On amplified and tiny signatures of neutrino oscillations

"ν oscillations in Venice", 25. july 2001

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Topics

1) (ν, N) association of ν-flavors : minimal extension in SO10

2) mass and mixing structure

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix}_{6\times 6} \; ; \; \mathcal{U} = T \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix}$$

$$\mathfrak{a} = (1+\varepsilon\varepsilon^{\dagger})^{-1/2}\, u$$

sub unitary

3) consequences : CPT, T

15 CP-parameters.

lepton flavor violation.

4) conclusions and outlook.

SO10 extension of the standard model

$$\left(\nu, N\right)^{\dot\gamma}_{(J)} \qquad (J) = 1, 2, 3$$

(3 parity)

$$\{f\}^{\dot\gamma}_{J} : \begin{pmatrix} u^{1,2,3} & \nu_e & N_e & \hat{u}^{1,2,3} \\ d^{1,2,3} & e^- & e^+ & \hat{d}^{1,2,3} \end{pmatrix} \qquad \dot\gamma\,(=1,2) \quad J\,(=1,2,3)$$

B−L : could it be conserved for $m_{N_J} \to \infty$? no

B−L (1 family)

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -1 & (+1) & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -1 & +1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \end{pmatrix}$$

including $N^{\dot\gamma}_{(J)}$ the (B−L) current is of vectorial form

(B−L)(+, −) pairing throughout.

all anomalies of

$$j_\mu^{B-L} \,/\, = \sum_{(J)} f^{*\alpha}_{(J)r} \,(B-L)_r\, \sigma_{\mu\alpha\beta}\, f^{\beta}_{Jr}$$

(no N_J)

$\left(g \times g,\; W \times W,\; Z \times Z, \ldots \right)$.

$su3_c \qquad su2_L, \; u1_y$

$$\boxed{\begin{aligned} \partial^\mu J_\mu^{(B-L)} \Big/ &= 3\, c_{2/2}\left(\mathrm{Spin}/\mu\right) \, P_{2/2}(R) \\ \text{no } \# \left\{ \begin{pmatrix} -(B-L) \\ \text{anyons} \end{pmatrix} \right\} & \qquad \searrow -\frac{1}{24} \end{aligned}}$$

$$\sum_n \lambda^{2n} \, P_{2n/2}(R) = \mathrm{Det}\left(1 - \lambda \bar{R}\right)$$

Pontryagin classes

$$= \exp \operatorname{tr} \log\left(1 - \lambda \bar{R}\right)$$

$$= \exp - \sum_{\nu=1}^{\infty} \lambda^{2\nu} \left(\operatorname{tr} \bar{R}^{2\nu}\right)\left(\frac{1}{2\nu}\right)$$

$$\bar{R}^a{}_b = \frac{1}{2\pi} \, \frac{1}{2} \, dx^\mu \wedge dx^\nu \left(R^a{}_b\right)_{\mu\nu}$$

curvature 2-form

$$c_{2/2}\left(\mathrm{Spin}/\mu\right) = -\frac{1}{24}$$

$$P_{2/2}(R) = \frac{1}{16\pi^2} \, R^{ab}{}_{\mu\nu} \, \tilde{R}_{ab}{}^{\mu\nu} \Leftarrow -\frac{1}{2} \operatorname{tr}\left(\bar{R}\right)^2$$

$$\tilde{R}_{ab}{}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\sigma\tau} R_{ab;\sigma\tau}$$

pontryagin class — spin class

Hirzebruch-
Atiyah.

$$\mathrm{Det}\left(1-\lambda\bar{R}\right) \qquad \mathrm{Det}\left(\frac{\lambda\bar{R}/2}{\sin\bar{R}/2\,\lambda}\right)^{1/2}$$

$$= \sum \lambda^{2\nu} P_{2\nu/2}(R) \qquad = \sum \lambda^{2n} c_{2n/2}(\mathrm{spin}, R)$$

$\downarrow$

pontryagin
characters

spin
characters.

$c_n\left(\mathrm{spin}/p\right)$ express one through the other

$$P_1 = -\frac{1}{2}\,tr\,\bar{R}^2$$

$$P_2 = -\frac{1}{4}\,tr\,\bar{R}^4 + \frac{1}{8}\left(tr\,\bar{R}^2\right)^2$$

....

$$c_1 = \frac{1}{48}\,tr\,\bar{R}^2 = -\frac{1}{24}P_1$$

$$c_2 = \frac{1}{128}\,\frac{1}{45}\left[\underbrace{tr\,\bar{R}^4 + \frac{5}{4}\left(tr\,\bar{R}^2\right)^2}_{-4P_2+7(P_1)^2}\right]$$

$d=4$ $\qquad\qquad$ $d=8$

$$p_1 = -\frac{1}{2}\,\mathrm{tr}\,\bar{R}^2 \qquad\qquad p_2 = -\frac{1}{4}\,\mathrm{tr}\,\bar{R}^4 + \frac{1}{8}\left(\mathrm{tr}\,\bar{R}^2\right)^2$$

$$c_1 = \frac{1}{48}\,\mathrm{tr}\,\bar{R}^2 \qquad\qquad c_2 = \frac{1}{2^7 3^2 \cdot 5}\left[\mathrm{tr}\,\bar{R}^4 + \frac{5}{4}\left(\mathrm{tr}\,\bar{R}^2\right)^2\right]$$

$$P(R) : \sum p_{2\nu/2}\,\lambda^{2\nu} = \mathrm{Det}\left(1 - \lambda\,\bar{R}\right)$$

$$\hat{A}(R) : \sum c_{2\nu/2}\,\lambda^{2\nu} = \mathrm{Det}\left(\frac{\lambda\,\bar{R}/2}{\sin\lambda\bar{R}/2}\right)^{1/2}.$$

- the classes are simple
- the relations are obvious but not clearly simple

$$c_1 = -\frac{1}{24}\,p_1$$

$$c_2 = \frac{1}{2^7 3^2 5}\left[-4p_2 + 7\left(p_1\right)^2\right]$$

the consequence is subtle:

there does not exist a symmetry which preserves chiral ν's massless! in SM $\otimes$ gravity.

$\hat{A}$ is the index of the (Euclidean) Dirac operator $\otimes$... in gravitational background.

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$\therefore$ B-L. $\left\{\begin{matrix} \nu^{\delta} \\ \varepsilon_{\alpha\beta}\, N^{*}\, \beta \end{matrix}\right\}_{A,J}$ can form

a Dirac like (doubled) and exactly degenerate fermion

$$\left\{\begin{matrix} (f\,\nu N)_A \\ (e^-)_A \end{matrix}\right\}_{J=1,2,3} \qquad A = 1, \cdots 4.$$

with a (ν, N) mass matrix of similar structure like the $\bar{q}\, q$ mass matrix

and then

$$\left[\bar{f}^{(\nu)}_J\, \gamma^\alpha\, f^\nu_J + (\bar{e}^-_J\, \gamma^\alpha\, e^-_J) + \frac{1}{3}\left(\bar{q}^c_J\, \gamma^\alpha\, q^c_J\right)\right] = J_\mu^{V}\,(B-L)$$

$(u\,\&\,d)_J$

is exactly conserved & vector current and further not spontaneously broken (and un-gauged then)

Let us 'fix ideas' as to the light ν-flavor mass scales

assuming

$m_{\nu_3} \gg m_{\nu_2} > (\text{or} \gg)\ m_{\nu_1}$

$$\underset{\substack{\text{atm.} \\ (\bar{\nu})_\mu \leftrightarrow (\bar{\nu})_\tau}}{\Delta^2_{32}} = 3 \cdot 10^{-3}\,(eV)^2 \leftrightarrow \left[\text{eventually to be modified}\right]$$

$$\Delta_{32} = 0.055 \text{ eV} = 632\,°K.$$

$$\underset{\substack{\text{solar} \\ \nu_e \leftrightarrow \nu_\mu \ldots}}{\Delta^2_{21}} = 3.5 \cdot 10^{-5}\,(eV)^2$$

LMA solution

$$\Delta_{21} = 0.0059\,\text{eV} = 68.5\,°K.$$

$m_{\nu_3} \simeq 0.055\,\text{eV} \longleftrightarrow 632\,°K$

$m_{\nu_2} = 0.0059\,\text{eV} \longleftrightarrow 68.5\,°K.$

m_{ν_1} $\quad (\gtrsim) \quad T_\gamma^\circ \sim 2.73\,°K$

$T_\nu^\circ \sim 2.0\,°K.$

hot dark matter $\rightarrow$ but 'cold' today. $\sqrt[3]{4/11} \cdot T_\gamma^\circ$

the general (ν, N) mass term
is of the form (compatible
with SM renormalizability
& exclusion

$$-\mathcal{L}_M = \frac{1}{2} \underbrace{\nu_i^{\delta} \Sigma_{\dot\gamma\dot\delta}}_{\nu_{i\dot\gamma}} M_{ik} \nu_k^{\dot\gamma} + h.c.$$

$$\nu_k^{\dot\gamma} = \begin{cases} \nu_1^{\dot\gamma}, \nu_2^{\dot\gamma}, \nu_3^{\dot\gamma} & \text{for } k = 1,2,3 \\ N_1^{\dot\gamma}, N_2^{\dot\gamma}, N_3^{\dot\gamma} & \text{for } k = 4,5,6 \end{cases}$$

$$\Sigma = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

$$M_{ik} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} = M_{ki}$$

$M \longrightarrow |\Delta(B-L)| = 2.$

$$\frac{v}{\sqrt{2}} = 174.1\,\text{GeV}$$

$$\mu = g^{N\nu} \cdot v_{ch} = g_{(u)}^{N\nu} \frac{1}{\sqrt{2}} v^{(u)}$$

one scalar doublet | two scalar doublet.

$M = 0$ for exact $B-L$ symmetry

So we can make the following
Cedar kaneexperiment
for $M=0$ only:

Just after ν decoupling and $\gamma, e^\pm, N$
reheating

$T_\nu \gg m_{\nu\,\mathrm{eight}}$ 3 hot $\nu+\bar\nu$ flavors.

$$\frac{N_\nu + N_{\bar\nu}}{N_\gamma} = \frac{\overset{3\times n_{fam}}{9}}{11}$$

all (almost) ν are left handed
$\bar\nu$ right handed.

N^δ admixture: $$\frac{N_N}{N_\nu} = \frac{1-\langle v\rangle}{2}$$

$$= \left\langle \frac{m_\nu^{2\ \mathrm{light}}}{4E_\nu} \right\rangle = \frac{\langle m_\nu^{2\ \mathrm{light}} \rangle}{36\,T^2} \ll 1$$

but after drop of $T < m_{\nu\,\mathrm{eight}}$ (i.e. today)

only $\frac{1}{2}$ of ν & $\bar\nu$ are weak interaction
(3, 11) active!

if we accelerate by gravity todays relic
neutrinos → centers of galaxies .....
and contact $\nu, \bar\nu$ by (W, ?) interactions

accelerate through [illegible] back
to relativistic energies. (near a
Schwarzschild
radius!)

$\frac{N_\nu + N_{\bar{\nu}}}{N_\gamma}$ (reaccelerated) (today)

$\rightarrow \frac{1}{2} \cdot \frac{9}{11}$ instead of $\frac{9}{11}$

half of the $\nu + \bar{\nu}$ (in the era of
electron weak W, Z - induced reactions
would be lost.

We consider the clearly 'orthogonal'
alternative: $\frac{N_\nu + N_{\bar{\nu}} \text{ reaccelerated today}}{N_\nu + N_{\bar{\nu}} \text{ expected}} = 1$

$\Rightarrow |M| \gg m_{\text{weight}}$ $\mathcal{M} = \begin{pmatrix} c & \mu^T \\ \mu & M \end{pmatrix}$

$\Rightarrow$ (leptonic) numbers are violated
(& $B-L$

The Dirac-like (Majorana) equations, right- and left-chiral fields:

$$(\nu^{\dot\gamma})_J \quad , \quad (\nu^*)_{\alpha j} = \varepsilon_{\alpha\gamma} (\nu_J^{\dot\gamma})^*$$

$$(\nu_{J+3})^{\dot\gamma} = (N^{\dot\gamma})_J \quad , \quad (N^*)_{\alpha j} = \varepsilon_{\alpha\gamma} (N_J^{\dot\gamma})^* = (\nu^*)_{\alpha}$$

$$i D^{\dot\gamma\alpha} \begin{pmatrix} \nu^* \\ N^* \end{pmatrix}_\alpha = \underbrace{\begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \nu^{\dot\gamma} \\ N^{\dot\gamma} \end{pmatrix}$$

$$\boxed{\begin{aligned} i D^{\dot\gamma\alpha} (\nu^*)_\alpha &= \mathcal{M} \, (\nu)^{\dot\gamma} \\ i D_{\alpha\dot\gamma} (\nu)^{\dot\gamma} &= \mathcal{M}^+ \, (\nu^*)_\alpha . \end{aligned}}$$

$$\mathcal{M}^+ = \bar{\mathcal{M}} ; \; \mathcal{M} = \mathcal{M}^T$$

$$\begin{pmatrix} (\nu^*)_\alpha \\ (\nu)^{\dot\gamma} \end{pmatrix}_k = f_{A,k} \qquad k = 1, \dots, 6$$

$$i \gamma^\mu D_\mu f = \left(\mathcal{M}^+ \frac{1+\gamma_5^R}{2} + \mathcal{M} \frac{1-\gamma_5^R}{2} \right) f$$

$$f = f^{(c)} = C \gamma^0 f^*$$

Diagonalization of $\mathcal{M}$ (6x6)

— no basic changes if we introduce further singlets (e.u.)

simple and straightforward but...

No. s beyond the pairing

$(\nu, N)_{J=1,2,3} \to \nu_k \quad k=1,\cdots \underline{6}.$

$$\mathcal{M} = \mathcal{U}\, \mathcal{M}_D\, \mathcal{U}^T \qquad (6\times 6).$$

$\mathcal{U}^{-1}/ \;\; = \bar{\mathcal{U}}^T.$

$$i\, D^{\dot\gamma\alpha} (\nu^*)_\alpha = \mathcal{U}\, \mathcal{M}_D\, (\mathcal{U}^T \nu)^{\dot\gamma}$$

$$\mathcal{U}^{-1} (\nu^*)_\alpha$$

$\mathcal{M}^+ = \bar{\mathcal{U}}\, \mathcal{M}_D\, \bar{\mathcal{U}}^{-1}$

$$= \left[(\mathcal{U}^T \nu)^* \right]_\alpha . \quad \text{unitary}$$

$$i\, D^{\dot\gamma\alpha} (\mathcal{U}^T \nu)^*_\alpha = \mathcal{M}_D\, (\mathcal{U}^T \nu)^{\dot\gamma}$$

$$i\, D_{\alpha\dot\gamma} (\mathcal{U}^T \nu)^{\dot\gamma} = \mathcal{M}_D\, (\mathcal{U}^T \nu)^*_\alpha .$$

$$\mathcal{M}_D = \begin{pmatrix} m_1 & & & & & \\ & m_2 & & & 0 & \\ & & m_3 & & & \\ & & & M_1 & & \\ & 0 & & & M_2 & \\ & & & & & M_3 \end{pmatrix} \to m_k;\; k=1,\cdots 6 \;\; \geq 0$$

Transformation

between weak int. ⟷ mass fields

$$\mathrm{I} \qquad \nu_K = \overline{U}_{Ke}\,\hat{\nu}_e + \overline{U}_{K\,e+3}\,\hat{N}_e \Big)^{\dot{\alpha}}$$

$$\mathrm{II} \qquad \nu^*_K = U_{Ke}\,\hat{\nu}^*_e + U_{K\,e+3}\,\hat{N}^*_e \Big)_{\alpha}$$

$$\mathrm{I}_{\text{heavy}} \qquad N_K = \overline{U}_{K+3\,e}\,\hat{\nu}_e + \overline{U}_{K+3\,e+3}\,\hat{N}_e \Big)^{\dot{\alpha}}$$

$$\mathrm{II}_{\text{heavy}} \qquad N^*_K = U_{K+3\,e}\,\hat{\nu}^*_e + U_{K+3\,e+3}\,\hat{N}^*_e \Big)_{\alpha}$$

Low-low oscillation physics:

only $\hat{\nu}$, $\hat{\nu}^*$ are involved

in ↗ production below m_N thresholds

↘ rescattering at distance $\sqrt{s} < m_N$

$$(\mathrm{I}, \mathrm{II}) \rightarrow U^{(6)}_{Ke} = (\tau\, U_0)_{\substack{3\times 3\\ Ke}}$$

not a unitary matrix

τ hermitian positive ($\leq \mathbb{1}$)

'Low-Low' oscillation states and amplitudes

$$u^{(e)}_{\kappa e} = \alpha_{\kappa e} \qquad \alpha : 3 \times 3$$

production

unnormalized state of ν-type κ

$$|\bar{p} ; t=0 ; \kappa \rangle_0 = \alpha_{\kappa e} \, |\bar{p} ; m_e ; 0 \rangle$$

$$= N_\kappa \, |\bar{p} ; t=0 , \kappa \rangle .$$

normalized
unitarily orthogonal
(no heavy N-s)

$$N_\kappa^2 = \left(\alpha \alpha^+ \right)_{\kappa\kappa} = \left(T^2 \right)_{\kappa\kappa} \quad (\leq 1)$$

$$\alpha = T \, U_0$$

independent of U_0 3×3 unitary.

$$T \leftarrow \left(1 + \tau\tau^+ \right)^{-1/2}$$

some subtle specifications →

$\nu \rightarrow |\vec{p}, t=0; k\rangle = \rho_k \, U_{ke} \, |\vec{p}, m_e, 0\rangle$

$$\rho_k = \frac{1}{N_k} = \left(\left(\mathbf{I}^2 \right)_{kk} \right)^{-1/2} \geq 1.$$

subtle things :

- $\rho_k U_{ke}$ can be enforced by properly tagging ν^k in production (countour only) irrespective of decay rates e.g. $\pi^+ \to \mu^+ \nu$ $(+n\gamma)$

- Spin: probabilities for helicities

$$\begin{cases} -1 & 1 - \dfrac{1-\langle v\rangle}{2} \\ +1 & \dfrac{1-\langle v\rangle}{2} \sim \left\langle \dfrac{m_\nu^2}{2E(E+p)} \right\rangle \end{cases}$$

induces $|\Delta L| = 2$ transitions,

↳ neglect for now.

does depend on decay amplitudes (or production)

– from $\nu \to \bar{\nu}$

$$\rho_K \to \rho_K \,; \quad \mathcal{U} \to \overline{\mathcal{U}}$$

neglecting 'wrong' helicity parts

$$\nu : \quad A_{K' \leftarrow K} = \rho_{K'} \rho_K \left(\mathcal{U} \, U_{diag}(t) \, \mathcal{U}^{+} \right)_{K'K}$$

$$\bar{\nu} : \quad \bar{A}_{K' \leftarrow K} = \rho_{K'} \rho_K \left(\overline{\mathcal{U}} \, U_{diag}(t) \left(\overline{\mathcal{U}} \right)^{+} \right)_{K'K}.$$

$$U_{diag}(t) = \begin{pmatrix} e^{-iE_1 t} & & 0 \\ & e^{-iE_2 t} & \\ 0 & & e^{-iE_3 t} \end{pmatrix}.$$

$$E_e = \sqrt{\bar{p}^2 + m_e^2} \sim p + \frac{1}{2} m_e^2 \frac{1}{p}.$$

T

$\downarrow$ matter effect independent of $\mathcal{U}$, S, τ, U_0.

$(1 + \tau\tau^{+})^{-1/2}$ $\qquad \mathcal{U} - \tau U_0 \quad$ for $k = k' = e$

Just modifies $\underline{\mathcal{U}}$. (energy dependent)

$\hat{T}$ and $\hat{\Theta} = \hat{C}\hat{P}\hat{T}$

$$A_{\substack{K' \leftarrow K \\ \nu - s}}(t) = \rho_{K'}\rho_K \left(a \, U_{diag}(t) \, a^+ \right)_{K'K}$$

$$\bar{A}_{\substack{K' \leftarrow K \\ \bar{\nu} - s}}(t) = \rho_{K'}\rho_K \left(\bar{a} \, U_{diag}(t) \, \bar{a}^+ \right)_{K'K}$$

$$\hat{C}\hat{P}\hat{T} \left[\bar{A}_{K' \leftarrow K}(-t) \right]^* = A_{K' \leftarrow K}(t)$$

$$\downarrow$$

$$(\rho_{K'}\rho_K) \times \left(\bar{a}_{K'e} \, U_{diag\,e}(-t) \, a_{Ke} \right)^*$$

$$= a_{K'e} \, \bar{U}_{\substack{diag \\ e}}(-t) \, \bar{a}_{Ke}$$

$$= a_{K'e} \, U_{\substack{diag \\ e}}(t) \, \bar{a}_{Ke}$$

$$\updownarrow \qquad \text{id.} + a.$$

$$(\rho_{K'}\rho_K) \times \qquad a_{K'e} \, U_{\substack{diag \\ e}}(t) \, \bar{a}_{Ke}$$

T : $A_{K' \leftarrow K}(-t)^* = A_{K' \leftarrow K}(t)$ and $A \to \bar{A}$

$\to a = \bar{a}$.

$\hat{\Theta}$ = CPT : no phase redundancies in (ν, N) sector.

$$\hat{\Theta}\, \nu^{\gamma}_{J}(x)\, \hat{\Theta}^{-1} = i\, F_{JK} (\nu^{*})^{\gamma}_{K}(-x)$$

$J, K = 1, \ldots, n$

$$\hat{\Theta}\, \nu^{*\gamma}_{J}(x)\, \hat{\Theta}^{-1} = -i\, \bar{F}_{JK}\, \nu^{\gamma}_{K}(-x)$$

$$\mathcal{L}_{kin} = \frac{1}{2} \nu^{+}\, i \overleftrightarrow{\partial}_{t}\, \nu$$

simple extension to gauge fields

$\mathcal{L}_{kin}(x) \to \mathcal{L}_{kin}(-x)$

$\hookrightarrow \quad F F^{+} = \mathbb{1}$. ; F unitary

$$\mathcal{H}_m = \frac{1}{2} \nu^{T} \underset{\text{spin.}}{(-\varepsilon)}\, \underset{\text{flavor}}{\mathcal{M}}\, \nu + \frac{1}{2} \nu^{+} \varepsilon\, \mathcal{M}^{+} \nu^{*}$$

$$\hat{\Theta} : \frac{1}{2} \nu(-x)\, \bar{F}^{T} (-\varepsilon)\, \underbrace{\bar{\mathcal{M}}^{+}}_{\mathcal{M}}\, \bar{F}\, \nu(-x)$$

$\rightarrow$.

$$\rightarrow \quad \bar{F}^T M \bar{F} = M. \; (= M^T).$$

$$\bar{F}^T = V^{-1} \qquad V V^{+} = \mathbb{1}.$$

$$M = U \, M_{diag} \, U^T$$

$$\underbrace{(V^{-1} U)}_{U_1} M_{diag} (V^{-1} U)^T = M_{diag}.$$

for general $M_{diag} = \begin{pmatrix} m_1 \cdots m_3 & & & 0 \\ & & M_1 & \\ 0 & & & \cdots M_3 \end{pmatrix}$
(nondegenerate masses).

$$U_1 M_{diag}^2 U_1^{-1} = M_{diag}^2$$

$$\rightarrow U_1 = U_{1\,diag.} = \begin{pmatrix} e^{i\varphi_1} & & 0 \\ & \ddots & \\ 0 & & e^{i\varphi_6} \end{pmatrix} = U_1^T$$

$$\mathcal{U}_1 M_{diag}\, \mathcal{U}_1^T = M_{diag}$$

$\rightarrow$

$$\mathcal{U}_1 = \begin{pmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_6 \end{pmatrix} \quad (\sigma_k = \pm 1)$$

So the redundancy of $\mp \mathcal{U}$

is the same as the redundancy of $\mathcal{U}$.

$$M \rightarrow \mathcal{U}(M) \;:\; \mathcal{U}\, M_{diag}\, \mathcal{U}^T = M.$$

$\Rightarrow$ all genuine phases in M

as transferred to $\mathcal{U}$ $(= \mathcal{U}(M))$

become observable in principle.

through CP $\leftrightarrow$ T violation. (not $\hat{\Theta}$!)

locality

Counting phases $\rightarrow$

Counting phases: $\#\varphi$ $6 \to n$ flavors

$\#$ imaginary parameter $(\mathcal{M}) = \#c = \frac{n(n+1)}{2} = \#\varphi$

from $\mathcal{U}$:

$$\#\varphi = \dim \left(\mathcal{U}(n) \Big/ O(n) \right).$$

$$= n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$$ $\checkmark$: general $\mathcal{M}$ $\to 21$ for $n=6$.

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however going back to lepton flavour symmetry

$\#\varphi = 15$ $\quad \mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \to \begin{pmatrix} 0 & \mu^T \\ \mu & 0 \end{pmatrix}.$

and staying in SM (N extended)

$\#\varphi = 21 \to 1$ leptonic CKM matrix

$\hookrightarrow 0$ for $m_\nu \to 0$ [or degenerate]

$\Longleftrightarrow$ small effects if ν-masses are involved.

mass and mixing

$$\longleftrightarrow \quad \mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix}$$

(through higgs-vacuum → Lagrangean masses. ⟹ ptl masses.)

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assume M is in some sense

$\gg \mu, \mu^T$

e.g. $\|M\|^2 = \operatorname{tr} MM^+ = \sum_{i,k} |M_{ik}|^2.$

$\|\mu\| \ll \|M\|$

then diagonalization goes in two steps:

$$\mathcal{U} = \mathcal{T} \cdot \begin{pmatrix} U_0 & 0 \\ 0 & V_0 \end{pmatrix} \; ; \; \mathcal{T} \; 6\times6.$$

$t \rightarrow \tau$ is the main feature:

$$M' = \tau^{-1} M (\tau^{-1})^T = \begin{pmatrix} \{11\} & \{12\} \\ \{21\} & \{22\} \end{pmatrix}$$

$$\{11\} = \tau_{11} \left[-t_\mu - \mu^T t^T + t M t^T \right] \tau_{11}^T$$

$$\{12\} = \tau_{11} \left[\mu^T - t M - t_\mu \bar{t} \right] \tau_{22}^T$$

$$\downarrow$$

$$0$$

$$\{21\} = \{12\}^T$$

$$\{22\} = \tau_{22} \left[\mu \bar{t} + t^+ \mu^T + M \right] \tau_{22}^T$$

can solve recursively

$$\mu^T - t M - t_\mu \bar{t} = 0 \Rightarrow t = \mu^T M^{-1} - t_\mu \bar{t} M^{-1}$$

$$\mathcal{T} = \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} \begin{pmatrix} c_d & s_d \\ -s_d & c_d \end{pmatrix} \begin{pmatrix} u & \\ & v \end{pmatrix}^{-1}.$$

u, v unitary

$$c_d = \begin{pmatrix} \cos\alpha_1 & & 0 \\ & \ddots & \\ 0 & & \cos\alpha_3 \end{pmatrix} \; ; \; s_d = \begin{pmatrix} \sin\alpha_1 & & \\ & \ddots & \\ & & \sin\alpha_3 \end{pmatrix}$$

$$\mathcal{T} = \begin{pmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} \\ \mathcal{T}_{21} & \mathcal{T}_{22} \end{pmatrix} \; ; \; t = u\, t_d\, v^{-1}$$

$$t_d = s_d (c_d)^{-1} (tg \ldots)$$

$$\mathcal{T}_{11} = (1 + t t^+)^{-1/2}. \qquad \mathcal{T}_{22} = (1 + t^+ t)^{-1/2}$$

$$\mathcal{T}_{12} = \mathcal{T}_{11}\, t = t\, \mathcal{T}_{22}$$

$$\mathcal{T}_{21} = -t^+ \mathcal{T}_{11} = -\mathcal{T}_{22}\, t^+.$$

t serves to reach M block diagonal
<u>3 × 3.</u>

$$\to \{11\} = -t\{22\}t^T = M'_{11}$$ symmetric 3×3.

$$\{22\} = M'_{22}.$$

light masses

$$U = T'\begin{pmatrix} U_0 & 0 \\ 0 & V_0 \end{pmatrix} \to M'_{11} = U_0\, m_{diag}\, U_0^T$$

$$M'_{22} = V_0\, M_{diag}\, V_0^T$$

$\to$ heavy masses.

$$t = \mu^T M^{-1} - t\mu \bar{t} M^{-1}$$

subtle $\to$ but t is **not** diagonalized by U_0, V_0^{-1}

$$t = u\, t_d\, w^{-1} \neq U_0\, t_d\, V_0^{-1}$$

recursive solution simple (just tedious)

$$t_{n+1} = \mu^T M^{-1} - t_n \mu \bar{t}_n M^{-1}$$ odd power series in $\|\mu\|/\|M\|$

$$t = u\, t_d\, w^{-1} \neq U_0\, t_d\, V_0^{-1}$$

some relations exist

Counting complex phases:

$\#_4(M) = 15$

$\#_c(t) = 9 \rightarrow \#_4(u, v)$ modulo

$u = u\, u_d \;,\; v \rightarrow v\, u_d$

$= 6 + 6 - 3$

so 6 phases remain in U_0, V_0
with only one of them three independent.

. . . . .

$$t_{n+1} = \mu^T M^{-1} - t_n \mu\, \bar{t}_n\, M^{-1} \;;\; t_0 = 0$$

$$t_1 = \mu^T M^{-1} \;,\quad t_2 = \mu^T M^{-1} - \mu^T M^{-1} \mu\, \mu^{+} M^{+-1} M^{-1}$$

. . . . .

$O(1)$ $\qquad\qquad$ $O(3) \ldots O(5)$

$t^{(1)}$

$$\mathcal{M}_{11}^{(1)(4)} = -\left(1 + \Delta^{(2)}\right)\left(\mu^T M^{-1} \mu\right)\left(1 + \Delta^{(2)T}\right)$$

$$\Delta^{(2)} = \frac{1}{2}\, t^{(1)} t^{(1)+} = \frac{1}{2}\, \mu^T M^{-1} M^{-1+} \bar{\mu}$$

. . . .

$$\mathcal{U}_0 = \mathcal{U}_0^{(2)} \cdot \mathcal{U}_0^{(0)} \quad \ldots$$ up to including 2nd order.

but in general $\mathcal{U}_0^{(0)}$ will be large because of 'misalignment' of

$$\mu^T \leftrightarrow M \quad \leftrightarrow \mu^T M^{-1} \mu \ldots$$

$\rightsquigarrow$ in many analyses: LMA ν_e solar $\nu_\mu \rightsquigarrow \nu_\tau$ / ϑ_{mix} large

$$\mathcal{U} = \tau\, \mathcal{U}_0 \qquad \tau \simeq 1 - \Delta^{(2)}$$

$$\Delta^{(2)} = \frac{1}{2} \mu^T M^{-1} M^{-1+} \mu^-$$

$$\tau = \tau^+ \qquad \Delta^{(2)} = \left(\Delta^{(2)}\right)^+$$

$\hookrightarrow$ hermitian positive ≤ 1

$\rightarrow$ end with generic estimate

$$\| \Delta^{(2)} \| = O\left(\frac{m_\nu^{1/3}}{m_N^{1/3}}\right) \sim 10^{-21}$$

$$= \left(\frac{\mu^{1/3}}{m_N^{1/3}}\right)^2$$

but generic estimate maybe too severe!

from SO 10 mass relation

only 10-s of scalars.

$\mu_\ell \cong \mu_{u,c,t}$ at the unification scale.

$\mu \simeq \frac{1}{3}\mu_{u,c,t}$ at m_Z, m_W scales.

$$\mu_{1/3} = \left|\mathrm{Det}\,\mu\right|^{1/3} \simeq \frac{1}{3}\left(m_u m_c m_t\right)^{1/3}$$

~ 0.35 GeV

$m_{\nu\,1/3} = \left(m_{\nu_1} m_{\nu_2} m_{\nu_3}\right)^{1/3}$ to fix ideas

let $\sim 10^{-2}\,\mathrm{eV} = 10^{-11}\,\mathrm{GeV}$.

$$\mathcal{M} \rightarrow \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \rightarrow \boxed{(\mu_\nu)_{1/3}\, M_{1/3} = \left(\mu_{1/3}\right)^2}$$

$M_{1/3} = \left(m_{N_1} m_{N_2} m_{N_3}\right)^{1/3} \longrightarrow$ for $\mu_{\nu\,1/3} = 10^{-11}$ GeV

$M_{1/3} = 10^{10}$ GeV

very crude estimate, but many ways to obtain similar $M_{1/3}$.

- $\| t^{(1)} t^{(1)+} \| \Rightarrow O(10^{-21})$.

with some caution

on the other hand $-t^{(1)}\mu = -\mu^T M^{-1} \mu$

is the essential driving term

generating light ν-masses.

Conclusions and outlook

- the major accessible features

$m_{\nu_{1,2,3}}$ and U_0 (large mixings)

hopefully will establish the specific structure of ν-N dynamics

- the small and subtle effects

1) $\Omega = \tau U_0 \quad \tau \neq 1$

2) lepton flavor violation

$\longleftrightarrow \nu, \bar{\nu}$ helicities

2-β decays prove much harder to give experimentally clear and interpretable signals

- in particular CP ⟺ T violating effects in the $\nu, \bar{\nu}$ associated leptonic sector are much richer than for $q, \bar{q}$. They are tried to smallness of ν mass
  Despite this 'low' energy obstruction it seems to me worth looking for other effects especially at 'maximally reachable energies $\nu \to N$
- "... wenig ist mehr!"

elementary matrix relations

$$\mathcal{M} = \mu_T = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \; ; \quad \begin{array}{l} \mathcal{M} : 6 \times 6 \\ \mu, M = M^T : 3 \times 3 \end{array}$$

$$\mathcal{M} = \mathcal{U} \, \mu_D \, \mathcal{U}^T \longleftarrow \mathcal{M} = \mathcal{M}^T$$

$$\rightarrow \quad \mathcal{U} = T \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} \; ; \quad \begin{array}{ll} \mathcal{U}, T & \text{unitary} : 6 \times 6 \\ u, v & \text{unitary} : 3 \times 3 \end{array}$$

$$T = \begin{pmatrix} (1+\tau\tau^+)^{-1/2} & (1+\tau\tau^+)^{-1/2} \tau = \tau (1+\tau^+\tau)^{-1/2} \\ -\tau^+ (1+\tau\tau^+)^{-1/2} = -(1+\tau^+\tau)^{-1/2} \tau^+ & (1+\tau^+\tau)^{-1/2} \end{pmatrix}$$

$$\tau = x \, tg_D \, y^{-1} \; ; \quad x, y \text{ unitary } 3 \times 3$$

$tg_D : (tg\,\alpha_1 \, ; tg\,\alpha_2 \, ; tg\,\alpha_3)$ orthogonal rotation angles $\left(0 \leq \alpha_k \leq \pi/2 \right.$

$$T = \begin{pmatrix} (1+\tau\tau^+)^{-1/2} & (1+\tau\tau^+)^{-1/2}\tau \\ & = \tau(1+\tau^+\tau)^{-1/2} \\ -\tau^+(1+\tau\tau^+)^{-1/2} & (1+\tau^+\tau)^{-1/2} \\ = -(1+\tau^+\tau)^{-1/2}\tau^+ & \end{pmatrix}$$

$\tau =$ generically $O(\mu/M)$.

$$\tau = x \, tg\theta \, y^{-1}$$

$$T = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x^{-1} & 0 \\ 0 & y^{-1} \end{pmatrix}.$$

$\cos\theta = (\cos\alpha_1, \cos\alpha_2, \cos\alpha_3) \to \sin\theta \; (\cos\alpha_k \to \sin\alpha_k$

$\tau = \tau(\mu, M)$ satisfies the equation

$$\tau M = \mu^T - \tau\mu\bar{\tau}$$

or $\tau = \mu^T M^{-1} - \tau\mu\bar{\tau} M^{-1}$

solution by iteration $\to$ convergent for $|\mu/M| \ll$

$$\tau_{n+1} = \mu^T M^{-1} - \tau_n \mu \bar{\tau}_n M^{-1}$$

$\tau_0 = 0 \qquad \tau_1 = \mu^T M^{-1}; \; \tau_2 = \mu^T M^{-1} - \mu^T M^{-1} \mu \mu^+ \times \bar{M}^{-1} M^{-1}$ ....

$$\tau = \mu^T M^{-1} - \tau \mu \bar{\tau}$$

$\mu \to \lambda\mu \; / \lambda \to 1.$

$$\tau_0 = 0; \quad \tau_1 = \mu^T M^{-1} \cdot \lambda$$

$$\tau_2 = \mu^T M^{-1} \lambda$$

$$- \mu^T M^{-1} \mu \mu^+ \bar{M}^{-1} M^{-1} \lambda^3$$

only odd powers. + like $tg\, A \sim A + .. A^3$ expansion.

for $|\mu / M| < 1$ generally sequence converges

τ is uniquely given by $\mu, M.$

$T^{-1} M T$ becomes block diagonal 3×3

$$T^{-1} M T = \begin{pmatrix} M_{eight\,(e)} & 0 \\ 0 & M_{heavy\,(h)} \end{pmatrix}$$

$$M_{e,h} = M_{e,h}^T$$

otherwise arbitrary complex in general!

in terms of τ, μ, M

$$\mathcal{M}_e = (1+\tau\tau^+)^{-1/2} \left\{ \begin{array}{l} -\tau\mu - \mu^T\tau^T\lambda^2 \\ +\tau M \tau \lambda^2 \end{array} \right\} \left[(1+\tau\tau^+)^{-1/2} \right]^T$$

$$\mathcal{M}_h = (1+\tau^+\tau)^{-1/2} \left\{ \begin{array}{l} \tau^+\mu^T + \mu\bar{\tau}\lambda^2 \\ + M \end{array} \right\} (1+\tau^+\tau)^{-1/2}$$

only even λ powers!

---

lowest order (λ^2) for $\mathcal{M}_e$

$$\mathcal{M}_e^{(2)} = -\mu^T M^{-1} \mu$$

$\mu_D^{\text{light}} = O[(\mu/M)^2]$

seesaw

through second order mixing $\longleftrightarrow$ sea-saw (and higher even orders) there exists (a) relation between M_e, M_h (of course).

$$M_e = (1+\tau\tau^+)^{-1/2}\underbrace{\left\{-\tau\mu - \mu^T\tau^T + \tau M \tau^T\right\}}_{\{N_e\}}\left((1+\tau\tau^+)^{-1/2}\right)^T$$

$$M_h = (1+\tau^+\tau)^{-1/2}\underbrace{\left\{\tau^+\mu^T + \mu\bar{\tau} + M\right\}}_{\{N_h\}}\left((1+\tau^+\tau)^{-1/2}\right)^T$$

$$N_e = -\tau N_h \tau^T$$

$$(1+\tau\tau^+)^{-1/2}\tau = \tau(1+\tau^+\tau)^{-1/2}$$

$$\rightarrow \quad M_e = -\tau M_h \tau^T$$

$$\tau M = \mu^T - \tau\mu\bar{\tau}$$

T: 9 real + 9 imaginary
CP+ CP−
parameters

M_ℓ 6 real + 6 imaginary
CP+ CP−
parameters

→ total of 15 CP− parameters.
all observable in principle
but tied impact to small
ν-masses!

generic orders:

$$T = O(\mu/M) = O\left[\left(\frac{m_{\nu e}}{m_{\nu h}}\right)^{1/2}\right]$$

$$(1+T^{+}T)^{-1/2} - 1 = O((\mu/M)^2) = O\left(\frac{m_{\nu e}}{m_{\nu h}}\right)$$

→ generally
$\sim O(10^{-21})$

having established the primary (but small) heavy-light mixing

$$\hookrightarrow \quad \tau = \underset{\lambda}{\mu^T M^{-1}} - \underset{\lambda^3}{\mu^T M^{-1} \mu \mu^+ \bar{M}^{-1} M^{-1}} \quad \cdots \lambda^5.$$

$$\mathcal{U} = T \begin{pmatrix} u & 0 \\ 0 & V \end{pmatrix}$$

$\hookrightarrow$ main i.e. amplified mixing matrix u

$$\mathcal{M}_e = u\, \mathcal{M}_e^D\, u^T \qquad u = O(\lambda^0)$$
$$(V)$$

$$\mathcal{M}_h = V\, \mathcal{M}_h^D\, V^T$$

$$\mathcal{M}_e^D = (m_{\nu_1}, m_{\nu_2}\ m_{\nu_3}) \quad \sim 10^{-10}\,\mathrm{GeV}$$

$$\mathcal{M}_h^D = (M_1, M_2, M_3) \quad \sim 10^{(10)\div 11}\,\mathrm{GeV}$$

generally to fix ideas.

$$\mathcal{M}_e^D = -\left(u^{-1}\tau V\right) \mathcal{M}_h^D \left(u^{-1}\tau V\right)^T$$

$\to$ giving no immediate relation between $[u, V$ and $\tau]$. except $\longrightarrow$.

$$M_e^D = (i u^{-1} \tau V) M_n^D (i u^{-1} \tau V)^T$$

$$i u^{-1} \tau V = g.$$

$$M_e^D = g M_n^D g^T$$

$$\rightarrow g = (M_e^D)^{1/2} O (M_n^D)^{-1/2}.$$

$O O^T = \mathbb{1}_{3\times 3}$ O is an orthogonal in general complex 3×3 matrix.

$\rightarrow$ gives one relation for the heavy light mixing angles

$$\tau = x\, tg_D\, g^{-1} \rightarrow |\mathrm{Det}\, \tau| = \prod_{k=1}^{3} tg\, \alpha_K$$

$$= \left(\frac{\prod^{3} m_{\nu K}}{\prod^{3} M_J} \right)^{1/2}.$$

with the geometric mean light and heavy ν-masses:

$$\bar{m}_e = \left(m_{\nu_1} m_{\nu_2} m_{\nu_3}\right)^{1/3}.$$

$$\bar{M}_h = \left(M_1 M_2 M_3\right)^{1/3}$$

$$\overline{tg\alpha} = \left(\prod^{3} tg\,\alpha_K\right)^{1/3}.$$

$$\overline{tg\alpha} = \left(\frac{\bar{m}_e}{\bar{M}_h}\right)^{1/2}.$$ generally $\sim (3 \cdot 10^{-10})$.

This compares to the

$$|Det\,\mathcal{M}| = |Det\,\mu|^2$$ 'sea-saw'

second order mixing & renormalizability

so with

$$|\bar{\mu}| = |Det\,\mu|^{1/3}.$$

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix}$$

$$\bar{m}_e \bar{M}_h = |\bar{\mu}|^2.$$

$$|\bar{\mu}| \sim \frac{\left(m_u m_c m_t\right)^{1/3}}{3}$$

SO_{10} tree level relation

$[16 \times 16 \times 10]$ coupling
$f \quad f \quad \varphi$

$|\mu| \sim .35\,GeV$

$m_u = 5.75\,MeV$

$m_c = 1.2\,GeV$

$m_t = 175\,GeV$

so for $\overline{|\mu|} \sim 0.35\,\text{GeV}$

$$\overline{|\mu|}^2 \sim 0.1\,(\text{GeV})^2.$$

$$\overline{m}_e \sim 10^{-2}\,\text{eV}$$

$$10^{-11}\,\text{GeV}$$

to obtain an order of magnitude estimate

$$\overline{m}_e\, \overline{M}_n = \overline{|\mu|}^2 \sim 0.1\,\text{GeV}^2$$

$$\rightarrow \quad \overline{M}_n = 10^{10}\,\underline{\text{GeV}}.$$

---

but there can be deviations from the (geometric) <u>mean</u>. (nongeneric if extreme) low energy ν oscillations

$$\rightarrow \quad \mathcal{U} = \begin{pmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} \\ \mathcal{U}_{21} & \mathcal{U}_{22} \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} u \\ 0 \end{pmatrix}$$

involve $\mathcal{U}_{11}$ only. no heavy mass eigenstates are produced in primary beam and in the secondary reactions ν_μ sa(?)

$$\nu_\mu \to \nu_\tau : \nu_\tau N \to \underline{\tau^-} X \text{ say.}$$

$$\mathcal{U}_{11} = (1+\tau\tau^+)^{-1/2} \cdot u \simeq u - \frac{1}{2}\tau\tau^+ u$$

'sub' unitary.

$$\|\mathcal{U}_{11}\| < 1.$$

This is not a loss of probability but the rows or columns of $(\mathcal{U}_{11})_{3\times 3}$ are not (to the tiny order of 10^{-21}!) unitarily orthogonal.

→ distance independent $\overset{(-)}{\nu}_x \to \overset{(-)}{\nu}_y$ transformation.

→ induced by $\tau\tau^+$ $(O(\lambda^2))$.

On simple and subtle properties of neutrinos

Epiphany conf., Cracow, 6.01.2001
Peter Minkowski

Topics

1) SO10 extension of the standard model

→ viz: $(\nu ; N)^{\gamma}_{(j)}$ ([illegible] family [illegible] $j = 1, 2, 3$ [illegible] left-handed [illegible])

Simple → modifs possible

2.) $\hat{\Theta} = CPT$; redundancies and their removal

subtle → modifs. impossible.

3) mass and mixing properties of

$(\nu, N)^{\gamma}_{(j)}$, $(\nu^{*}, N^{*})^{\gamma}_{(j)}$

4) counting CP-violating parameters

5) outlook and conclusions