The theory of the Neutrino Mass

IV International Workshop on:
"Neutrino Oscillations in Venice

Un altro modo di guardare il cielo

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solution to sol and atm neutrino anomalies was the simplest

ν propagation studied with 4 independent sources
- sun
- cosmic rays
- reactors
- accelerators

spanning > 12 order of magnitudes in L/E

neutrinoss propagate as massive neutral fermions with specific mixing angles between mass and interaction eigenstates:

ν **oscillations**

+ possibly, a number of (still undetected) subleading effects results can be encoded in a [Lorentz x SU(2)xU(1)]-invariant Lagrangian

$$L = L_{SM} + \delta L(m_\nu) + \ldots$$

1st evidence of the incompleteness of the SM

additional operators giving negligibly small contributions to ν propagation in present experiments

L_{SM} invariant under global, non-anomalous

$$\underbrace{\frac{B}{3} - L_e, \qquad \frac{B}{3} - L_\mu, \qquad \frac{B}{3} - L_\tau}$$

broken individually by $\delta L(m_\nu)$
possible exception: (B - L)

[see Zwirner talk]

From the theory view point the simplest and more appealing (though still unconfirmed) possibility for $\delta L(m_\nu)$ is the leading non-renormalizable SU(2)xU(1) invariant operator

Weinberg's list

$$L = L_{SM} + \frac{c_5}{\Lambda} L_5 + \frac{c_6}{\Lambda^2} L_6 + \ldots$$

[80 independent d=6 operators]
Λ= scale of new physics

a unique d=5 operator (up to flavour combinations)

$$\frac{\mathrm{L}_5}{\Lambda_L} = \frac{(\tilde{H}^+ l)(\tilde{H}^+ l)}{\Lambda_L} = \frac{1}{2}\frac{v^2}{\Lambda_L}\nu\nu + \ldots$$

$$m_\nu = y\frac{v^2}{\Lambda_L} \longleftrightarrow m_f = \frac{y_f}{\sqrt{2}}v$$

smallness of m_ν due to $\frac{v}{\Lambda_L} << 1$

[for a different scenario see Shaposhnikov's talk]

$$m_\nu \approx \sqrt{|\Delta m^2_{32}|} \approx 0.05\ \mathrm{eV} \rightarrow \quad \Lambda_L \approx 10^{15}\ \mathrm{GeV}$$ not that far from GUT scale

the effective theory is "nearly" renormalizable
the first effect of New Physics: neutrino masses and mixing angles!

L_5 violates B-L by two units

- B-L violated, in general, when attempting to unify particle interactions (GUTs)
- global quantum numbers expected to be violated at some level by quantum gravity effects

ν as a window on GUT physics

$\Lambda_L \approx 10^{15}\ GeV$ independent indication of a new physical threshold around the GUT scale

- many GUTs contain ν^c
- heavy ν^c exchange produces a specific version of L_5

$$\frac{L_5}{\Lambda_L} = -\frac{1}{2}(\tilde{H}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{H}^+ l) + h.c. + \ldots$$

see-saw mechanism

see-saw can enhance small mixing angles in M and in y_ν into the large ones observed in ν oscillations

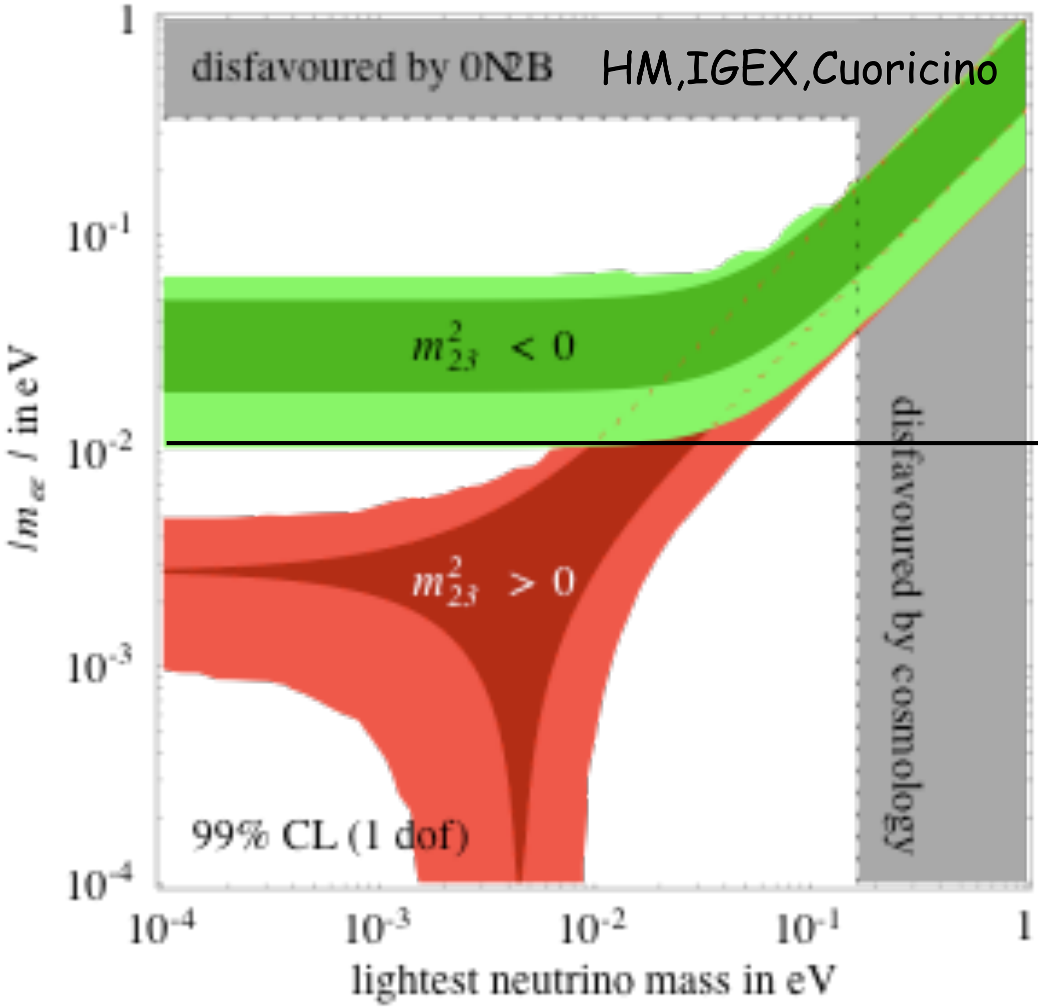
interesting link to baryogenesis

- B-L violation welcome in baryognesis
- out-of-equilibrium, CP violating decay of ν^c can drive baryogenesis through leptogenesis

if $\delta L(m_\nu)=L_5$ we expect $0\nu\beta\beta$ at some level, through the combination

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12} e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13} e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



[Strumia,Vissani]

from the current knowledge of $(\Delta m^2_{ij}, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

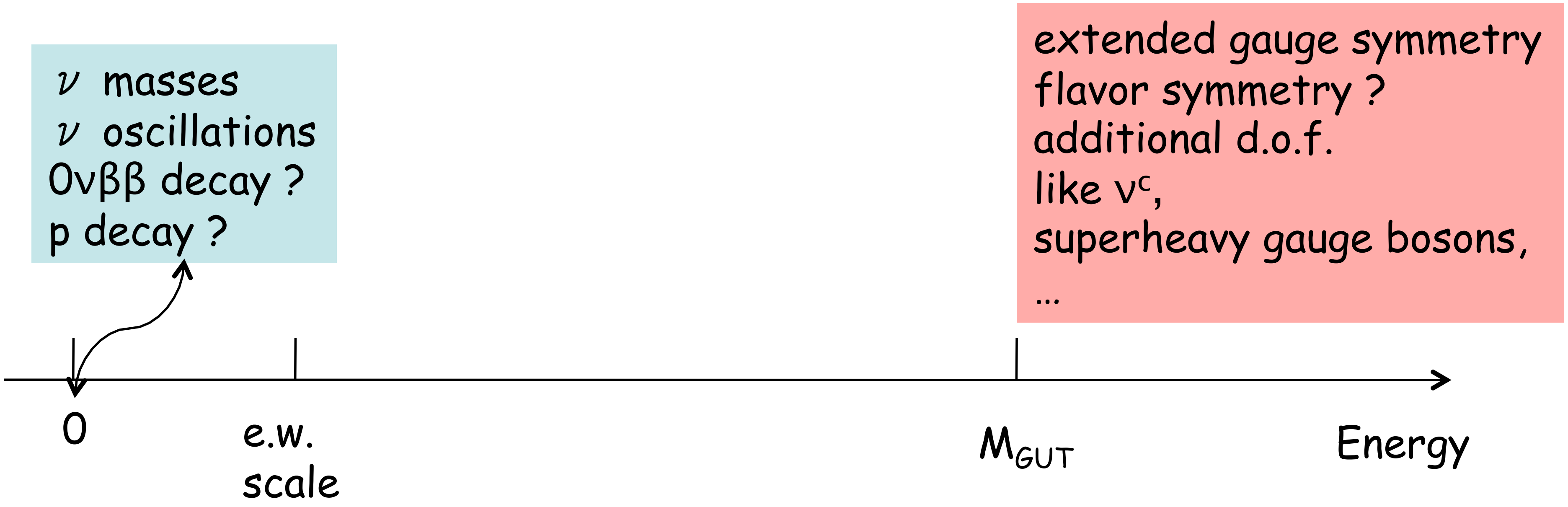
future expected sensitivity on $|m_{ee}|$

10 meV

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

CUORE	^{130}Te	(30-50) meV
Majorana	^{76}Ge	(20-70) meV
GERDA	^{76} Ge	(90-290) meV (phase II)
		10 meV (phase III ?)

without any extra assumptions



difficult to realize additional tests of the high-energy theory

e.g. the (type I) see-saw
depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

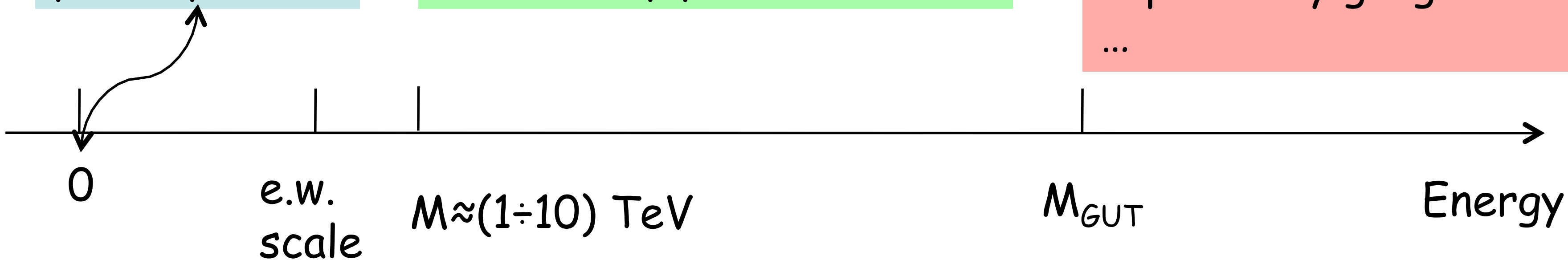
the double of those
describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles
and 3 phases

1st assumption:
there is new physics at a scale $M \approx (1 \div 10)$ TeV << M_{GUT}

ν masses
ν oscillations
$0\nu\beta\beta$ decay ?
p decay ?

- (g-2) discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

extended gauge symmetry
flavor symmetry ?
additional d.o.f.
like ν^c,
superheavy gauge bosons,
…



the energy region close to M will be explored by LHC soon

highly desirable to investigate the impact of L_5 on the physics at the scale M

2st assumption: high-energy theory invariant under a flavor symmetry G_f broken by a set of <φ> << 1 (in units of Λ)

in the lepton sector

$$L_{mass} = -e^c H^+ y_e(\varphi) l + \frac{(\tilde{H}^+ l) Y(\varphi) (\tilde{H}^+ l)}{\Lambda_L} + h.c. + \ldots$$

G_f-invariant

after G_f breaking from <φ>, masses of charged leptons and of neutrinos are generated

$$m_l = \frac{v}{\sqrt{2}} y_e(\langle\varphi\rangle) \qquad\qquad m_\nu = \frac{v^2}{\Lambda_L} Y(\langle\varphi\rangle)$$

at energies E<M, after integrating out the d.o.f. associated to the scale M

$$L_{eff} = L_{mass} + i\frac{e}{M^2} e^c H^+ \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{M}(\langle\varphi\rangle) l + h.c. + \ldots$$

[4-fermion operators]

L_{eff} local operator, still invariant under G_f [by treating <φ> as spurions]

[neglecting RGE effects, still controlled by <φ>, but not local in <φ>]

- effects with $1/M^2$ suppression can be observable

- flavor pattern in L_{eff} controlled (up to RGE effects) by the same SB parameters $\langle\varphi\rangle$ that control m_e and m_ν

- in the basis where charged leptons are diagonal

$\mathrm{Im}\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$	d_i	electric dipole moments
$\mathrm{Re}\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$	$a_i = \frac{(g-2)_i}{2}$	anomalous magnetic moments
$\left\lvert\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}\right\rvert^2 \quad (i \neq j)$	$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)}$	LFV transitions $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$
[4-fermion operators]	$\mu \to eee$, $\tau \to \mu\mu\mu$, $\tau \to eee$, ...	

- bounds on the scale M, from the present limits on d_i, a_i, R_{ij}
- correlations among d_i, a_i, R_{ij} and ϑ_{13} from the pattern $\langle\varphi\rangle$

here: 2 examples

[D'Ambrosio, Giudice, Isidori, Strumia 2002
Cirigliano, Grinstein, Isidori, Wise 2005]

Minimal Flavor Violation [MFV]

$$G_f = SU(3)_l \times SU(3)_{e^c} \times \ldots$$

the largest G_f

$$l = (\bar{3},1) \qquad e^c = (1,3)$$

$$\varphi \equiv \begin{cases} y_e = (3,\bar{3}) \\ Y = (6,1) \end{cases}$$

G_f broken only by the Yukawa coupling of L_{SM} and L_5

y_e and Y can be expressed in terms of lepton masses and mixing angles

$$y_e = \sqrt{2}\frac{m_e^{diag}}{v} \qquad Y = \frac{\Lambda_L}{v^2}U^* m_\nu^{diag} U^+$$

$G_f = A_4 \times Z_3 \times U(1)_{FN}$

explicitly tailored to reproduce a nearly tri-bimaximal (TB) mixing

[Ma, Rajasekaran 2001; Ma 0409075; Altarelli & F. 0504165 & 0512103 Altarelli, F, Lin 0610165]

$$l = (3, \omega, 0)$$
$$e^c = (1, \omega^2, +2)$$
$$\mu^c = (1'', \omega^2, +1)$$
$$\tau^c = (1', \omega^2, 0)$$

$$\varphi \equiv \begin{cases} \varphi_T/\Lambda = (3,1,0) \\ \varphi_S/\Lambda = (3,\omega,0) \\ \xi/\Lambda = (1,\omega,0) \\ \vartheta/\Lambda = (1,1,-1) \end{cases}$$

can also be extended to the quark sector
[F, Hagedorn, Lin, Merlo 0702194, Altarelli,F, Hagedorn 08020090]

TM mixing requires a specific vacuum alignment

$$\langle\varphi_T\rangle/\Lambda = (u,0,0) + O(u^2)$$
$$\langle\varphi_S\rangle/\Lambda = (u,u,u) + O(u^2)$$
$$\langle\xi\rangle/\Lambda = u + O(u^2)$$
$$\langle\vartheta\rangle/\Lambda \equiv t$$

tau Yukawa coupling < 4π

$$0.001 < u < \lambda^2$$

corrections to TB mixing

$$t \approx \lambda^2 \qquad \lambda \approx 0.22$$
$$\vartheta_{13} = O(u)$$

$$y_e(\langle\varphi\rangle) = \begin{pmatrix} c_e t^2 u & 0 & 0 \\ 0 & c_\mu t\, u & 0 \\ 0 & 0 & c_\tau u \end{pmatrix} + O(u^2) \qquad Y(\langle\varphi\rangle) = \begin{pmatrix} a+2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a-b/3 \\ -b/3 & a-b/3 & 2b/3 \end{pmatrix} u + O(u^2)$$

$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}$ in MFV

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii} = \alpha\,(y_e)_{ii} + \ldots$$

O(1) (complex) coefficient

$$L_{eff} = i\alpha \frac{e}{M^2} e^c H^+ \left(\sigma^{\mu\nu} F_{\mu\nu}\right) y_e\, l + \ldots$$

$d_e < 1.6 \times 10^{-27}\ e\ cm$	$M > 80\ TeV$
$d_\mu < 2.8 \times 10^{-19}\ e\ cm$	$M > 80\ GeV$
$\delta a_e < 3.8 \times 10^{-12}$	$M > 350\ GeV$
$\delta a_\mu \approx 30 \times 10^{-10}$	$M \approx 2.7\ TeV$

α approximately real?

[from recent review by Raidal et al 08011826]

[warning: relation between the scale M and new particle masses M' can be not trivial. In a weakly interacting theory g M/4π≈M']

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij} = \beta\,(y_e Y^+ Y)_{ij} + \ldots$$

$$= \sqrt{2}\beta \frac{(m_l)_{ii}}{v}\frac{\Lambda_L^2}{v^4}\left[\Delta m^2_{sol} U_{i2}U^*_{j2} \pm \Delta m^2_{atm} U_{i3}U^*_{j3}\right] + \ldots$$

+ for normal hierarchy
- for inverted hierarchy

a positive signal at MEG $10^{-11} < R_{\mu e} < 10^{-13} \div 10^{-14}$ always be accommodated [but for a small interval around $\vartheta_{13} \approx 0.02$ where $R_{\mu e}=0$]

non-observation of R_{ij} can be accommodated by lowering Λ_L

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left|\frac{2}{3} r \pm \sqrt{2}\sin\vartheta_{13} e^{i\delta}\right|^2 < 1 \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

[Cirigliano, Grinstein, Isidori, Wise 2005]

both $\mu \to e\gamma$ and $\tau \to \mu\gamma$ could be above future sensitivity

$R_{\mu e} < 1.2 \times 10^{-11}$ implies $R_{\tau\mu} < 10^{-9}$

0.02

ϑ_{13}

0 0.1 0.2

here $\mu \to e\gamma$ vanishes

$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}$ in $A_4 \times \ldots$

[F, Hagedorn, Lin, Merlo, in preparation]

$$\mathcal{M}(\langle\varphi\rangle) = \begin{pmatrix} O(t^2 u) & \cdot & \cdot \\ O(t u^2) & O(t u) & \cdot \\ O(u^2) & O(u^2) & O(u) \end{pmatrix}$$

in the basis where charged leptons are diagonal; operators contribute to both $\mathcal{M}_{ii}$ and $\mathcal{M}_{ij}$ (i≠j)

diagonal elements $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$ are of the same size as in MFV: similar lower bounds on the scale M

up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from ϑ_{13}

$\tau \to \mu\gamma \quad \tau \to e\gamma$ below expected future sensitivity

$$R_{\mu e} < 1.2\times 10^{-11}(10^{-13}) \Rightarrow \frac{u}{M^2} < 1.2\times 10^{-11}(1.1\times 10^{-12})\ GeV^{-2}$$

$$u > 0.001 \Rightarrow M > 10(30) \quad TeV$$

$$u \approx 0.05 \Rightarrow M > 70(200) \quad TeV$$

probably above the region of interest for the μ (g-2) and for LHC

$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}$ in $A_4 \times \ldots$

with high-energy theory both A_4 and SUSY invariant [preliminary]

$$\mathcal{M}(\langle\varphi\rangle) = \begin{pmatrix} O(t^2 u) & \cdot & \cdot \\ O(t u^3) & O(t u) & \cdot \\ O(u^3) & O(u^3) & O(u) \end{pmatrix}$$

in the basis where charged leptons are diagonal

we have preliminary indications that off-diagonal elements $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij}$ are down by a factor of O(u) compared to generic non-SUSY case

up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from ϑ_{13}

$$R_{\mu e} < 1.2 \times 10^{-11} (10^{-13}) \Rightarrow \frac{u^2}{M^2} < 1.2 \times 10^{-11} (1.1 \times 10^{-12})\ GeV^{-2}$$

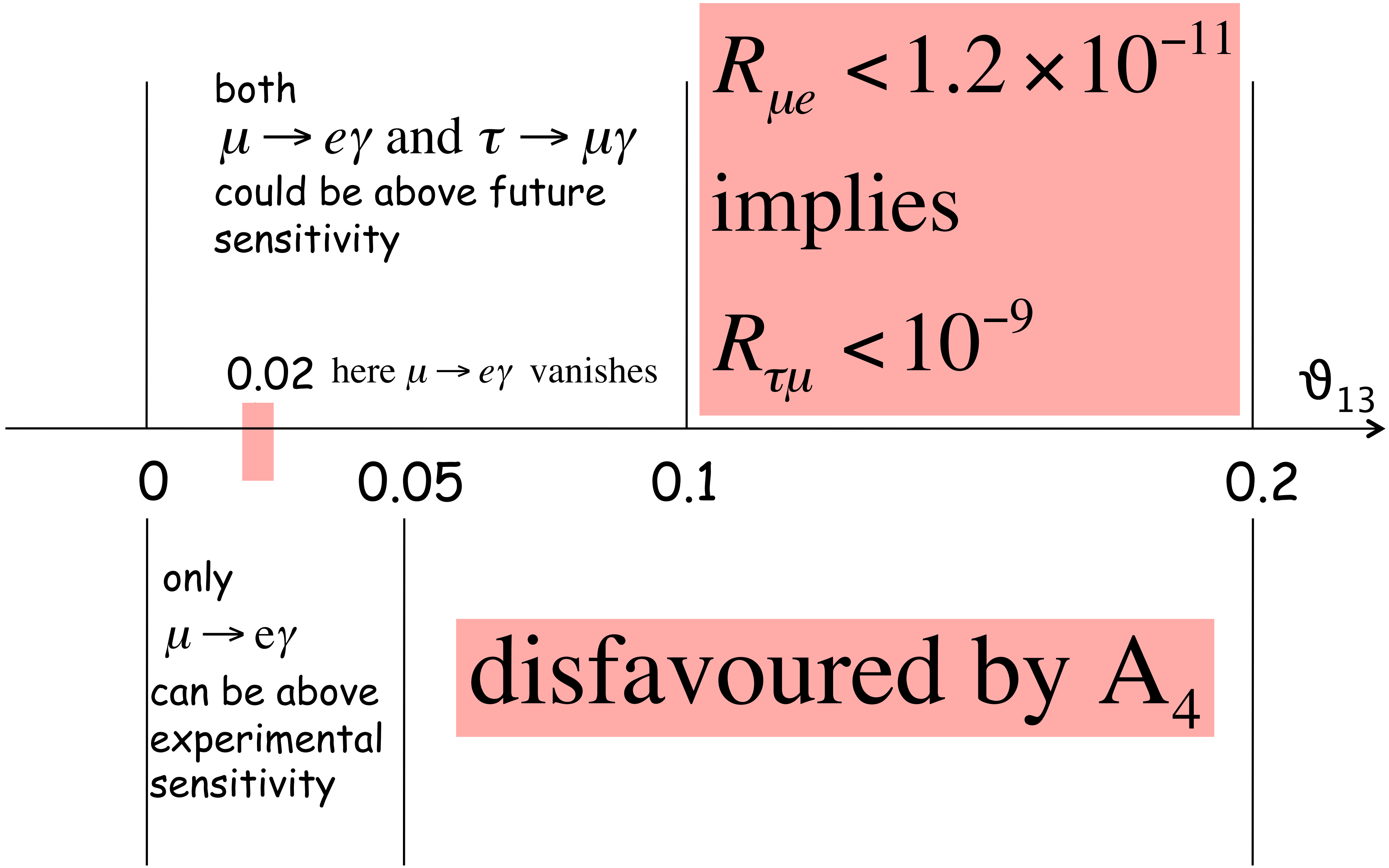
$$u > 0.001 \Rightarrow M > 0.3(1) \quad TeV$$

$$u \approx 0.05 \Rightarrow M > 2(7) \quad TeV$$

O(1) coefficient

$$BR(\mu \to e\gamma) = \underbrace{\frac{12\pi^3 \alpha_{em}}{G_F^2 m_\mu^4} (\delta a_\mu)^2}_{0.0014 \times \left(\frac{\delta a_\mu}{30 \times 10^{-10}}\right)^2} [\gamma \vartheta_{13}]^4$$

MFV [scale M can be of order 1 TeV]



SUSYxA_4 [scale M can be of order 1 TeV (preliminary)]

Conclusion

theory of neutrino masses

it does not exist! Neither for neutrinos nor for charged fermions. We lack a unifying principle.

like weak interactions before the electroweak theory

$SU(2)_L \otimes U(1)_Y$ gauge invariance

all fermion-gauge boson interactions in terms of 2 parameters: g and g'

?

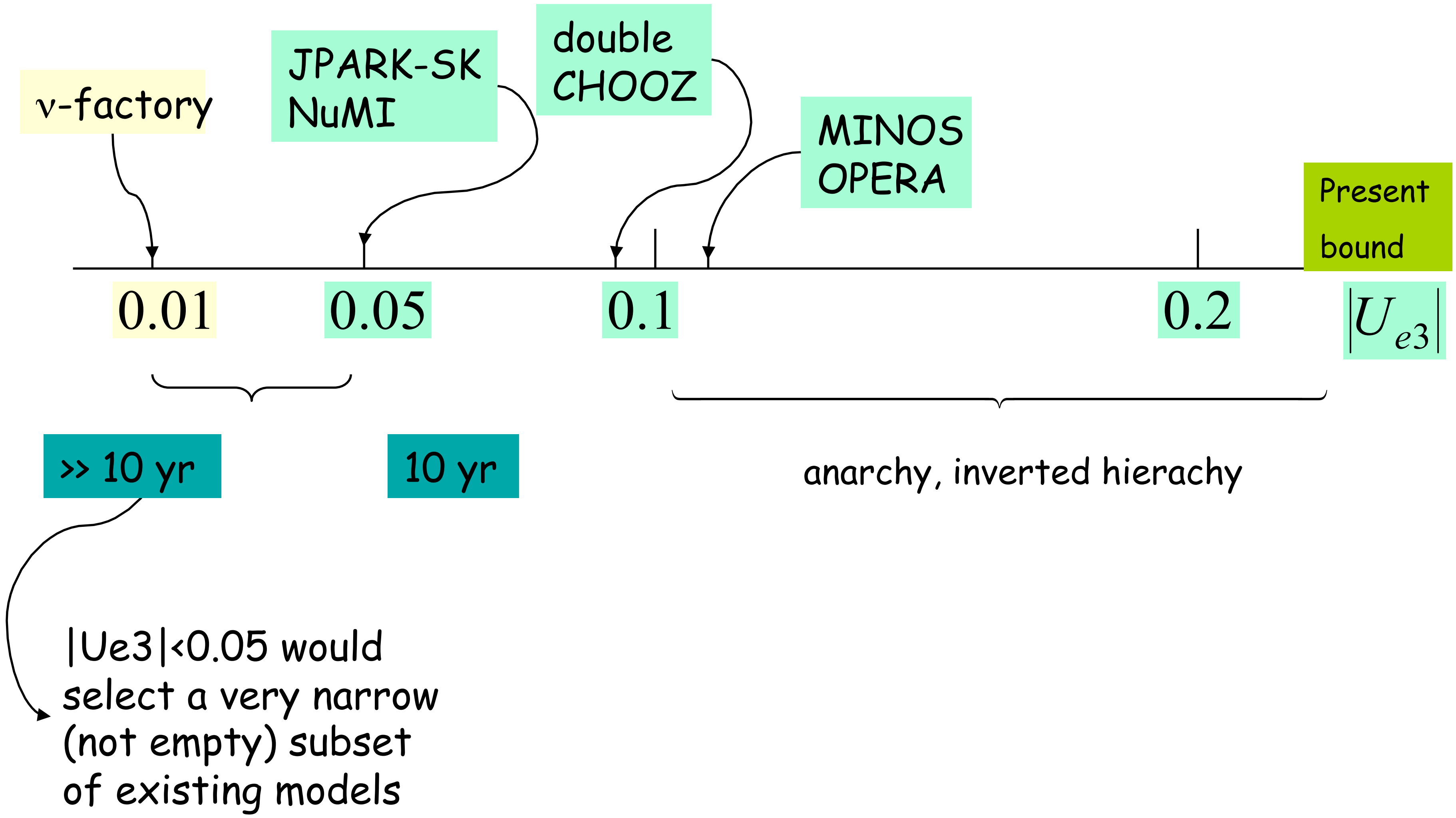
Yukawa interactions between fermions and spin 0 particles: many free parameters (up to 22 in the SM!)

only few ideas and prejudices about neutrino masses and mixing angles

caveat: several prejudices turned out to be wrong in the past!

- $m_\nu \approx 10$ eV because is the cosmologically relevant range
- solution to solar is MSW Small Angle
- atmospheric neutrino problem will go away because it implies a large angle

- Most of plausible range for Ue3 explored in 10 yr from now



	current precision	future < 10 yr
Δm^2_{12}	$(8.0 \pm 0.3)\times 10^{-5}$ eV^2 [$\approx 4\%$]	few percent [KamLAND]
$\left\lvert \Delta m^2_{23} \right\rvert$	$(2.5 \pm 0.3)\times 10^{-3}$ eV^2 [$\approx 12\%$]	0.15×10^{-3} eV^2 LBL conventional beams 0.05×10^{-3} eV^2 [$\approx 2\%$] superbeams
ϑ_{12}	$\tan^2\vartheta_{12} = 0.45^{+0.09}_{-0.08}$ $\vartheta_{12} = 33^0 \pm 2^0$	$\delta \tan^2\vartheta_{12} \approx 2\delta \sin^2\vartheta_{12}$ ν_e scattering rate down by about of pp neutrinos to 1% a factor 2: challenging
ϑ_{13}	$< 0.23\ (13^0)$ 90% C.L.	0.10 rad LBL, ChoozII 0.05 rad superbeams
ϑ_{23}	$\sin^2\vartheta_{23} = 0.52^{+0.07}_{-0.08}$ $\vartheta_{12} = 46^{0}{}^{+4^0}_{-5^0}$	$\delta \sin^2\vartheta_{23} \approx \delta\vartheta_{23}$ down by about superbeams a factor 2
sign Δm^2_{23}	---	> 10 yr
δ	---	> 10 yr

non-oscillation "solutions"

ν decay	$P_{ff} = c + c' e^{-\frac{mL}{\tau E}} + \ldots$	wrong E dependence
ν decoherence	$P_{ff} = 1 - \frac{1}{2}\sin^2 2\vartheta (1 - e^{-\frac{\gamma L}{E}} \cos\frac{\Delta m^2 L}{2E})$	wrong E dependence
spin flavour precession (for solar ν)	$\mu_{ij} \approx 10^{-11}\mu_B \quad B \approx 80\mathrm{KGauss}$	rejected by KamLAND no such large B in Earth
Lorentz invariance violation	$P_{ff} = 1 - \sin^2 2\vartheta \sin^2(\delta c\, LE/2)$	wrong E dependence
non-standard ν interactions	$\delta L = \varepsilon\, G_F \psi\psi\nu\nu$ E-independent P_{ff}	sol: clash between solar and KamLAND data atm: wrong E dependence
mass varying neutrinos	$\delta m_\nu = \frac{\lambda\lambda'}{m^2} N_e$	sol: clash between solar and KamLAND data
ν oscillations with a non unitary mixing matrix U	non-canonical ν kinetic terms in flavour basis from dim=6 operator	ν oscillations, W,Z decays universality tests, LFV UU⁺=1 at the percent level

all these effects can play, at most, a **subleading** role

Flavor symmetries II (the lepton mixing puzzle)

$$\text{why} \quad U_{PMNS} \approx U_{TB} \equiv \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad ?$$

[TB=TriBimaximal]

$$U_{PMNS} = U_e^+ U_\nu$$

Consider a flavor symmetry G_f such that G_f is broken into two different subgroups: G_e in the charged lepton sector, and G_ν in the neutrino sector. m_e is invariant under G_e and m_ν is invariant under G_ν. If G_e and G_ν are appropriately chosen, the constraints on m_e and m_ν can give rise to the observed U_{PMNS}.

$$\begin{tikzcd} & G_f \arrow[dl] \arrow[dr] & \\ G_e \arrow[d] & & G_\nu \arrow[d] \\ m_e \text{ diagonal} & & U_{TB}^T\, m_\nu\, U_{TB} = (m_\nu)_{diag} \end{tikzcd}$$

The simplest example is based on a small discrete group, $G_f=A_4$. It is the subgroup of SO(3) leaving a regular tetrahedron invariant. The elements of A_4 can all be generated starting from two of them: S and T such that

$$S^2 = T^3 = (ST)^3 = 1$$

S generates a subgroup Z_2 of A_4
T generates a subgroup Z_3 of A_4

simple models have been constructed where $G_e=Z_3$ and $G_\nu=Z_2$ and where the lepton mixing matrix U_{PMNS} is automatically U_{TB}, at the leading order in the SB parameters. Small corrections are induced by higher order terms.

the generic predictions of this approach is that θ_{13} and $(\theta_{23}-\pi/4)$ are very small quantities, of the order of few percent: testable in a not-so-far future.

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