Neutrino Mass Models

- Why BSM?
- Neutrino mass models decision tree
- Survey of approaches
- TBM, A_4, Form Dominance, CSD
- Family symmetry and GUTs
- Mixing Sum Rules



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Why Beyond Standard Model?

1. There are no right-handed neutrinos ν_R
2. There are only Higgs doublets of $SU(2)_L$
3. There are only renormalizable terms

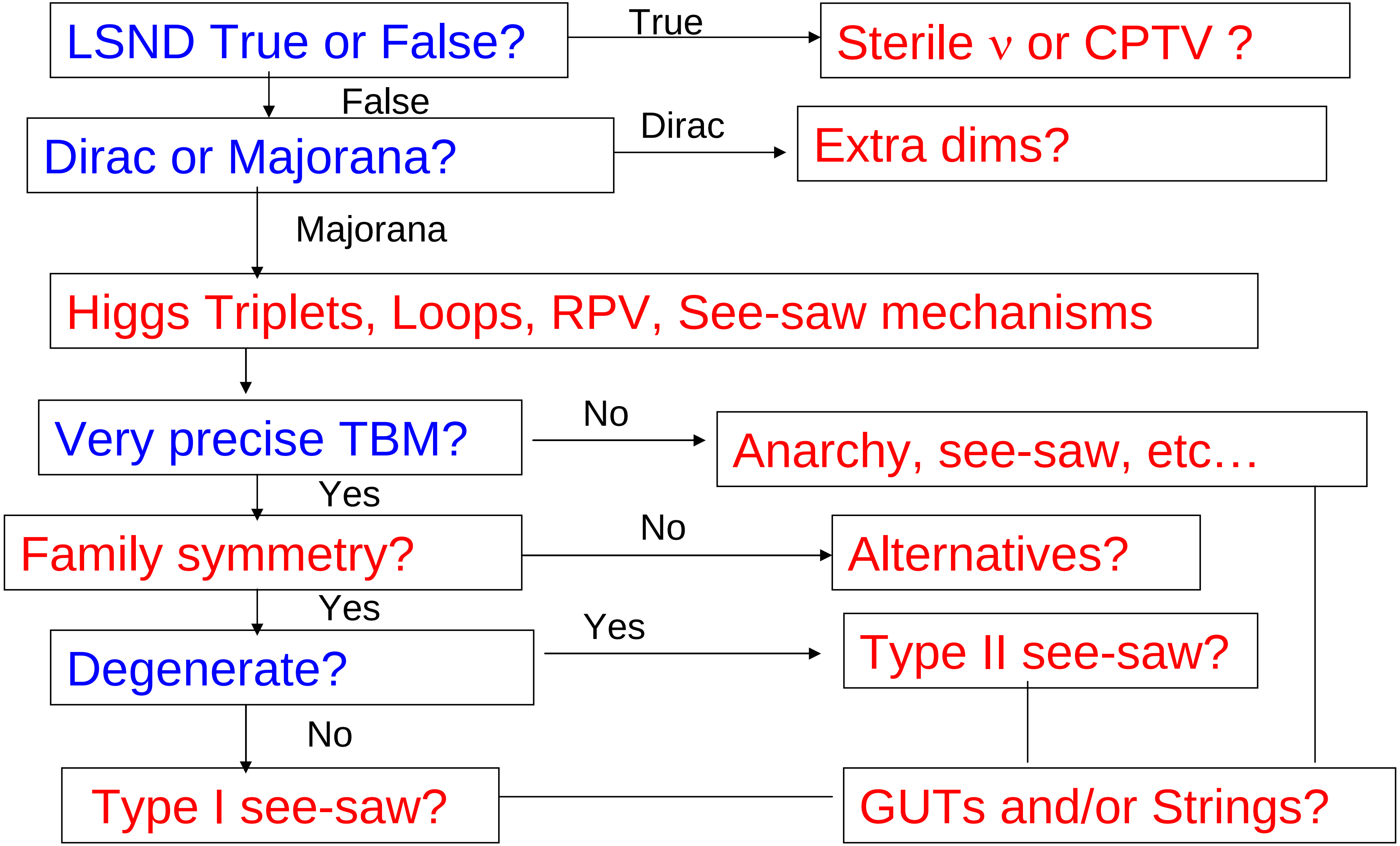
In the Standard Model these conditions all apply so neutrinos are massless, with ν_e , ν_μ , ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ

Neutrinos and anti-neutrinos are distinguished by the total conserved lepton number $L=L_e+L_\mu+L_\tau$

To generate neutrino mass we must relax 1 and/or 2 and/or 3

Staying within the SM is not an option – but what direction?

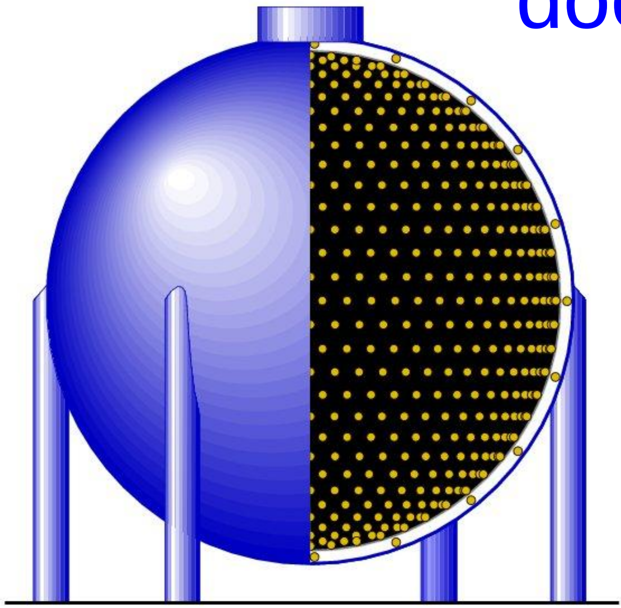
Neutrino mass models decision tree

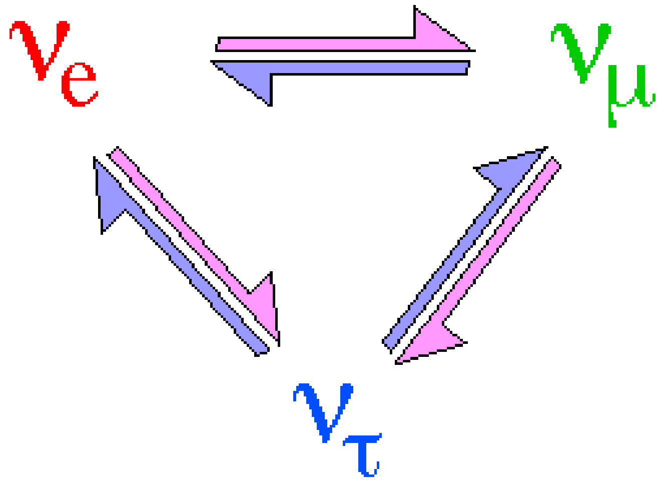


LSND True or False?

MiniBoone does not support LSND result

does support three neutrinos





In this talk we assume that LSND is false

Dirac or Majorana?

Petcov talk

Majorana masses

CP conjugate

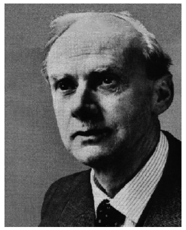
$$m_{LL}\bar{\nu}_L\nu_L^c$$

$$M_{RR}\bar{\nu}_R\nu_R^c$$

Violates L

Violates L_e, L_μ, L_τ

Neutrino=antineutrino



$$m_{LR}\bar{\nu}_L\nu_R$$

Conserves L

Violates L_e, L_μ, L_τ

Neutrino$\neq$antineutrino

Dirac mass

1st Possibility: Dirac

Recall origin of electron mass in SM with $L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad e_R^-, \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

$$\lambda_e \bar{L} H e_R^- = \lambda_e \langle H^0 \rangle \overline{e_L^-} e_R^-$$

Yukawa coupling λ_e must be small since $\langle H^0 \rangle = 175$ GeV

$$m_e = \lambda_e \langle H^0 \rangle \approx 0.5\, MeV \Leftrightarrow \lambda_e \approx 3.10^{-6}$$

Introduce right-handed neutrino ν_{eR} with zero Majorana mass

$$\lambda_\nu \bar{L} H^c \nu_{eR} = \lambda_\nu \langle H^0 \rangle \overline{\nu_{eL}} \nu_{eR}$$

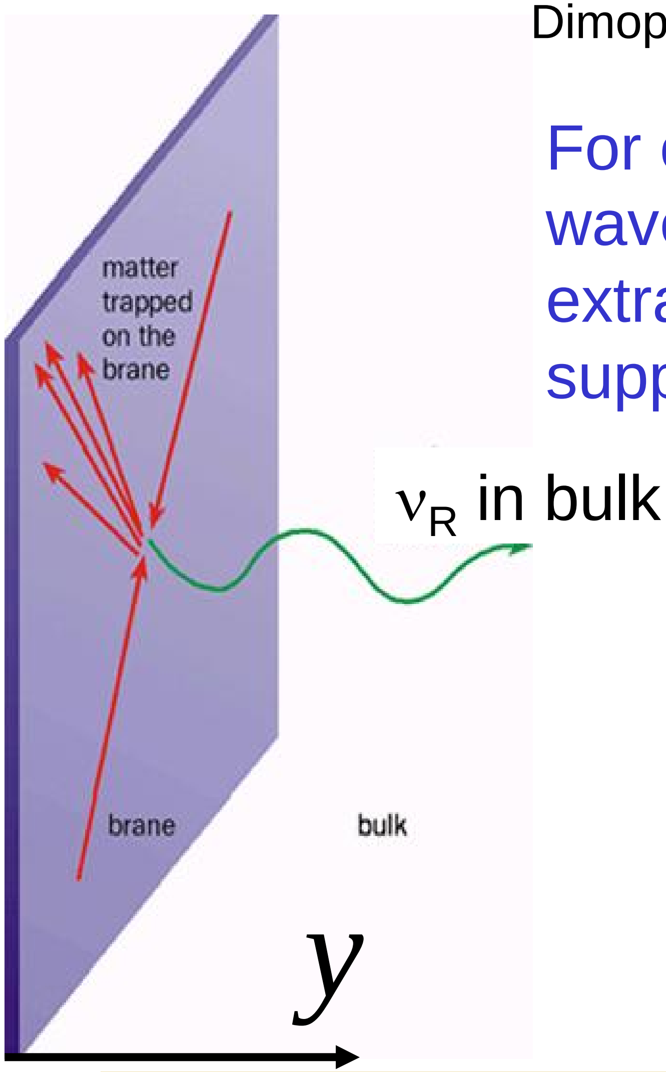
then Yukawa coupling generates a Dirac neutrino mass

$$m_{LR}^\nu = \lambda_\nu \langle H^0 \rangle \approx 0.2\, eV \Leftrightarrow \lambda_\nu \approx 10^{-12}$$

Why so small?
– extra dimensions

Flat extra dimensions with RH neutrinos in the bulk

Dienes, Dudas, Gherghetta; Arkhani-Hamed, Dimopoulos, Dvali, March-Russell



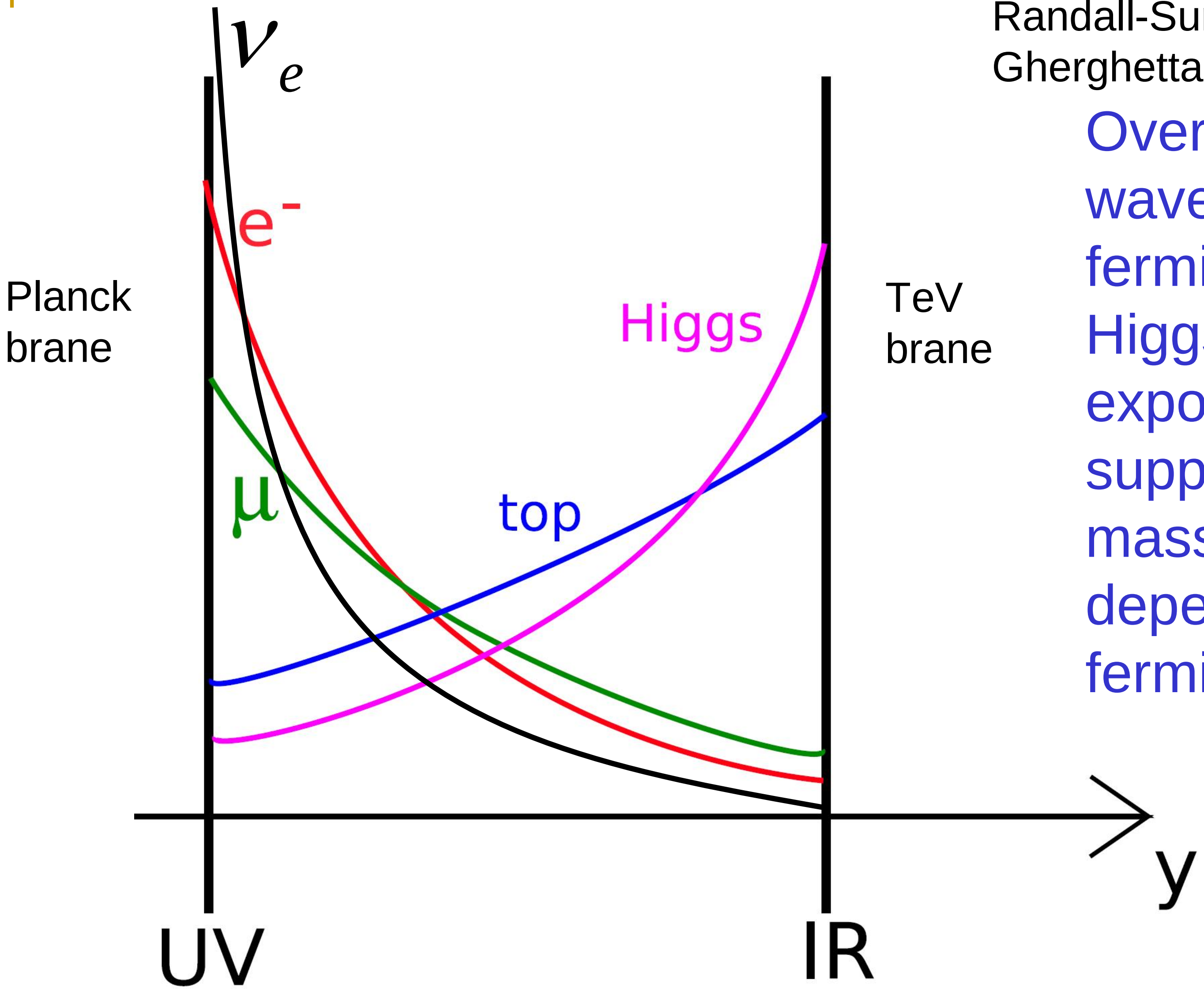
For one extra dimension y the ν_R wavefunction spreads out over the extra dimension, leading to a volume suppressed Yukawa coupling at y=0

$$\rightarrow m_{LR}^{\nu} = \frac{\lambda \langle H^0 \rangle}{\sqrt{V}} = \lambda \langle H^0 \rangle \frac{M_{string}}{M_{Planck}}$$

$$e.g. \quad \frac{M_{string}}{M_{Planck}} = \frac{10^7}{10^{19}} = 10^{-12}$$

Warped extra dimensions with SM in the bulk

Randall-Sundrum; Rubakov, Gherghetta, Binetruy,…

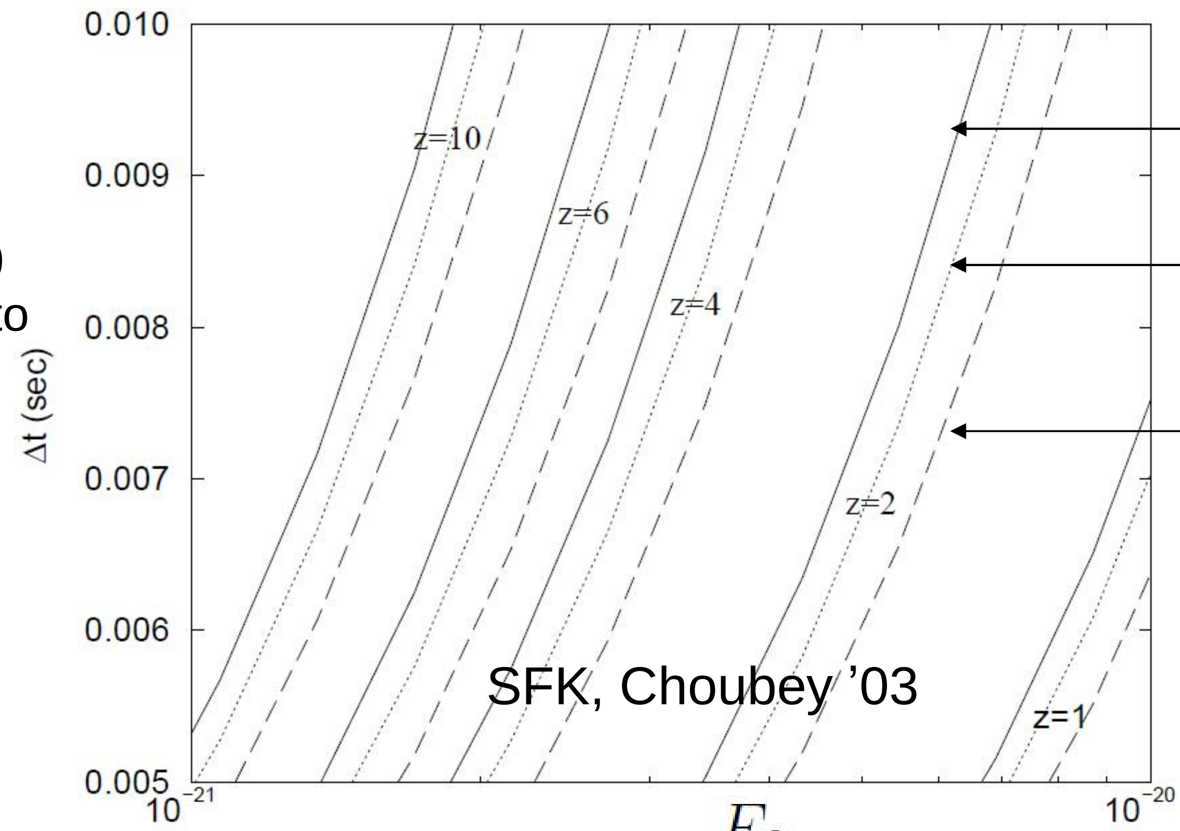


Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

Aside: some models with warped extra dimensions address the problem of dark energy in the Universe

Neutrino Telescopes studying neutrinos from GRBs may be able to shed light on Neutrino Mass, Quantum Gravity and Dark Energy

Time delay (s) relative to low energy photons



Cosmological Constant

SUGRA Quintessence

INV Quintessence

$$\epsilon'_0 = \frac{E_0}{E_{QG}}$$

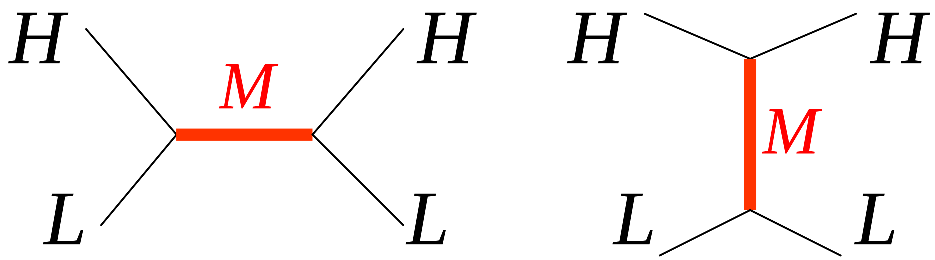
2nd Possibility: Majorana

Renormalisable
ΔL =2 operator $\lambda_\nu LL\Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable
ΔL =2 operator $\frac{\lambda_\nu}{M} LLHH = \frac{\lambda_\nu}{M} \langle H^0 \rangle^2 \overline{\nu}_{eL} \nu^c_{eL}$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0 \rangle^2 / M$ where M is some high energy scale

The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)

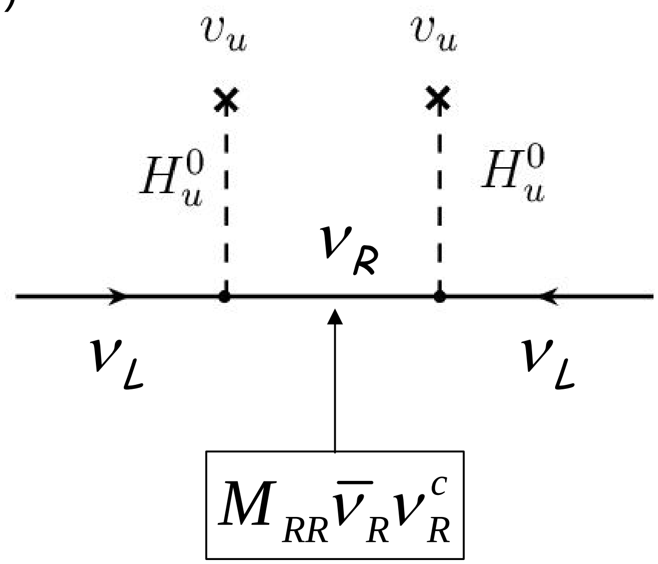


- Loop models
- RPV SUSY
- See-saw mechanisms

•Type I and II see-saw mechanism

Type I see-saw mechanism

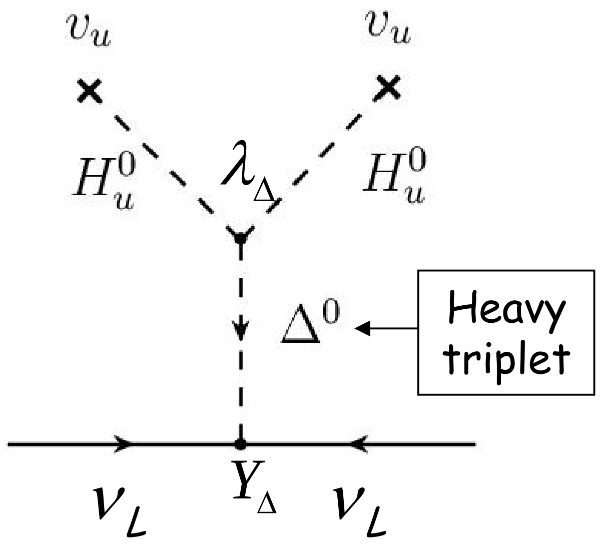
Minkowski (1977)



$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type II see-saw mechanism

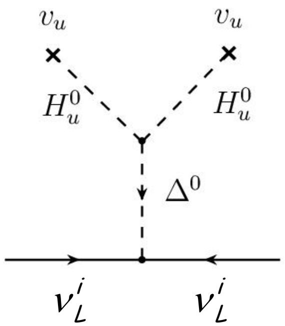
Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

•Type II upgrade of type I models

Antusch, SFK

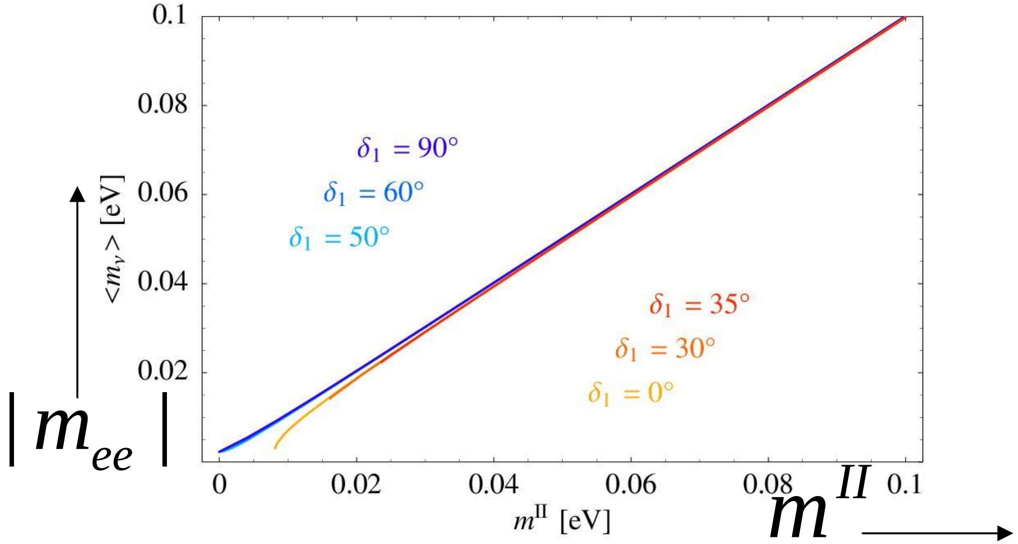


$$m_{LL}^{\nu} = m^{II}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} - m_{LR}M_{RR}^{-1}m_{LR}^{T}$$

Unit matrix type II contribution from an SO(3) family symmetry

Hierarchical type I contribution controls the neutrino mixings and mass splittings

Type II contribution governs the neutrino mass scale and renders neutrinoless double beta decay observable



Very precise Tri-bimaximal mixing (TBM) ?

Harrison, Perkins, Scott

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\theta_{12} = 35^\circ, \qquad \theta_{23} = 45^\circ, \qquad \theta_{13} = 0^\circ.$$

c.f. data

$$\theta_{12} = 33.8^\circ \pm 1.4^\circ, \theta_{23} = 45^\circ \pm 3^\circ, \ \theta_{13} < 12^\circ$$

← See other talks at this workshop for more up to date values

- Current data is consistent with TBM

Consider the TB neutrino mass matrix in the flavour basis

i.e. diagonal charged lepton basis

$$M^{\nu\ \mathrm{diag}}_{eff} = U^T_{\mathrm{TBM}} (M^\nu_{eff})^{TBM} U_{\mathrm{TBM}} = (m_1,\ m_2,\ m_3)$$

$$(M^\nu_{eff})^{TBM} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T$$

$$\Phi_1 \Phi_1^T = \frac{1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}, \quad \Phi_2 \Phi_2^T = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \Phi_3 \Phi_3^T = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\Phi_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Columns of U_{TBM}

$$(M^{\nu}_{eff})^{TBM} = \begin{pmatrix} a & b & c \\ . & d & e \\ . & . & f \end{pmatrix}$$

$$a = \frac{2}{3}m_1 + \frac{1}{3}m_2,$$
$$b = c = -\frac{1}{3}m_1 + \frac{1}{3}m_2,$$
$$d = f = \frac{1}{6}m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_3,$$
$$e = a + b - d.$$

How to achieve these relations in a model?

The most elegant models involve $\leq$ 3 parameters which satisfy these relations

Low, Volkas

$$(M^{\nu}_{eff})^{TBM} = \begin{pmatrix} a & b & b \\ . & d & (a+b-d) \\ . & . & d \end{pmatrix}$$

Such a mass matrix is called form diagonalizable since it is diagonalized by the TBM matrix for all values of a,b,d

$$a, b, d \rightarrow m_1, m_2, m_3$$

hence for all values of neutrino masses

Form Dominance

Chen,SFK

Form Dominance is a mechanism for achieving a form diagonalizable effective neutrino mass matrix starting from the type I see-saw mechanism

Work in diagonal M_{RR} basis $M_{RR} = \mathrm{diag}(M_A, M_B, M_C)$

M_D is LR Dirac mass matrix $M_D = (A, B, C)$ A,B,C are column vectors

$$M^{\nu}_{eff} = M_D M^{-1}_{RR} M^T_D \longrightarrow M^{\nu}_{eff} = \frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C}$$

Form Dominance assumption: columns of Dirac mass matrix $\propto$ columns of U_{TBM}

$$A = a\Phi_1 = \frac{a}{\sqrt{6}}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad B = b\Phi_2 = \frac{b}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad C = c\Phi_3 = \frac{c}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\longrightarrow (M^{\nu}_{eff})^{TBM} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T$$

with $m_1 = a^2/M_A, \ m_2 = b^2/M_B, \ m_3 = c^2/M_C$

N.B. Only three parameter combinations

Basis Invariance and the R matrix

In FD a particular RH neutrino mass eigenstate is associated with a particular light neutrino mass eigenstate

i.e. in FD the basis invariant Casas-Ibarra matrix R is unit matrix

$$\left(A_i M_A^{-1/2} \;\; B_i M_B^{-1/2} \;\; C_i M_C^{-1/2} \right) = \left(U_{i1} m_1^{1/2} \;\; U_{i2} m_2^{1/2} \;\; U_{i3} m_3^{1/2} \right) R^T$$

This means that FD may be defined in a basis invariant way as R=1

Family Symmetry

Clearly TBM suggests a family symmetry, but one that is badly broken in the charged lepton sector

Diagonal charged lepton basis Lagrangian $L = L^{\nu} + L^{E}$

$$L^{\nu} = \begin{pmatrix} L_e & L_\mu & L_\tau \end{pmatrix} \begin{pmatrix} a & b & b \\ . & d & (a+b-d) \\ . & . & d \end{pmatrix} \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix}$$

Respects $L_\mu \leftrightarrow L_\tau$

Lepton doublets

$$L^{E} = \begin{pmatrix} L_e & L_\mu & L_\tau \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$m_e \ll m_\mu \ll m_\tau$

does not respect $L_\mu \leftrightarrow L_\tau$

Flavons and Vacuum Alignment

To achieve different symmeties in the neutrino and charged lepton sectors we need to align the Higgs fields which break the family symmetry (flavons) along different symmetry preserving directions (vacuum alignment)

e.g. consider $A_4 = \Delta_{12} = Z_3 \otimes Z_2 \times Z_2$ with reps 3,1,1',1''

Altarellli, Feruglio

$Z_3 \downarrow T$, $Z_2 \downarrow S$, $Z_2 \downarrow TST^2$

Note that Z_2^S respects $L_\mu \leftrightarrow L_\tau$ but Z_3^T violates it

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

$A_4 \rightarrow Z_2^S$ via the triplet flavon ϕ_S $\quad \frac{\langle \phi_S \rangle}{\Lambda} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \alpha_s \longrightarrow$ ϕ_S only occurs in L^ν

$A_4 \rightarrow Z_3^T$ via the triplet flavon ϕ_T $\quad \langle \phi_T \rangle = \begin{pmatrix} v_T \\ 0 \\ 0 \end{pmatrix} \longrightarrow$ ϕ_T only occurs in L^E

A_4 see-saw models satisfy form dominance

Chen,SFK

$$N = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} \sim 3 \qquad L = \begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} \sim 3\,, \quad e_R \sim 1\,, \quad \mu_R \sim 1''\,, \quad \tau_R \sim 1'$$

Model 1

$$M_{RR} = \overline{N^c}N(\langle\phi_S\rangle + \langle u\rangle) = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} \Lambda$$

Altarellli talk

$$M_D = yH\overline{L}N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} yv \xrightarrow{\text{Diagonal RHN basis}} yv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Model 2

$$M_{RR} = M_R\overline{N^c}N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} M_R$$

Both satisfy Form Dominance → R=1

$$M_D = H\overline{L}N\left(\frac{\langle\phi_S\rangle}{\Lambda} + \frac{\langle u\rangle}{\Lambda}\right) = \begin{pmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{pmatrix} v \xrightarrow{\text{Diag RHN}} \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

Natural Form Dominance

Chen,SFK

The A_4 see-saw models are very economical since the neutrino sector only involves two flavon VEVs

$$\frac{\langle \phi_S \rangle}{\Lambda} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \alpha_s \,, \quad \frac{\langle u \rangle}{\Lambda} = \alpha_0$$

Model 1

$$\mathrm{diag}(m_1, m_2, m_3) = \left(\frac{1}{3\alpha_s + \alpha_0}, \frac{1}{\alpha_0}, \frac{1}{3\alpha_s - \alpha_0} \right) \frac{y^2 v^2}{\Lambda} \rightarrow \frac{1}{m_1} - \frac{1}{m_3} = \frac{2}{m_2}$$

Model 2 $$(m_1,\ m_2,\ m_3) = \left((3\alpha_s + \alpha_0)^2,\ \alpha_0^2,\ (3\alpha_s - \alpha_0)^2 \right) \frac{v^2}{M_R}$$

However some cancellations of VEVs are required to obtain Δm^2_{atm} and Δm^2_{sol}

This suggests natural form dominance in which a different flavon is associated with each physical neutrino mass → 3 flavons $\Phi_{1,2,3}$

$$A = a\Phi_1 = \frac{a}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad B = b\Phi_2 = \frac{b}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad C = c\Phi_3 = \frac{c}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$\langle \Phi_1 \rangle \rightarrow m_1$

$\langle \Phi_2 \rangle \rightarrow m_2$

$\langle \Phi_3 \rangle \rightarrow m_3$

Constrained Sequential Dominance

SFK

A special case of Natural Form Dominance for $|m_1| \ll |m_2| < |m_3|$

$$<\Phi_3> = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} v \qquad <\Phi_2> = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tilde{v} \qquad <\Phi'> = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} V$$

$$M_D \sim \begin{pmatrix} 0 & \tilde{v} & 0 \\ v & \tilde{v} & 0 \\ -v & \tilde{v} & V \end{pmatrix}$$

$$F\ .\Phi_3 \nu_R^1 h \qquad F\ .\Phi_2 \nu_R^2 h \qquad F\ .\Phi' \nu_R^3 h$$

Note for negligible m_1 the flavon Φ_1 is irrelevant and can be replaced by the flavon Φ'

Several examples of suitable non-Abelian Family Symmetries:

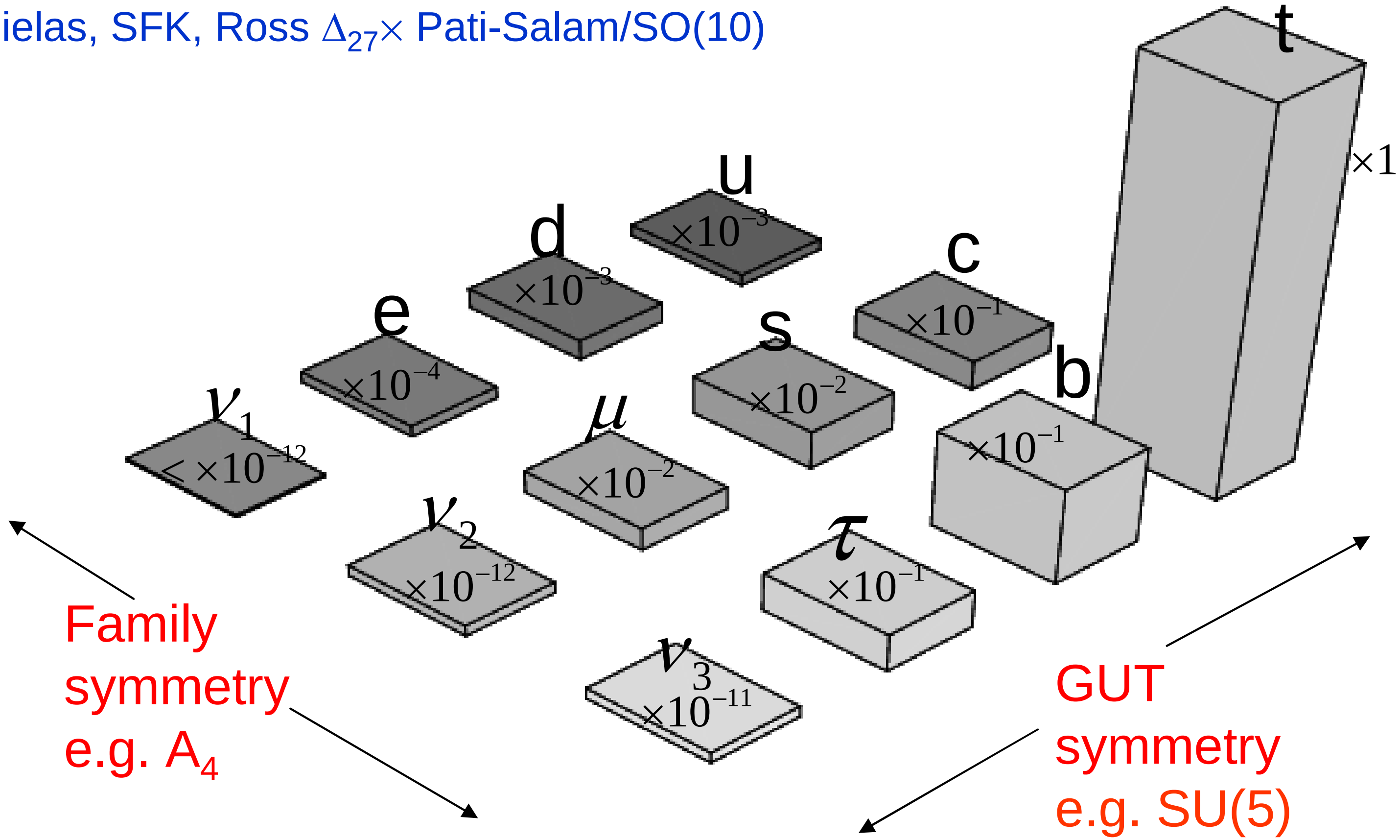
SFK, Ross; Velasco-Sevilla; Varzelias $SU(3)$ Δ_{27}

SFK, Malinsky $SO(3)$ A_4

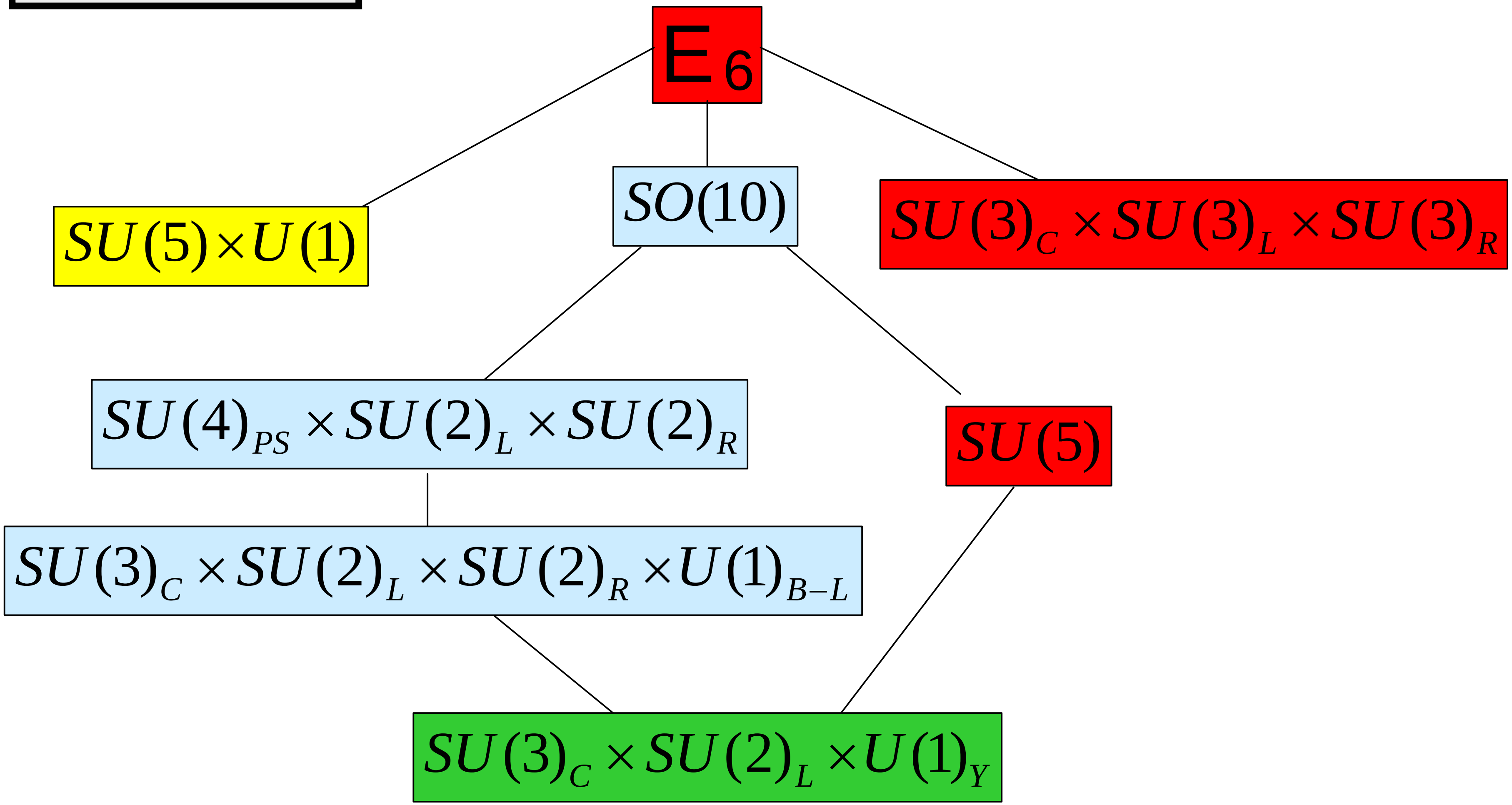
Discrete subgroups preferred by vacuum alignment

Family × GUT symmetry

e.g. Chen and Mahanthappa T'× SU(5)
Altarelli, Feruglio, Hagedorn $A_4 \times SU(5)$ (in 5d)
SFK, Malinsky $A_4 \times$ Pati-Salam
Varzielas, SFK, Ross $\Delta_{27} \times$ Pati-Salam/SO(10)

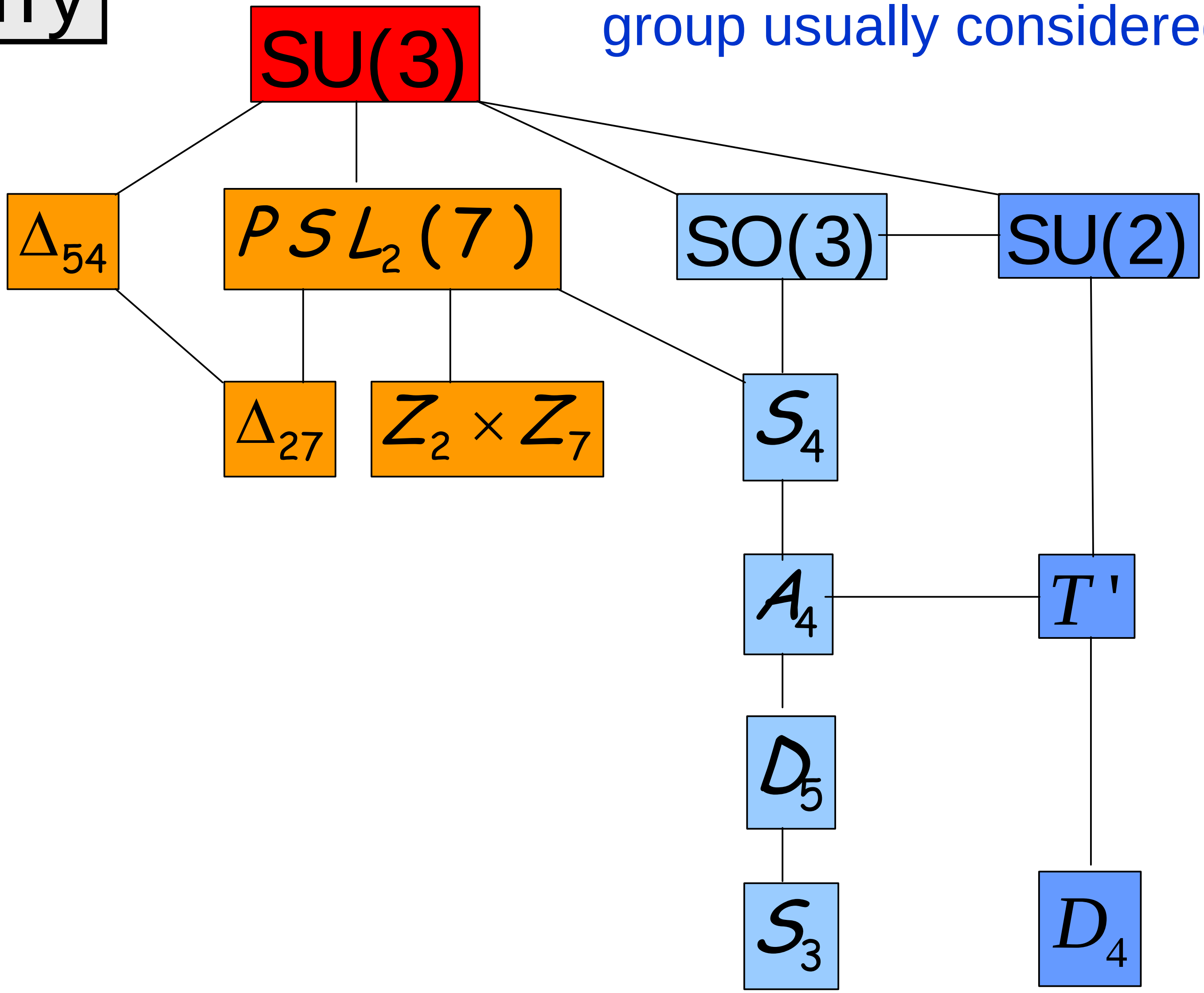


GGUT

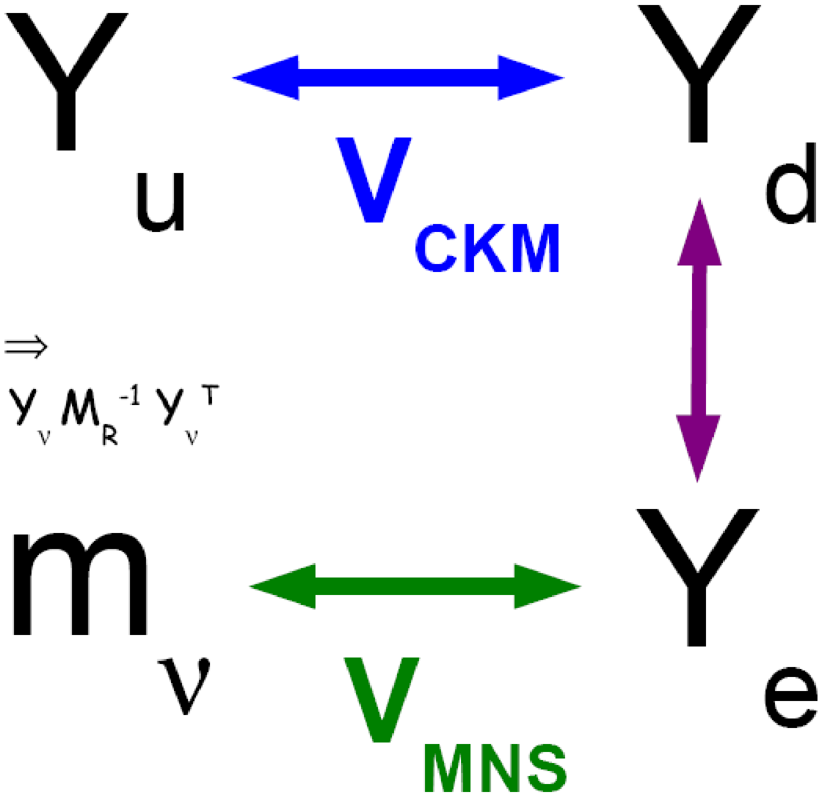

E_6
$SU(5)\times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

G_{Family}

SU(3) is the largest family group usually considered



GUT relations



Class of models: $\theta_{12}^{d} \gg \theta_{12}^{u}$

$\theta_{12}^{d} \approx \theta_C$

Georgi-Jarlskog

GUT relation

$$\theta_{12}^{e} \approx \frac{\theta_{12}^{d}}{3} \approx \frac{\theta_C}{3}$$

See-saw ⇒

$m_\nu = v_{EW}^2 Y_\nu M_R^{-1} Y_\nu^T$

Mixing Sum Rule

$$U_{PMNS} = V^{E_L} V^{\nu_L \dagger}$$

Cabibbo-like

Tri-bimaximal

Bjorken; Ferrandis, Pakvasa; SFK

$$\theta_{13} \approx \frac{\theta_{12}^{e}}{\sqrt{2}} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^{\circ},$$

$$\theta_{12} = 35^{o} + \frac{\theta_C}{3\sqrt{2}} \cos\delta$$

SFK

Oscillation phase

$$\theta_{12}^{o} = 35^{o} + \theta_{13}^{o} \cos\delta$$

Mixing Sum Rule

SFK; Antusch,SFK; Masina

$$s \approx r\cos\delta + \eta\left(\frac{1}{6} - \frac{1}{3}\frac{m_2^{\nu}}{m_3^{\nu}}\cos\alpha_2\right)$$

RG correction < 1^{o}

Antusch,SFK,Malinsky, SFK, Boudjemaa

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

SFK

Conclusion

- **Neutrino mass and mixing requires new physics BSM**
- **Many roads for model building, but answers to key experimental questions will provide the signposts**
- **If TBM is accurately realised this may imply a new symmetry of nature: family symmetry broken by flavons**
- **See-saw naturally leads to TBM via Form Dominance**
- **GUTs $\times$ family symmetry with see-saw + FD is very attractive framework for TBM $\rightarrow$ sum rule prediction**
- **The sum rule underlines the importance of showing that the deviations from TBM r,s,a are non-zero and measuring them and CP phase δ**
- **Neutrino Telescopes may provide a window into neutrino mass, quantum gravity and dark energy**