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Supernova neutrinos:
strong-coupling effects of weak interactions

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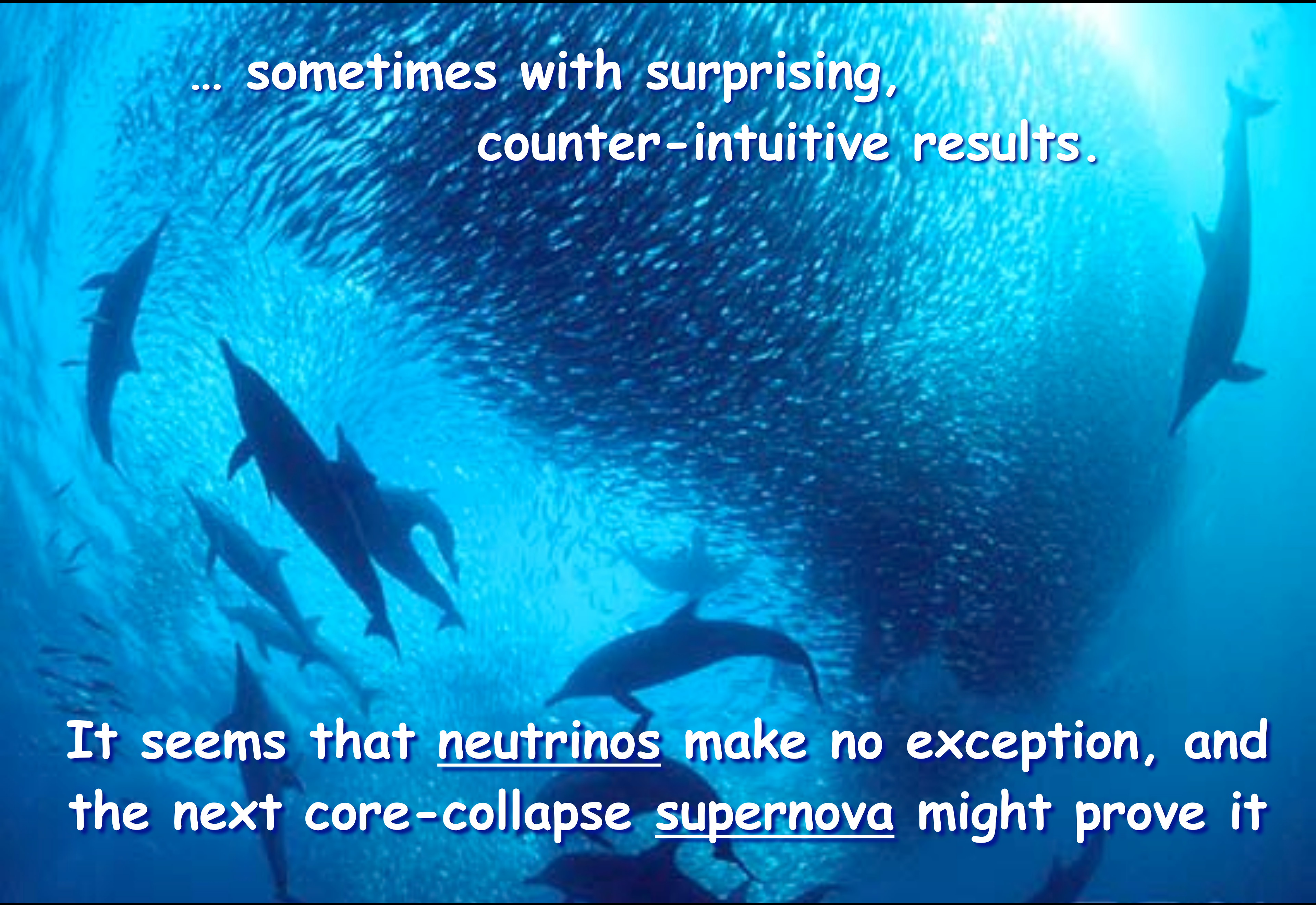
PROLOGUE

Densely packed individuals often show a collective behavior ...



… sometimes with surprising,
counter-intuitive results.

It seems that neutrinos make no exception, and the next core-collapse supernova might prove it

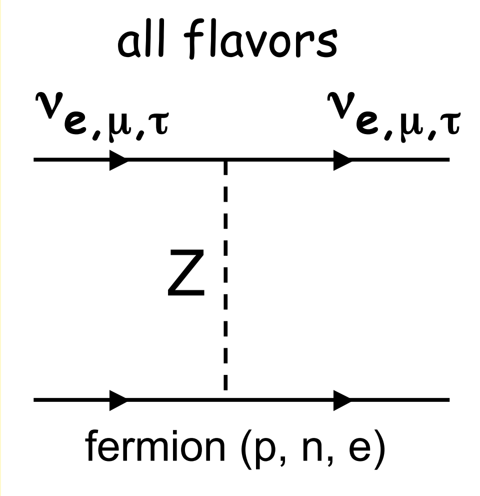
Outline

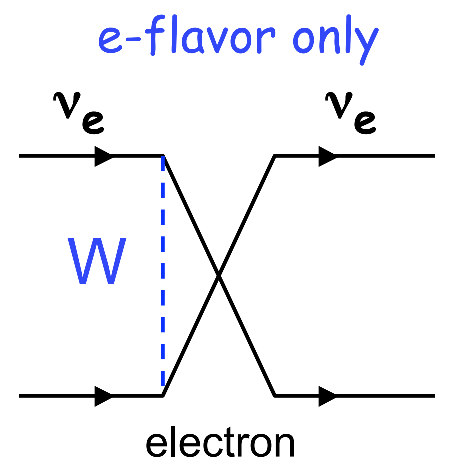
- Intro on (SN) ν properties
- Coupled equations of motion
- Flavor pendulum analogy
- Spectral split and swap
- Discussion of our work(*)
- Recent/open issues *(if time allows)*
- Conclusions

(*) **G.L. Fogli, E.L., A. Marrone & A. Mirizzi,** "Collective neutrino flavor transitions in supernovae and the role of trajectory averaging" **arXiv:0707.1998 [hep-ph]**, JCAP 0712:010,2007.

INTRODUCTION

In SN matter, neutrino flavor transitions are sensitive to the difference in ν_e forward scattering amplitude ($\propto G_F$) over background fermions (Mikheyev-Smirnov-Wolfenstein effect)



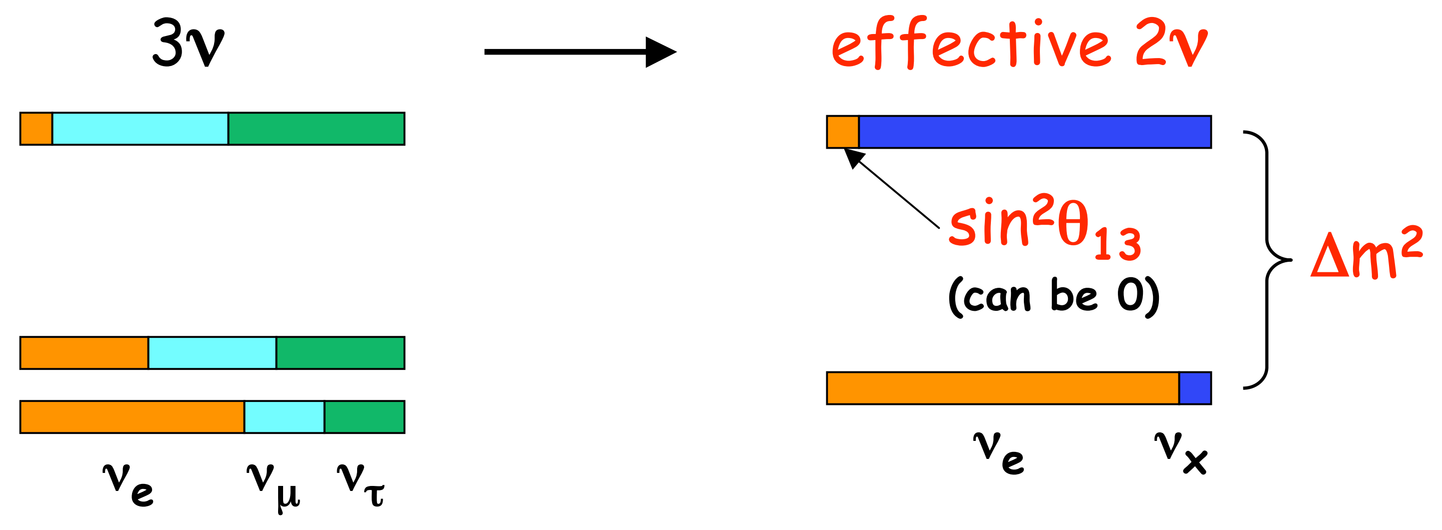


Interaction energy difference depends on electron density N_e:

$$\lambda = \sqrt{2}\, G_F\, N_e$$

Typical SN ν energies, E ~ O(10) MeV, are below threshold for μ and τ and production via CC. The ν_μ and ν_τ behave in a similar way during production, propagation, detection, and are often denoted by a common symbol ν_x.

Reasonable approximation in the context of SN neutrinos:



3ν → effective 2ν

$\sin^2\theta_{13}$

(can be 0)

Δm^2

ν_e ν_μ ν_τ

ν_e ν_x

Hamiltonian in $(\nu_e, \nu_x)^T$ basis:

$$H = \pm\frac{\omega}{2}\begin{bmatrix}-\cos 2\theta_{13} & +\sin 2\theta_{13} \\ +\sin 2\theta_{13} & +\cos 2\theta_{13}\end{bmatrix} + \begin{bmatrix}\lambda & 0 \\ 0 & 0\end{bmatrix}$$

$\omega = \Delta m^2/2E$

$\pm = \mathrm{hierarchy}$

Becomes diagonal (no flavor change) for $\theta_{13}=0$ or very large λ

Flavor changes induced by this "MSW" hamiltonian: studied for >20 y.

Well-known MSW effects can occur in a SN envelope when the ν potential $\lambda=\sqrt{2}\, G_F\, N_e$ is close to osc. frequency $\omega=\Delta m^2/2E$ ($\Delta m^2=|m^2_3-m^2_{1,2}|$, $\theta_{13}\neq 0$).

For t~few sec after bounce,

$\lambda\sim\omega$ at **$x>>10^2$ km** (large radii).

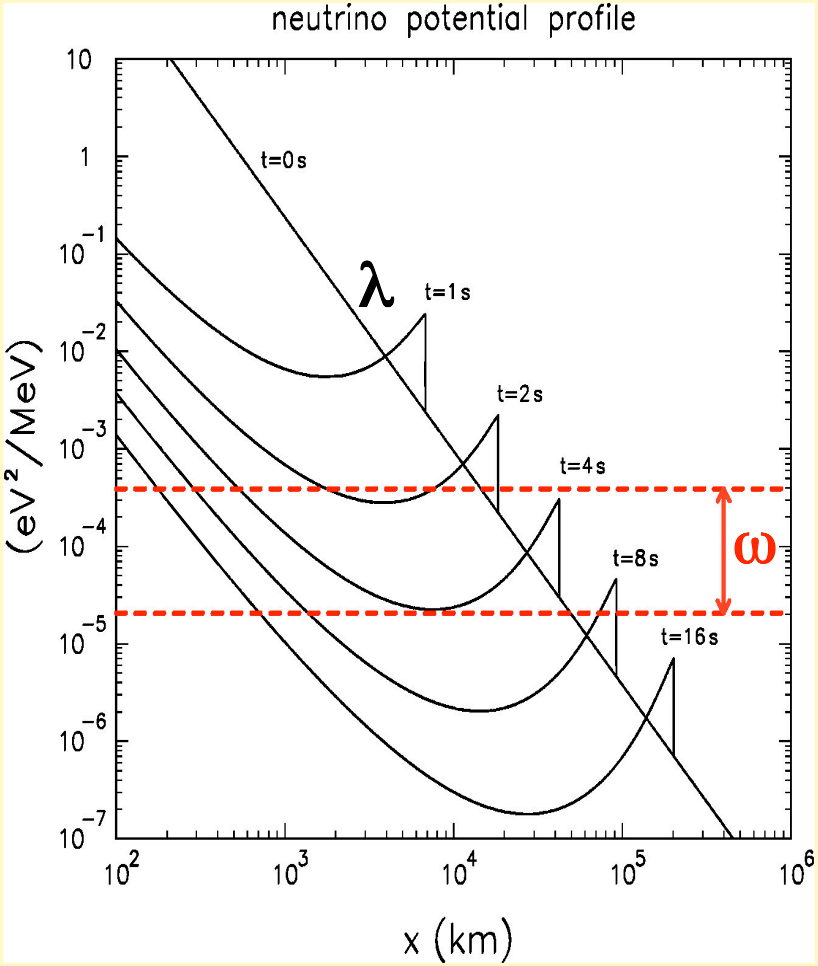
What about small radii?

Popular wisdom:

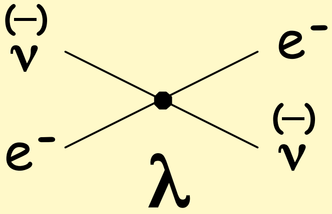
$\lambda>>\omega$ at **$x<O(10^2)$ km**,

thus flavor transitions suppressed.

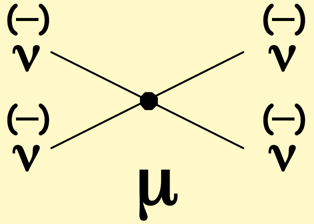
Incorrect!



Besides



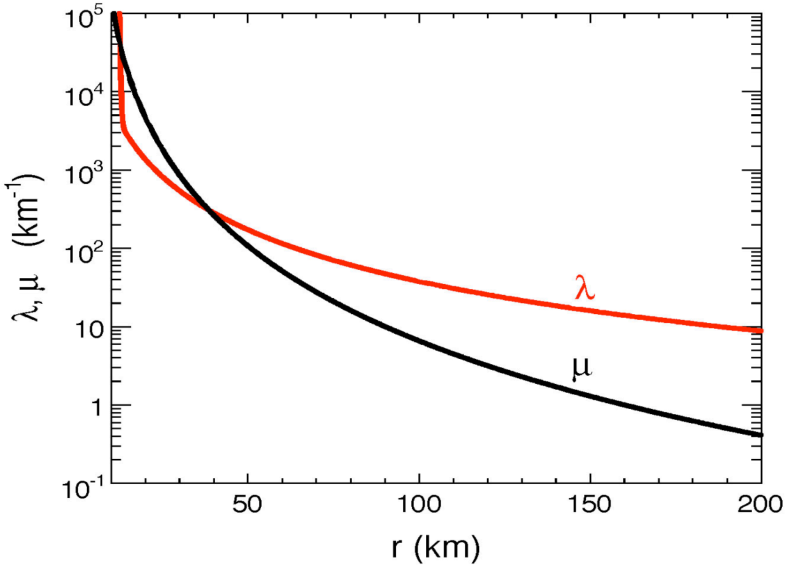
(CC) one has to include



(NC)

At small r, neutrino and antineutrino density (n and $\bar{n}$) high enough to make self-interactions important. Strength: **$\mu=\sqrt{2}\,G_F\,(n+\bar{n})$**

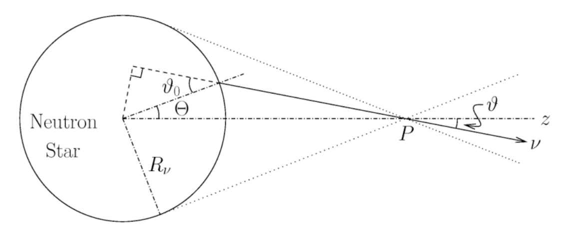
Angular modulation factor: **$(1-\cos\Theta_{ij})$**
If averaged: **"single-angle"** approxim.
Otherwise : **"multi-angle"** (difficult)



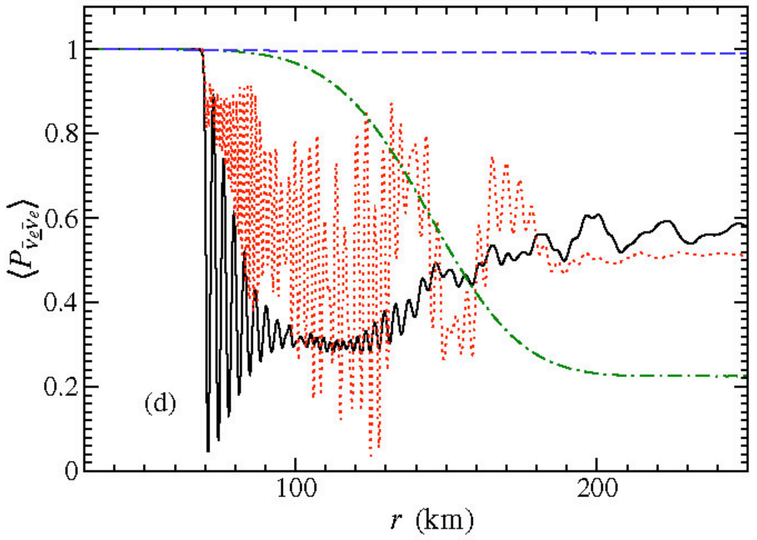
Self-interaction effects known for ~20 y in SN. But, recent boost of interest after new crucial results by Duan, Fuller, Carlson, Qian '05-'06

Lesson: self-interactions (μ) can induce large, non-MSW flavor change at small radii, despite large matter density λ

E.g.: multi-angle numerical simulations from
Duan, Fuller, Carlson, Qian, astro-ph/0606616
obtained by solving O(10^6) coupled differential eqs.
in a cilindrically symmetric “bulb model” for the SN



energy-averaged anti-ν_e survival
probability in inverted hierarchy



Emerging collective effects:
flavor oscillations of both neutrinos
and antineutrinos with similar features
at all energies (quite unlike MSW)

- - - - - - - - - “synchronized” regime

——————— “bipolar” oscillations along
- - - - - - - - - two different trajectories

-·-·-·-·-·-·- no self-inter. (pure MSW)

Clearly, something interesting is going on. But: DFCQ scenario, formalism, numerics & results were initially quite complicated to understand.
Significant effort to clarify these complex phenomena in the last ~2 years.

Recent “wave” of papers on SN neutrino self interactions (time-ordered):

[01] Fuller & Qian	astro-ph/0505240	
[02] Duan, Fuller & Qian	astro-ph/0511275	<— The “synchronized” and “bipolar” regimes
[03] Duan, Fuller, Carlson & Qian	astro-ph/0606616	<— Large-scale multi-angle calculations
[04] Balantekin & Pehlivan	astro-ph/0607527	
[05] Duan, Fuller, Carlson & Qian	astro-ph/0608050	
[06] Hannestad, Raffelt, Sigl & Wong	astro-ph/0608695	<— The “flavor pendulum” analogy
[07] Raffelt & Sigl	hep-ph/0701182	
[08] Duan, Fuller, Carlson & Qian	astro-ph/0703776	
[09] Raffelt and Smirnov	hep-ph/0705.1830	<— The “spectral split”
[10] Esteban, Pastor, Tomas, Raffelt & Sigl	astro-ph/0706.2498	
[11] Duan, Fuller & Qian	astro-ph/0706.4293	
[12] Duan, Fuller, Carlson & Qian	astro-ph/0707.0290	<— The “spectral split”
[13] Fogli, Lisi, Marrone & Mirizzi	hep-ph/0707.1998	<— Our work **(this talk)**
[14] Raffelt & Smirnov	hep-ph/0709.4641	
[15] Duan, Fuller, Carlson & Qian	astro-ph/0710.1271	
[16] Esteban, Pastor, Tomas, Raffelt & Sigl	astro-ph/0712.1137	<— The mu-tau flavor difference
[17] Dasgupta & Dighe	hep-ph/0712.3798	<— The three neutrino formalism
[18] Duan, Fuller & Qian	hep-ph/0801.1363	<— Three-neutrino spectral split
[19] Dasgupta, Dighe, Mirizzi, Raffelt	hep-ph/0801.1660	<— Three-neutrino spectral split

.... + a few other recent papers ....

Our contribution [13]:

- **Exploration of self-interaction effects for “typical” matter profile with no MSW effects at small radii (unlike the shallow profile in [03])**
- **Test of robustness of effects when passing from (simple) single-angle calculations to (difficult but more realistic) multi-angle ones.**

COUPLED EQUATIONS OF MOTION
FOR SN (ANTI)NEUTRINOS

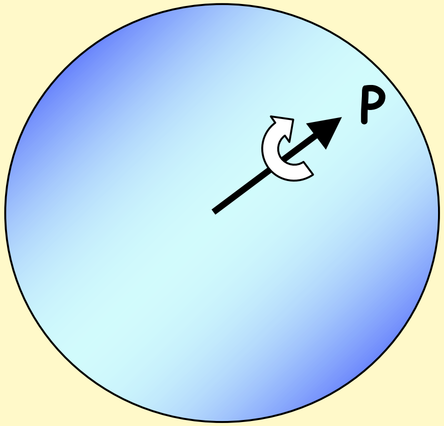
Coupled equations of motion (for 2 flavors, e and $x=\mu,\tau$)

Neutrino wavefunction sensitive to neutrino density -> use density matrix.

Liouville equations: $i\partial_t \rho=[H,\rho]$ (for each neutrino mode)

Decompose 2x2 (anti)neutrino density matrix over Pauli matrices to get a "**polarization**" (Bloch) 3-vector $\mathbf{P}=(P_1,P_2,P_3)=(P_x,P_y,P_z)$. [Ditto for H.]

Bloch equations: $\partial_t \mathbf{P} = \mathbf{V}\times\mathbf{P}$ (precession-like, $|P|$=const)



Any mode P moves on a Bloch sphere (abstract "flavor space").

"up" direction : ν_e flavor
"down" direct. : ν_x flavor
generic direct. : mixed flavor state

Probability P_{ee} related to $P_3=P_Z$

Coupled equations of motion (cont'd)

The problem is that there are lots of kinematical neutrino modes: continuous distributions over energy and angle(s) -> no less than ∞^2 !

Discretize over energy spectrum (**N_E bins**), and over angular distribution in multi-angle simulations (**N_Θ bins**) -> Get discrete index (indices), **P_i** .

Evolution governed by **$6 \mathrm{x} N_E \mathrm{x} N_\Theta$ coupled Bloch equations** of the form:

$$
\begin{aligned}
\dot{\mathbf{P}}_i &= \mathbf{V}_{\text{ector}}[+\omega,\ \lambda,\ \mu,\ \mathbf{P}_j,\ \overline{\mathbf{P}}_j] \times \mathbf{P}_i \\
\dot{\overline{\mathbf{P}}}_i &= \mathbf{V}_{\text{ector}}[-\underset{\substack{\uparrow \\ \text{vacuum}}}{\omega},\ \underset{\substack{\uparrow \\ \text{matter}}}{\lambda},\ \underset{\substack{\uparrow \\ \text{self-interaction}}}{\mu},\ \underbrace{\mathbf{P}_j,\ \overline{\mathbf{P}}_j}_{\text{ij couplings}}] \times \overline{\mathbf{P}}_i
\end{aligned}
$$

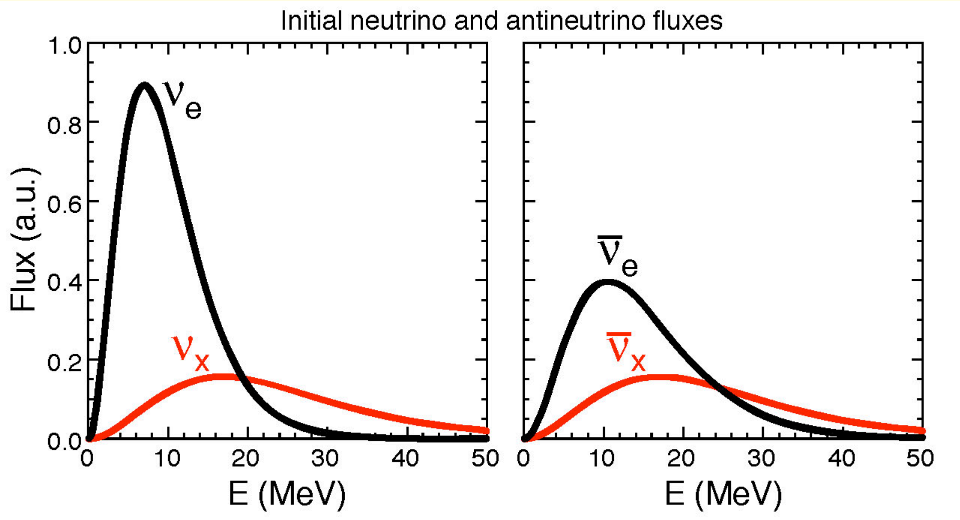
Large, "stiff" set of (strongly) coupled differential equations.

EOM in single-angle approximation:

$$\dot{\mathbf{P}} = \left[+\omega\mathbf{B} + \lambda\mathbf{z} + \mu\textstyle\sum_E(\mathbf{P} - \overline{\mathbf{P}})\right] \times \mathbf{P}$$
$$\dot{\overline{\mathbf{P}}} = \left[-\omega\mathbf{B} + \lambda\mathbf{z} + \mu\textstyle\sum_E(\mathbf{P} - \overline{\mathbf{P}})\right] \times \overline{\mathbf{P}}$$

$$\mathbf{P} = \mathbf{P}_E$$
$$\mathbf{z} = (0,\ 0,\ 1)^T$$
$$\mathbf{B} = (\sin 2\theta_{13},\ 0,\ \pm\cos 2\theta_{13})$$

Initial conditions at neutrinosphere:



$$\mathbf{P}^i = \frac{n_e - n_x}{n_e + n_x}\,\mathbf{z}$$
$$\overline{\mathbf{P}}^i = \frac{\overline{n}_e - \overline{n}_x}{\overline{n}_e + \overline{n}_x}\,\mathbf{z}$$

$$L_\nu/\mathrm{flavor} \sim 10^{51}\ \mathrm{erg/s}$$

Get 6x$\mathrm{N_E}$ equations (numerically manageable)

Try to gain physical and analytical understanding

Subcases and analogies:

$$(1)\ \dot{\mathbf{P}} = [+\omega \mathbf{B}] \times \mathbf{P}$$ Vacuum

$$(2)\ \dot{\mathbf{P}} = [+\omega \mathbf{B} + \lambda \mathbf{z}] \times \mathbf{P}$$ MSW

$$(3)\ \dot{\mathbf{P}} = \left[+\omega \mathbf{B} + \lambda \mathbf{z} + \mu \textstyle\sum_E (\mathbf{P} - \overline{\mathbf{P}})\right] \times \mathbf{P}$$ **Self-int.**

(1) Precessing "spin" **P** in external "magnetic" field **B** with frequency ω

- *trivial*

(2) Precession in magnetic field with variable frequency and direction

- *well-understood in most cases*

(3) Precession in variable m.f. plus additional fields from other spins

- *shows surprising features, often discovered numerically before being understood analytically. E.g., the "large" matter term λ**z** is ~irrelevant for self-interaction effects. Matter ~doesn't matter!*

Strong couplings between polarization vectors make the problem difficult, but also make an analytical understanding possible after all ! Key tool of “**near-alignment**” or “**strong polarization**”, e.g.:

$\mathbf{P}_i$'s

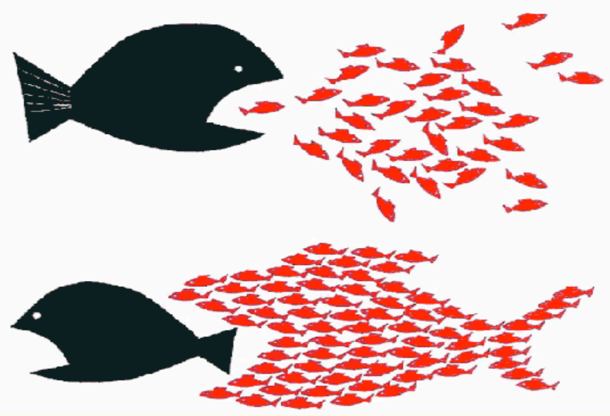
$\mathbf{J} = \Sigma_i\, \mathbf{P}_i$

(global polarization vector)

(In general, the P’s seem to be “pinned” to some global vector)

To some extent, the flavor evolution can be understood in terms of:

$$\mathbf{J}=\Sigma\,\mathbf{P},\qquad \overline{\mathbf{J}}=\Sigma\,\overline{\mathbf{P}},\qquad \mathbf{S}=\mathbf{J}+\overline{\mathbf{J}},\qquad \mathbf{D}=\mathbf{J}-\overline{\mathbf{J}}$$

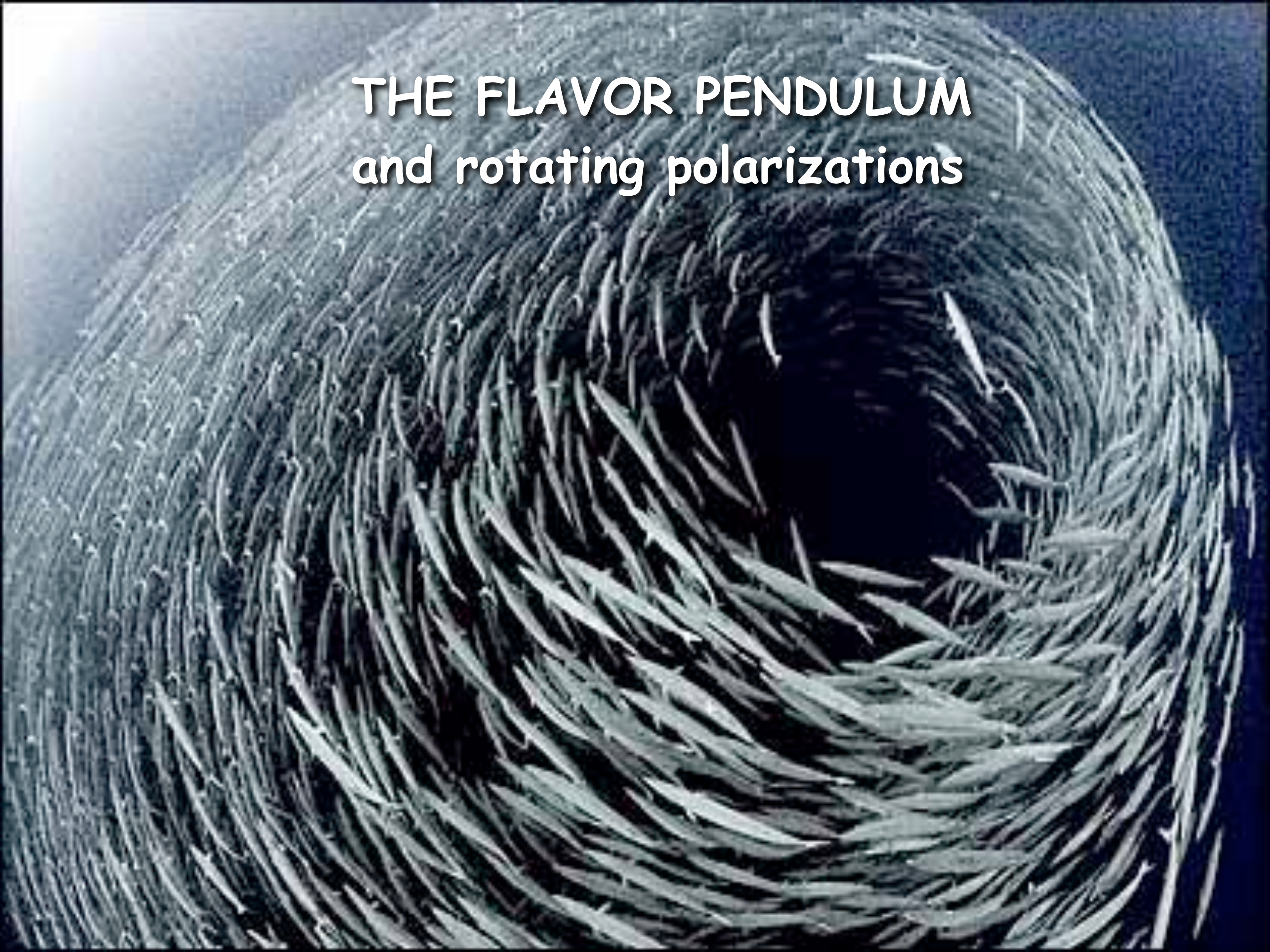


Decrease of |J| and |$\underline{J}$| then signals a loss of polarization ("kinematical decoherence")

Interesting: the EOM imply that D_Z is an exact constant of motion, corresponding to net lepton number conservation (far from MSW):

$$(\nu_e\overline{\nu}_e \to \nu_x\overline{\nu}_x,\ \text{but}\ \#(\nu_e-\overline{\nu}_e)=\text{const})$$

(another "collective" aspect of self-induced flavor transformations)



THE FLAVOR PENDULUM

and rotating polarizations

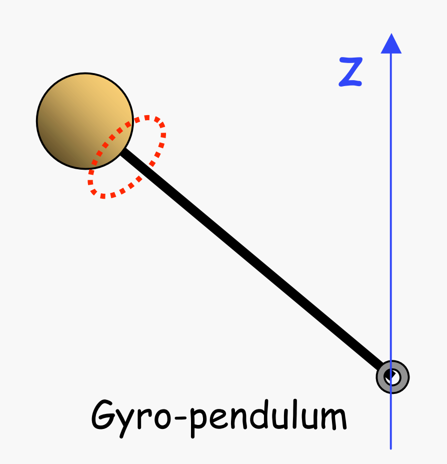
The flavor pendulum (Hannestad, Raffelt, Sigl, Wong 2006)

A surprising link between self-interacting neutrinos and classical mechanics.

It turns out that a linear combination Q of the global polarization vectors of neutrinos and antineutrinos obeys the same dynamics of a gyroscopic pendulum (=spherical pendulum with radially spinning mass) with unit length:

$$\mathbf{Q} = \mathbf{S} - (\omega_{\rm ave}/\mu)\mathbf{B}$$

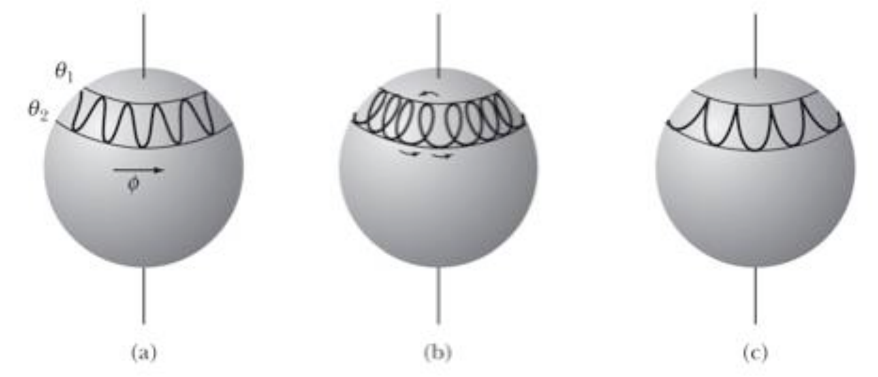
$$\begin{array}{lcl}
\mathbf{Q}/Q & \Leftrightarrow & \mathbf{r}\ \text{(unit length vector)} \\
\mathbf{D} & \Leftrightarrow & \mathbf{L}\ \text{(total angular momentum)} \\
\mu^{-1} & \Leftrightarrow & m\ \text{(mass)} \\
\mathbf{D}\cdot\mathbf{Q}/Q & \Leftrightarrow & \sigma\ \text{(spin)} \\
\omega_{\rm ave}\,\mu\,Q\,\mathbf{B} & \Leftrightarrow & -\mathbf{g}\ \text{(gravity field)} \\
 & & \\
\mathbf{L} & = & m\mathbf{r}\times\dot{\mathbf{r}}+\sigma\mathbf{r} \\
\dot{\mathbf{L}} & = & m\mathbf{r}\times\mathbf{g}
\end{array}$$



Roughly speaking:

Mass^{-1} ~ (anti)neutrino density

Spin ~ #neutrino - #antineutrino



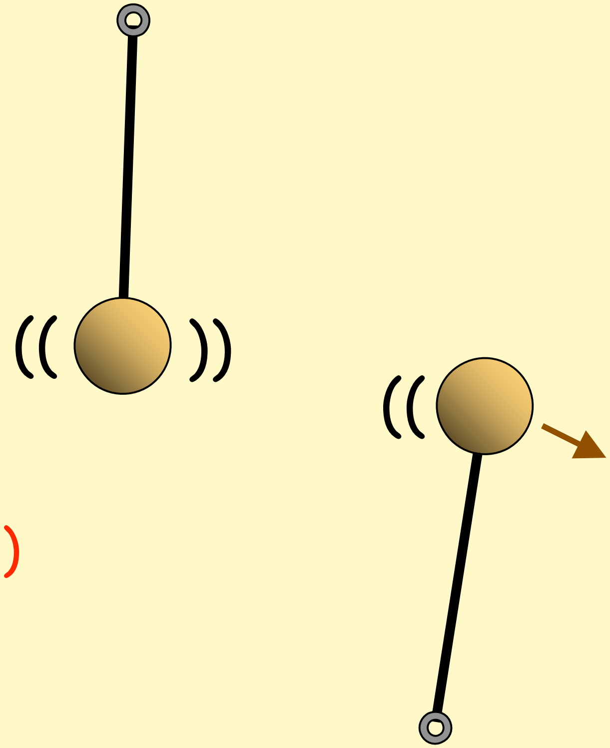
Generic motion is a combination of

Precession (around z)

Nutation (along rotation axis)

…but with slowly increasing mass!

Neutrino mass hierarchy (and θ_{13}) sets initial conditions and fate.



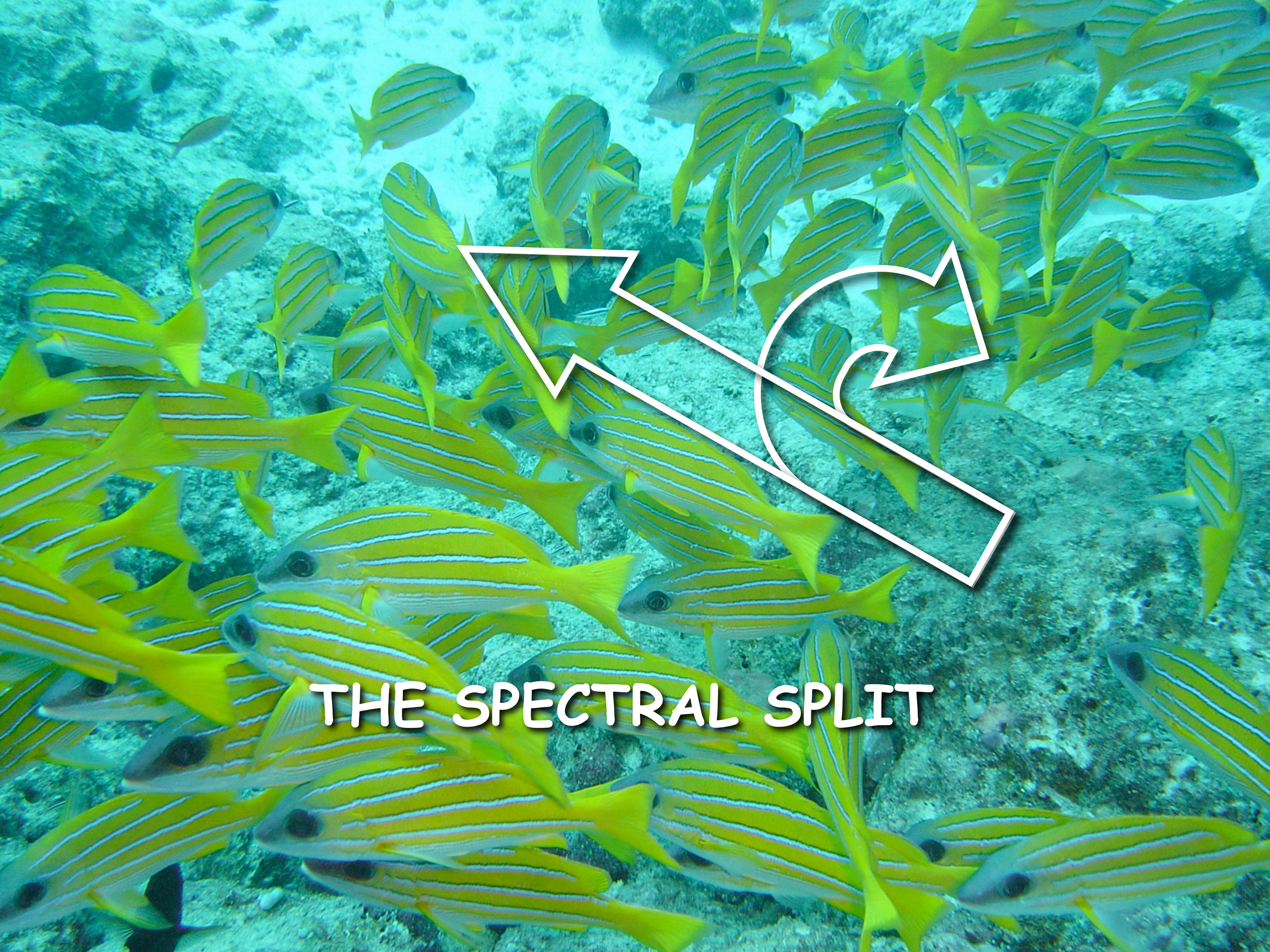
Normal hierarchy:

Pendulum starts in ~downward (stable) position and stays nearby. No significant flavor change.

Inverted hierarchy:

Pendulum starts in ~upward (unstable) position and eventually falls down. Significant flavor changes.

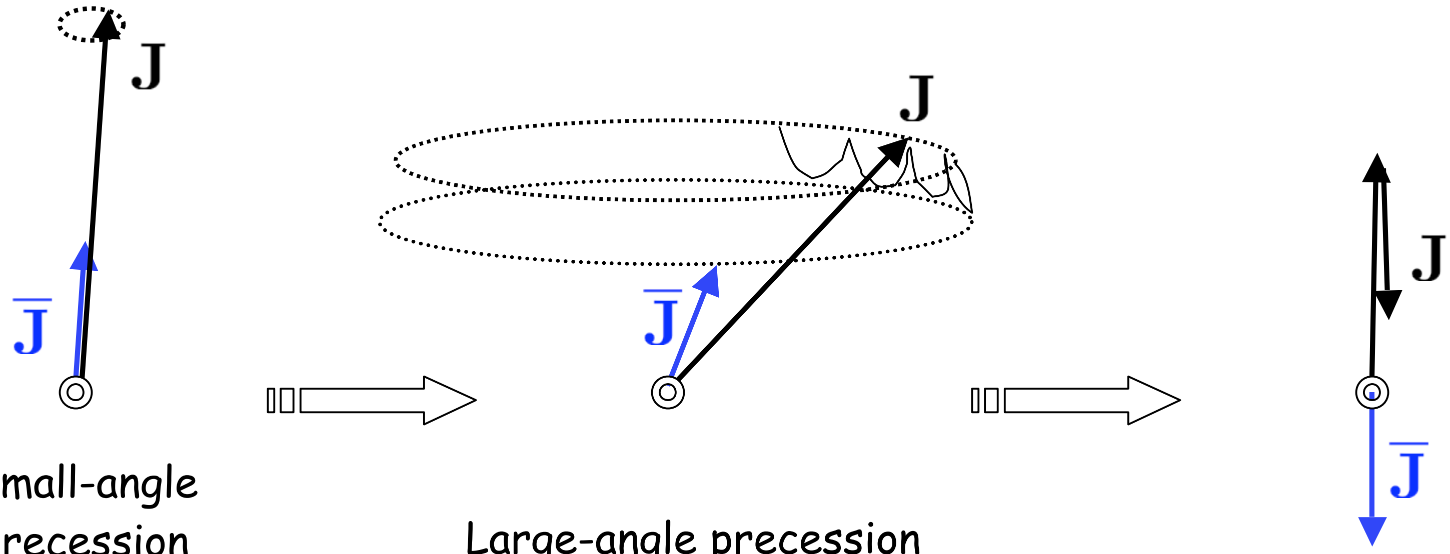
θ_{13} sets initial misalignment with vertical. Specific value not much relevant (provided that $\theta_{13} > 0$).
Only for $\theta_{13} = 0$ exactly, initial conditions are "frozen".



THE SPECTRAL SPLIT

The spectral split (hereafter, inv. hierarchy and $\theta_{13}>0$ assumed)

Global polarization vectors (**J** and $\underline{J}$, with $|\mathbf{J}|>|\underline{J}|$) follow pendulum motion as far as near-alignment holds. Eventually $\underline{J}$ reaches the stable downward position, while **J** can't, to preserve lepton number conservation ($\sim J_z-\underline{J}_z$)



Small-angle precession
("synchronized oscillations")

Large-angle precession +nutations
("bipolar oscillations ")

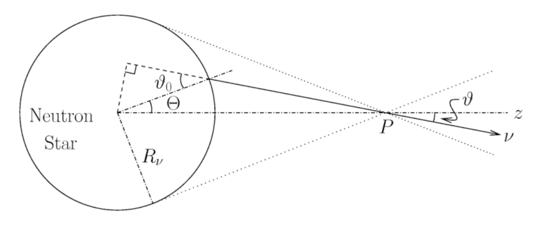
Final stable configuration
(**J** "splits")

Final state: whole $\underline{J}$ and **high-E part of J** inverted (**spectral split/swap**)

(Inversion = complete flavor change)

Warning

All the previous analytical tools have been developed in single-angle approx. Extension to multi-angle case: difficult. More complicated equations of motion:



$$\begin{aligned}\dot{\mathbf{P}}_\vartheta &= \left[+\omega\mathbf{B}+\lambda\mathbf{z}+2\pi\sqrt{2}G_F\int dc_{\vartheta'}\,dE\,(1-c_\vartheta c_{\vartheta'})(j\mathbf{P}_{\vartheta'}-\bar{j}\bar{\mathbf{P}}_{\vartheta'})\right]\times\mathbf{P}_\vartheta\\ \dot{\bar{\mathbf{P}}}_\vartheta &= \left[-\omega\mathbf{B}+\lambda\mathbf{z}+2\pi\sqrt{2}G_F\int dc_{\vartheta'}\,dE\,(1-c_\vartheta c_{\vartheta'})(j\mathbf{P}_{\vartheta'}-\bar{j}\bar{\mathbf{P}}_{\vartheta'})\right]\times\bar{\mathbf{P}}_\vartheta\end{aligned}$$

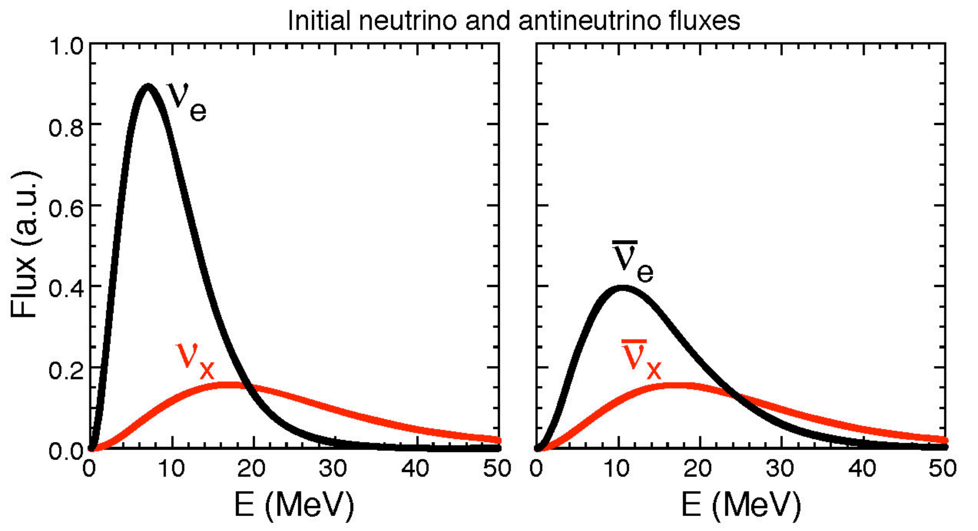
So far, no better way than using numerical calculations (with tricks to save computer time in the above 2D integrals) Need to solve $6\text{x}N_E\text{x}N_\Theta$ equations via stiff ODE integrators.

We use the GAMD software developed by colleagues in the Math Department of the Bari U. (F. Iavernaro and F. Mazzia), over a 32x80 grid (=15360 coupled equations).

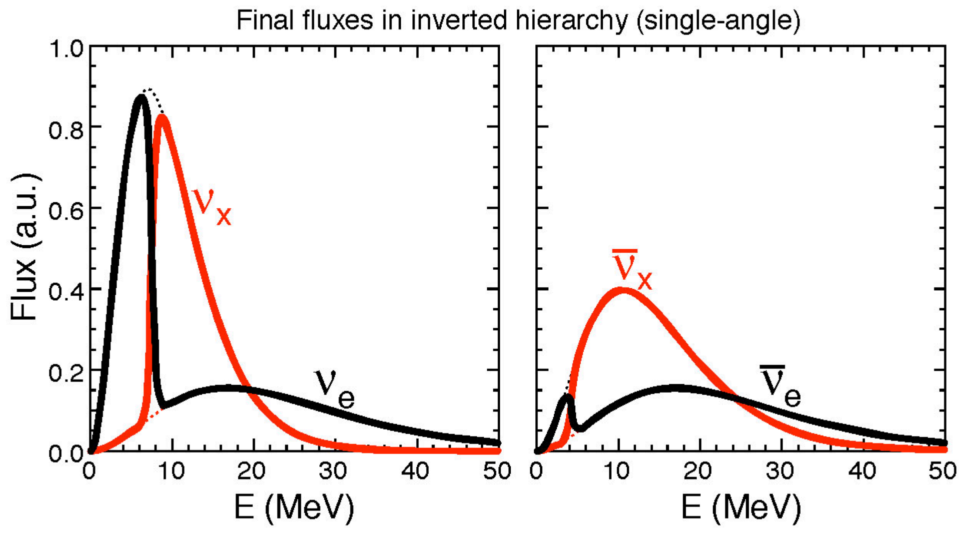
OUR EXPLORATION OF SELF-INTERACTION EFFECTS

arXiv:0707.1998 [hep-ph]

Results for the spectral split/swap (inv. hier.)


Initial neutrino and antineutrino fluxes

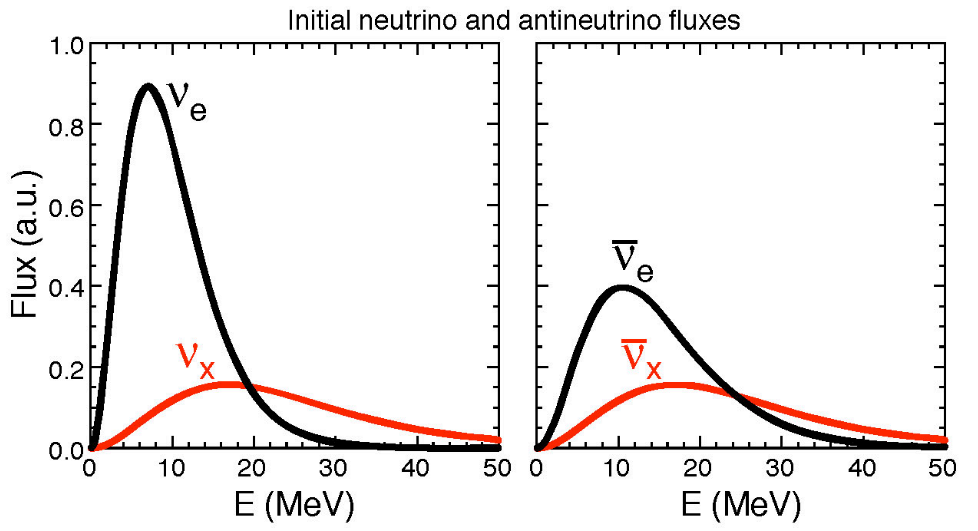
Initial fluxes at the neutrinosphere (r~10 km)


Final fluxes in inverted hierarchy (single-angle)

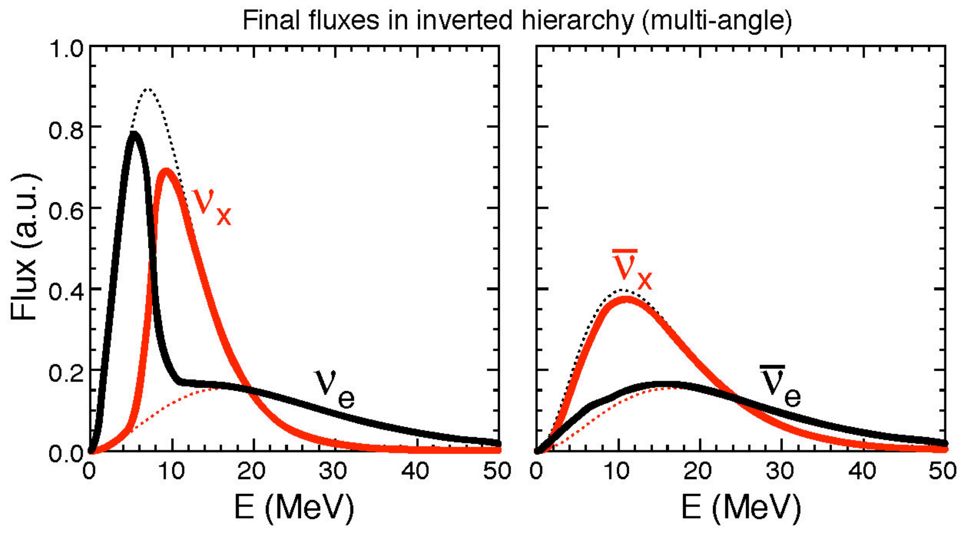
Final fluxes at the end of collective effects (r~200 km)

[Single-angle approximation]

Results for the spectral split/swap (inv. hier.)



Initial fluxes at the neutrinosphere (r~10 km)



Final fluxes at the end of collective effects (r~200 km)

[Multi-angle calculation, note smearing effect]

Spectral split/swap of neutrino spectra appears to be a robust signature of self-interaction effects in SN for inverse hierarchy

(Not much happens in normal hierarchy.)

It needs nonzero θ_{13} to build up, but specific value of θ_{13} is of little relevance (for definiteness, θ_{13}=0.01 in our work)

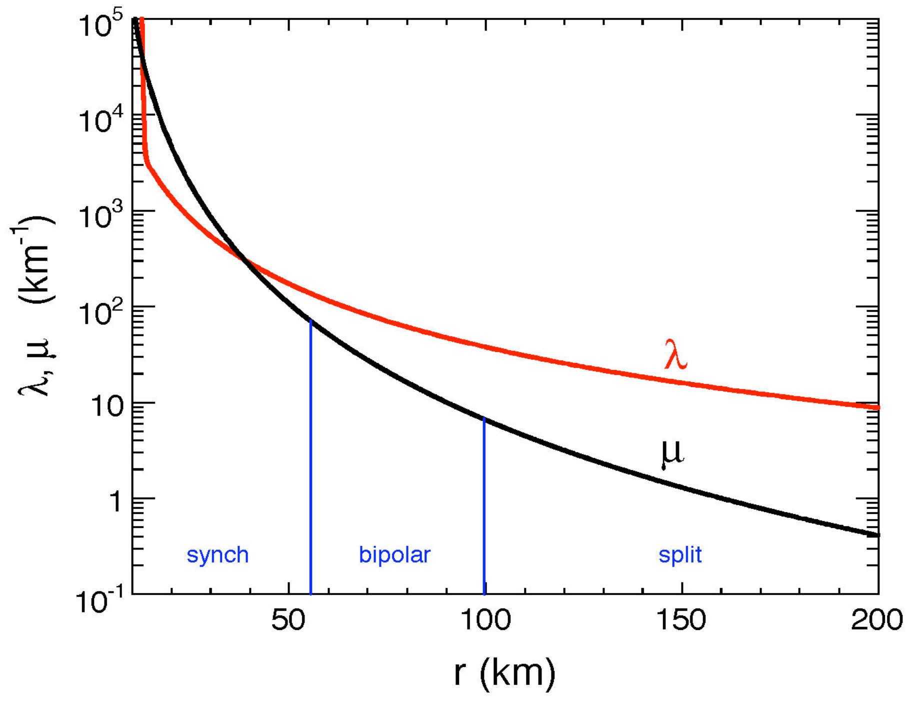
Might be the "ultimate test" of $\theta_{13} > 0$ & of inverted hierarchy

The neutrino splitting energy (~7 MeV in our case) is determined only by lepton number conservation (1 equation in 1 unkown)

"Final" spectra at r~200 km represent the new "initial conditions" for the subsequent MSW evolution (if any) at larger radii

Oscillations between ~10 and ~200 km

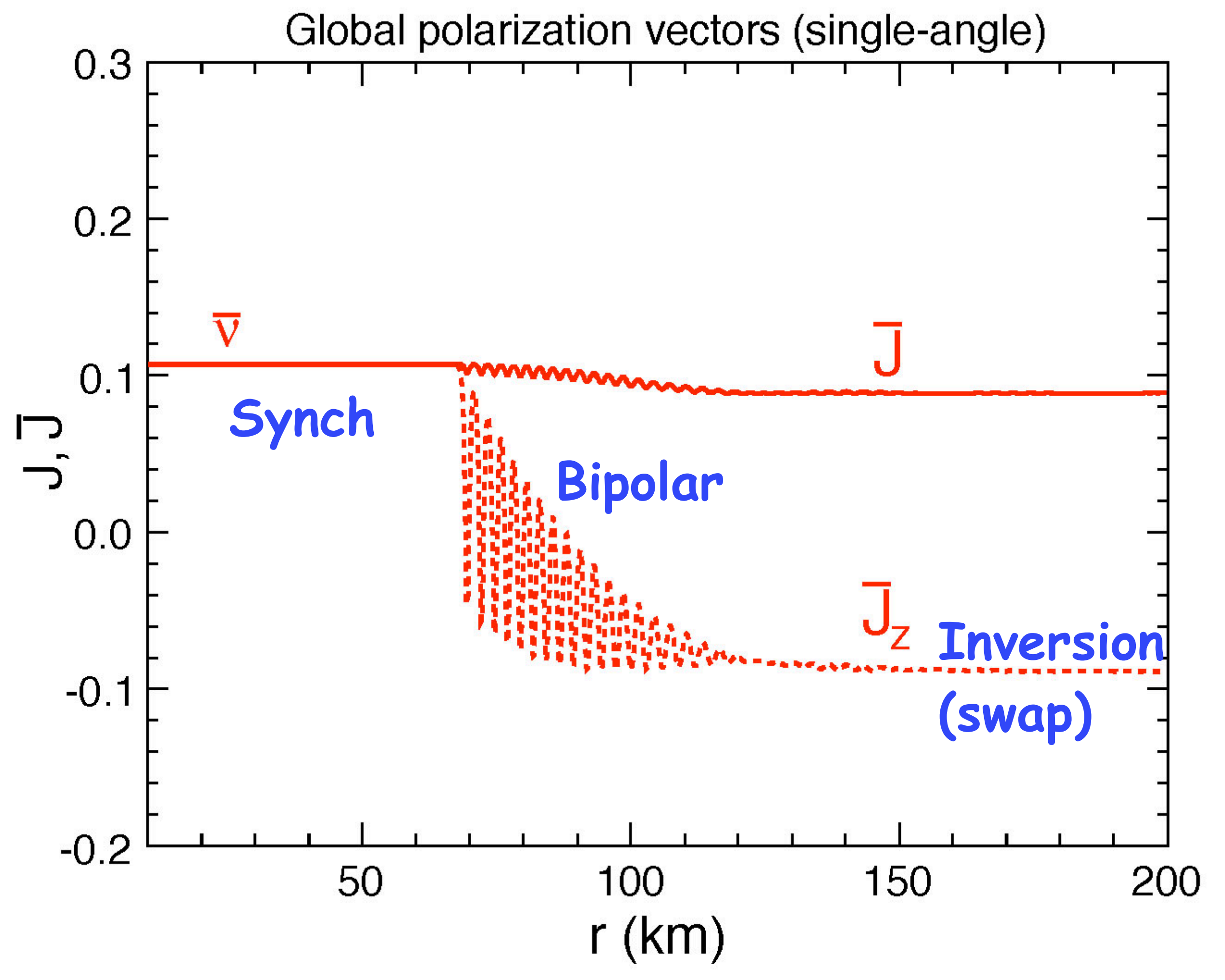
Analytical expectations for characteristic ranges:



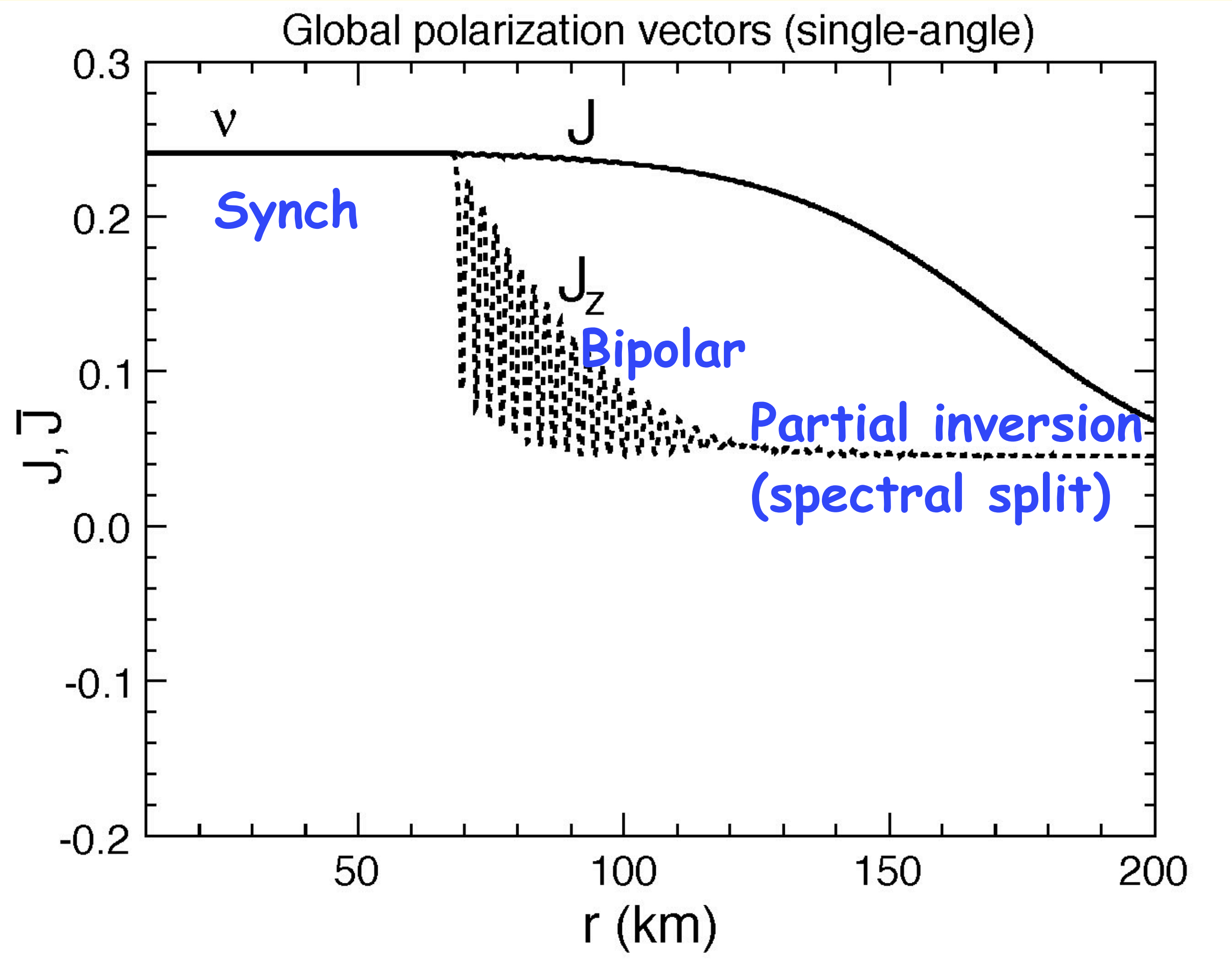
Confirmed by our numerical simulations in single and multi-angle cases.

Main difference between “single-angle” and “multi-angle” results: smearing of bipolar oscillations. Basic features remain robust.

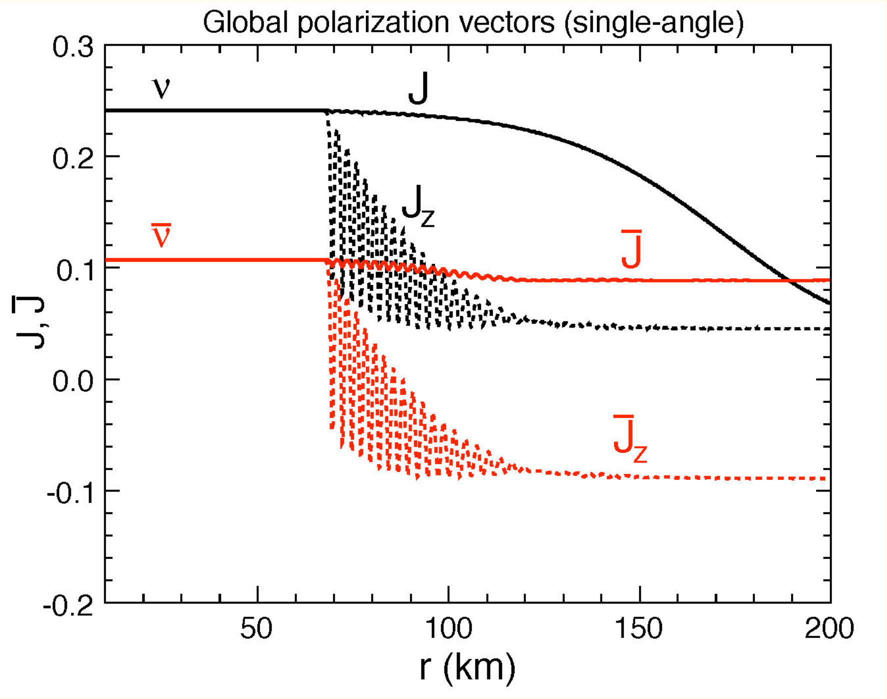
Antineutrinos: numerical results (single-angle)

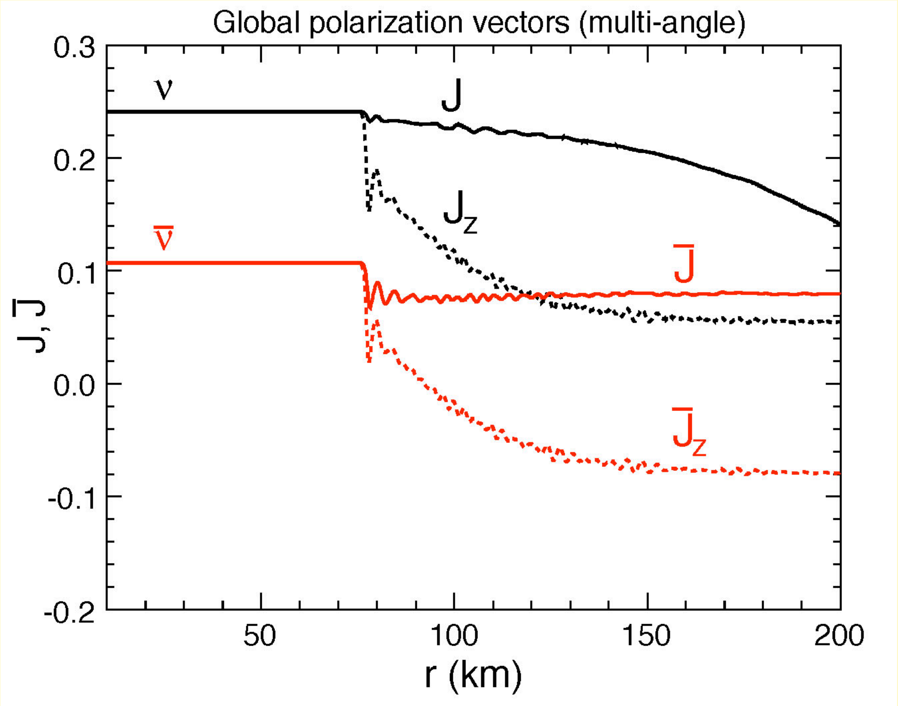


Neutrinos: numerical results (single-angle)



Single-angle vs Multi-angle

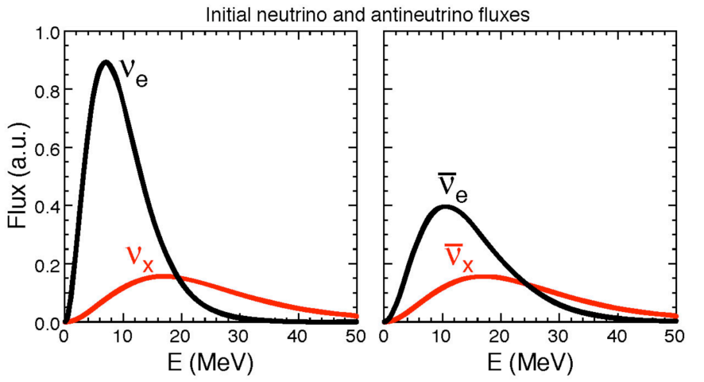


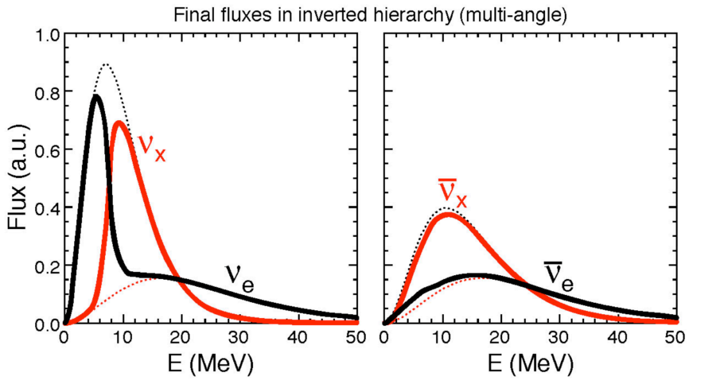


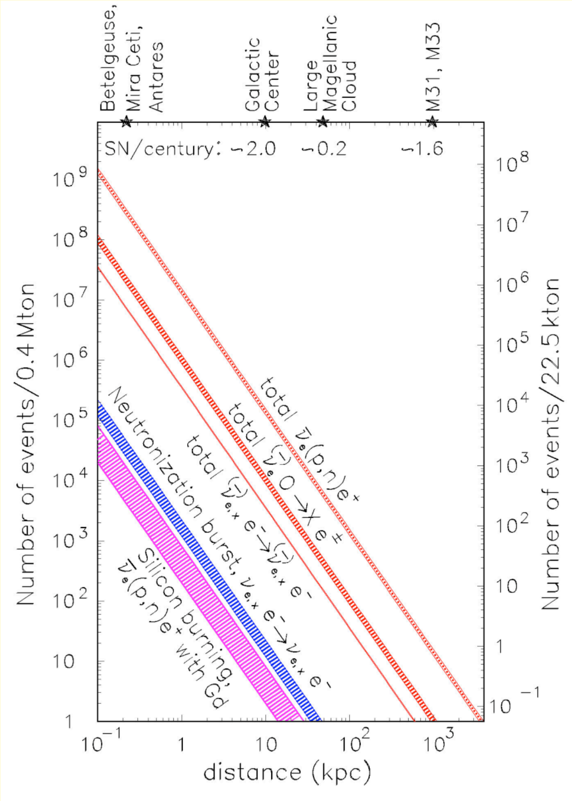
Note smearing of bipolar oscillations.
Other features are qualitatively similar.

Main message:

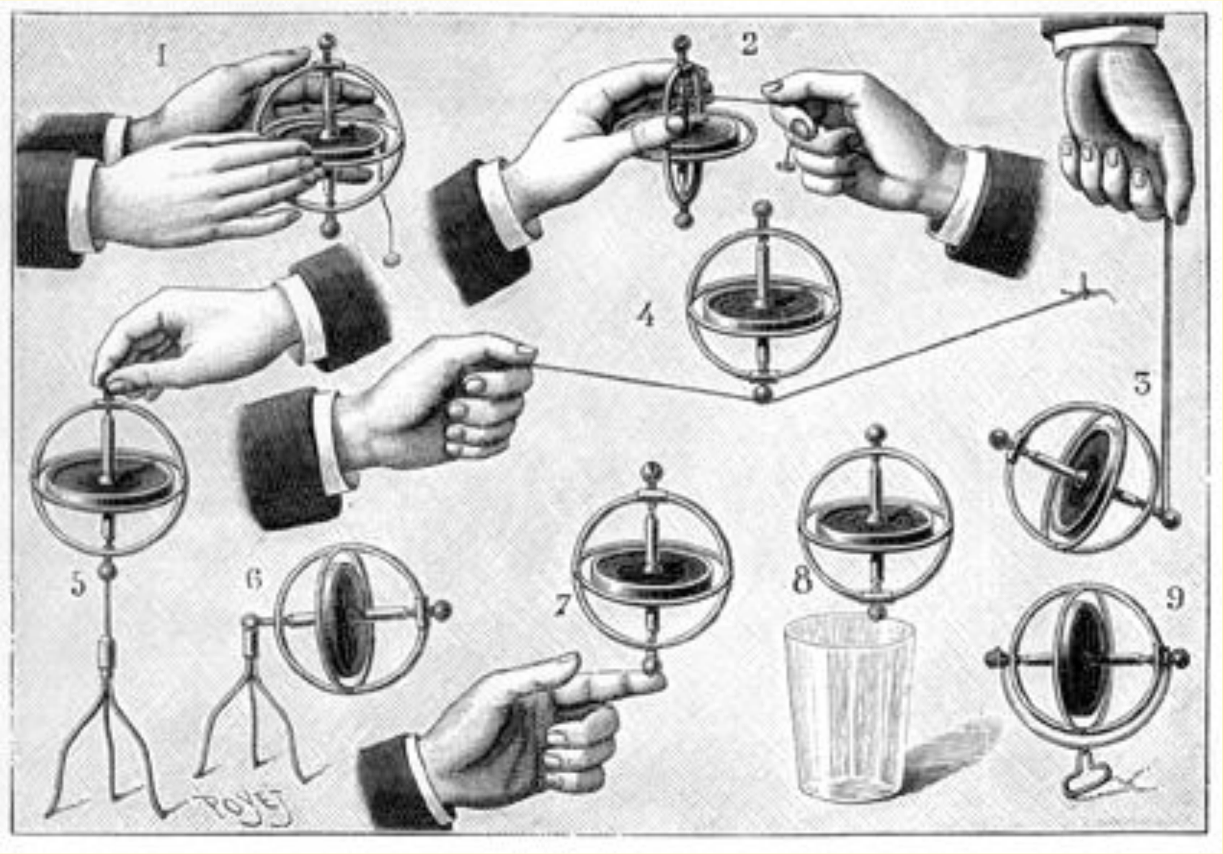
For experimentalists: Spectral split/swap seems to be a robust, well-understood and observable signature of SN neutrino self interactions in inverted hierarchy (provided that θ_{13} is nonzero)







For theorists: Playing with the flavor *gyro*-pendulum is fun!



Many formal aspects of the EOM yet to be explored

RECENT / OPEN ISSUES
(TROUBLES?)

The previous results show that the limit $\theta_{13}\to 0$ is tricky. Some phenomena occur (in inverted hierarchy) only for $\theta_{13}\neq 0$, no matter how small. Are there other tricky limits?

Single-angle → multi-angle.

We have observed modest oscillation smearing (decoherence) when passing from single- to multi-angle approximation. However, complete decoherence takes place for hypothetically small asymmetry between neutrinos and antineutrinos [Esteban et al., astro-ph/07062498]. The whole subject is not really understood theoretically.

$2\nu \to 3\nu$

The effective two-family approach sets $\delta m^2=0$. It is important to remove this approximation to test the robustness of the results. [Recent papers.] Also, there are known (1-loop) differences between ν_μ and ν_τ propagation. Need to check evolution for ν_e , ν_μ, ν_τ flavors separately. [Seems important at highest luminosities, $t \ll 1$ sec].

Bulb model → Realistic model

Asphericities, inhomogeneities, turbulence during SN explosion might influence self-interaction effects. Hard to manage from any viewpoint (with the possible exception of small-amplitude density fluctuations).

No feedback → feedback on SN explosion simulations

SN explosion simulations typically do not account for neutrino oscill. (including self-induced ones). Removal of this approximation is hard.

Standard Model → Beyond the SM

We have assumed standard electroweak interactions between neutrinos and background matter+neutrinos. Possible new interactions beyond SM (e.g., leptonic FCNC) might profoundly change the results.

etc.

CONCLUSIONS

In the dense supernova core, neutrinos are a nontrivial background to themselves - perhaps more important than the matter background

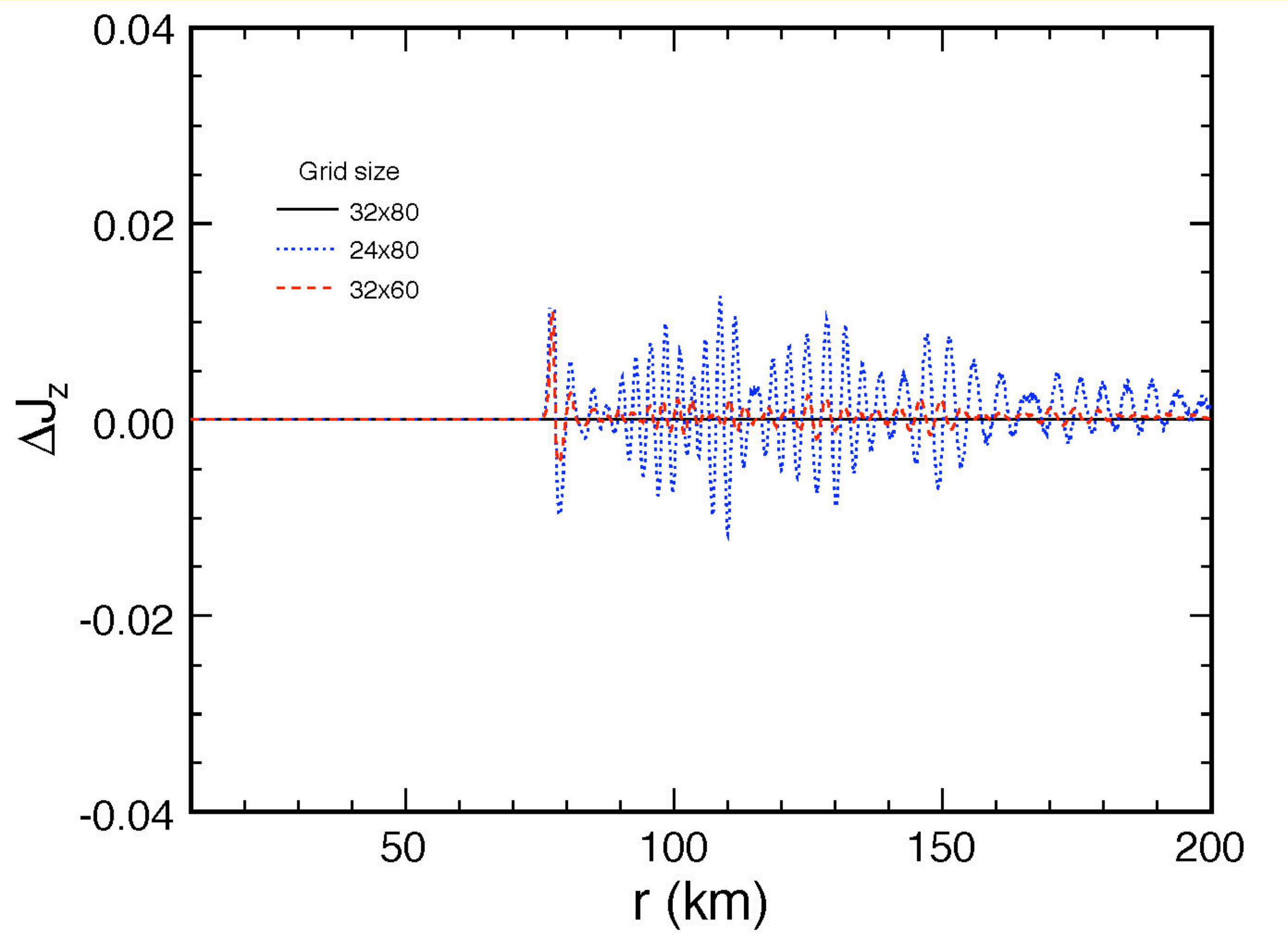
As a consequence, collective flavor transformation phenomena occur.
The spectral split/swap seems to provide an observable signature.

Much remains to be explored, both analytically and numerically.
After 21 years from SN 1987A, SN ν's continue to surprise us.

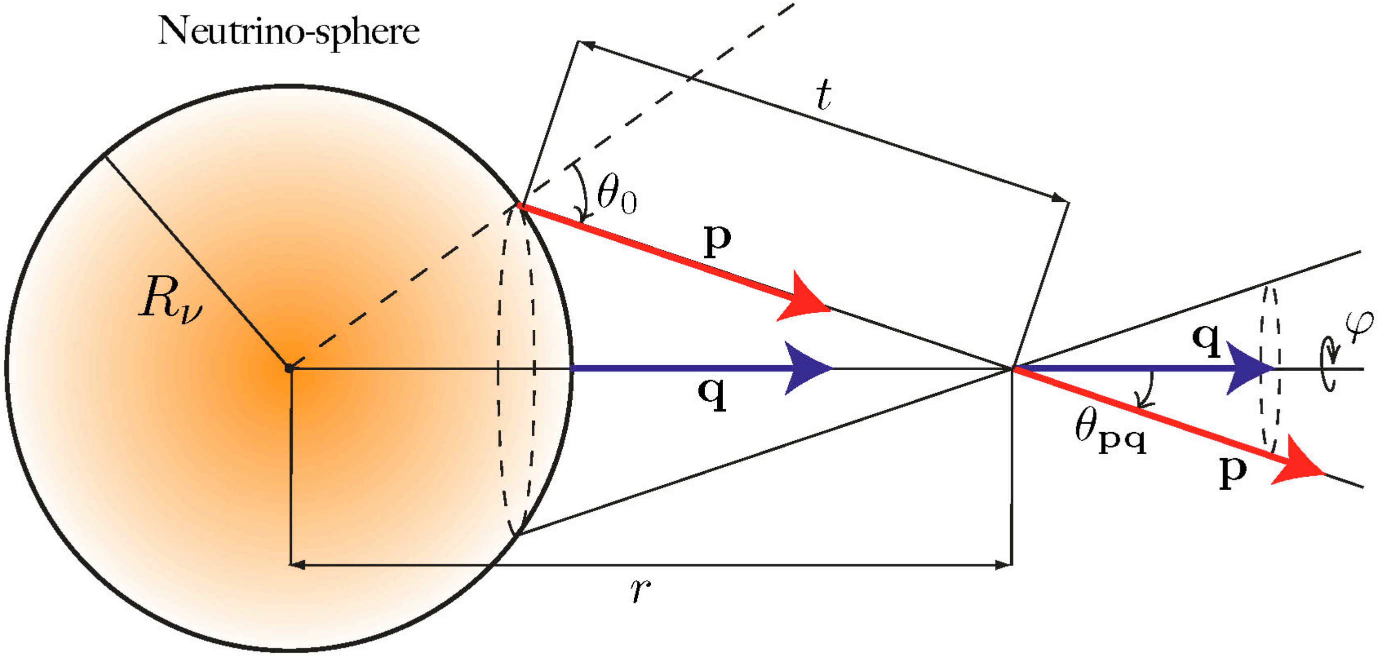
Thank you for your attention

Backup slides

Test of numerical convergence



Geometry: the neutrino bulb model



$$r \sin\theta_{\mathbf{pq}} = R_\nu \sin\theta_0 \qquad t = \sqrt{r^2 - R_\nu^2 \sin^2\theta_0} - R_\nu \cos\theta_0$$

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Pauli and Bohr interested in a spinning top

Single-angle vs Multi-angle (individual components P_i)