Introduzione alle masse e ai mescolamenti dei neutrini (I)

Eligio Lisi
INFN, Bari, Italy

The lectures are intended for a broad audience of students or researchers from different fields in particle physics

The goal is to "get you interested" in neutrino physics, by recalling basic neutrino properties and phenomena, which will be further discussed in more specialized lectures

Some simple exercises are also proposed (with solutions)

People interested in further reading can usefully browse the "Neutrino Unbound" website: www.nu.to.infn.it , or just mail me for advice about specific topics: eligio.lisi@ba.infn.it

Feel free to stop me and ask questions at any time!

Outline:

Pedagogical Introduction
Neutrino masses and spinor fields
Neutrinoless double beta decay
2ν, 3ν… $N\nu$ vacuum oscillations
[Homework]

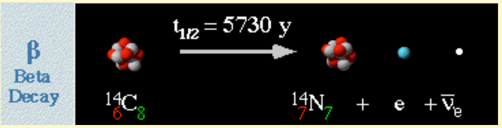
I

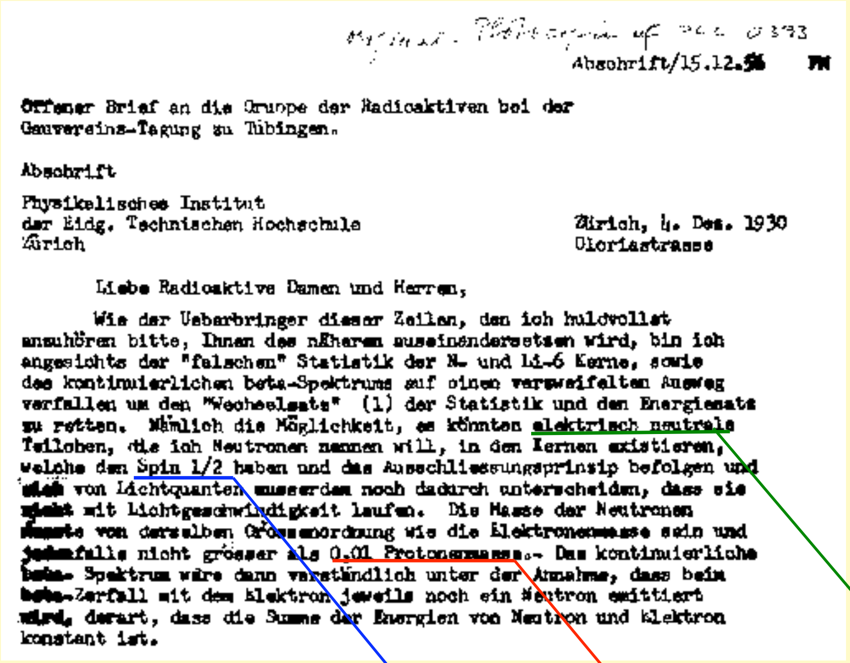
Recap
2ν oscillations in matter
Solar and KamLAND oscillations
Absolute neutrino masses
[Homework]

II

The past year (2010) was the 80th Neutrino Birthday!

The neutrino was invented in 1930 by Wolfgang Pauli as a "desperate remedy" to explain the continuous β-ray spectrum via a 3-body decay, e.g.,





Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

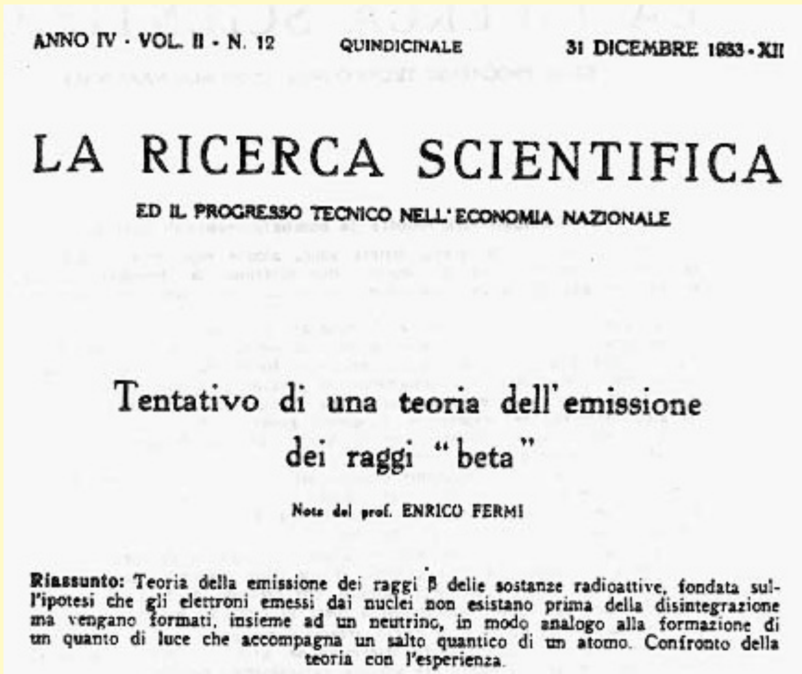
Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0.01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Kinematics: spin 1/2, tiny mass, zero electric harge

The name "neutrino" (="little neutral one", in Italian) was actually invented by Enrico Fermi, who first proposed in 1933-34 a theory for its **dynamics** (weak interactions)


ANNO IV · VOL. II · N. 12 QUINDICINALE 31 DICEMBRE 1933 · XII

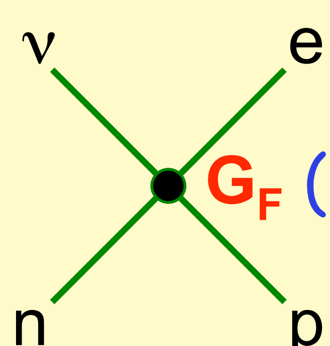
LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.



G_F (Fermi constant)

Short detour! … let's go back in time.
A Latin saying:

Nomen [est] Omen

"Name [is] Destiny"

Neutrino - What's in a name?

The root of the name [neutrino] … is a [kwa]stion

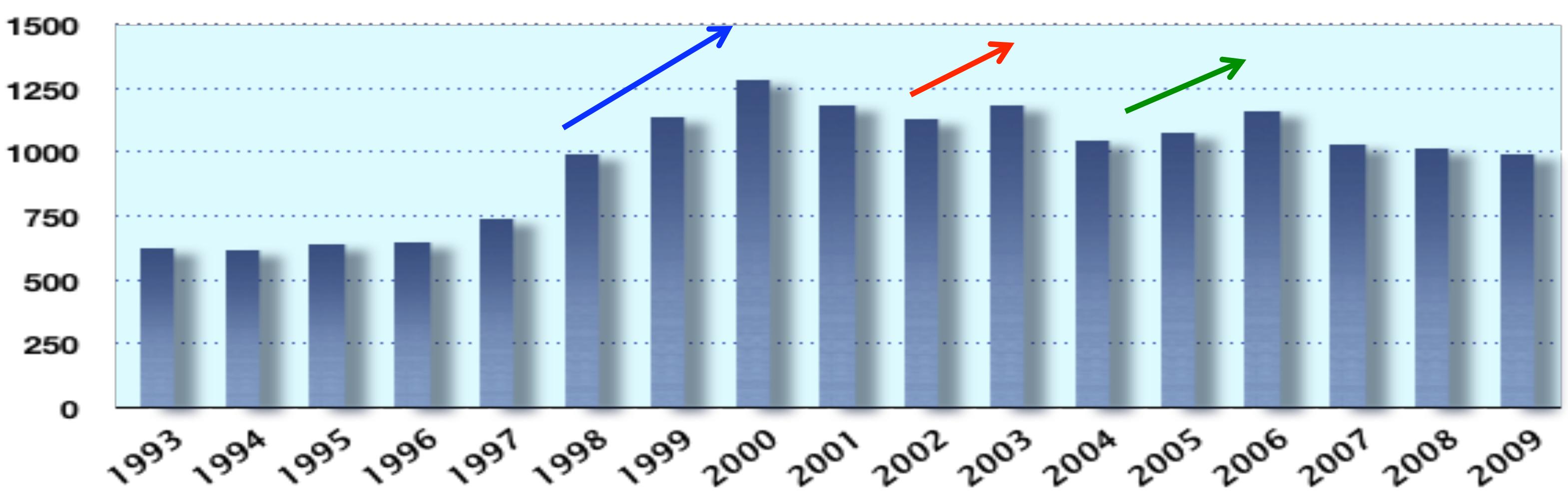
Language	**Word tree**	*…Some branches*	**Meaning**
Physics (Fermi 1934)	**NEUTR-INO**		**Little neutral one**
Italian	**NEUTRO**		**Neutral**
Latin	**NE-UTER**		**Not either; neutral**
Latin	**UTER**		**Either**
Greek	↑	*OUDETEROS*	*Neutral*
Old High German	↗	*HWEDAR*	*Which of two; whether*
Phonetic change/loss	[K]**UOTER**[US]		**Which of the two?**
Ionic Greek	**KOTEROS**		**Which of the two?**
Sanskrit	**KATARAS**		**Which of the two?**
Latin	↑	*QUANTUS*	*How much?*
Sanskrit		*KATAMAS*	*Which out of many?*
Sanskrit		*KATHA*	*How?*
Sanskrit	↗	*KAS*	*Who?*
Indo-European root	**KA** *or* **KWA**		**Interrogative base**

If "name is destiny," then … neutrino's destiny is to raise questions!

Answers to a major "which of…" question have dramatically raised the interest in neutrino physics in recent years:

Q. Which of the three neutrinos have mass ?

A. …at least one! …at least two! […osc. cycles!] + cosmology!

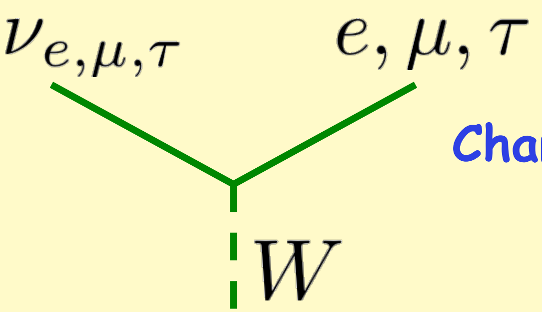


papers with "neutrino(s)" in title (from SPIRES)

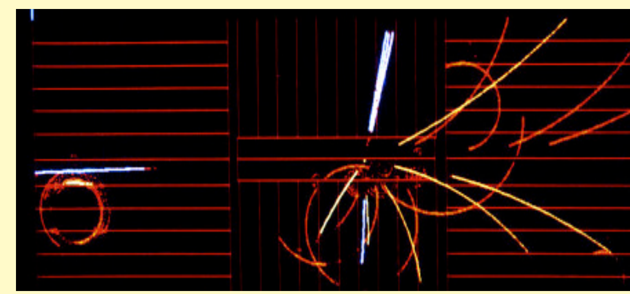
Many decades of research have revealed relevant properties of the neutrino. For instance, there are 3 different neutrino "flavors"

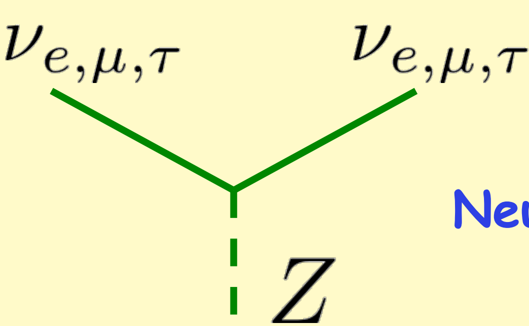
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad \begin{array}{l} \leftarrow \quad q = 0 \\ \leftarrow \quad q = -1 \end{array} \quad (\Delta q = 1)$$

and their Fermi interactions are mediated by a charged vector boson W, with a neutral counterpart, the Z boson



Charged Current (Δq=1)



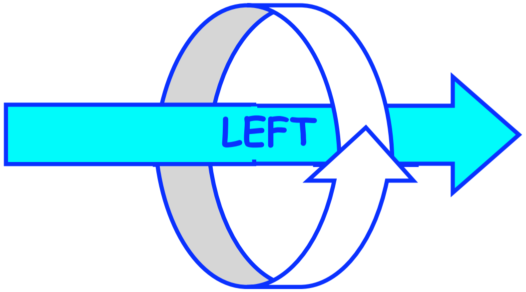


Neutral Current (Δq=0)

Such interactions are chiral (= not mirror-symmetric):

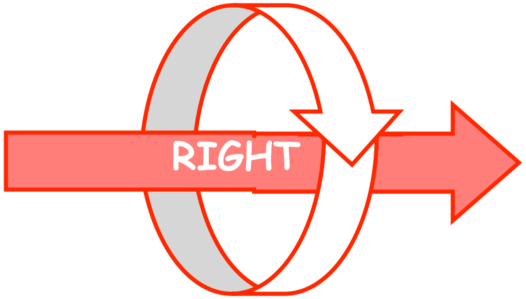
Neutrinos are created in
a left-handed (LH) state

ν



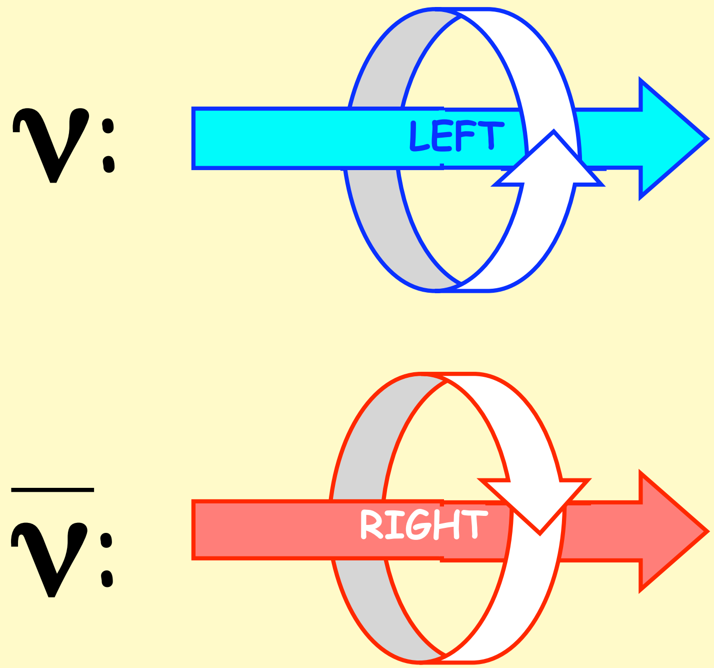
Anti-nus are created in
a right-handed (RH) state

$\bar{\nu}$



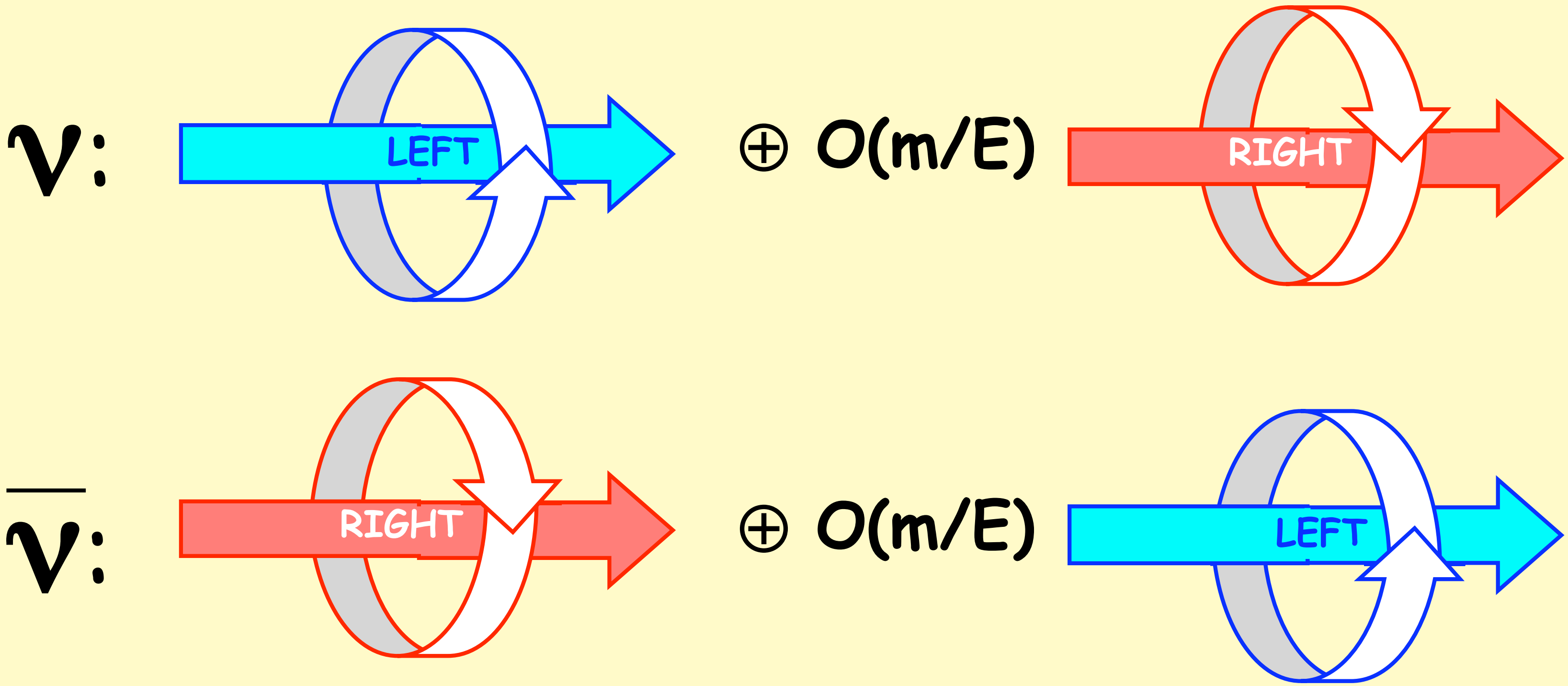
Neutrinos couldn't see themselves in a mirror... like vampires!

For massless neutrinos: handedness is a constant of motion



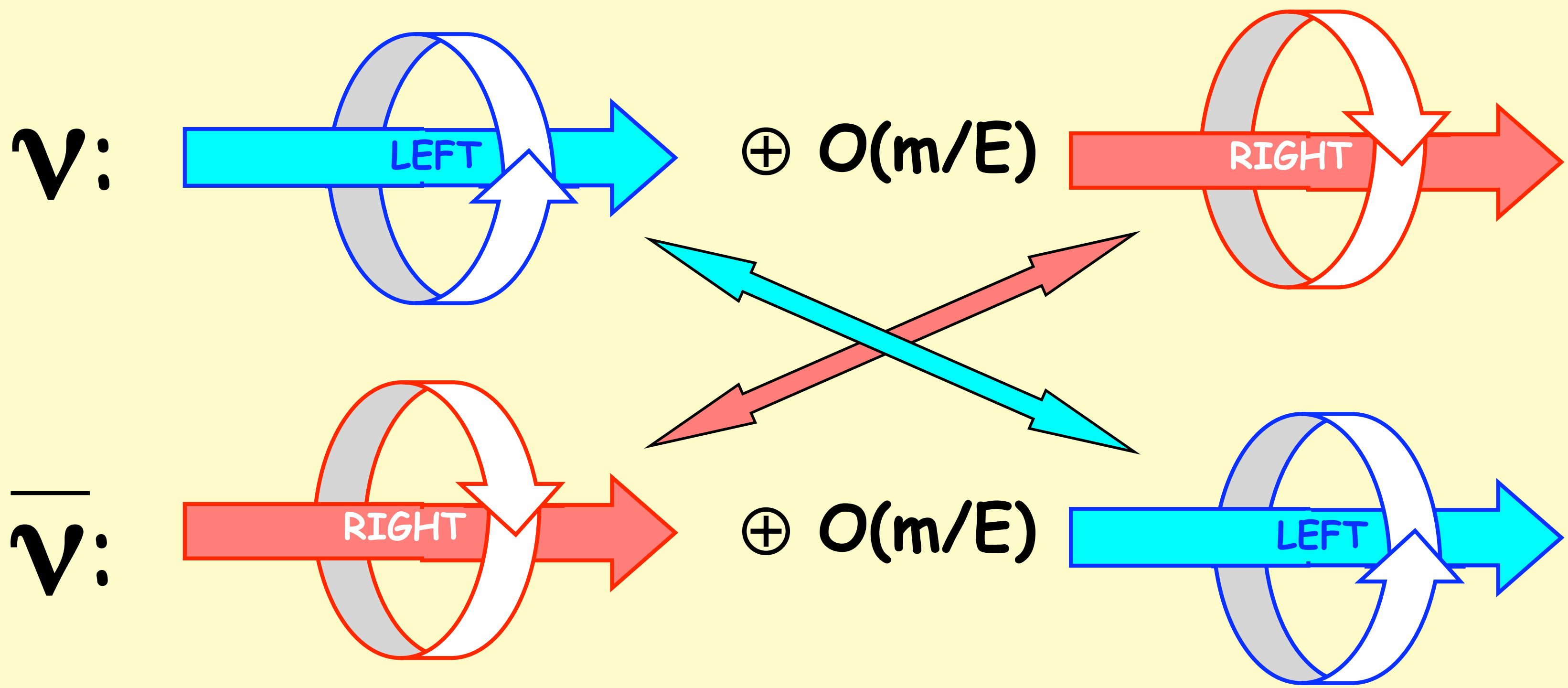
2 independent d.o.f.: massless ("Weyl") 2-spinor

But: massive ν can develop the "wrong" handedness at $O(m/E)$
(the Dirac equation mixes RH and LH states for $m_\nu \neq 0$):



If these 4 d.o.f. are independent: massive ("Dirac") 4-spinor
[→ Distinction between neutrinos and antineutrinos, as for electrically charged fermions. Can define a "lepton number"]

But, for neutral fermions, 2 components might be identical !



Massive ("Majorana") 4-spinor with 2 independent d.o.f.

[No distinction between neutrinos and antineutrinos, up to a phase:
A *very* neutral particle: no electric charge, no leptonic number…]

Exercise 1. Define the electron neutrino as the neutral particle emitted in β+ decay, and the electron antineutrino as the neutral particle emitted in β- decay. Reactions which have been observed:

$$\nu_e + n \rightarrow p + e^- \qquad\qquad \overline{\nu}_e + p \rightarrow n + e^+$$

while the following reactions have not been observed:

$$\overline{\nu}_e + n \rightarrow p + e^- \qquad\qquad \nu_e + p \rightarrow n + e^+$$

If neutrinos and antineutrinos are different (Dirac case), that's easy to understand. Try to understand the same (non)observations in the case of Majorana neutrinos.

Summary of options for neutrino spinor field:

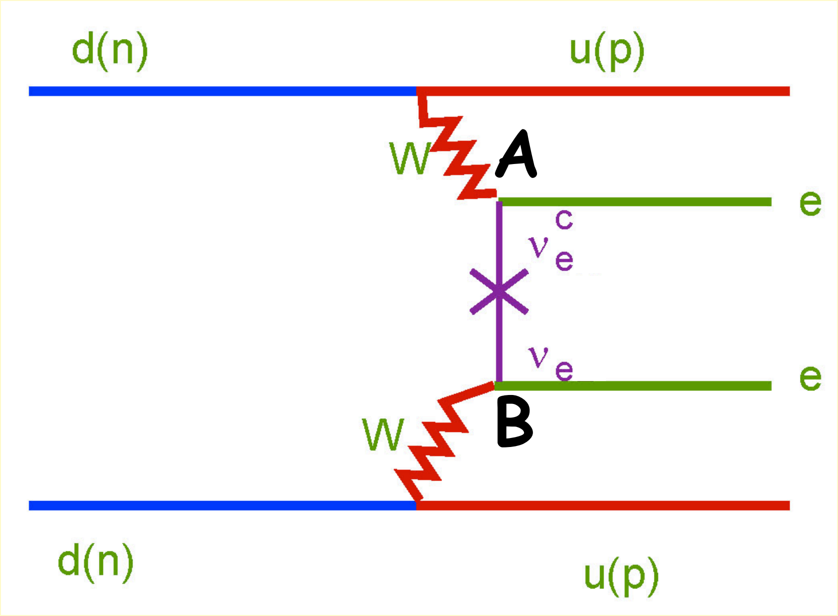
m=0, Weyl:	$\psi = \psi_R$ or $\psi = \psi_L$	massless field with 2 d.o.f.
m≠0, Majorana:	$\psi = \psi_R + \psi_R^c = \psi^c$ or $\psi = \psi_L + \psi_L^c = \psi^c$	massive field with 2 d.o.f.
m≠0, Dirac:	$\psi = \psi_R + \psi_L \neq \psi_c$	massive field with 4 d.o.f.

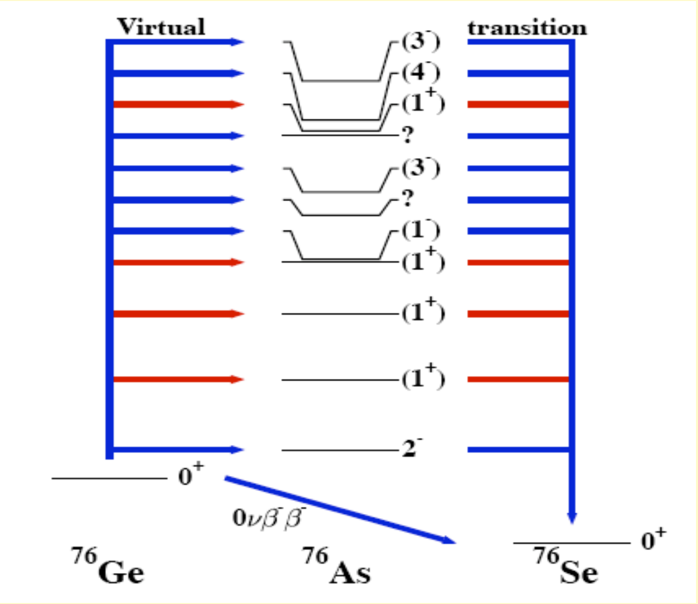
Conjugation operator: $\psi^c = \mathcal{C}(\psi) = i\gamma^2\psi^*$, $\psi_{\mathrm{antiparticle}} = \mathcal{C}(\psi_{\mathrm{particle}})$

Appendix: Majorana masses and "see-saw" mechanism to explain their smallness

Experiments: A unique experimental handle →

Neutrinoless double beta decay: (A,Z) → (A,Z+2)+2e



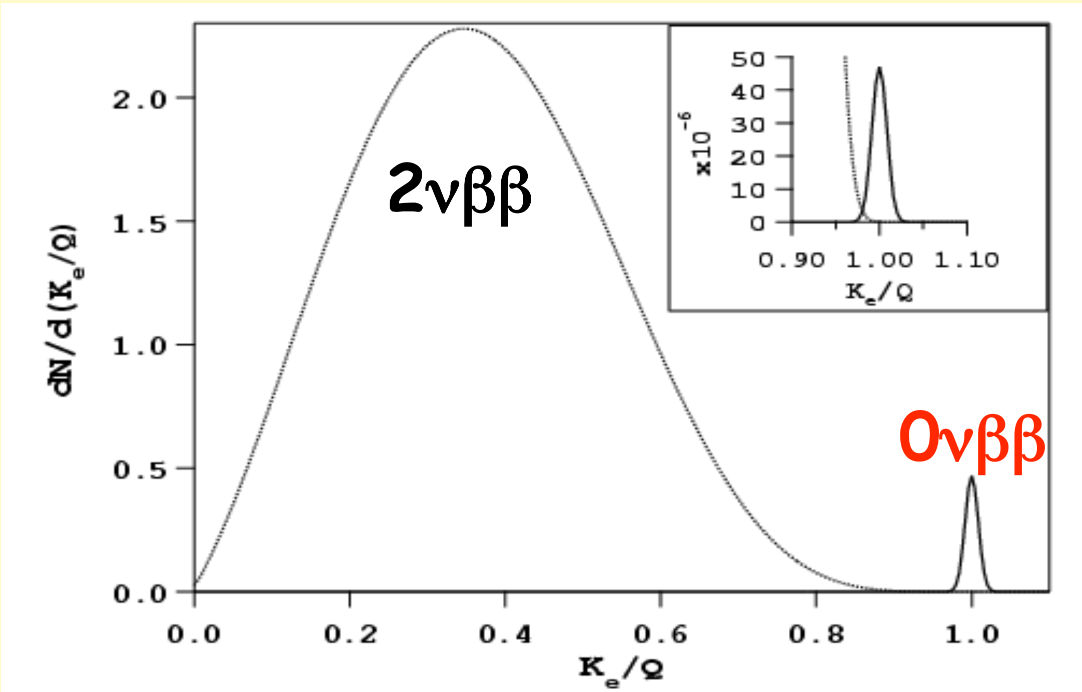


Can occur only for Majorana neutrinos. Intuitive picture:

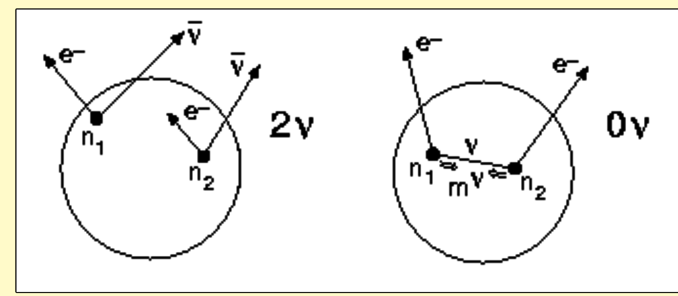
1) A RH antineutrino is emitted at point "**A**" together with an electron
2) If it is massive, at O(m/E) it develops a LH component (not possible if Weyl)
3) If neutrino=antineutrino, this component is a LH neutrino (not possible if Dirac)
4) The LH (Majorana) neutrino is absorbed at "**B**" where a 2nd electron is emitted

[EW part is "simple". Nuclear physics part is rather complicated and uncertain.]

Experimentally: Look at sum energy of both electrons



Need to see the 0νββ line emerge above background, at the endpoint of spectrum from "conventional" (and observed) **2νββ** decay.



Very rare to detect (if it occurs): doubly-weak and suppressed by m/E.

Need to be tenacious… like O. Cremonesi (see next lecture)

Recap: if neutrinos have mass, they can develop the **"wrong handedness"** with **amplitude of O(m_{ass}/E_{nergy}).** The only known chance to observe this tiny effect is **$0\nu\beta\beta$ decay.**

But, if neutrinos are not only massive but mixed, they can also develop in the **"wrong flavor"** as a major consequence (**"neutrino flavor oscillations"**). This effect, despite being only of **$O(m^2/E)$ in the phase**, can become observable over macroscopic distances (similar to optical interferometry).

Flavor oscillations have proven that neutrinos have mass and mix, just as quarks do. Let's temporarily take for granted these facts and discuss their implications for **$0\nu\beta\beta$** and absolute ν masses

3ν masses and mixings

- 3 flavor and mass states: $$(\nu_e,\, \nu_\mu,\, \nu_\tau)^T = U\,(\nu_1,\, \nu_2,\, \nu_3)^T$$

Unitary matrix $\mathbf{U_{\alpha i}}$ depends on: 3 rotation angles θ_{ij} + 1 complex CP phase. Conventionally, same ordering of the CKM quark matrix used for neutrinos:

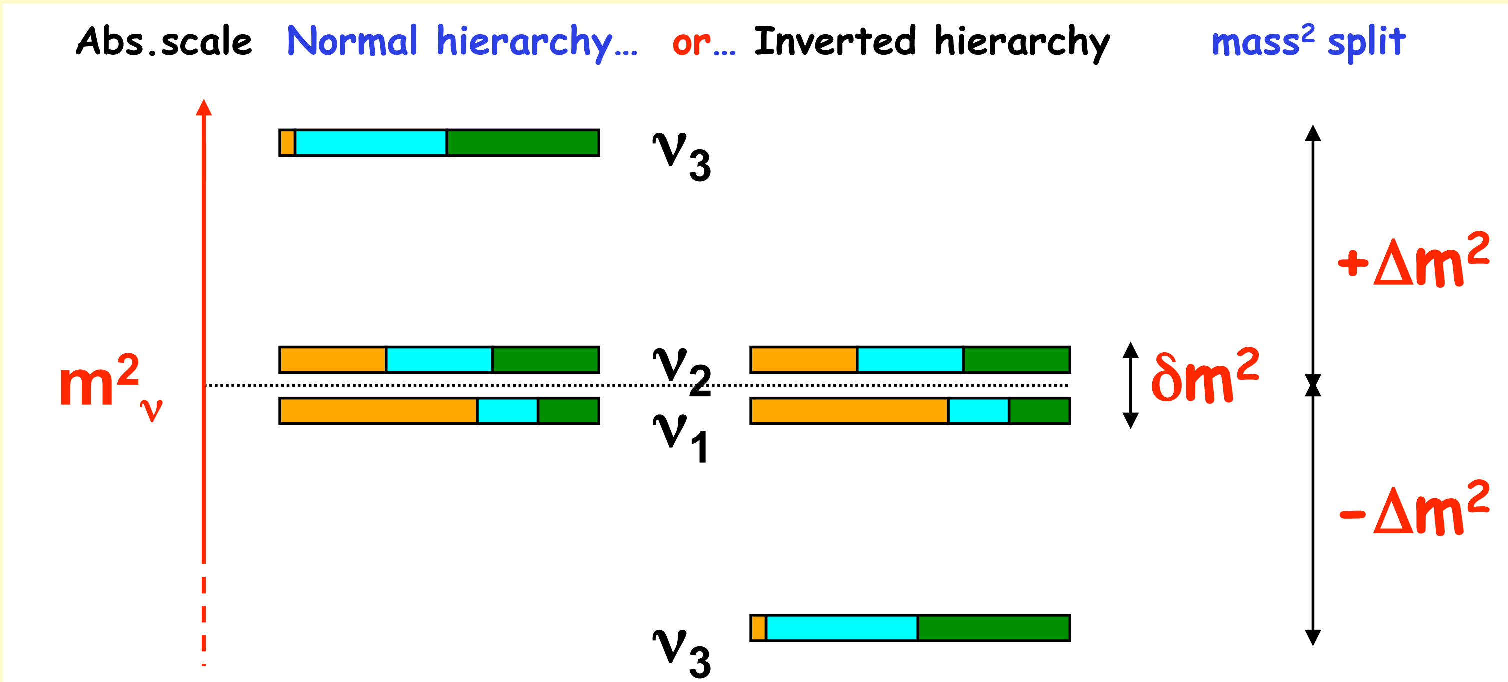
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij}=\cos(\theta_{ij})$ etc. Such ordering happens to be very useful for approxim.
[Note: For antineutrinos: $U \rightarrow U^*$]

Neutrino masses: $\mathbf{m_1}$, $\mathbf{m_2}$, $\mathbf{m_3}$. Oscillations constrain $\mathbf{m_i^2 - m_j^2}$ (see later).

Take these results for granted! 3ν mass-mixing overview

(here, with 1 digit accuracy). Flavors = e μ τ



$$\delta m^2 \sim 8 \times 10^{-5}\ \mathrm{eV}^2 \qquad \sin^2\theta_{12} \sim 0.3$$

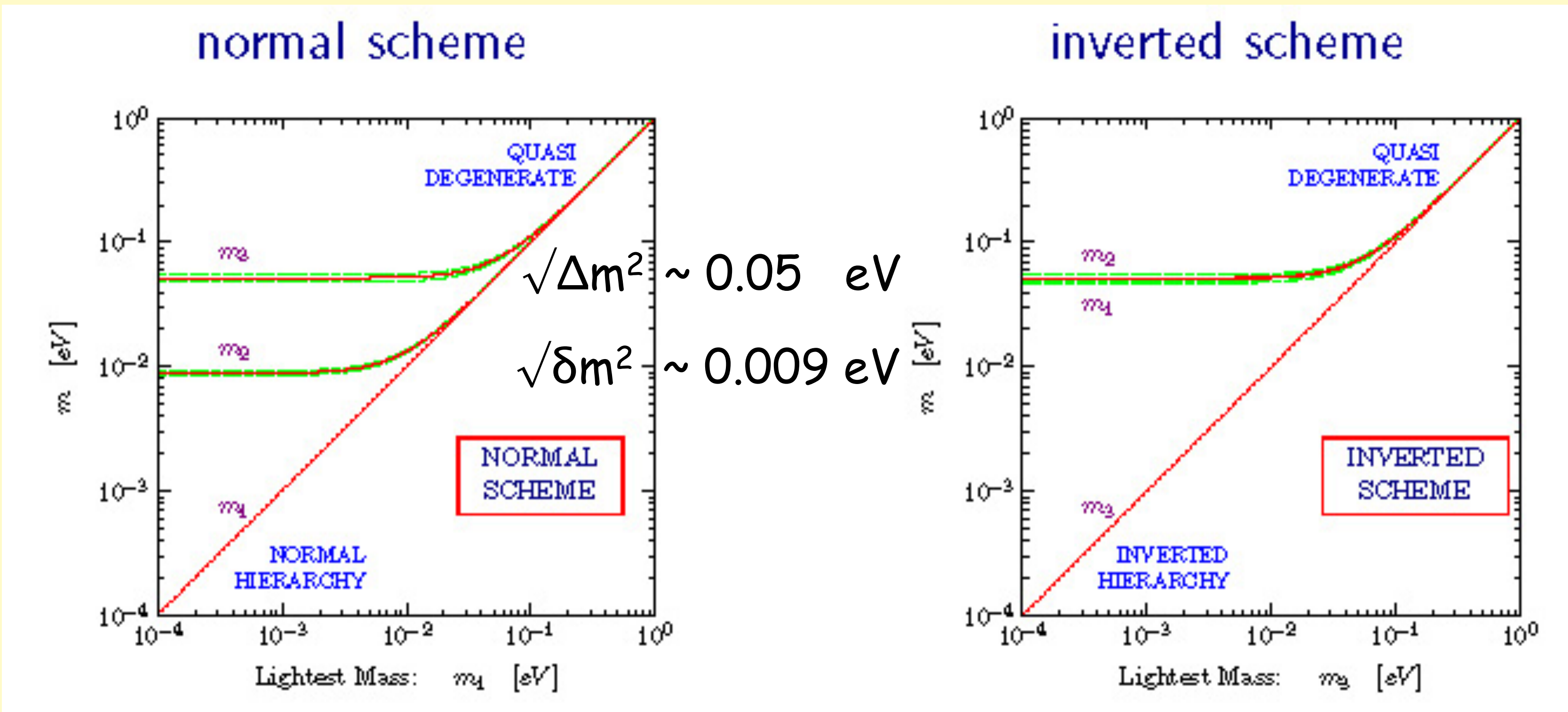
$$\Delta m^2 \sim 3 \times 10^{-3}\ \mathrm{eV}^2 \qquad \sin^2\theta_{23} \sim 0.5$$

$$m_\nu < O(1)\ \mathrm{eV} \qquad \sin^2\theta_{13} < \mathrm{few}\%$$

$$\mathrm{sign}(\pm\Delta m^2)\ \mathrm{unknown} \qquad \delta\ (\mathrm{CP})\ \mathrm{unknown}$$

Oscillations constrain neutrino mixings and mass splittings but not the absolute mass scale.

E.g., can take the lightest neutrino mass as free parameter:

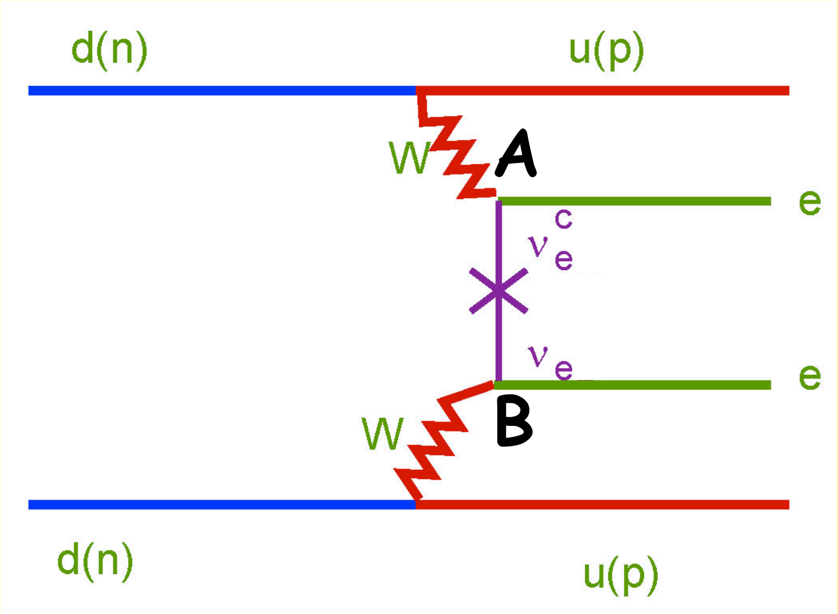


(However, the lightest neutrino mass is not really an "observable")

We know three realistic observables to attack ν masses →

Observable #1: 0νββ decay (iff Majorana!)

For each mass state ν_i, 0νββ amplitude proportional to:



… mixing of ν_e with ν_i

… mass of ν_i

… mixing of ν_i with ν_e

(times an unknown ν_i phase)

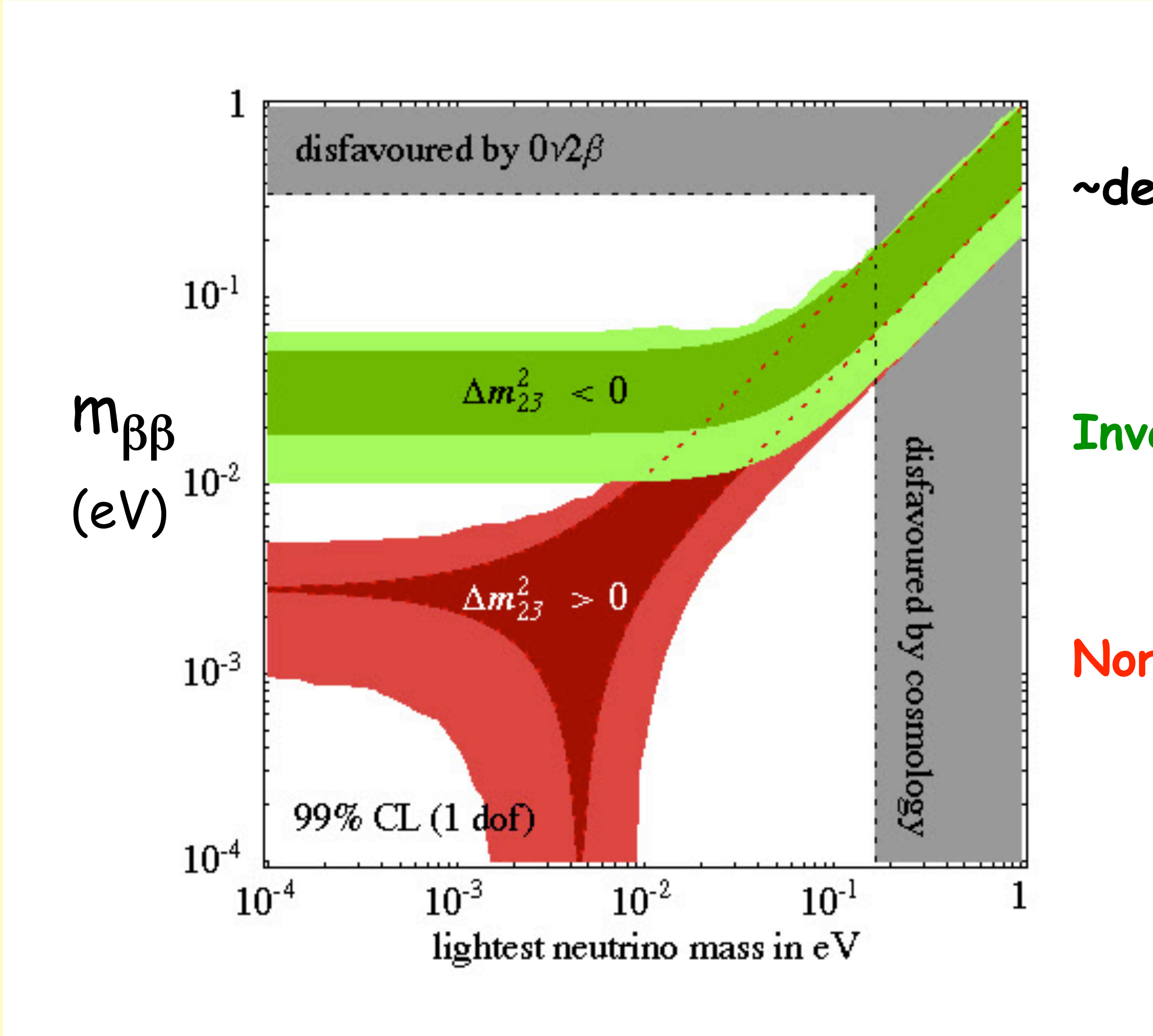
Summing up for three massive neutrinos:

Amplitude ~ "effective Majorana mass"

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

[complex linear combination of masses; c_{ij} = cos θ_{ij} etc.]

Typical plot of $m_{\beta\beta}$ versus lightest neutrino mass, including constraints from oscillation data:



~degenerate masses

Inverted hierarchy

Normal hierarchy

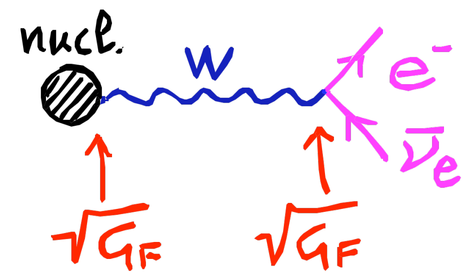
$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

Observable #2: beta decay

Classic kinematic search for neutrino mass:
look at high-energy endpoint Q of spectrum.

β-decay

rate: $d\Gamma \propto G_F^2 \times (\text{phase sp.})$

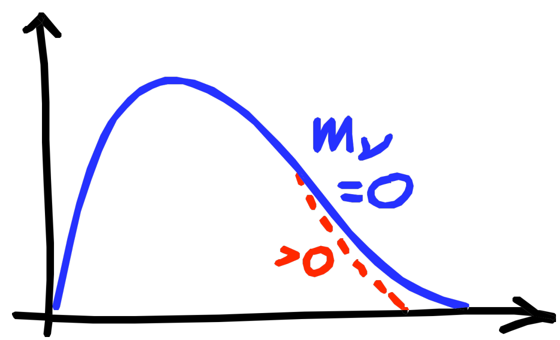


$|\ \ |^2 \propto G_F^2$

energy spectrum:

$$\frac{d\Gamma}{dE_e} \propto G_F^2\, p_e E_e (Q - E_e)^2 \qquad (m_\nu = 0)$$

$$G_F^2\, p_e E_e (Q - E_e)\sqrt{(Q-E_e)^2 + m_\nu^2} \qquad (>0)$$



μ-decay

$$\Gamma_\mu = \frac{1}{\tau_\mu} \propto G_F^2 m_\mu^5$$

"defines" G_F

For just one (electron) neutrino family: sensitivity to $m^2(\nu_e)$ (obsolete)

For three neutrino families ν_i, and individual masses experimentally <u>unresolved</u> in beta decay: sensitivity to the sum of $m^2(\nu_i)$, weighted by squared mixings $|U_{ei}|^2$ with the electron neutrino. Observable:

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

(so-called "effective electron neutrino mass")

Note: mass state with largest electron flavor component is ν_1:

$$|U_{e1}|^2 \approx \cos^2\theta_{12} \approx 0.7$$

… and we can't exclude that ν_1 is ~massless in normal hierarchy.

Observable #3: neutrino mass in cosmology

Standard big bang cosmology predicts a relic neutrino background with total number density $336/cm^3$ and temper. $T_\nu \sim 2$ K $\sim 1.7 \times 10^{-4}$ eV $\ll \sqrt{\delta m^2}, \sqrt{\Delta m^2}$.

→At least two relic neutrino species are nonrelativistic today (we can't exclude the lightest to be ~ massless)

→Their total mass contributes to the normalized energy density as $\Omega_\nu \approx \Sigma/50$ eV, where

$$\Sigma = m_1 + m_2 + m_3$$

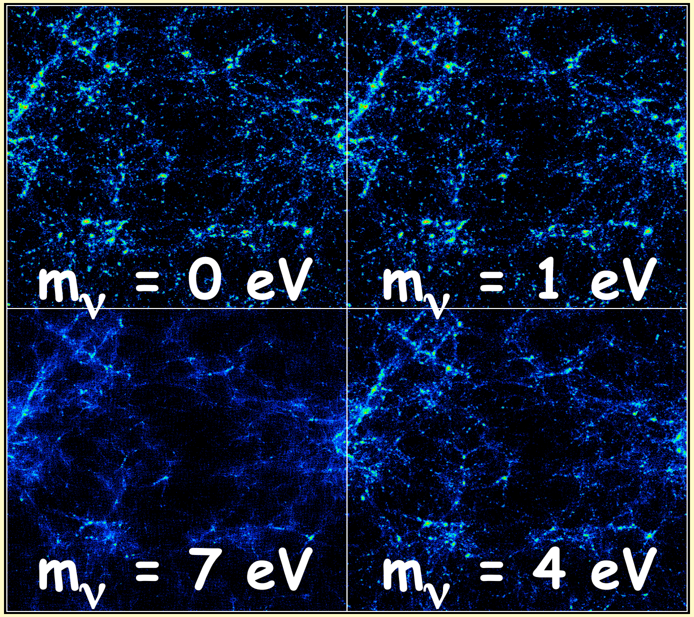
→So, if we just impose that neutrinos do not saturate the total matter density, $\Omega_\nu < \Omega_m \approx 0.25$, we get

$m_i < 4$ eV – not bad!

Much better bounds can be derived from neutrino effects on structure formation (including constraints coming from CMB data).

Massive neutrinos are difficult to cluster because of their relatively high velocities: they suppress matter fluctuations on scales smaller than their mass-dependent free-streaming scale.

→ Get mass-dependent suppression of small-scale structures



(E..g., Ma 1996)

[See detailed talks by A. Melchiorri & M. Viel]
[+ first & prospective Planck results by Bersanelli, Bartolo]

Summary of absolute mass observables (m_β, $m_{\beta\beta}$, Σ)

1) β decay: $m^2_i \neq 0$ can affect spectrum endpoint. Sensitive to the "effective electron neutrino mass":

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

2) $0\nu\beta\beta$ decay: Can occur if $m^2_i \neq 0$ and $\nu=\bar{\nu}$ (Majorana, not Dirac) Sensitive to the "effective Majorana mass" (and phases):

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

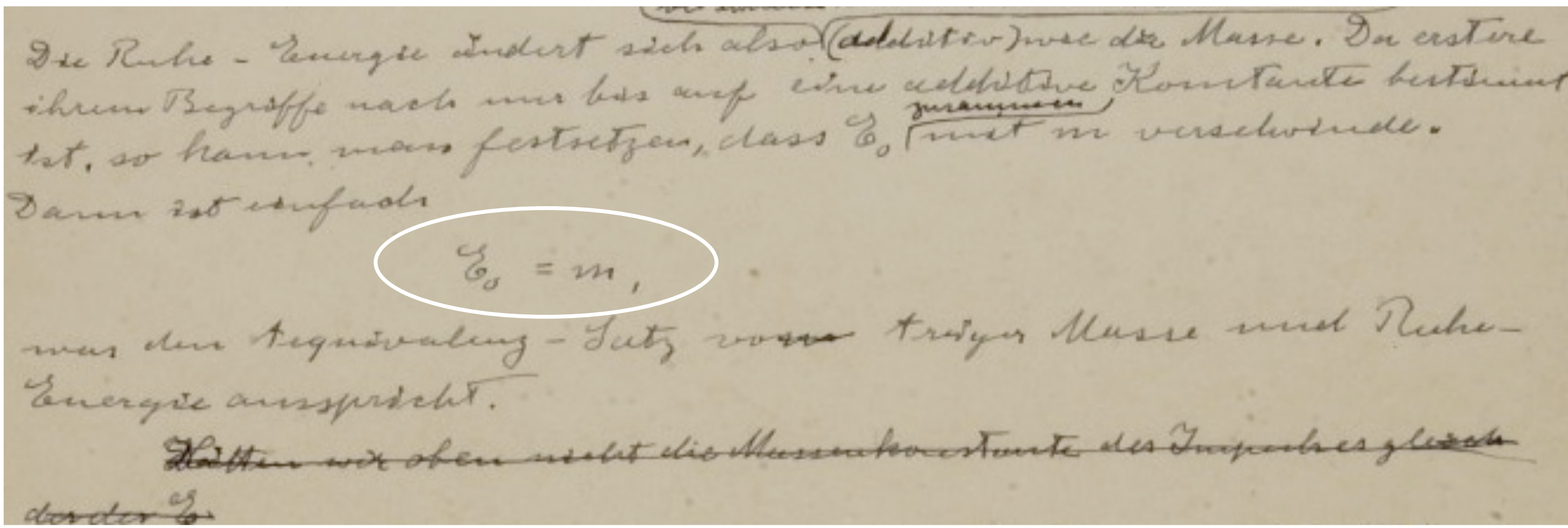
3) Cosmology: $m^2_i \neq 0$ can affect large scale structures in (standard) cosmology constrained by CMB + other data. Sensitive to:

$$\Sigma = m_1 + m_2 + m_3$$

But… how do we know that indeed neutrinos have mass and mix?

Neutrino flavor oscillations in vacuum (2ν)

The starting point is a century-old equation …



Die Ruhe-Energie ändert sich also (additiv) wie die Masse. Da erstere ihrem Begriffe nach nur bis auf eine additive Konstante bestimmt ist, so kann man festsetzen, dass $\mathcal{E}_0$ zusammen mit m verschwinde. Dann ist einfach

$$\mathcal{E}_0 = m,$$

was den Aequivalenz-Satz von träger Masse und Ruhe-Energie ausspricht.

~~Hätten wir oben nicht die Massenkonstante des Impulses gleich~~ ~~der der $\mathcal{E}_0$~~

… namely, for p≠0:

$$E = \sqrt{m^2 + p^2}$$

(in natural units)

Our ordinary experience takes place in the limit: $p \ll m$

$$E \simeq m + \frac{p^2}{2m}$$

... while for neutrinos the proper limit is: $p \gg m$

$$E \simeq p + \frac{m^2}{2p}$$

Energy difference between two neutrinos ν_i e ν_j with mass m_i e m_j in the same beam $(p_i = p_j \simeq E)$:

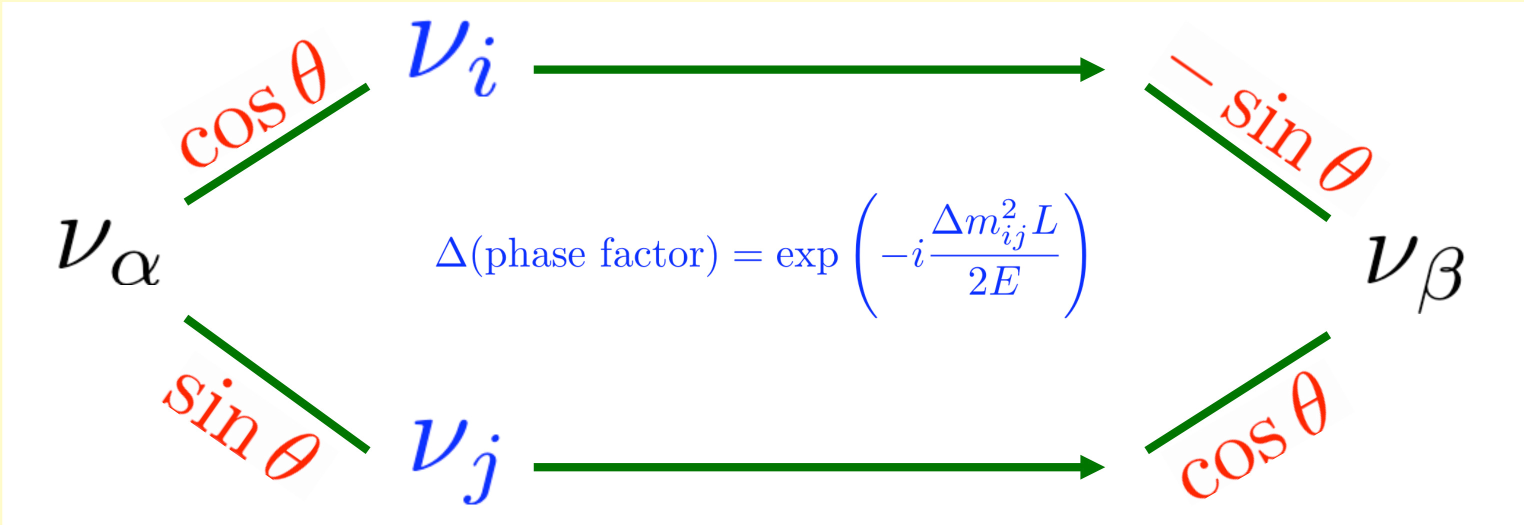
$$\Delta E \simeq \frac{\Delta m_{ij}^2}{2E}$$

PMNS*: neutrinos with definite mass (ν_i and ν_j) might have NO definite flavor (ν_α e ν_β), e.g.,

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_j \end{pmatrix}$$

*Pontecorvo; Maki, Nakagawa & Sakata

Analogy with a two-slit interference experiment in vacuum:



This is the simplest case (only 2 neutrinos involved, no interactions with matter). It shows that, if neutrinos are massive and mixed (like quarks), then flavor is not a good quantum number during propagation. Indeed, it changes ("oscillates") significantly over a distance L (≈Δt) dictated by the uncertainty relation:

$$1 \sim \Delta E \Delta t \simeq \frac{m_i^2 - m_j^2}{2E}\, L$$

Exercise 2. Prove that a neutrino created with flavor α can develop a different flavor β with a periodical oscillation probability in **L/E**:

$$P(\nu_\alpha \to \nu_\beta) = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

(B. Pontecorvo)

Amplitude
(vanishes for θ=0 or $\pi/2$)

Phase difference
(vanishes for degenerate masses)

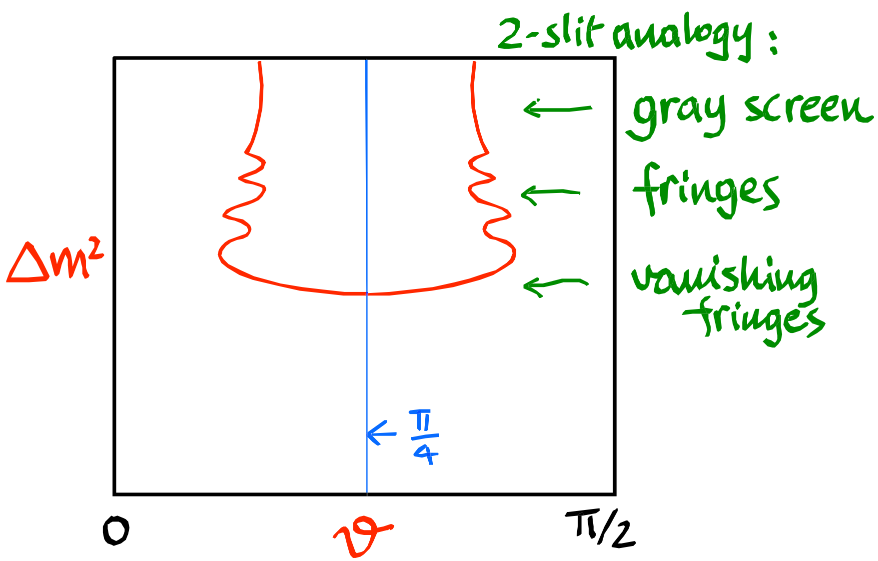
Note : This is the flavor "appearance" probability.
The flavor "disappearance" probability is the complement to 1.

Exercise 3. The oscillation effect depends on the **difference** of (squared) masses, not on the **absolute masses**. Why?

Exercise 4 . Show that:

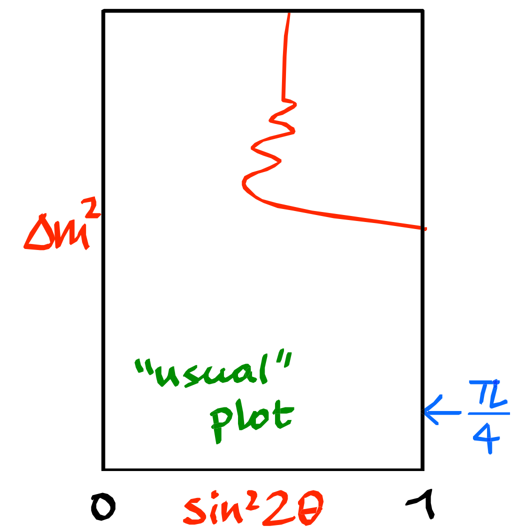
$$\frac{\Delta m^2 L}{4E} = 1.267\left(\frac{\Delta m^2}{\mathrm{eV}^2}\right)\left(\frac{L}{\mathrm{km}}\right)\left(\frac{\mathrm{GeV}}{E}\right)$$

Typical iso-$\langle P_{\alpha\beta} \rangle$ contours



Octant symmetry: $\theta \to \frac{\pi}{2} - \theta$ in $P_{e\mu}$

If 2nd octant folded onto the 1st one:

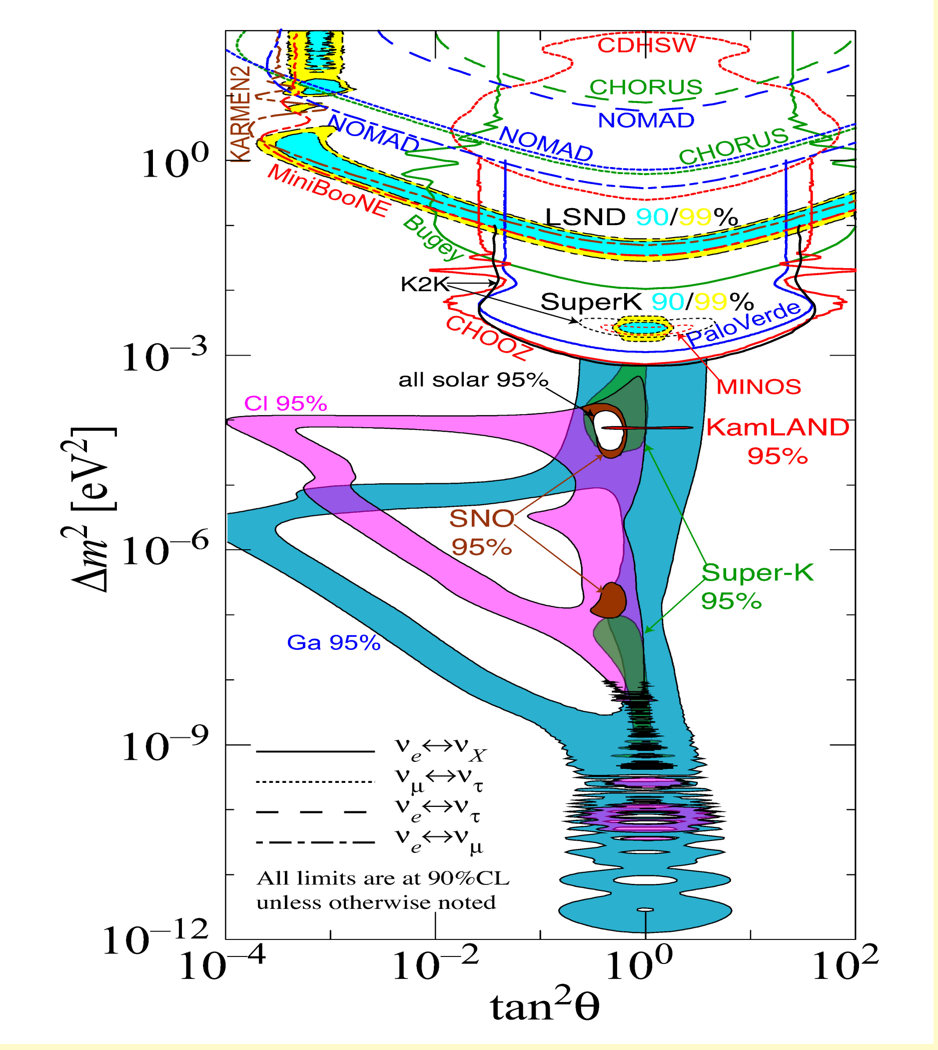


Basically obsolete

In general, better to use $\log \tan^2\theta$
(preserve octant-symmetry) or $\sin^2\theta$

(Note: Octant symmetry broken by 3ν and/or matter effects)

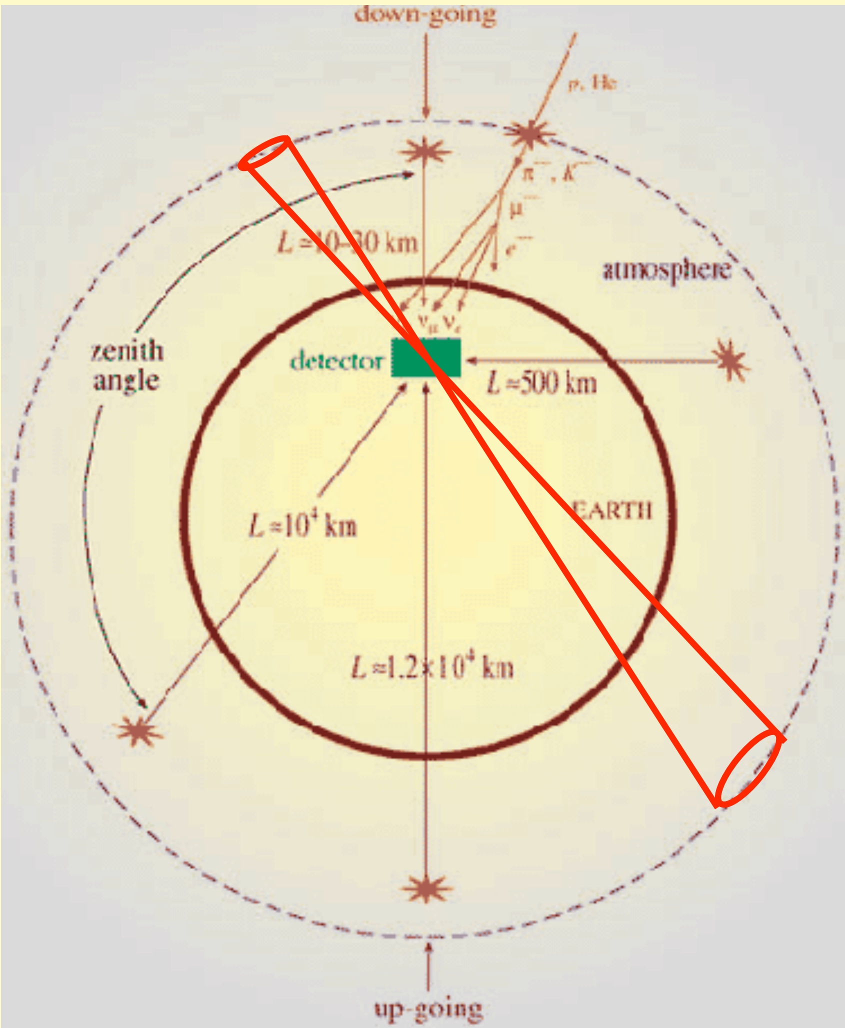
Octant (a)symmetric contours:



[Particle Data Group]

Observation of "effective 2ν" oscillations of atmospheric ν's

Cosmic rays hitting the atmosphere can generate secondary (anti)neutrinos with electron and muon flavor via meson decays (→ μ/e flavor ratio ~ 2). Energies: E~ 0.1 - 100 GeV. Pathlengths: L~ 10 - 10000 km

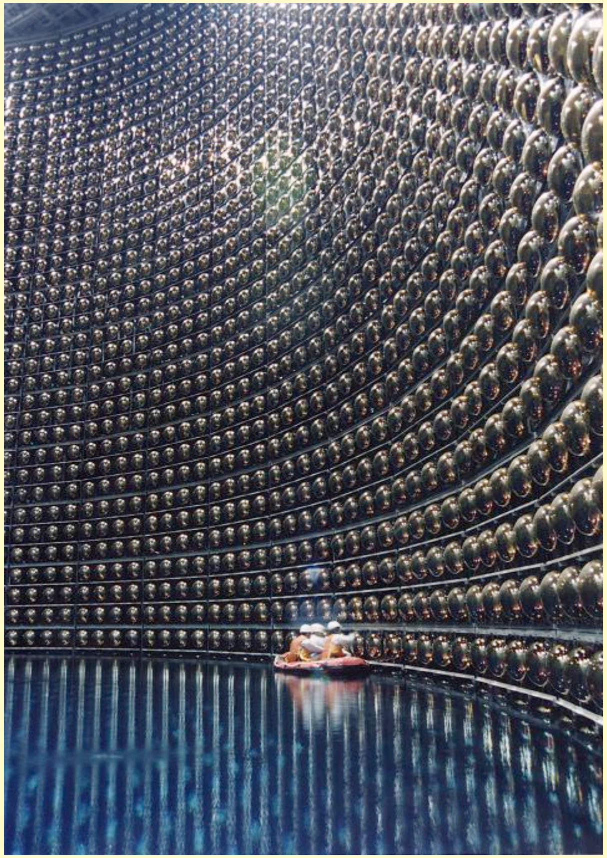


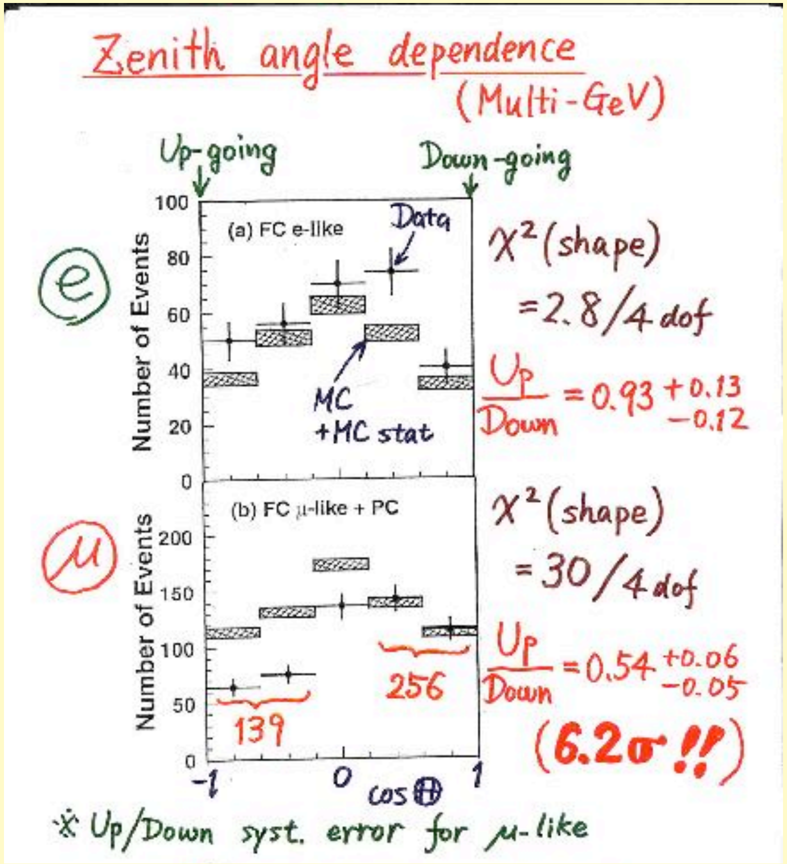
Same ν flux expected from opposite solid angles (up-down symmetry)

[Flux dilution (~1/r²) is compensated by larger production surface (~r²)]

Should be reflected in symmetry of event zenith spectra, if energy & angle can be reconstructed well enough

The breakthrough (1998)





T. Kajita for Super-Kamiokande, at Neutrino'98, Takayama

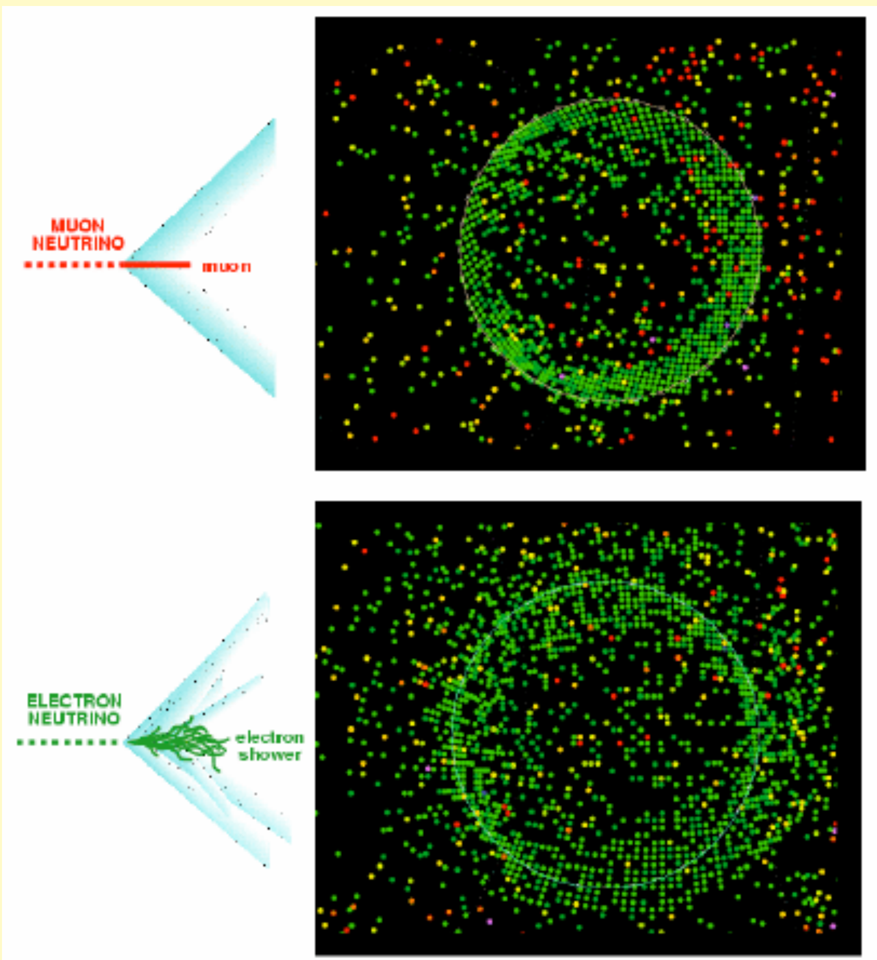
(Also: F. Ronga for MACRO at Neutrino'98)

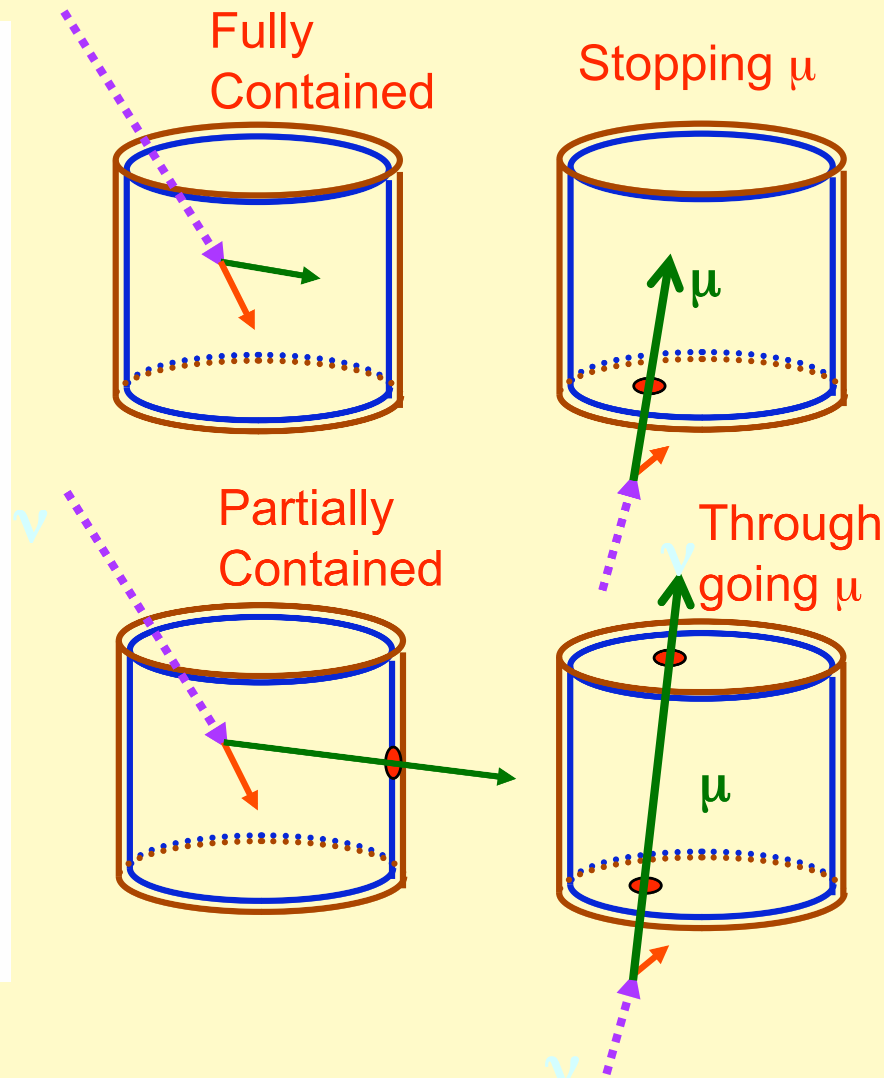
Detection in SK

Parent neutrinos detected via CC interactions in the target (water).

Final-state μ and e distinguished by ≠ Cherenkov ring sharpness.

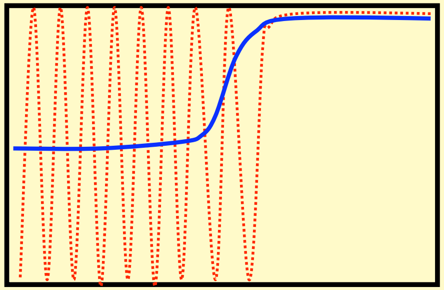
(But: no charge discrimination, no τ event reconstruction). Topologies:



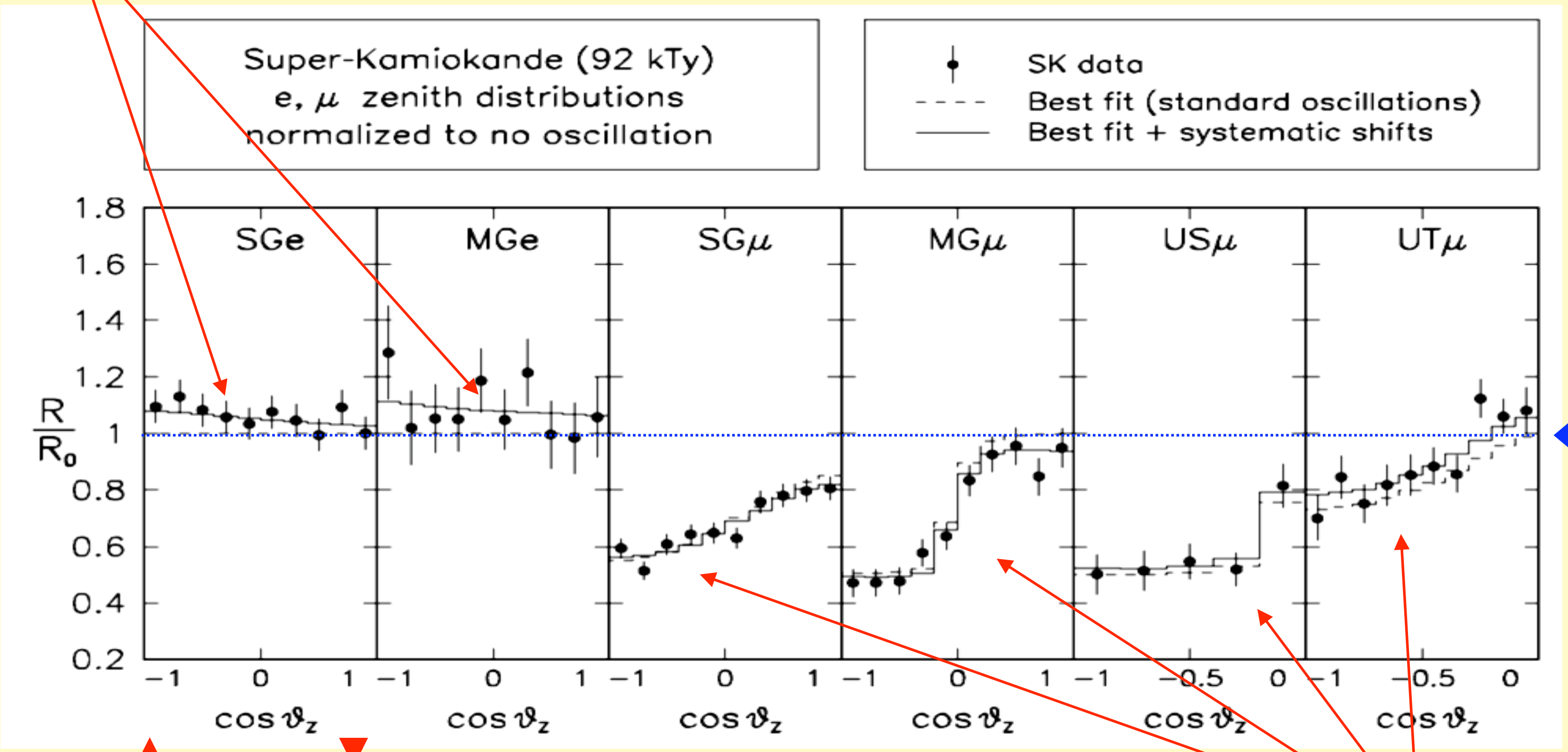


RESULTS SK zenith distributions

SGe	Sub-GeV electrons
MGe	Multi-GeV electrons
SGμ	Sub-GeV muons
MGμ	Multi-GeV muons
USμ	Upward Stopping muons
UTμ	Upward Through-going muons


$\cos\theta_Z$

electrons ~OK



no osc.

up

down

muon deficit from below

Observations over several decades in L/E:

ν_e induced events: ~ as expected

ν_μ induced events: disappearance from below

Interpretation in terms of oscillations:

Channel $\nu_\mu \rightarrow \nu_e$? No (or subdominant)

Channel $\nu_\mu \rightarrow \nu_\tau$? Yes (dominant)

2ν–like approximation works well over five L/E decades…

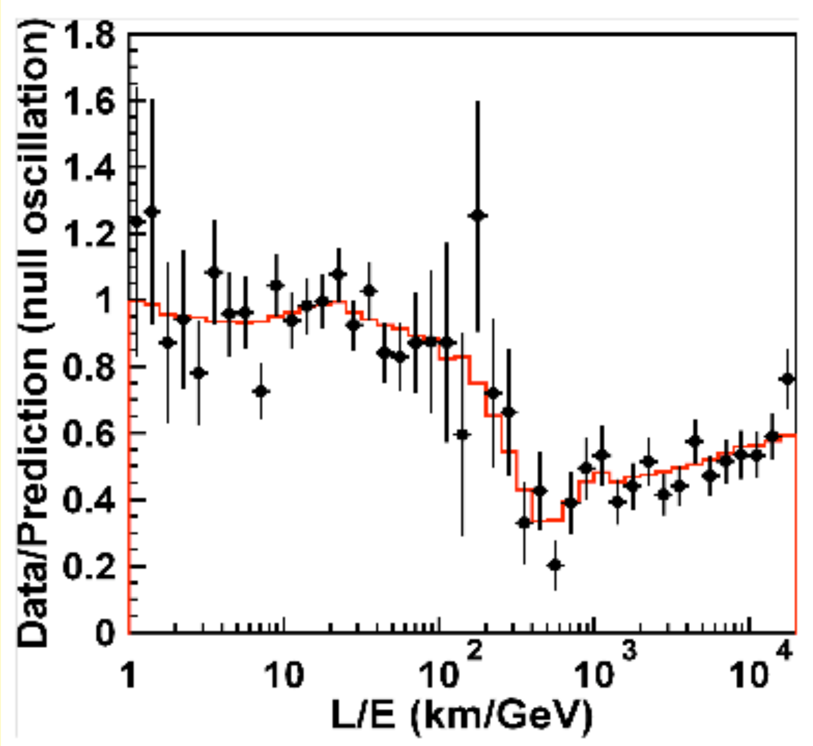
$$P_{\mu\tau} = \sin^2(2\theta)\,\sin^2(\Delta m^2 L/4E_\nu)$$

[In this channel, oscillations are ~vacuum-like,
despite the presence of Earth matter]

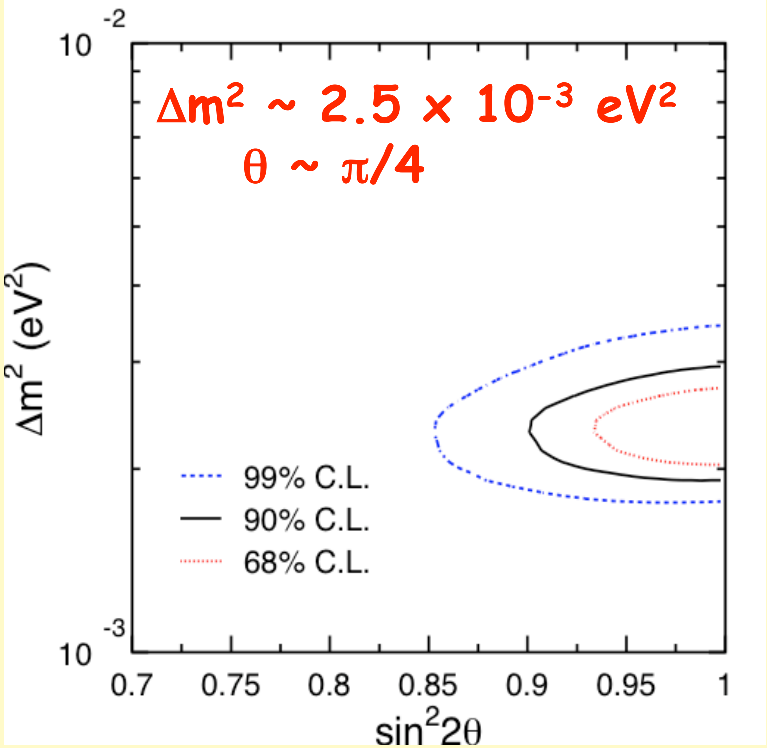
… but where are the “oscillations” ?

Dedicated L/E analysis to "see" half-period of oscillations

1st oscillation dip still visible despite large L & E smearing



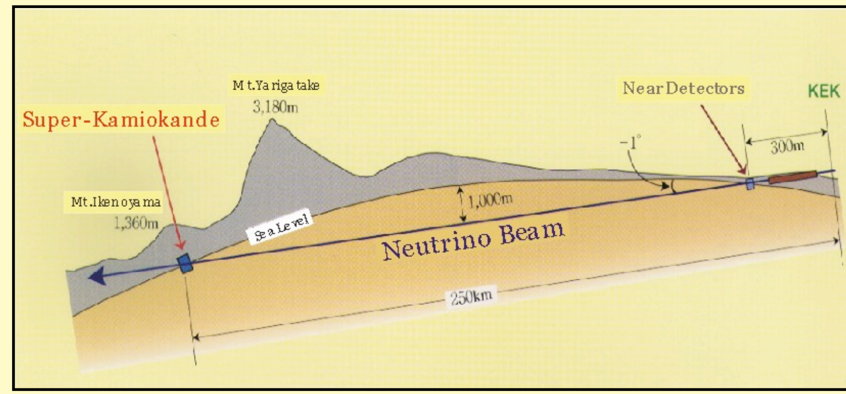
Strong constraints on the parameters (Δm^2, θ)

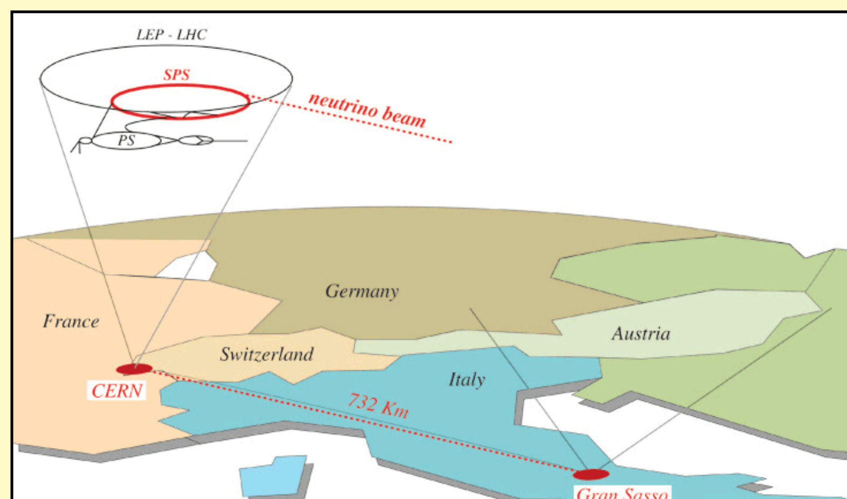


Same mass/mixing parameters confirmed in disappearance mode ($\nu_\mu \rightarrow \nu_\mu$) by other atmospheric expts (MACRO, Soudan2) and by long-baseline expts with controlled source (accelerator beams)

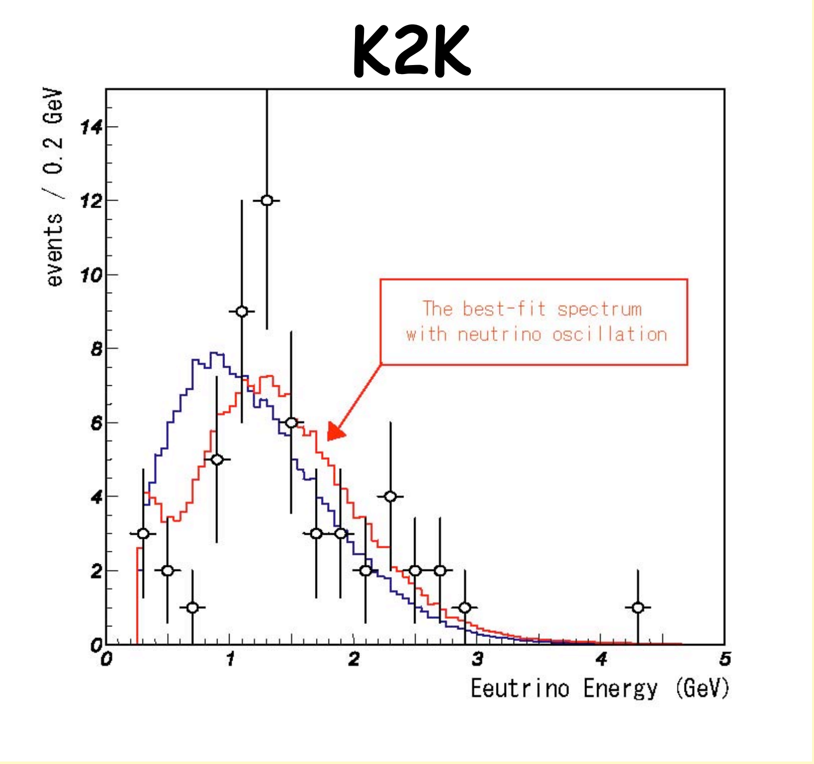
Long-baseline neutrino experiments (K2K, MINOS, CNGS)

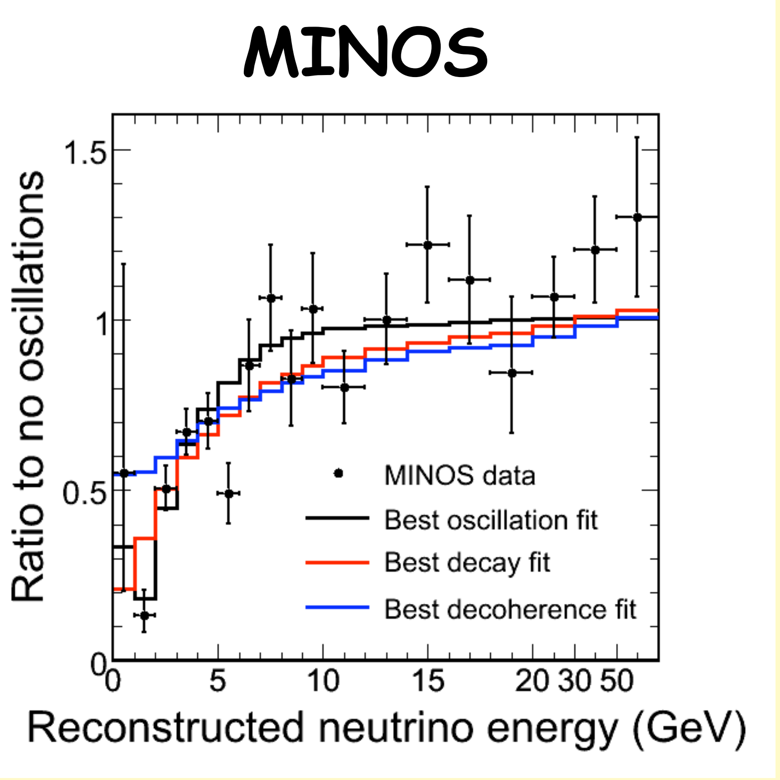
"Reproducing atmospheric ν_μ physics" in controlled conditions





Accelerator Results (muon disappearance mode)

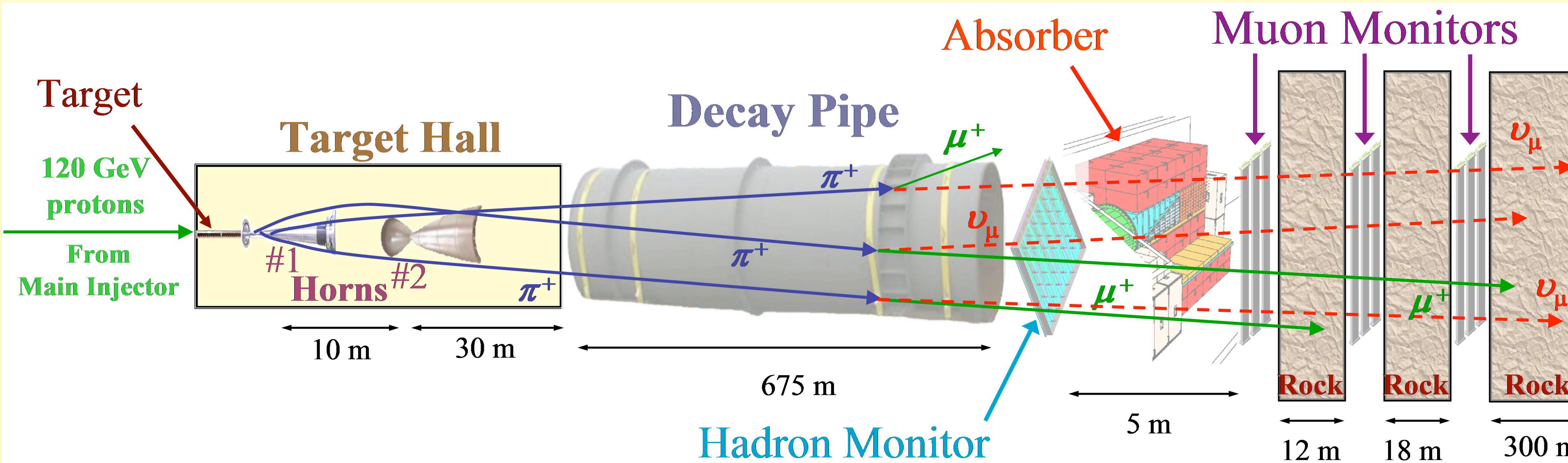




1^{st} oscillation dip also observed.

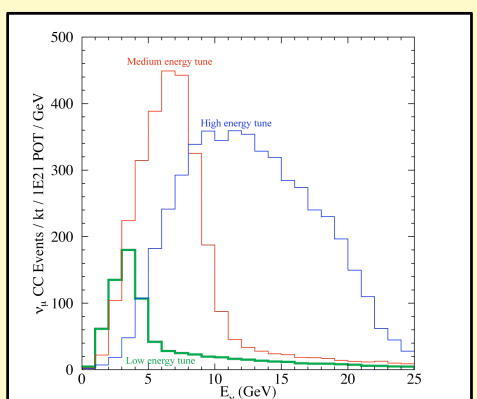
[Exotic explanations without dip (decay, decoherence) disfavored]

Production (e.g., MINOS)

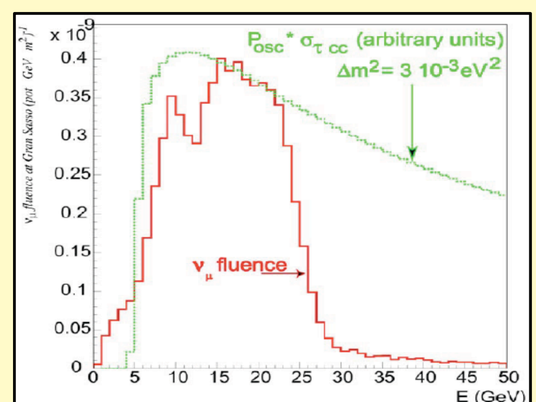


π decay: ν energy is only function of $\nu\pi$ angle and π energy

Spectra:



MINOS



OPERA

Open questions for Δm^2-driven ν_μ oscillations:

The quest for hierarchy and octant: Is the sign of Δm^2 positive ("normal hierarchy") or negative ("inverted hierarchy")? Is θ > or < $\pi/4$?

The quest for ν_τ appearance: We expect dominant $\nu_\mu \rightarrow \nu_\tau$ transitions, but haven't seen the τ flavor directly – the hunt is going on with the CNGS beam (1 candidate so far). See talk by F. Terranova, A. Guglielmi

The quest for ν_e appearance: We haven't seen $\nu_\mu \rightarrow \nu_e$ transitions; are they absent or just suppressed? This is a crucial problem for its implications on leptonic CP violation. See talk by M. Mezzetto

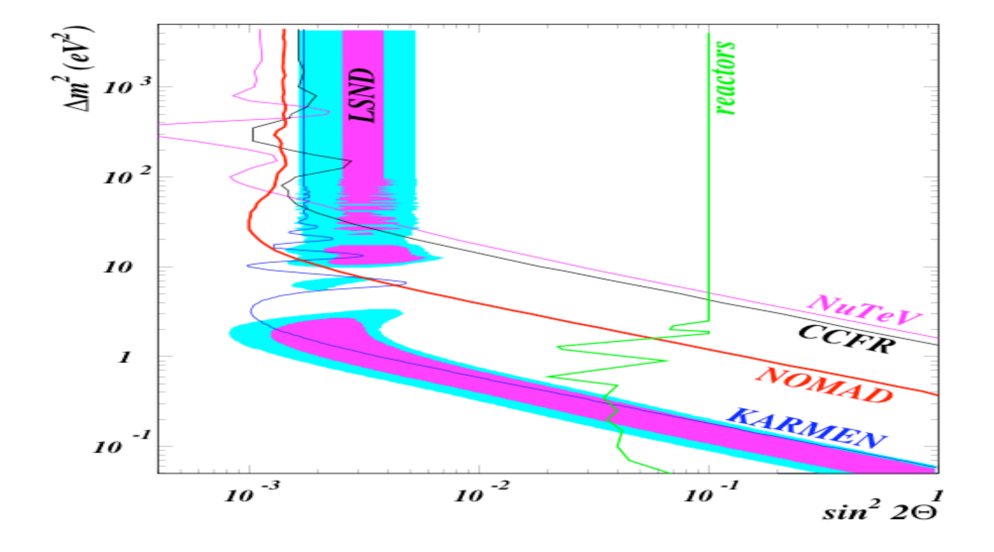
The quest for sterile neutrinos: Besides the known neutrinos $\nu_{e\mu\tau,L}$ (LH, gauge doublets) there might be new "sterile" states $\nu_{s,R}$ (RH, gauge singlets) leading to further disappearance $\nu_{\mu L} \rightarrow (\nu_{s,R})^c$
See talk by C. Giunti

Useful to rephrase some of these questions in 3ν language (tomorrow)

Short baseline accelerator expts: Beyond 3 neutrinos?

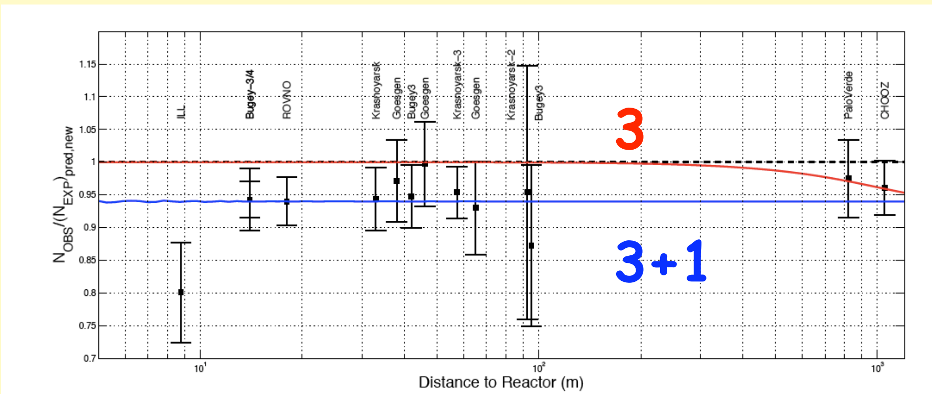
In principle, the thee flavor states may be mixed with N>3 neutrino states, in which case there must be N-3 "sterile" states. One than talks of 3+1, 3+2, 3+x models in current jargon. The 3x3 mixing matrix U becomes a submatrix of a NxN matrix, with "leaks" to sterile ν mixing (expected to be small).

Long ago, the **LSND** experiment found a signal of possible $\nu_\mu \rightarrow \nu_e$ oscillations at (preferentially) small mixing and relatively high ΔM^2 scale of O(0.1-1) eV^2



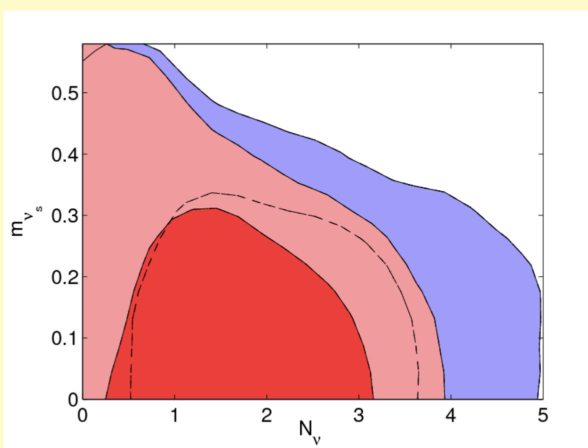
At least 3+1 model needed. Large literature on attempts to reconcile LSND with other data, by using new (sterile) states and/or new interactions. **But: No compelling data nor convincing model emerged so far.**

Situation about sterile neutrino somewhat confusing, but also exciting, since other "hints" in favor of extra (sterile) neutrinos have appeared recently, although still at low (~2σ) confid. level:



"Reactor neutrino anomaly"

Reanalysis of old data/fluxes: Electron flavor disappearance at very small L/E (= high ΔM^2) **?**
[Talk by C. Giunti]



"Extra radiation"

Room for 1-2 extra relativistic dof from precision cosmology; sub-eV sterile neutrino(s) **?**
[Talk by A. Melchiorri]

In the following, I shall not further consider sterile neutrinos, and will focus on active ones and their oscillations.

So far:

2ν oscillations in vacuum

$$P(\nu_\alpha \to \nu_\beta) = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$

To conclude:

3ν oscillations in vacuum

- For the 3 masses, let's assume for the moment a single dominant splitting:

$$m_1 \simeq m_2 \quad \text{and} \quad \Delta m^2 = |m_3^2 - m_{1,2}^2|$$

which is a reasonable approx. for all experiments where $\Delta m^2 L/4E \sim O(1)$ namely, atmospheric, long-baseline accelerator, short-baseline reactor expts.

Then, the vacuum oscillation probabilities are generalized as (2ν → 3ν):

$$P_{\alpha\beta} \simeq \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \longrightarrow P_{\alpha\beta} \simeq 4|U_{\alpha 3}|^2|U_{\beta 3}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

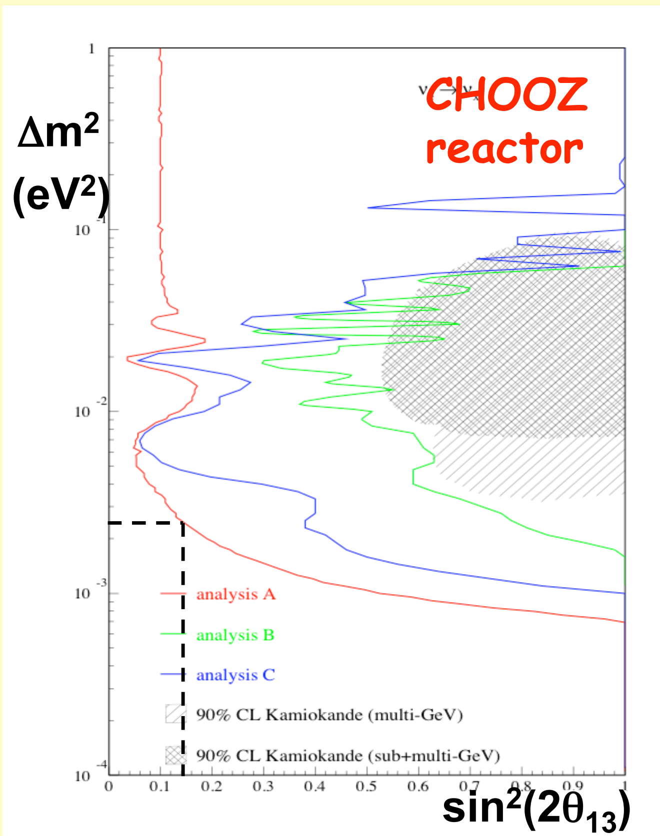
$$P_{\alpha\alpha} \simeq 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \longrightarrow P_{\alpha\alpha} \simeq 1 - 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

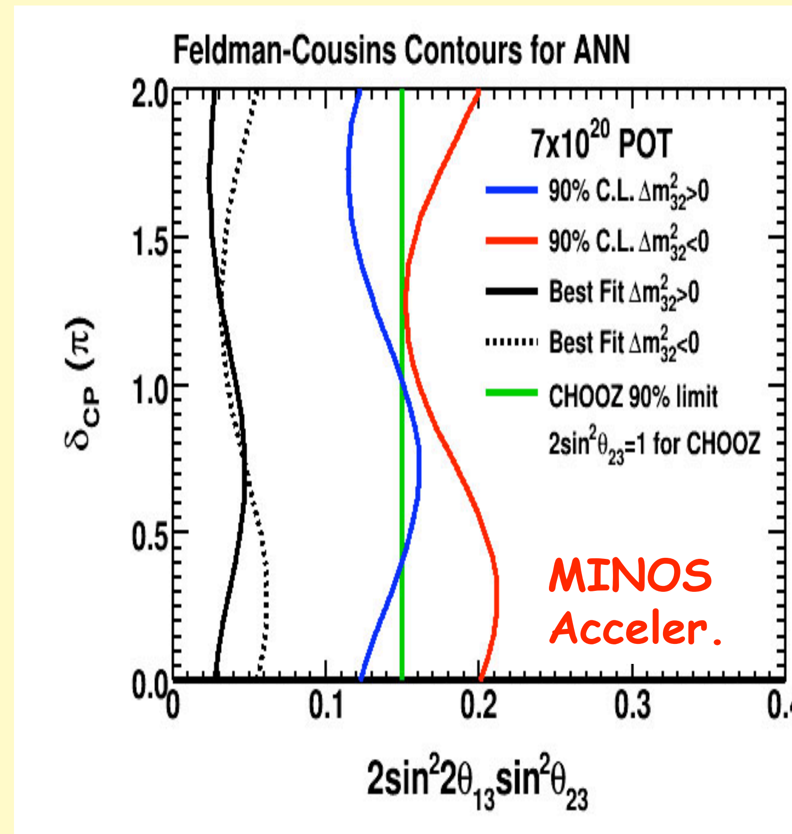
The amplitudes now differ in different oscillation channels, yet they do not depend on the hierarchy or the CP phase.
Also, they do not depend on θ_{12}, due to the assumed degeneracy $m_1 \approx m_2$

In such notation, the previous "$\nu_\mu \rightarrow \nu_\tau$" mixing angle is $\theta_{23} \sim \pi/4$, while θ_{13} modulates the oscillation amplitude in the $\nu_e \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_e$ channels where, unfortunately, no signal has been found so far…

$$P_{ee} = 1-\sin^2(2\theta_{13})\sin^2(\Delta m^2 L/4E_\nu)$$

$$P_{\mu e}=\sin^2\theta_{23}\sin^2(2\theta_{13})\sin^2(\Delta m^2 L/4E_\nu)$$

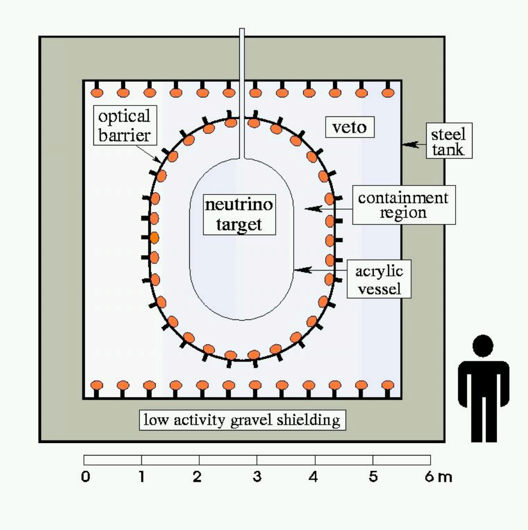




World data consistent with $\sin^2\theta_{13}$ < few %.

More about the short-baseline reactor experiment CHOOZ

~1 km →

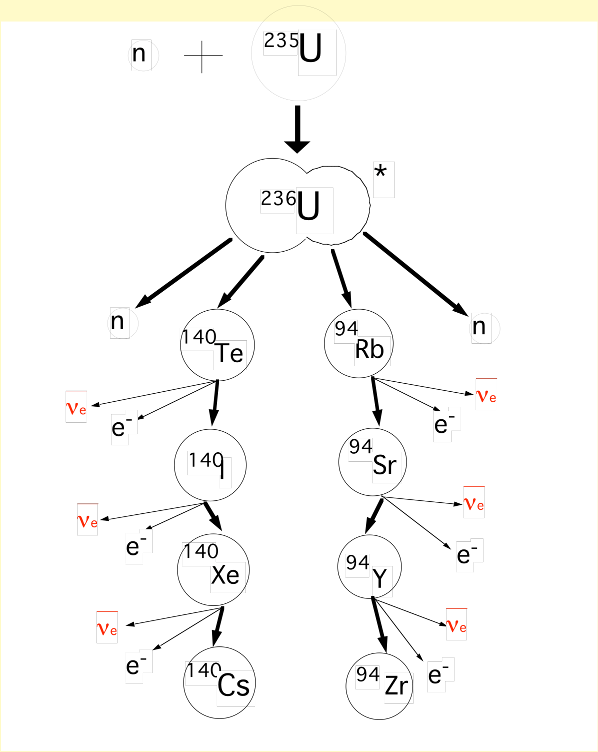


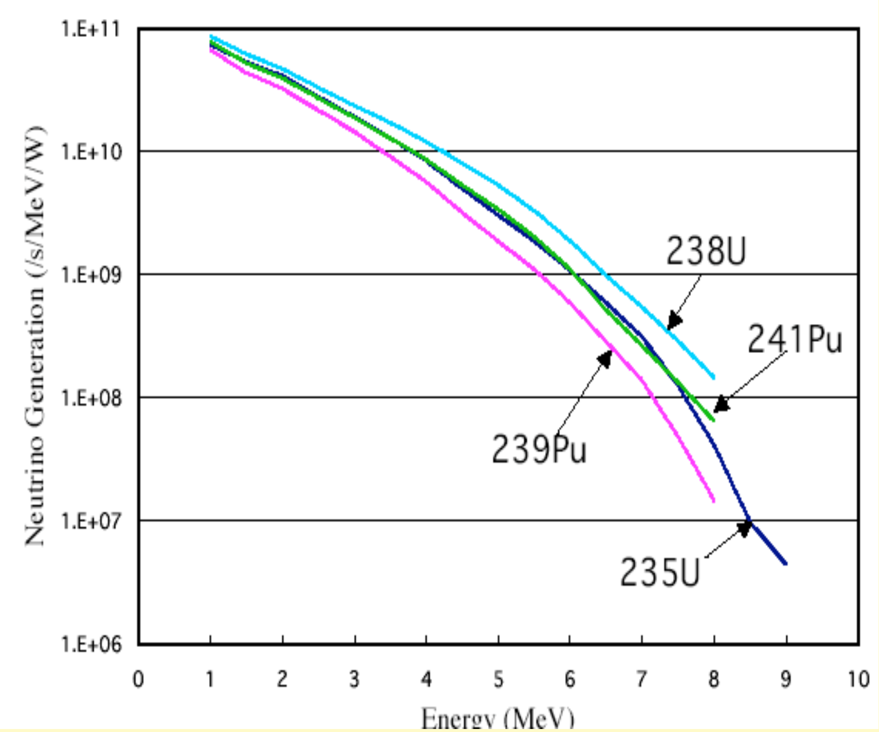
Production

Reactors: Intense sources of anti-ν_e (~6×10^{20}/s/reactor)

Typically, 6 neutron decays to reach stable matter from fission:

~200 MeV per fission / 6 decays: Typical available neutrino energy is E~ few MeV



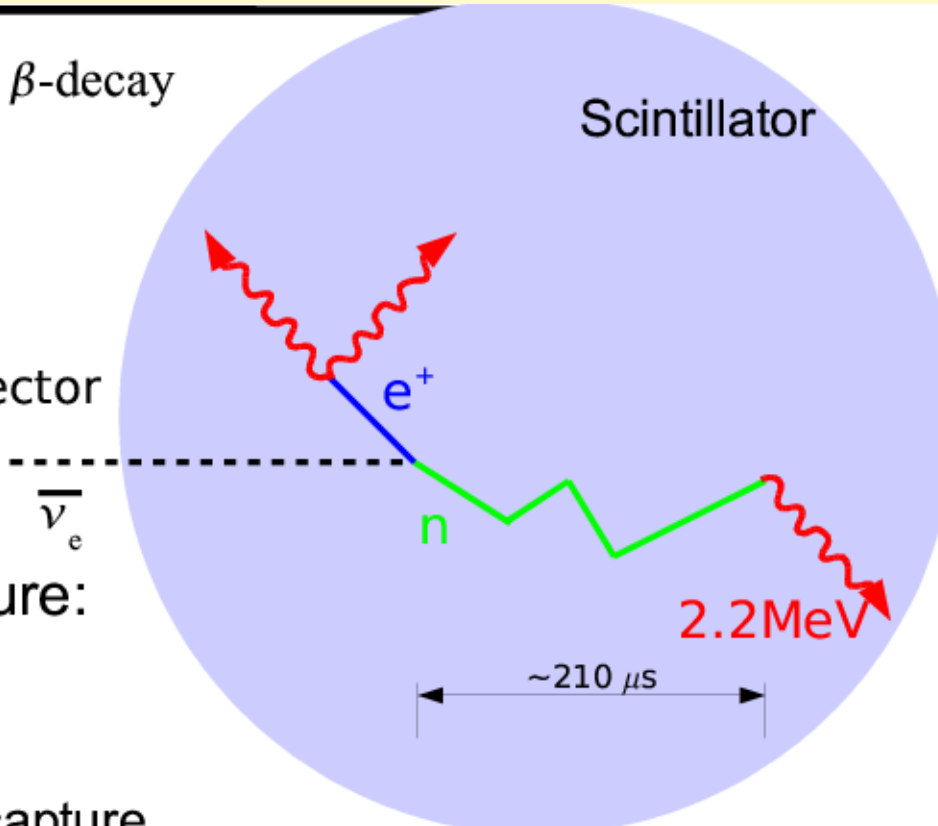


Detection

Reaction Process: inverse β-decay

$$\overline{\nu_e} + p \rightarrow e^+ + n$$
$$n + p \rightarrow d + \gamma$$

Scintillator is target and detector

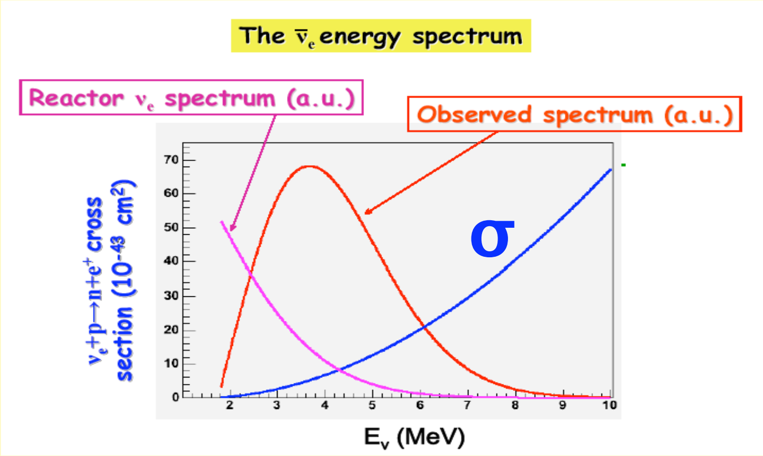


- Distinct two-step signature:
  - prompt event: positron
    $E_\nu \approx E_{e^+} + 0.8\ MeV$
  - delayed event: neutron capture after ~210μs
    - 2.2 MeV gamma

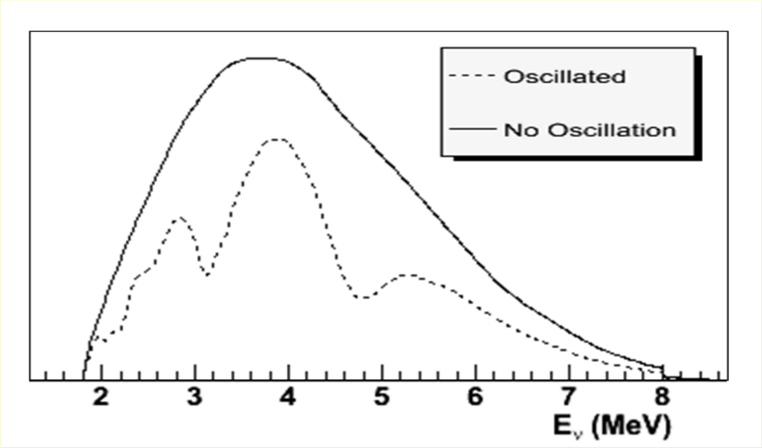
Delayed coincidence: good background rejection

Results

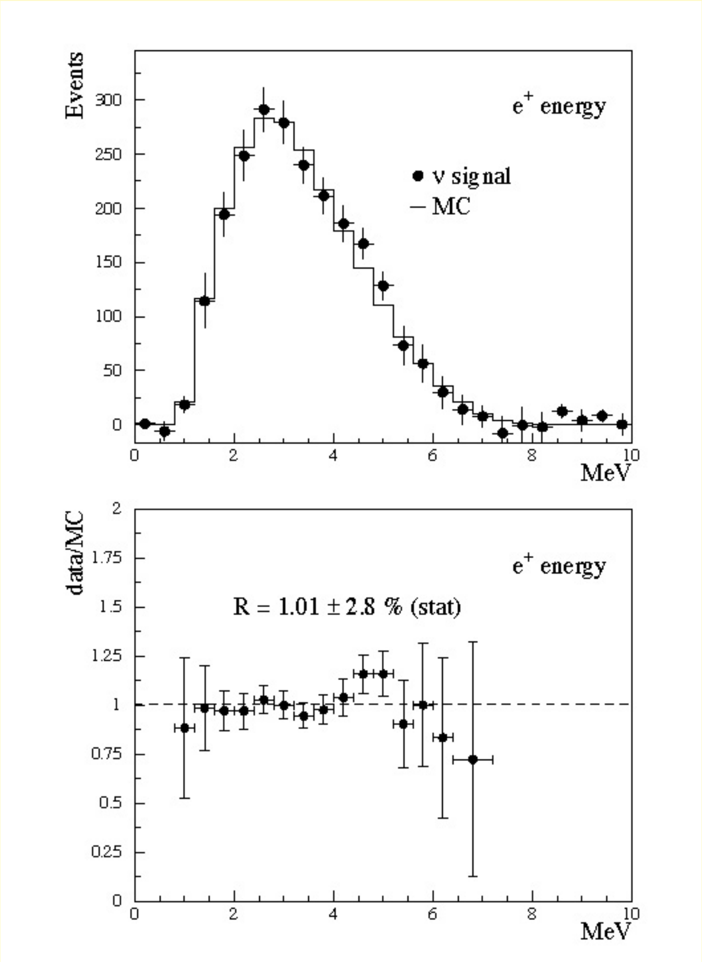
Expected spectrum (no oscill.):



With oscillations (qualitative):



CHOOZ: no oscillations within few % error



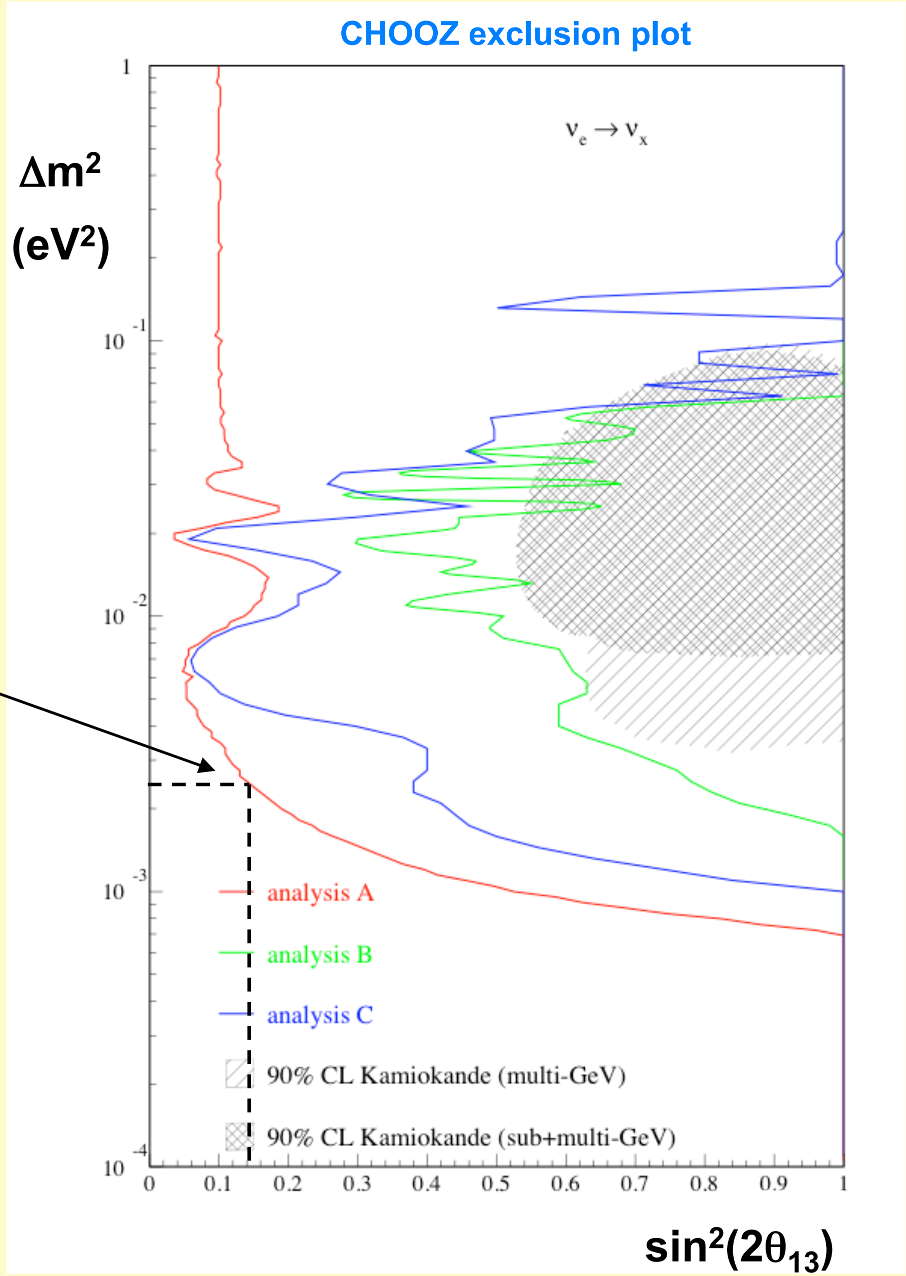
Interpretation

One mass scale dominance:
$P_{ee} = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L/4E_\nu)$

For any value of Δm^2 in the range allowed by atmospheric data, get stringent upper bound on θ_{13}

$\sin^2 \theta_{13} < \text{few } \%$

Feverish world-wide activity to build new reactor experiment with higher θ_{13} sensitivity → need to use a second (close) detector to reduce systematics



3ν, 2nd step: two mass splittings

We have seen that atmospheric (and long-baseline accelerator) experiments have established the mass splitting of ν_3 with respect to $\nu_{1,2}$, with oscillation parameters:

$$\Delta m^2 = |m_3^2 - m_{1,2}^2| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$
$$\sin^2\theta_{23} \simeq 0.5$$

We shall see tomorrow that solar and long-baseline reactors, sensitive to much larger L/E, have established the splitting between ν_1 and ν_2 with oscillation parameters:

$$\delta m^2 = m_2^2 - m_1^2 \simeq 8 \times 10^{-5}\ \mathrm{eV}^2$$
$$\sin^2\theta_{12} \simeq 0.3$$

This opens the door to leptonic CP violation, iff $\theta_{13}>0$!

In a full 3ν scenario, a CP violating difference may arise between neutrino and antineutrino oscillation probabilities,

$$P_{\alpha\beta}(\nu) - P_{\alpha\beta}(\bar{\nu}) = 2\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$
$$\times \sin\left(\frac{\Delta m^2 - \frac{\delta m^2}{2}}{4E} L\right) \sin\left(\frac{\Delta m^2 + \frac{\delta m^2}{2}}{4E} L\right) \sin\left(\frac{\delta m^2}{4E} L\right)$$

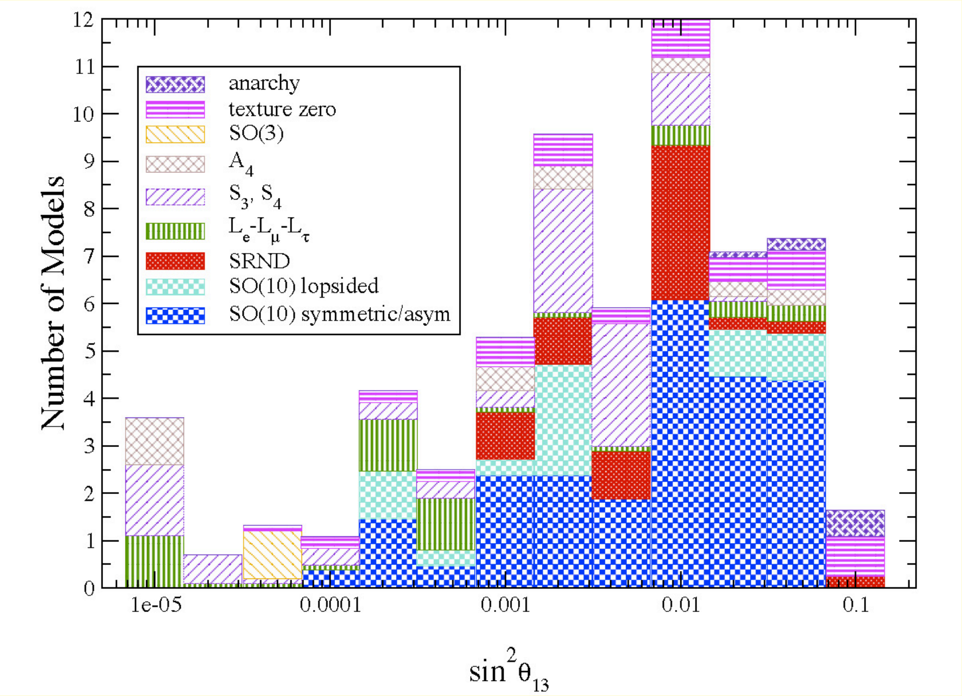
provided that:

- $\sin 2\theta_{13}$ is nonzero
- $\sin\delta$ is nonzero
- the oscillation phases are neither too small nor too large

Hunt for θ_{13} crucial in current neutrino research, in order to plan future CP-violation searches!

[see talk by M. Mezzetto]

Also: θ_{13} important to restrict theoretical models for ν masses



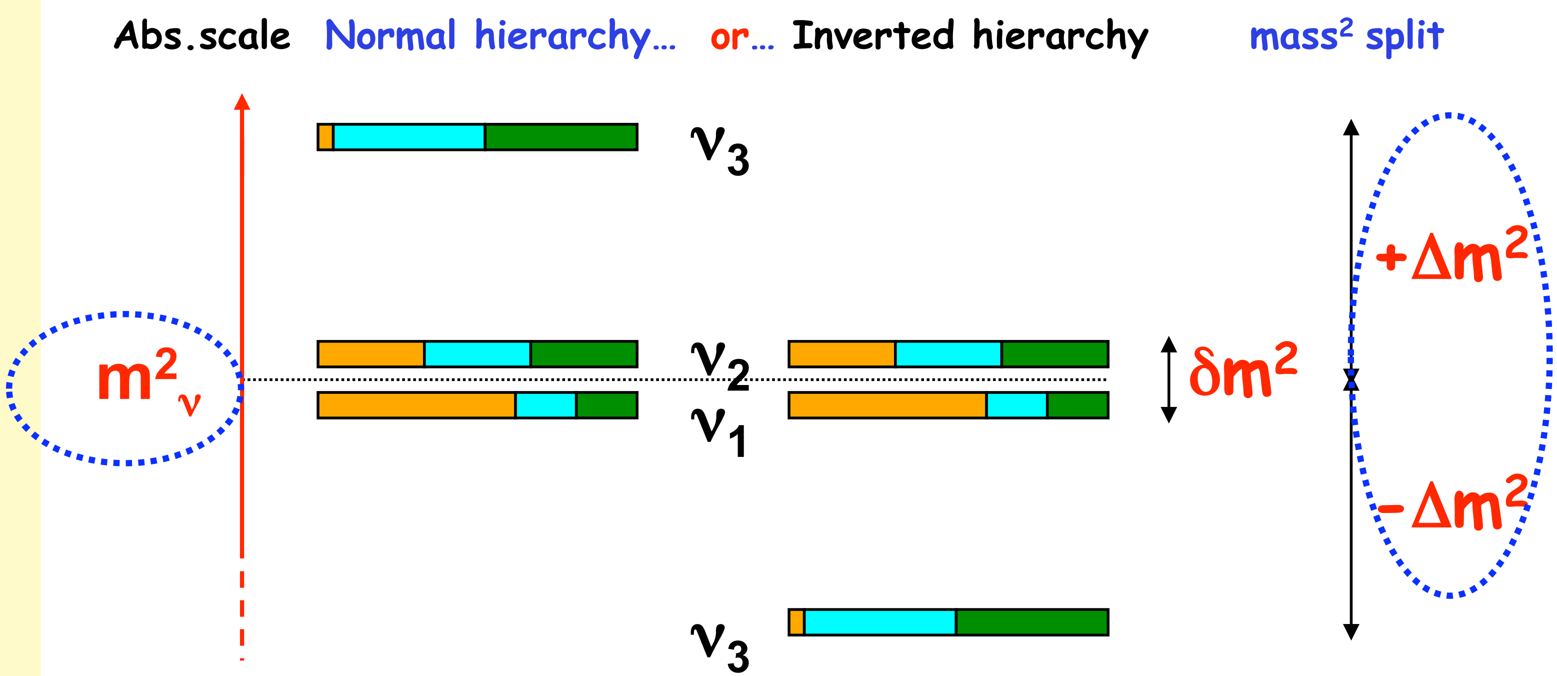
E.g.: CH Albright, 2008, "distribution" of published predictions

RECAP and end of LECTURE I

Poster of the Neutrino Oscillation Workshop 2004 (NOW 2004, Otranto, Italy)

3ν mass-mixing overview

(here, with 1 digit accuracy). Flavors = e μ τ



$$\delta m^2 \sim 8 \times 10^{-5}\ \mathrm{eV}^2 \qquad \sin^2\theta_{12} \sim 0.3$$

$$\Delta m^2 \sim 3 \times 10^{-3}\ \mathrm{eV}^2 \qquad \sin^2\theta_{23} \sim 0.5$$

$$m_\nu < O(1)\ \mathrm{eV} \qquad \sin^2\theta_{13} < \mathrm{few}\%$$

$$\mathrm{sign}(\pm\Delta m^2)\ \mathrm{unknown} \qquad \delta\ (\mathrm{CP})\ \mathrm{unknown}$$

[...+sterile? Giunti]

Recap of absolute mass observables (m_β, $m_{\beta\beta}$, Σ)

1) β decay: $m^2_i \neq 0$ can affect spectrum endpoint. Sensitive to the "effective electron neutrino mass":

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

2) $0\nu\beta\beta$ decay: Can occur iff Majorana (not Dirac)! Sensitive to the "effective Majorana mass" (and phases):

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

[Cremonesi]

3) Cosmology: $m^2_i \neq 0$ can affect large scale structures in (standard) cosmology constrained by CMB + other data. Sensitive to:

$$\Sigma = m_1 + m_2 + m_3$$

[Melchiorri]

THANK YOU FOR YOUR ATTENTION

HOMEWORK

Solution 1

- If ν's are Dirac, then $\nu_e \neq \bar{\nu}_e$, and one can attach a leptonic number to the doublets (ν_e, e^-) and $(\bar{\nu}_e, e^+)$, which is conserved in the observed reactions $(\Delta L = 0)$ and would be violated in the other two $(\Delta L = 2)$.

- If ν's are Majorana, then $\nu_e = \bar{\nu}_e$, and we are just naming:

  "ν_e" = LH component of ν state

  "$\bar{\nu}_e$" = RH component of ν state

  The initial "ν_e" is LH, being produced in a weak (β^+) decay. While propagating, it remains dominantly LH, but can develop a small RH component ("$\bar{\nu}_e$") at $\mathcal{O}(m/E)$. Then also the reaction $\bar{\nu}_e + n \to p + e^-$ can take place in principle, but is so suppressed to be practically unobservable. Lepton number violation $(\Delta L = 2)$ is allowed in principle, but suppressed at $\mathcal{O}(m/E)$ in practice.

Solution 2

- Mass basis $\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$ and flavor basis $\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$ are related by:

$$\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \qquad \text{with } \Delta m^2 = m_2^2 - m_1^2$$

- Evolution equation in mass basis (mb):

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \mathcal{H}_{mb}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \qquad \leftarrow \text{Schrödinger eq. in natural units } (\hbar = c = 1)$$

where the Hamiltonian is simply

$$\mathcal{H}_{mb} = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix} \simeq \begin{pmatrix}p+\frac{m_1^2}{2E} & 0\\ 0 & p+\frac{m_2^2}{2E}\end{pmatrix} = \underbrace{\left(p+\frac{m_1^2+m_2^2}{4E}\right)\mathbb{1}}_{\propto\,\mathbb{1}} + \underbrace{\begin{pmatrix}-\frac{\Delta m^2}{4E} & 0\\ 0 & +\frac{\Delta m^2}{4E}\end{pmatrix}}_{\text{Traceless}}$$

Final results do not depend on the part proportional to $\mathbb{1}$ – check it.
(Reason: it gives an overall phase which disappears in observable real quantities). So we take:

$$\mathcal{H}_{mb} = \frac{\Delta m^2}{4E}\begin{pmatrix}-1 & 0\\ 0 & +1\end{pmatrix}$$

Solution 2 (ctd)

- Evolution operator in mass basis:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_t = S_{mb} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_0 \quad \text{where}$$

$$S_{mb} = e^{-i\mathcal{H}_{mb} t} \simeq e^{-i\mathcal{H}_{mb} x} = \begin{pmatrix} e^{i\frac{\Delta m^2}{4E}x} & 0 \\ 0 & e^{-i\frac{\Delta m^2}{4E}x} \end{pmatrix}$$

← $x \simeq t$ for ultrarelativistic neutrinos

- Evolution operator in flavor basis (fb):

$$S_{fb} = U\, S_{mb}\, U^T$$

$$= \cos\left(\frac{\Delta m^2 x}{4E}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\sin\left(\frac{\Delta m^2 x}{4E}\right)\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- Amplitudes for flavor transitions:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_t = S_{fb} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_0 \quad \rightarrow$$

Off-diagonal elements of S_{fb} give amplitudes for $\nu_\alpha \to \nu_\beta$ and $\nu_\beta \to \nu_\alpha$

Solution 2 (ctd)

- Probability of flavor transition is the square modulus of the amplitude:

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha) = \left| -i \sin 2\theta \sin\left(\frac{\Delta m^2 x}{4E}\right)\right|^2$$

$$= \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

($x = L$ in the lecture).

- The diagonal elements of the evolution operator would give the "flavor survival" amplitude. Check that

$$1 - P(\nu_\alpha \to \nu_\beta) = P(\nu_\alpha \to \nu_\alpha) = P(\nu_\beta \to \nu_\beta).$$

$$\rightarrow \quad P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

Solution 3

Oscillations depend only on the difference of phases, and thus of neutrino energies. Indeed, the results do not change by an overall shift of the Hamiltonian:

$$H \to H + \text{const} \cdot \mathbb{1}$$

Since the zero-point energy is irrelevant in this context, the absolute neutrino mass scale is unobservable (in oscillation searches).

Solution 4

$\hbar c = 197.327 \text{ MeV} \cdot \text{fm} = 1$ in natural units.

Therefore: $1 \text{ MeV} \cdot 1 \text{ m} = 5.0677 \times 10^{12}$

Then:
$$\frac{\Delta m^2 L}{4E} = \frac{1}{4}\left(\frac{\Delta m^2}{eV^2}\, eV^2\right)\left(\frac{L}{m}\cdot m\right)\left(\frac{MeV}{E}\cdot\frac{1}{MeV}\right)$$
$$= \frac{1}{4}\left(\frac{1 eV^2 \cdot 1m}{1 MeV}\right)\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right)$$

$$\frac{1}{4}\frac{eV^2 m}{MeV} = \frac{1}{4}\times 10^{-12}\,\frac{MeV^{\cancel{2}}\cdot 1m}{1\,\cancel{MeV}} = \frac{10^{-12}}{4}(MeV\cdot m) = 0.25\times 10^{-12}\times 5.0677\times 10^{12} = 1.267$$

$$\frac{\Delta m^2 L}{4E} = 1.267\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{m}\right)\left(\frac{MeV}{E}\right) = 1.267\left(\frac{\Delta m^2}{eV^2}\right)\left(\frac{L}{km}\right)\left(\frac{GeV}{E}\right)$$

Appendix on Majorana mass terms

Dirac and Majorana mass terms (1 family)

- Dirac mass terms are of the form $m\bar{\psi}\psi$ (4 dof ψ)
- Majorana " " " " " $\frac{1}{2}m\bar{\psi}\psi$ (2 dof ψ)

Three possibilities:

Dirac : $\psi = \psi_L + \psi_R \rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$

Majorana (L) : $\psi = \psi_L + \psi_L^c \rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_L^c + \bar{\psi}_L^c\psi_L$

Majorana (R) : $\psi = \psi_R + \psi_R^c \rightarrow \bar{\psi}\psi = \bar{\psi}_R\psi_R^c + \bar{\psi}_R^c\psi_R$

Most general mass term for <u>one</u> neutrino family:

$$m_D(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) + \frac{1}{2}m_L(\bar{\psi}_L\psi_L^c + \bar{\psi}_L^c\psi_L) + \frac{1}{2}m_R(\bar{\psi}_R\psi_R^c + \bar{\psi}_R^c\psi_R)$$

[Last two terms absent for charged fermions.]

- Previous mass term can be rewritten as:

$$\frac{1}{2}\left[\overline{\psi}_L + \overline{\psi}_L^c,\ \overline{\psi}_R + \overline{\psi}_R^c\right]\begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}\begin{bmatrix} \psi_L + \psi_L^c \\ \psi_R + \psi_R^c \end{bmatrix}$$

- Diagonalization provides fields with definite masses. (If mass < 0, redefine field $\psi \to \gamma_5 \psi$ so that $m \to -m$)

- Since the basis fields $(\overset{(-)}{\psi}_L + \overset{(-)}{\psi}{}_L^c)$ and $(\overset{(-)}{\psi}_R + \overset{(-)}{\psi}{}_R^c)$ are Majorana, diagonalization will generally produce mass eigenvectors which are also Majorana

Diagonalize $M = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix}$

$$M = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \qquad \begin{aligned} T &= \mathrm{Tr}\, M = m_L + m_R \\ D &= \det M = m_L m_R - m_D^2 \end{aligned}$$

Eigenvalues : $m_\pm = \frac{1}{2}\left(T \pm \sqrt{T^2 - 4D}\right)$

Diagonalization angle : $\sin 2\theta = \dfrac{m_D}{\sqrt{T^2-4D}}$ $\qquad \cos 2\theta = \dfrac{m_L - m_R}{\sqrt{T^2-4D}}$
(not a mixing angle!)

$$\begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} = \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix}$$

Eigenvectors

$$\begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1' & v_2' \end{bmatrix} \begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The see-saw mechanism

Many extensions of the Standard Model predict the existence of singlet neutrinos (ν_R).

E.g., in the $\underline{16}$ representation of SO(10):

$$\begin{pmatrix} u_L & u_L & u_L & \nu_L \\ d_L & d_L & d_L & e_L \\ u_R & u_R & u_R & \boldsymbol{\nu_R} \\ d_R & d_R & d_R & e_R \end{pmatrix}$$

→ can get a Majorana mass term $\sim m_R(\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c)$, where m_R is presumably a large mass scale characterizing the SM extension.

For $m \ll M$, diagonalization of $\begin{bmatrix} 0 & m \\ m & M \end{bmatrix}$ gives:

Eigenvectors (ν fields)	Eigenvalues (masses)
$\nu_{heavy} \cong (\nu_R + \nu_R^c) + \frac{m}{M}(\nu_L + \nu_L^c)$	M
$\nu_{light} \cong -(\nu_L + \nu_L^c) + \frac{m}{M}(\nu_R + \nu_R^c)$	$(-)\frac{m^2}{M} \ll m$

← see-saw

The light state is active (contains ν_L) and has a very small mass $\sim m^2/M$

Presumably: $m \sim \mathcal{O}(m_{quarks}, m_{leptons})$

$M \sim \mathcal{O}(\Lambda_{beyond\ SM})$

The see-saw mechanism might explain the smallness of neutrino masses