Implications of Future Precision Experiments

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Coming Improvements

MINOS: improved oscillation parameters MiniBOONE ←→ LSND L/E dependence of oscillations KATRIN Better 0v2β limits / signals

But why do we need precision measurements?

. . .

Solar Neutrinos: Learning About the Sun

 $p+p \mapsto {}^{2}H + e^{+} + v_{e}$

Observables:

- optical (total energy, surface dynamics, sun-spots, historical records, B, ...)

- composition

- neutrinos (rates, spectrum, ...)





 $p + e^{-} + p \mapsto {}^{2}H + v_{e}$

Learning from Atmospheric Neutrinos



New Physics Beyond the SM



Precison with New Neutrino Beams

- <u>conventional beams, superbeams</u>
 → MINOS, CNGS, T2K, NOvA, T2H,...
- <u>β-beams</u>

→ pure v_e and \overline{v}_e beams from radioactive decays; $\gamma \simeq 100$

• <u>neutrino factories</u>

 \rightarrow clean neutrino beams from decay of stored μ 's

$$P(\nu_e \to \nu_\mu) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2}$$

$$\pm \sin \delta_{\rm CP} \alpha \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta) \sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})}$$

$$+ \cos \delta_{\rm CP} \alpha \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta) \sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})}$$

$$+ \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}$$

correlations & degeneracies





Double Chooz





existing far detector hall

... + another existing big hall!

see talk by T. Lasserre

Double Chooz and Triple Chooz



Double Chooz and $O\nu 2\beta$

- m_{ee} versus m₁
 - for $\sin^2 2\theta_{13} = 0.2$

for $\sin^2 2\theta_{13} = 0.03$

➔ Double Chooz

Bilenky, Pascoli, Petcov Klapdor, Päs, Smirnov

ML,Merle, Rodejohann



• • •

precise neutrino parameters

why is this interesting?

- → unique flavour information
- → very precise: no hadronic uncertainties
- → different from quarks ←→ see-saw
- → tests models / ideas about flavour

History: Elimination of SMA



The Value of Precision for θ_{13}

- models for masses & mixings
- input: Known masses & mixings
 - \rightarrow distribution of θ_{13} "predictions"
- + θ_{13} often close to experimental bounds
 - ➔ motivates new experiments
 - θ₁₃ controls 3-flavour effects
 like leptonic CP-violation

for example: $\sin^2 2\theta_{13} < 0.01$ \Rightarrow

<u>physics question: why is θ₁₃ so small ?</u>
 → numerical coincidence
 → symmetry



Reference	$\sin\theta_{13}$	$\sin^2 2\theta_{13}$
50(10)		
Goh, Mohapatra, Ng [40]	0.18	0.13
Orbifold SO(10)		
Asaka, Buchmüller, Covi [41]	0.1	0.04
SO(10) + flavor symmetry		
Babu, Pati, Wilczek [42]	$5.5\cdot10^{-4}$	$1.2 \cdot 10^{-6}$
Blazek, Raby, Tobe [43]	0.05	0.01
Kitano, Mimura [44]	0.22	0.18
Albright, Dar [45]	0.014	7.8 - 10-1
Markenne [46]	0.99	0.18
Rog Velegeo Seville [47]	0.07	0.02
Chen, Mahanthappa [48]	0.15	0.09
Baby [49]	0.1	0.04
	0.1	0.01
Buchmüller, Wyler [50]	0.1	0.04
Bando Obara [51]	0.01 0.06	$4 \cdot 10^{-4}$ 0.01
Flavor summetries	0.01 11 0.000	1 10
Crimus Leveure [52, 52]	0	0
,,,,	-	-
Grimus Lavoura [52]	03	03
Crimus, Levoure [52] Babu, Ma, Valle [54]	0.3 0.14	0.3
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55]	0.3 0.14 0.08 0.4	0.3 0.08 0.03 0.5
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56]	0.3 0.14 0.08 0.4 0.07 0.14	0.3 0.08 0.03 0.5 0.02 0.08
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Monapatra [55] Ohlsson, Seidi [56] King, Ross [57]	0.3 0.14 0.08 0.4 0.07 0.14 0.2	0.3 0.08 0.03 0.3 0.02 0.08 0.15
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures	0.3 0.14 0.08 0.4 0.07 0.14 0.2	0.3 0.08 0.03 0.3 0.02 0.08 0.15
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohisson, Seidi [56] King, Ross [57] <i>Teatures</i> Honda, Kanako, Tanimoto [58]	0.3 0.14 0.08 0.4 0.07 0.14 0.2 0.08 0.20	0.3 0.08 0.03 0.3 0.02 0.38 0.15
Grimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohisson, Seidi [56] King, Ross [57] Textures Honda, Kaneko, Tanimoto [58] Lebed, Martin [59]	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.07 \dots 0.14 \\ 0.2 \\ 0.08 \dots 0.20 \\ 0.1 \end{array}$	0.3 0.08 0.03 0.3 0.02 0.08 0.15 0.03 0.15 0.04
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] <i>Textures</i> Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60]	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.07 \dots 0.14 \\ 0.2 \\ \hline 0.08 \dots 0.20 \\ 0.1 \\ 0.01 \dots 0.05 \end{array}$	$\begin{array}{r} 0.3 \\ 0.08 \\ 0.03 \dots 0.3 \\ 0.02 \dots 0.03 \\ 0.15 \\ \hline 0.03 \dots 0.15 \\ 0.04 \\ 4 \cdot 10 \stackrel{\bullet}{-} \dots 0.01 \end{array}$
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] <i>Textures</i> Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61]	$\begin{array}{c} 0.3 \\ \hline 0.14 \\ \hline 0.08 \dots 0.4 \\ \hline 0.07 \dots 0.14 \\ \hline 0.2 \\ \hline 0.08 \dots 0.20 \\ \hline 0.1 \\ \hline 0.01 \dots 0.05 \\ \hline 0.2 \end{array}$	$\begin{array}{r} 0.3 \\ 0.08 \\ 0.03 \dots 0.5 \\ 0.15 \\ 0.02 \dots 0.15 \\ 0.04 \\ 4 \cdot 10 \xrightarrow{4} \dots 0.01 \\ 0.15 \end{array}$
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61] $3 \times 2 \ see-saw$	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.2 \\ 0.08 \dots 0.20 \\ 0.1 \\ 0.01 \dots 0.05 \\ 0.2 \end{array}$	$\begin{array}{r} 0.3 \\ 0.08 \\ 0.03 \dots 0.5 \\ 0.02 \dots 0.06 \\ 0.15 \\ 0.03 \dots 0.15 \\ 0.04 \\ 4 \cdot 10^{-1} \dots 0.01 \\ 0.15 \end{array}$
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61] $3 \times 2 \ see-saw$ Appeleuist, Piai, Shrock [62, 63]	0.3 0.14 0.08 0.4 0.07 0.14 0.2 0.08 0.20 0.1 0.01 0.05 0.2 0.05	$\begin{array}{r} 0.3 \\ 0.08 \\ 0.03 \dots 0.5 \\ 0.15 \\ 0.02 \dots 0.15 \\ 0.04 \\ 4 \cdot 10^{-2} \dots 0.01 \\ 0.15 \\ 0.04 \\ \end{array}$
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61] $3 \times 2 \ see-saw$ Appelquist, Piai, Shrock [62, 63] Frompton, Clochew, Yanagida [64]	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.07 \dots 0.14 \\ 0.2 \\ 0.08 \dots 0.20 \\ 0.1 \\ 0.01 \dots 0.05 \\ 0.2 \\ \hline 0.85 \\ 0.1 \\ \end{array}$	$\begin{array}{r} 0.3 \\ 0.08 \\ 0.03 \dots 0.5 \\ 0.15 \\ 0.02 \dots 0.15 \\ 0.04 \\ 4 \cdot 10^{-2} \dots 0.01 \\ 0.15 \\ 0.04 \\ 0.01 \\ 0.01 \end{array}$
Crimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures Honda, Kanako, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61] $3 \times 2 \ see-saw$ Appelquist, Piai, Shrock [62, 63] Frempton, Clachew, Vanagida [64] Mei, Xing [65] (normal hierarchy)	0.3 0.14 0.08 0.4 0.07 0.14 0.2 0.08 0.20 0.1 0.01 0.05 0.2 0.05 0.1 0.07	$\begin{array}{r} 0.3 \\ 0.08 \\ 0.03 \dots 0.5 \\ 0.15 \\ 0.03 \dots 0.15 \\ 0.04 \\ 4 \cdot 10^{-1} \dots 0.01 \\ 0.15 \\ \hline 0.01 \\ 0.15 \\ \hline 0.01 \\ 0.02 \\ \hline \end{array}$
Grimus, Lavoura [52]Babu, Ma, Valle [54]Kuchimanchi, Mohapatra [55]Ohlsson, Seidi [56]King, Ross [57]TexturesHonda, Kaneko, Tanimoto [58]Lebed, Martin [59]Bando, Kaneko, Obara, Tanimoto [60]Ibarra, Ross [61] 3×2 see-sawAppelquist, Piai, Shrock [62, 63]Frampton, Clashow, Vanagida [64]Mei, Xing [65] (normal hierarchy)(inverted hierarchy)	$\begin{array}{c} 0.3 \\ \hline 0.14 \\ \hline 0.08 \dots 0.4 \\ \hline 0.2 \\ \hline 0.08 \dots 0.20 \\ \hline 0.1 \\ \hline 0.01 \dots 0.05 \\ \hline 0.2 \\ \hline 0.05 \\ \hline 0.1 \\ \hline 0.07 \\ > 0.006 \end{array}$	$\begin{array}{c} 0.3 \\ \hline 0.08 \\ \hline 0.03 \dots 0.5 \\ \hline 0.02 \dots 0.08 \\ \hline 0.15 \\ \hline 0.03 \dots 0.15 \\ \hline 0.04 \\ \hline 4 \cdot 10^{-1} \dots 0.01 \\ \hline 0.15 \\ \hline 0.04 \\ \hline 0.02 \\ > 1.6 \cdot 10^{-4} \end{array}$
Grimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61] 3 × 2 see-saw Appelquist, Piai, Shrock [62, 63] Frampton, Clashow, Vanagida [64] Mei, Xing [65] (normal hierarchy) (inverted hierarchy)	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.2 \\ 0.08 \dots 0.20 \\ 0.1 \\ 0.01 \dots 0.05 \\ 0.2 \\ \hline 0.05 \\ 0.1 \\ 0.07 \\ > 0.006 \end{array}$	$\begin{array}{c} 0.3 \\ \hline 0.08 \\ \hline 0.03 \dots 0.5 \\ \hline 0.02 \dots 0.08 \\ \hline 0.15 \\ \hline 0.03 \dots 0.15 \\ \hline 0.04 \\ \hline 0.04 \\ \hline 0.02 \\ > 1.6 \cdot 10^{-4} \end{array}$
Grimus, Lavoura [52]Babu, Ma, Valle [54]Kuchimanchi, Mohapatra [55]Ohlsson, Seidi [56]King, Ross [57]TexturesHonda, Kaneko, Tanimoto [58]Lebed, Martin [59]Bando, Kaneko, Obara, Tanimoto [60]Ibarra, Ross [61] 3×2 see-sawAppelquist, Piai, Shrock [62, 63]Frempton, Clachew, Yanagida [64]Mei, Xing [65] (normal hierarchy)(inverted hierarchy)Anarchyde Gouvéa, Murayama [66]	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.2 \\ 0.08 \dots 0.20 \\ 0.1 \\ 0.01 \dots 0.05 \\ 0.2 \\ \hline 0.01 \dots 0.05 \\ 0.2 \\ \hline 0.05 \\ 0.1 \\ 0.07 \\ > 0.006 \\ > 0.1 \end{array}$	$\begin{array}{c} 0.3 \\ \hline 0.08 \\ \hline 0.03 \dots 0.5 \\ \hline 0.02 \dots 0.08 \\ \hline 0.15 \\ \hline 0.03 \dots 0.15 \\ \hline 0.04 \\ \hline 4 \cdot 10 \xrightarrow{+} \dots 0.01 \\ \hline 0.15 \\ \hline 0.04 \\ \hline 0.02 \\ > 1.6 \cdot 10^{-4} \\ \hline > 0.04 \end{array}$
Grimus, Lavoura [52] Babu, Ma, Valle [54] Kuchimanchi, Mohapatra [55] Ohlsson, Seidi [56] King, Ross [57] Textures Honda, Kaneko, Tanimoto [58] Lebed, Martin [59] Bando, Kaneko, Obara, Tanimoto [60] Ibarra, Ross [61] 3 × 2 see-saw Appelquist, Piai, Shrock [62, 63] Frempton, Clashew, Yanagida [64] Mei, Xing [65] (normal hierarchy) (inverted hierarchy) Anarchy de Gouvéa, Murayama [66] Renormalization group enhancement	$\begin{array}{c} 0.3 \\ 0.14 \\ 0.08 \dots 0.4 \\ 0.2 \\ 0.08 \dots 0.20 \\ 0.1 \\ 0.01 \dots 0.05 \\ 0.2 \\ \hline 0.01 \dots 0.05 \\ 0.2 \\ \hline 0.07 \\ > 0.006 \\ > 0.1 \\ \end{array}$	$\begin{array}{c} 0.3 \\ \hline 0.08 \\ \hline 0.03 \dots 0.5 \\ \hline 0.02 \dots 0.08 \\ \hline 0.15 \\ \hline 0.03 \dots 0.15 \\ \hline 0.04 \\ \hline 4 \cdot 10^{-1} \dots 0.01 \\ \hline 0.15 \\ \hline 0.04 \\ \hline 0.02 \\ > 1.6 \cdot 10^{-4} \\ \hline > 0.04 \\ \hline \end{array}$

Further Implications of Precision

Precision allows to identify / exclude:

- special angles: $\theta_{13} = 0^{\circ}$, $\theta_{23} = 45^{\circ}$, ... $\leftarrow \rightarrow$ discrete f. symmetries?
- special relations: $\theta_{12} + \theta_C = 45^\circ$? $\leftarrow \rightarrow$ quark-lepton relation?
- quantum corrections
 renormalization group evolution

Provides also measurements or tests of:

- **MSW effect** (coherent forward scattering and matter profiles)
- cross sections
- 3 neutrino unitarity **< >** sterile neutrinos with small mixings
- neutrino decay (admixture...)
- decoherence
- NSI
- MVN, ...

The larger Picture: GUTs



GUT Expectations and Requirements

Quarks and leptons sit in the same multiplets

- → one set of Yukawa coupling for given GUT multiplet
- \rightarrow ~ tension: small quark mixings $\leftarrow \rightarrow$ large leptonic mixings
- → this was in fact the reason for the `prediction' of small mixing angles (SMA) – ruled out by data

Mechanisms to post-dict large mixings:

- → sequential dominance
- → type II see-saw
- ➔ Dirac screening
- → ...

Single right-handed Dominance

$$m_D = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & a & b \\ \cdot & c & d \end{pmatrix} \qquad M_R = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & x & 0 \\ \cdot & 0 & y \end{pmatrix}$$

If one right-handed neutrino dominates, e.g. y >> x

- \rightarrow small sub-determinant ~ m₂.m₃
- \rightarrow m₂ << m₃ i.e. a natural hierrachy
- \rightarrow tan $\theta_{23} \simeq a/c$ i.e. naturally large mixing

Sequential Dominance

$$m_D = \begin{pmatrix} a & b & c \\ d & e & f \\ g & e & h \end{pmatrix} \qquad M_R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$$

$$m_{\nu} = -m_D \cdot M_R^{-1} \cdot m_D^T = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

sequenatial dominance: z >> y >> x

→ small determinant ~ m₁ . m₂ . m₃
 → m₁ << m₂ << m₃ natural
 → naturally large mixings

King, ...

Large Mixings and See-Saw Type II

see-saw type II
$$m_v = M_L - m_D M_R^{-1} m_D^T$$

<u>m_D and M_R may posess small mixings and hierarchy</u> However: M_L can be numerically more important Example: Break GUT \rightarrow SU(2)_L x SU(2)_R x U(1)_{B-L} \rightarrow M_L from LR \rightarrow large mixings natural for almost degenerate case m₁~m₂~m₃ \rightarrow type I see-saw would only be a correction

type I – type II interference
 → M_L ~ m_DM_R⁻¹m_D^T
 → many possibilities

Dirac Screening

Question: Do neutrino masses always depend on the Dirac Yukawa couplings? **>** no

ML, Schmidt Smirnov

Assume:
$$\mathbf{v}_{\mathbf{L}}, \mathbf{v}_{\mathbf{R}}^{\mathbf{C}}, \mathbf{S} \rightarrow \qquad \mathcal{M} = \begin{pmatrix} 0 & Y_{\nu} \langle \phi \rangle & 0 \\ Y_{\nu}^{T} \langle \phi \rangle & 0 & Y_{N}^{T} \langle \sigma \rangle \\ 0 & Y_{N} \langle \sigma \rangle & M_{S} \end{pmatrix}$$

→ double seesaw

$$m_{
u}^{0} = \left[rac{\left\langle \phi
ight
angle}{\left\langle \sigma
ight
angle}
ight]^{2} Y_{
u} \left(Y_{N}
ight)^{-1} M_{S} \left(Y_{N}^{T}
ight)^{-1} Y_{
u}^{T}$$

fit fermions into GUT representations relation between Yukawa couplings, e.g. E6 $Y_{\nu} = c \cdot Y_{N}$

Consequences of Dirac Screening

Complete screening of Dirac structure

$$m_
u = c^2 \left[rac{\langle \phi
angle}{\langle \sigma
angle}
ight]^2 M_S$$

Outcome:

- Dirac Yukawa structure (small mixings) screened
- Hierarchical neutrino spectrum not required in see-saw
- Quark-lepton complimentarity possiblewith or without degenerate neutrino masses
- Double see-saw predics for M_R to be below M_{GUT} first see-saw $\rightarrow M_R \sim \langle s \rangle / M_S \simeq 10^{-3} M_{GUT} \simeq 10^{13} \text{ GeV}$

Flavour Unification

- so far no understanding of flavour, 3 generations
- apparant regularities in quark and lepton parameters
- ➔ flavour symmetries
- ➔ not texture zeros



Discrete Flavour Symmetries

Discrete Flavour Symmetries $\leftarrow \rightarrow$ flavour structure Example: Dihedral groups D_n

$$< A, B | A^n = 1, B^2 = 1, (AB)^n = 1 >$$





Specific Example: D₅ Hagedorn, ML, Plentinger

$$< A, B | A^n = 1, B^2 = 1, (AB)^n = 1 > .$$

complex generators

2₁:
$$A = \begin{pmatrix} e^{i\frac{2\pi}{5}} & 0\\ 0 & e^{-i\frac{2\pi}{5}} \end{pmatrix}$$
 $B = \begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix}$
2₂: $A = \begin{pmatrix} e^{i\frac{4\pi}{5}} & 0\\ 0 & e^{-i\frac{4\pi}{5}} \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix}$

character table

classes	\mathcal{C}_1	\mathcal{C}_2	\mathcal{C}_3	\mathcal{C}_4
G	1	В	А	A^2
$h_{\mathcal{C}_i}$	1	5	2	2
n_{C_i}	1	2	5	5
11	1	1	1	1
1_{2}	1	-1	1	1
2_1	2	0	$\frac{1}{2}(-1+\sqrt{5})$	$\frac{1}{2}(-1-\sqrt{5})$
2_{2}	2	0	$\frac{\overline{1}}{2}(-1-\sqrt{5})$	$\frac{\overline{1}}{2}(-1+\sqrt{5})$

Kronecker products

$$\begin{array}{rclrcl}
1_1 \times 1_1 &=& 1_1 \\
1_2 \times 1_1 &=& 1_2 \\
2_1 \times 1_1 &=& 2_1 \\
2_2 \times 1_1 &=& 2_2 \\
1_2 \times 1_2 &=& 1_1 \\
2_1 \times 1_2 &=& 2_1 \\
2_2 \times 1_2 &=& 2_2 \\
2_1 \times 2_1 &=& 1_1 + 1_2 + 2_2 \\
2_2 \times 2_1 &=& 2_1 + 2_2 \\
2_2 \times 2_2 &=& 1_1 + 1_2 + 2_1
\end{array}$$

Clebsch-Gordan Coefficients ...

D₅ Allowed Mass Terms

Task: search for mass terms which are for suitable Higges singlets under D₅

Notation: $L = \{L_1, L_2, L_3\}$ i_{th} generation fermions $\lambda_{ij}L_i^T(i\sigma_2)\phi L_j^c$ **Dirac mass terms:** $\lambda_{ij}L_{i}^{T} \equiv \phi L_{j}$ Majorana mass terms: $\Xi = \begin{pmatrix} \xi^0 & -\frac{\xi}{\sqrt{2}} \\ -\frac{\xi^+}{\sqrt{2}} & \xi^{++} \end{pmatrix}$ with

Resulting D₅ Symmetry Texture

L	L^C	Mass Matrix
$(1_2, 1_1, 1_1)$	$(2_1, 1_1)$	$\begin{pmatrix} \kappa_{1}\psi_{2}^{1} & -\kappa_{1}\psi_{1}^{1} & \kappa_{4}\phi^{2} \\ \kappa_{2}\psi_{2}^{1} & \kappa_{2}\psi_{1}^{1} & \kappa_{5}\phi^{1} \\ \kappa_{3}\psi_{2}^{1} & \kappa_{3}\psi_{1}^{1} & \kappa_{6}\phi^{1} \end{pmatrix}$

D5 singlet mass terms require the following quantum numbers for the scalars:

$$egin{aligned} \phi_1 &\sim \mathbf{1}_1 \ , \ \phi_2 &\sim \mathbf{1}_2 \ \mathrm{and} \ \psi_1 &\sim \mathbf{2}_1 \ . \end{aligned}$$

→ Check if phenomenological successful predictions arise

GUT *and* Flavour Unification

Example: SO(10) x SU(3)



GUT \otimes Flavour Unification

→ GUT group ⊗ continuous, gauged flavour group

- for example SO(10) \otimes SU(3)_{flavour}
- Generations are 3_F
- SSB of SU(3)_{flavour} between Λ_{GUT} and Λ_{Planck}
 - → all flavour Goldstone Bosons eaten
 - → discrete (ungauged) sub-group survives ←→ SSB potential
 - → e.g. Z2, S3, D5, A4, ...
 - → structures in flavour space

GUT \otimes Flavour Challenges

- GUT \otimes flavour is rather restricted
 - small quark mixings
 - large leptonic mixings
 - \rightarrow from unified GUT \otimes flavour representations
 - strong links between Yukawa couplings
- Difficulty grows with
 - size of flavour symmetry
 - size of the GUT group
 - → so far only a few viable models
 - → limited possibilities

Distinguish models by future precision





→v-parameters extremely valuable

→ long term: most precise flavour info