

$$m_L = \sqrt{\frac{\Delta m_{\rm A}^2}{2}} \left(\begin{array}{ccc} 0 & 1 & 1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{array} \right) \Rightarrow v_L = \sqrt{\Delta m_{\rm A}^2/2}$$

- singular $\Rightarrow m_L \propto M_R$ not possible
- perturbation matrix has to generate $\Delta m_\odot^2 \Rightarrow \epsilon \sim \sqrt{\Delta m_\odot^2/\Delta m_{\rm A}^2}$
- democratic perturbation does not work
- as usual in IH: tuned correction
- if μ–τ symmetric: $|U_{e3}| = \theta_{23} - \pi/4 = 0$ and

$$\Delta m_\odot^2/\Delta m_{\rm A}^2 \simeq \sqrt{2}\,(a+d+e)\epsilon \text{ and } \sin\theta_{12} \simeq \sqrt{\tfrac{1}{2} - \tfrac{1}{8}(a-d-e)\epsilon}$$

$$\Rightarrow (a+d+e) \stackrel{!}{\ll} (a-d-e)$$

Other possibility to perturb zeroth order matrix:

$$m_D = v \begin{pmatrix} a_D\,\lambda^4 & b_D\,\lambda^5 & c_D\,\lambda^5 \\ \cdot & d_D\,\lambda^2 & e_D\,\lambda^2 \\ \cdot & \cdot & f_D\,\lambda \end{pmatrix} \text{ and } M_R = v_R \begin{pmatrix} a_R\,\lambda^7 & 0 & 0 \\ \cdot & d_R\,\lambda^2 & 0 \\ \cdot & \cdot & f_R \end{pmatrix}$$

giving

$$m_\nu^I = -m_D^T\,M_R^{-1}\,m_D \simeq \frac{v^2}{v_R} \begin{pmatrix} \mathcal{O}(\lambda) & \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda^2) \\ \cdot & \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda^3) \\ \cdot & \cdot & \mathcal{O}(\lambda^2) \end{pmatrix}$$

and phenomenology okay once added to m_L

Quasi-degeneracy:

$$m_L = m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} \Rightarrow v_L = m_0$$

- all perturbations work (democracy needs RG)
- m_L can be inverted $\Rightarrow m_L \propto M_R$ works!

$$m_D = v \begin{pmatrix} a_D\,\lambda^3 & b_D\,\lambda^2 & c_D\,\lambda^2 \\ \cdot & d_D\,\lambda & e_D\,\lambda \\ \cdot & \cdot & f_D \end{pmatrix} \text{ and } M_R = v_R \begin{pmatrix} X & 0 & 0 \\ \cdot & 0 & Y \\ \cdot & \cdot & 0 \end{pmatrix}$$

m_D "up-quark like" and $N_{1,2,3}$ have charges 0, 1 and -1 under $L_\mu - L_\tau$

S. Choubey, W.R., Eur. Phys. J. C **40**, 259 (2005)

Deviations from Bimaximal Mixing via Type II See-Saw

$$U_{\rm bimax} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Vissani; Barger, Pakvasa, Weiler, Whisnant

Baltz, Goldhaber, Goldhaber; Georgi, Glashow;

Stancu, Ahluwalia

corresponding to $\theta_{12} = \theta_{23} = \pi/4$, $U_{e3} = 0$ and leading to a mass matrix

$$m_\nu^{\rm bimax} = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+D) & \frac{1}{2}(A-D) \\ \cdot & \cdot & \frac{1}{2}(A+D) \end{pmatrix} = m_L$$

where

$$A = \frac{m_1^0 + m_2^0}{2} \ , \ B = \frac{m_2^0 - m_1^0}{2\sqrt{2}} \ , \ D = m_3^0$$

Deviations from Bimaximal Mixing

Discrete LR symmetry:

$$M_R^{-1} = \frac{v_L}{v_R}\, m_L^{-1} = \frac{v_L}{v_R} \begin{pmatrix} \tilde{A} & \tilde{B} & \tilde{B} \\ \cdot & \frac{1}{2}(\tilde{A}+\tilde{D}) & \frac{1}{2}(\tilde{A}-\tilde{D}) \\ \cdot & \cdot & \frac{1}{2}(\tilde{A}+\tilde{D}) \end{pmatrix}$$

$$\tilde{A} = \frac{A}{A^2 - 2B^2}\ ,\ \tilde{B} = \frac{-B}{A^2 - 2B^2}\ ,\ \tilde{D} = \frac{1}{D}$$

for $(m_3^0)^2 \gg (m_{1,2}^0)^2$ and hierarchical $m_D \simeq \mathrm{diag}(0,0,m)$:

$$m_D^T\, M_R^{-1}\, m_D \simeq \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 0 & 0 \\ \cdot & 0 & s \end{pmatrix}$$

with $s \equiv v_L^2\ \frac{m^2}{4\gamma\, v^2} \left(\frac{1}{m_1^0} + \frac{1}{m_2^0} + \frac{2}{m_3^0} \right)$

Joshipura, Paschos, W.R., NPB **611**, 227 (2001)

W.R., PRD **70**, 073010 (2004)

Deviations from Bimaximal Mixing

$$m_\nu^{II} + m_\nu^I = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+D) & \frac{1}{2}(A-D) \\ \cdot & \cdot & \frac{1}{2}(A+D) - s \end{pmatrix}$$

magnitude of perturbation:

$$s \simeq \frac{0.02}{\gamma} \left(\frac{v_L}{10^{-2}\ \mathrm{eV}}\right)^2 \left(\frac{10^{-3}\ \mathrm{eV}}{m_1^0}\right)\ \mathrm{eV}$$

observables:

$$|U_{e3}| \simeq \frac{B\,s}{\sqrt{2}\,D^2}\ ,\ \sin^2\theta_{23} \simeq \frac{1}{2}\left(1 + \frac{s}{D}\right)\ ,\ \tan 2\theta_{12} \simeq 4\sqrt{2}\,\frac{B}{s}$$

- $s \ll D$ but $s \simeq A, B$
- $|U_{e3}|$ stays very small, $\theta_{23} - \pi/4$ can be sizable
- mimics QLC ($\theta_{12} + \theta_C = \pi/4$) if $s/B \simeq 8\sqrt{2}\sin\theta_C \simeq 2$

Deviations from Bimaximal Mixing

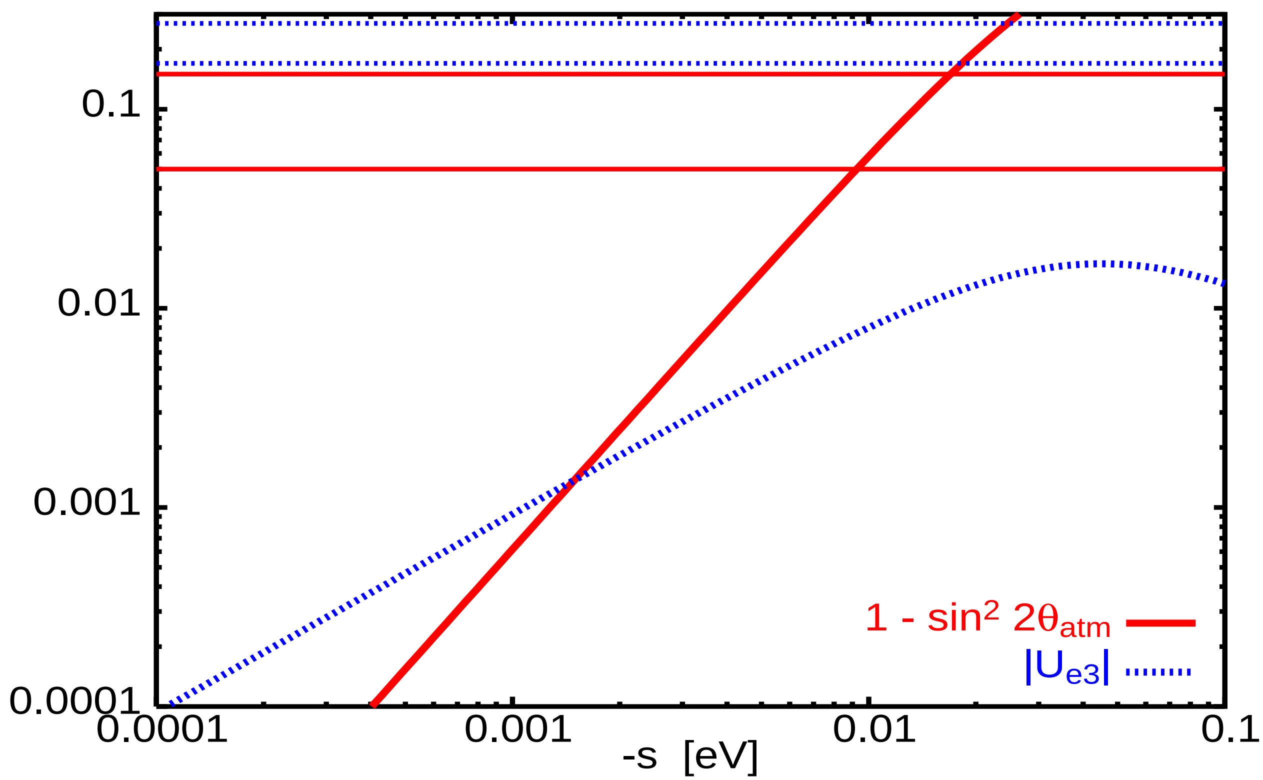
$$m_\nu^{II} + m_\nu^{I} = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+D) & \frac{1}{2}(A-D) \\ \cdot & \cdot & \frac{1}{2}(A+D)-s \end{pmatrix}$$

if initially $A = (m_1^0 + m_2^0)/2 = 0$ then $(\Delta m_\odot^2)^0 = 0$

$\Rightarrow$ perturbation from conventional term generates $\Delta m_\odot^2$ and deviation from maximal solar neutrino mixing!!

$\Rightarrow$ explains phenomenological observation

$$\pi/4 - \theta_{12} \sim \sqrt{\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2}}$$



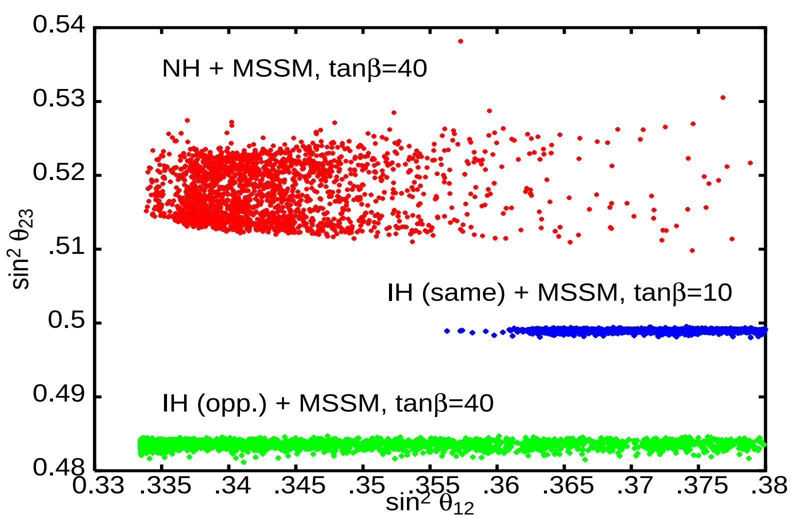
$$\frac{2\,|U_{e3}|}{\tan 2\theta_{12}} \simeq \left(\sin^2\theta_{23} - \frac{1}{2}\right)^2 \simeq \frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm A}}\,\cos 2\theta_{12}$$

How to break μ–τ symmetry: Renormalization

$$(m_\nu)_{\alpha\beta} \to (m_\nu)_{\alpha\beta}\,(1+\epsilon_\alpha)\,(1+\epsilon_\beta) \text{ where } \epsilon_\alpha = (1+\tan^2\beta)\,\frac{m_\alpha^2}{16\,\pi^2}\,\ln\frac{M_X}{M_Z}$$

this means for μ–τ symmetry:

$$\begin{pmatrix} A & B & B \\ \cdot & D & F \\ \cdot & \cdot & D \end{pmatrix} \longrightarrow \begin{pmatrix} A & B & B\,(1+\epsilon_\tau) \\ \cdot & D & F\,(1+\epsilon_\tau) \\ \cdot & \cdot & D\,(1+2\,\epsilon_\tau) \end{pmatrix} \text{ Broken!}$$



E.g: TBM, IH with same CP parities: $\tan\beta \lesssim 10$

Breaking of μ–τ symmetry: $U = U_\ell^\dagger \, U_\nu$

Charged lepton contribution to PMNS matrix

$$U = U_\ell^\dagger \, U_\nu \text{ with } U_i = e^{i\Phi} \, P \, \tilde{U} \, Q$$

In general, the PMNS matrix can always be written as:

$$U = \tilde{U}_\ell^\dagger \, P_\nu \, \tilde{U}_\nu \, Q_\nu$$

- $\tilde{U}_\ell$ and $\tilde{U}_\nu$ parametrized with 3 angles and 1 phase, each
- $P_\nu = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})$
- $Q_\nu = \mathrm{diag}(1, e^{i\rho}, e^{i\sigma})$ "Majorana–like"
- $\theta_{12,13,23}$ and Dirac (Majorana) phase depend on 4 (6) phases
- only one phase from charged leptons: suppressed if hierarchical mixing

Frampton, Petcov, W.R., Nucl. Phys. B **687**, 31 (2004)

Breaking of μ–τ symmetry: Neutrino Telescopes

Alternative example: high-energy neutrinos (IceCube)

$$P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 2\,\mathcal{R} \sum_{i>j} U_{\alpha j}\, U_{\alpha i}^*\, U_{\beta j}^*\, U_{\beta i} = \sum |U_{\alpha i}|^2\, |U_{\beta i}|^2$$

gives in leading order: $P \equiv \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ \cdot & P_{\mu\mu} & P_{\mu\tau} \\ \cdot & \cdot & P_{\tau\tau} \end{pmatrix}$

$$\simeq \begin{pmatrix} (1 - 2\, c_{12}^2\, s_{12}^2)\,(1 - 2\,|U_{e3}|^2) & c_{12}^2\, s_{12}^2 + \Delta & c_{12}^2\, s_{12}^2 - \Delta \\ \cdot & \frac{1}{2}(1 - c_{12}^2\, s_{12}^2) - \Delta & \frac{1}{2}(1 - c_{12}^2\, s_{12}^2) \\ \cdot & \cdot & \frac{1}{2}(1 - c_{12}^2\, s_{12}^2) + \Delta \end{pmatrix}$$

with $\Delta \equiv \frac{1}{4}\, \cos\delta\, \sin 4\theta_{12}\, |U_{e3}| + 2\, s_{12}^2\, c_{12}^2\, \left(\frac{1}{2} - \sin^2\theta_{23}\right)$

universal first order correction!

$$\frac{\text{measure of } \mu\text{–}\tau \text{ symmetry breaking}}{(\texttt{W.R., JCAP } \mathbf{0701}\texttt{, 029 (2007)})}$$