Introduzione alle masse e ai mescolamenti dei neutrini (II)

Eligio Lisi
INFN, Bari, Italy

Outline:

Pedagogical Introduction
Neutrino masses and spinor fields
Neutrinoless double beta decay
2ν, 3ν… Nν vacuum oscillations
[Homework]

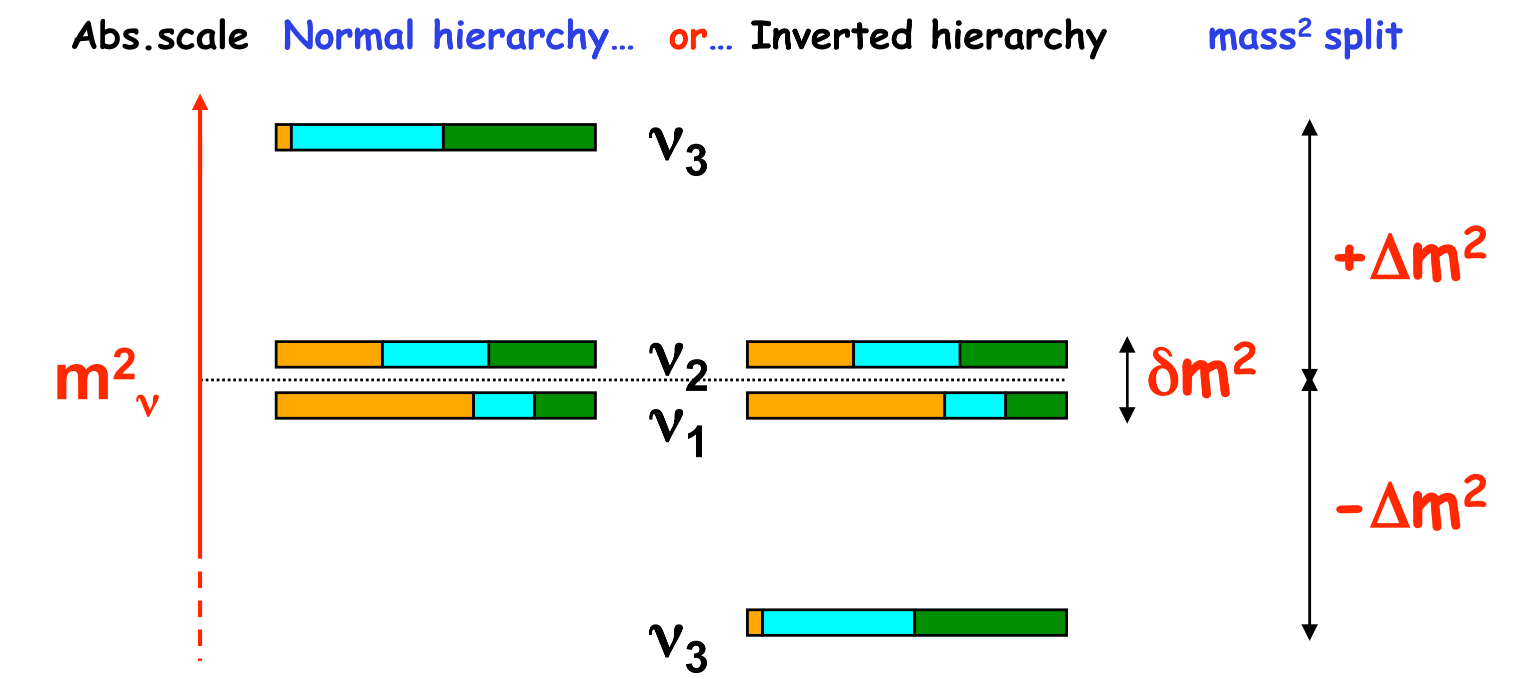
I

Recap
2ν oscillations in matter
Solar and KamLAND oscillations
Absolute neutrino masses
[Homework]

II

3ν mass-mixing overview in just one slide

(here, with 1 digit accuracy). Flavors = e μ τ



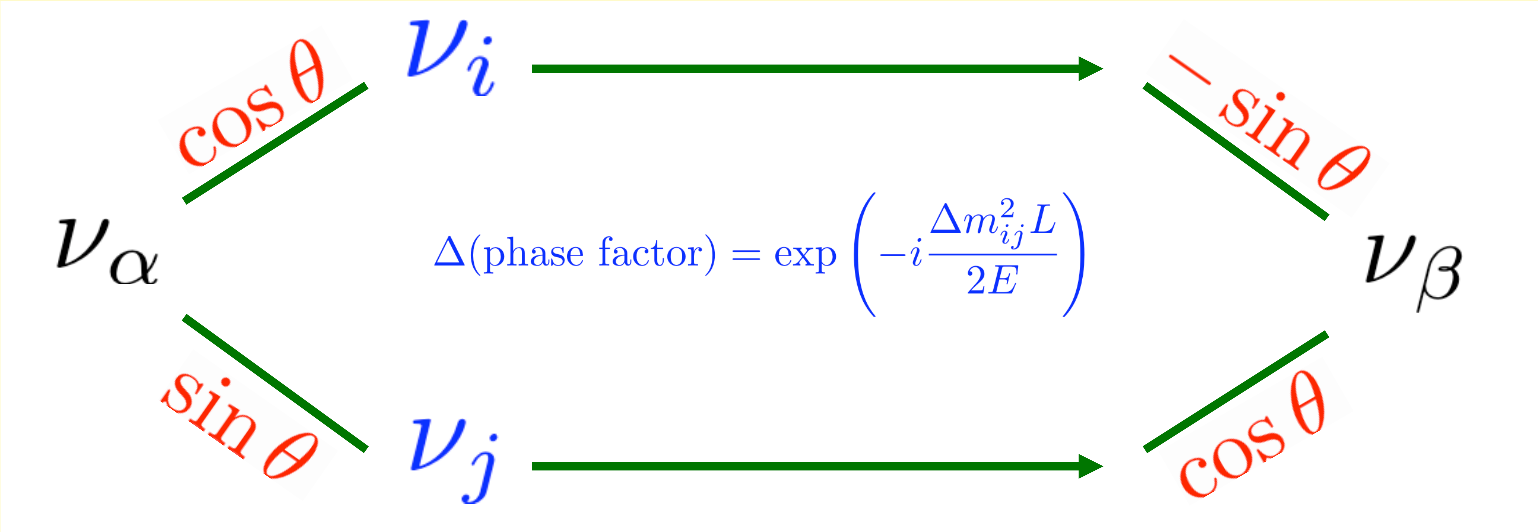
$$\delta m^2 \sim 8 \times 10^{-5}\ \mathrm{eV}^2 \qquad \sin^2\theta_{12} \sim 0.3$$

$$\Delta m^2 \sim 3 \times 10^{-3}\ \mathrm{eV}^2 \qquad \sin^2\theta_{23} \sim 0.5$$

$$m_\nu < O(1)\ \mathrm{eV} \qquad \sin^2\theta_{13} < \mathrm{few}\%$$

$$\mathrm{sign}(\pm\Delta m^2)\ \mathrm{unknown} \qquad \delta\ (\mathrm{CP})\ \mathrm{unknown}$$

Oscillations in vacuum: analogy with a two-slit experiment

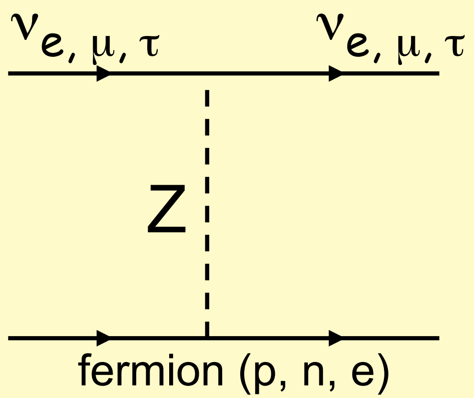


This is the simplest case (only 2 neutrinos involved, no interactions with matter). It shows that, if neutrinos are massive and mixed (like quarks), then flavor is not a good quantum number during propagation. Indeed, it changes ("oscillates") significantly over a distance L (≈Δt) dictated by the uncertainty relation:

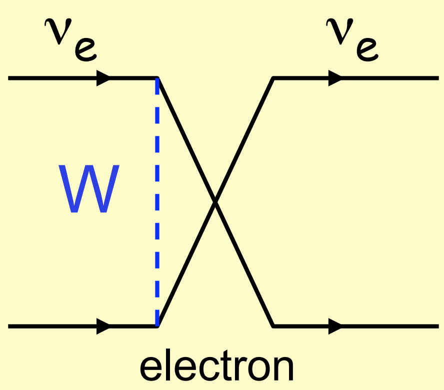
$$P(\nu_\alpha \to \nu_\beta) = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$

Neutrino flavor oscillations in matter

Neutrinos of all flavors ($\nu_{e,\ \mu,\ \tau}$) have the same amplitude for coherent forward scattering in matter via NC. However, only ν_e can further scatter via CC, since ordinary matter contains e, not μ or τ. This fact implies a difference in the relative propagation of ν_e versus $\nu_{\mu,\ \tau}$, (but not between ν_μ and ν_τ): the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

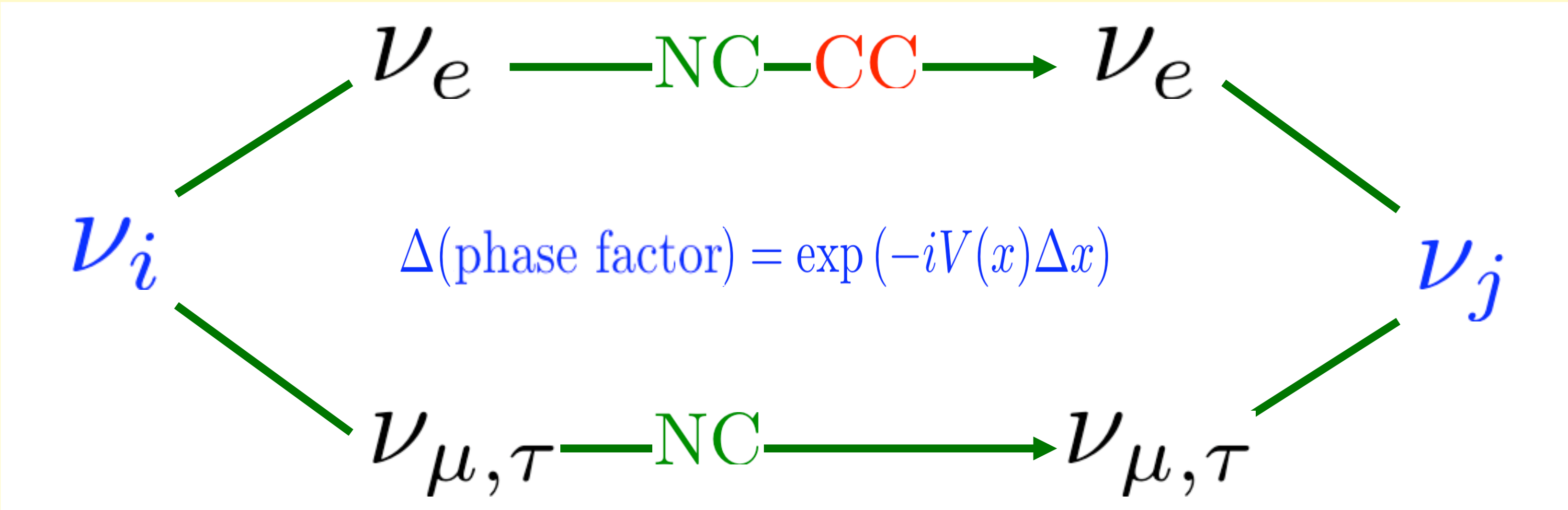


ν_μ & ν_τ (e.g., atmospheric) feel background fermions in the same way (through NC); no relative phase change while propagating (~ vacuum-like propagation, as anticipated)



But ν_e , in addition to NC, have CC interac. with background electrons (density N_e). Energy difference: $\mathbf{V = +\sqrt{2}\, G_F\, N_e}$ leads to a phase difference in matter

Again, analogy with the two-slit experiment:
one "arm" (flavor) feels a different "refraction index"



governed by the local (electron) density:

$$V(x) = V_e - V_{\mu,\tau} = \sqrt{2}\,G_F\,N_e(x) \quad [N_e = \text{electron density}]$$

(-V for antineutrinos)

Exercise 5. Prove that oscillations between ν_e and ν_x ($=\nu_\mu, \nu_\tau$) in matter with **constant** density lead to Pontecorvo's formula

$$P(\nu_e \to \nu_x) = \sin^2 2\tilde{\theta} \sin^2\left(\frac{\Delta\tilde{m}^2 L}{4E}\right)$$

with effective (tilde) parameters defined as

$$\frac{\Delta\tilde{m}^2}{\Delta m^2} = \frac{\sin 2\theta}{\sin 2\tilde{\theta}} = \sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + (\sin 2\theta)^2}$$

where $$A = 2VE = 2\sqrt{2}G_F N_e E$$

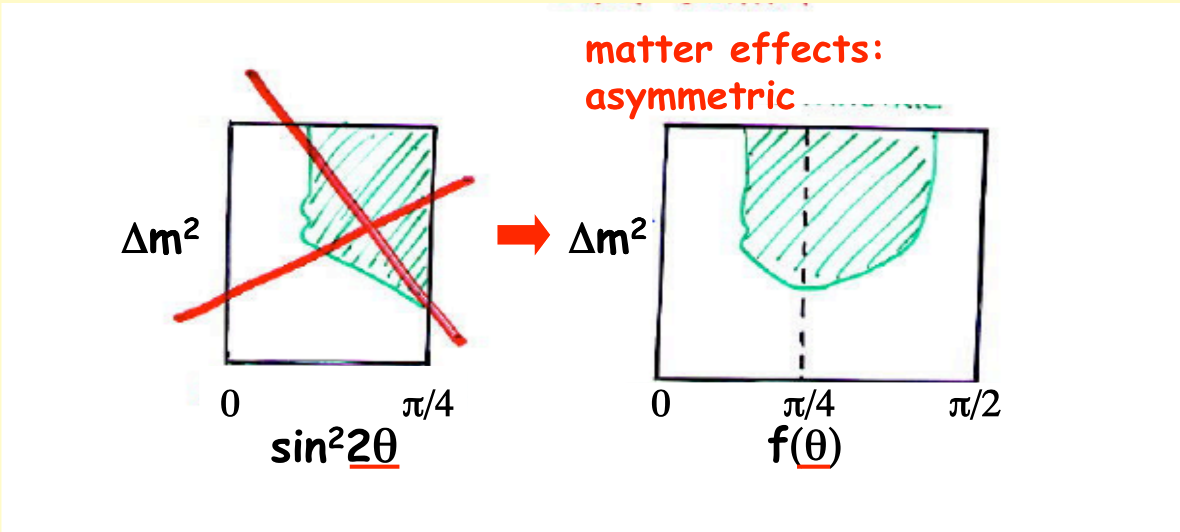
Exercise 6 (Conversion factors). Prove that

$$\frac{A}{\Delta m^2} = 1.526 \times 10^{-7}\left(\frac{N_e}{\mathrm{mol/cm^3}}\right)\left(\frac{E}{\mathrm{MeV}}\right)\left(\frac{\mathrm{eV}^2}{\Delta m^2}\right)$$

Rule of thumb (**~valid also for non-constant density**):

Expect strong matter effects when **$A/\Delta m^2 \sim O(1)$**.

Note: matter effects are octant-asymmetric;
need to unfold second octant.

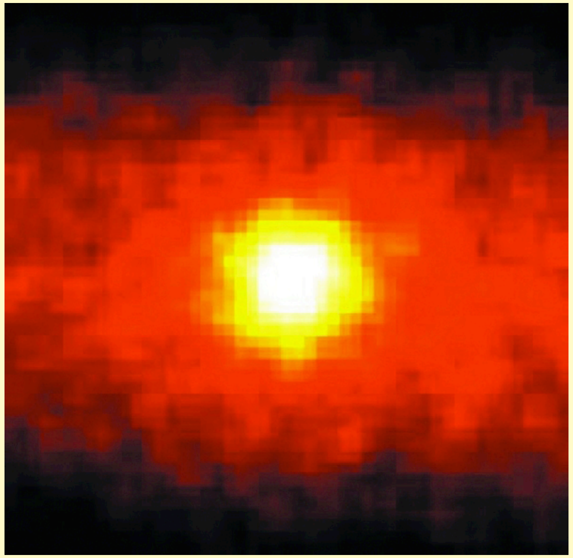


Asymmetry is particular pronounced for solar neutrinos, with mass-mixing parameters **(δm^2, θ_{12})**

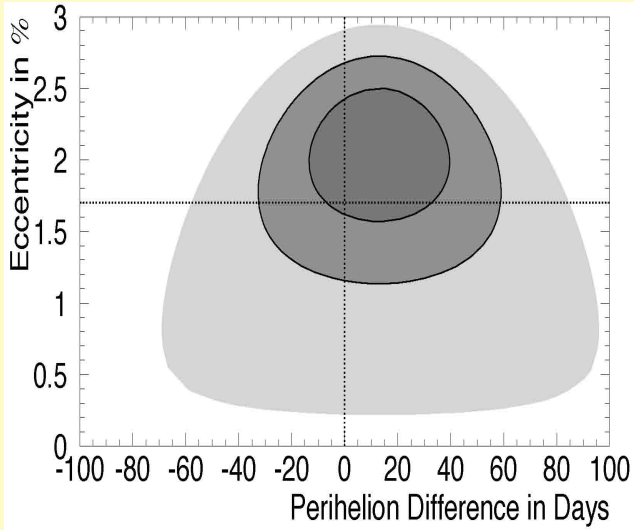
[N.B.: Effects also depend on sign of squared mass difference: Handle to hierarchy discrimination.]

Experiments sensitive to the "small" δm^2:

Solar neutrinos

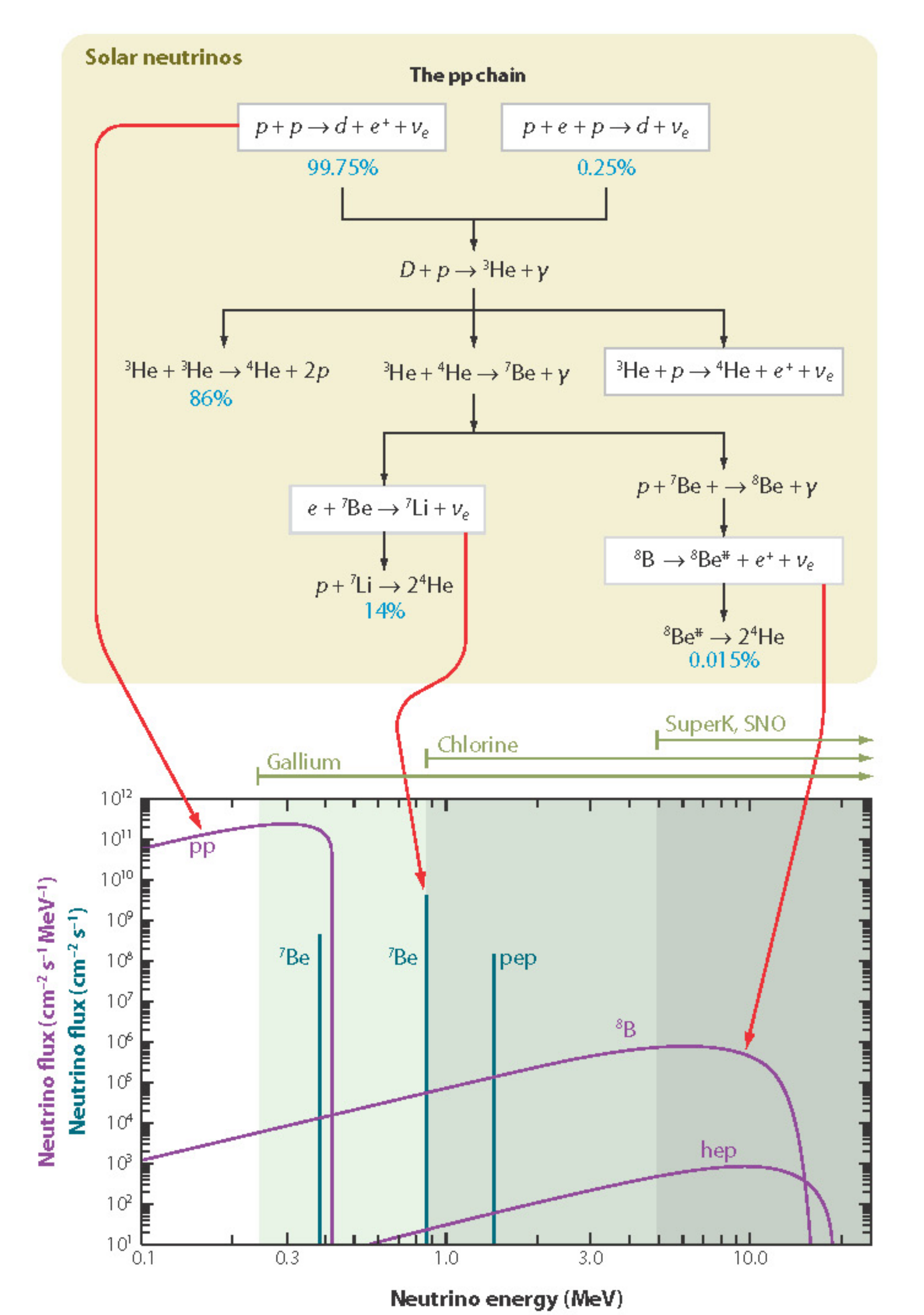


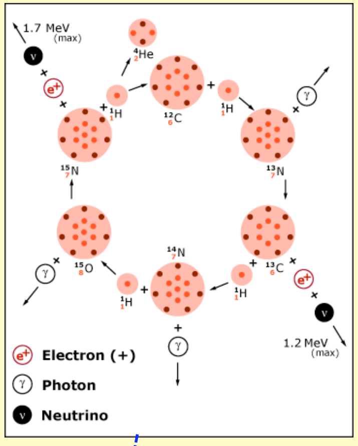
The Sun seen with neutrinos (SK)

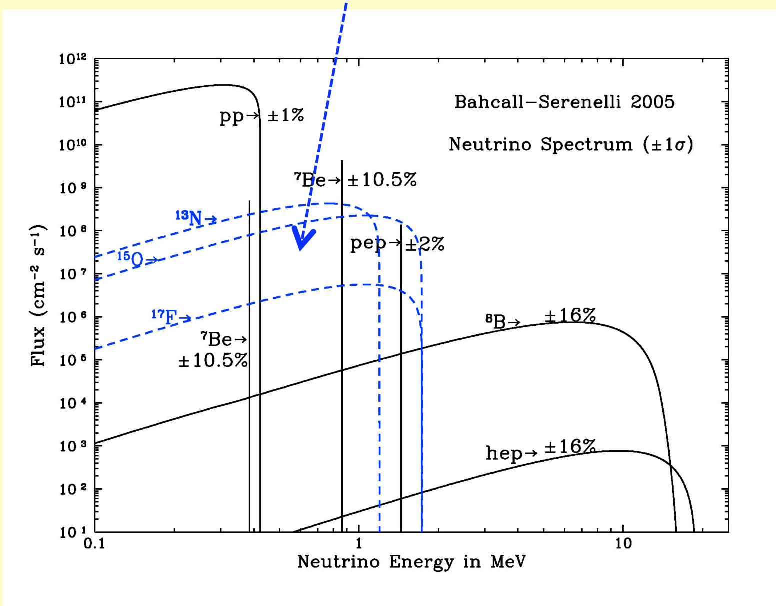


Earth orbit from solar ν (SK)

Production
pp (+CNO) cycle







Detection

Radiochemical: count the decays of unstable final-state nuclei.
(low energy threshold, but energy and time info lost/integrated)

$$^{37}\mathrm{Cl} + \nu_e \rightarrow {}^{37}\mathrm{Ar} + e \quad (\mathrm{CC})$$

Homestake

$$^{71}\mathrm{Ga} + \nu_e \rightarrow {}^{71}\mathrm{Ge} + e^- \quad (\mathrm{CC})$$

GALLEX/GNO, SAGE

Elastic scattering: events detected in real time with either "high" threshold (Č, directional) or "low" threshold (Scintillators)

$$\nu_x + e^- \rightarrow \nu_x + e^- \quad (\mathrm{NC}, \mathrm{CC})$$

SK, SNO, Borexino

Interactions on Deuterium: CC events detected in real time; NC events separated statistically + using neutron counters.

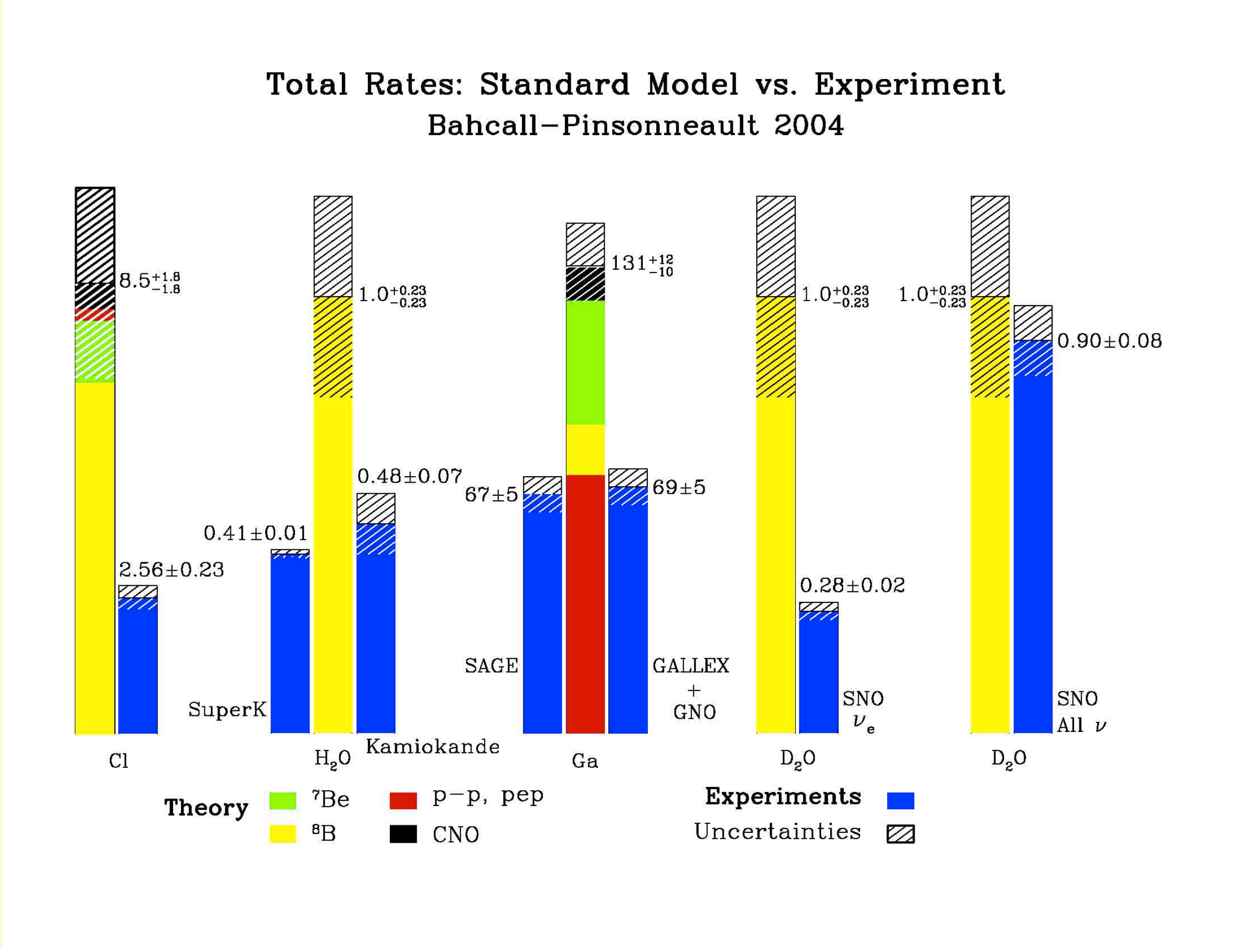
$$\nu_e + d \rightarrow p + p + e^- \quad (\mathrm{CC})$$

$$\nu_x + d \rightarrow p + n + \nu_x \quad (\mathrm{NC})$$

SNO (Sudbury Neutrino Observatory)

Experimental Results

All results in CC mode indicated a ν_e deficit…

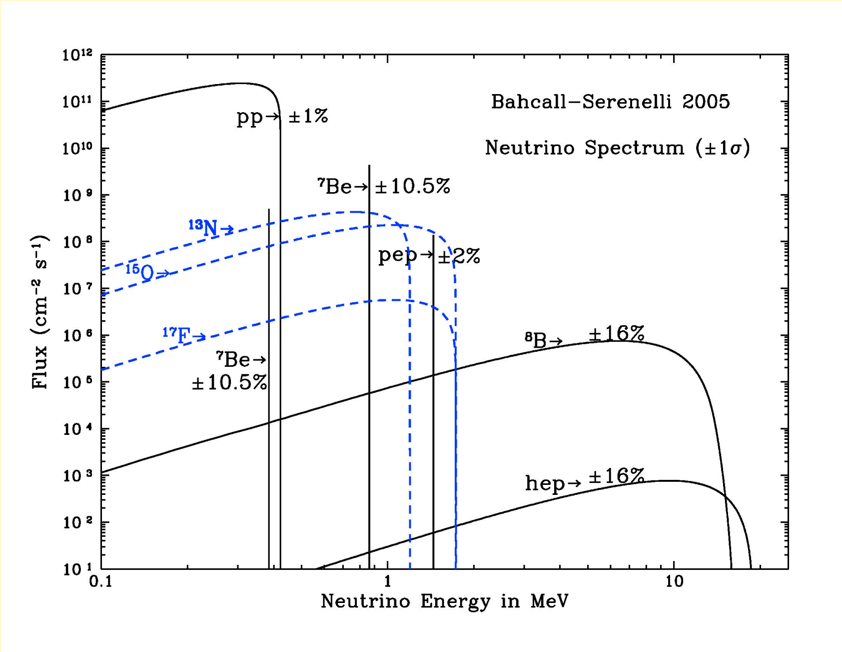


…as compared to solar model expectations

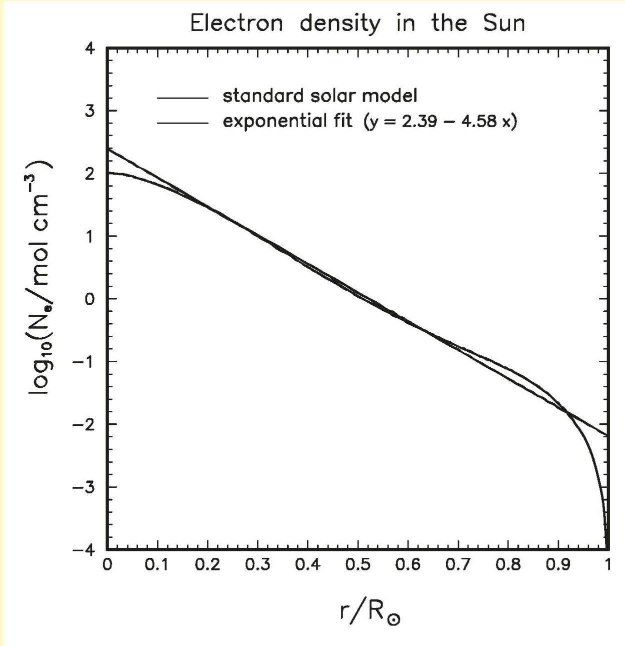
Latest confirmation: BOREXINO at Gran Sasso

Interpretation

The Sun is an intense source of ν_e with $E \sim O(10^{\pm1})$ MeV …



… and its electron density range is $\sim O(10^{\pm2})$ mol/cm^3

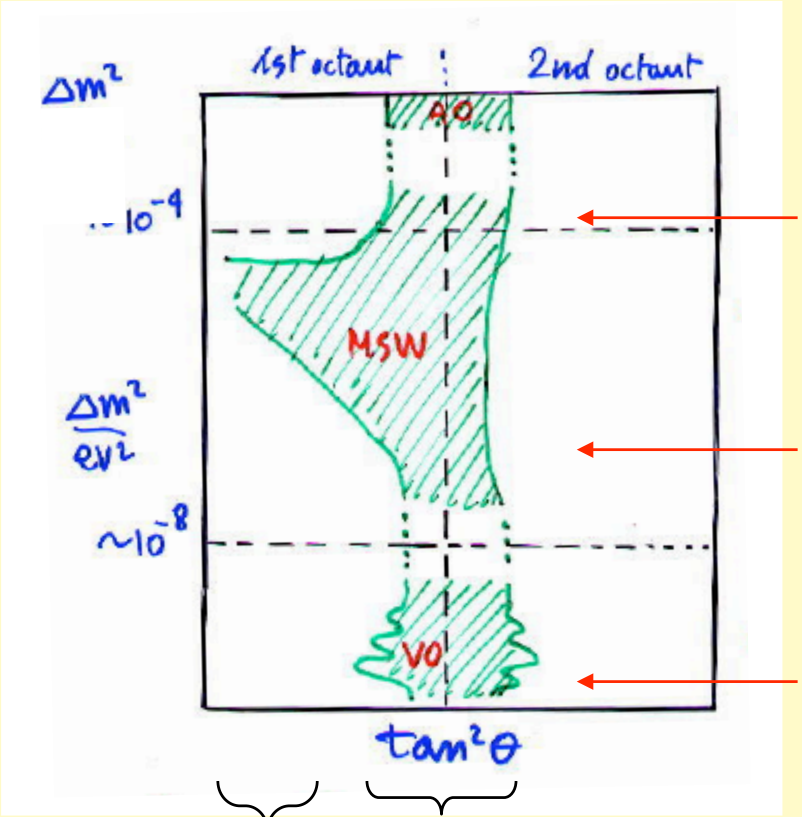


$$\ldots\text{therefore, } A/\delta m^2 \sim O(1) \text{ if } \delta m^2 \sim O(10^{-10} - 10^{-3})\ \text{eV}^2$$

The Sun is an ideal place to look for oscillations in matter, driven the **"small" squared mass difference** **δm^2** (not the "large" Δm^2), and Nature has been kind enough to fulfill these expectations! The corresponding (solar) mixing angle is θ_{12}

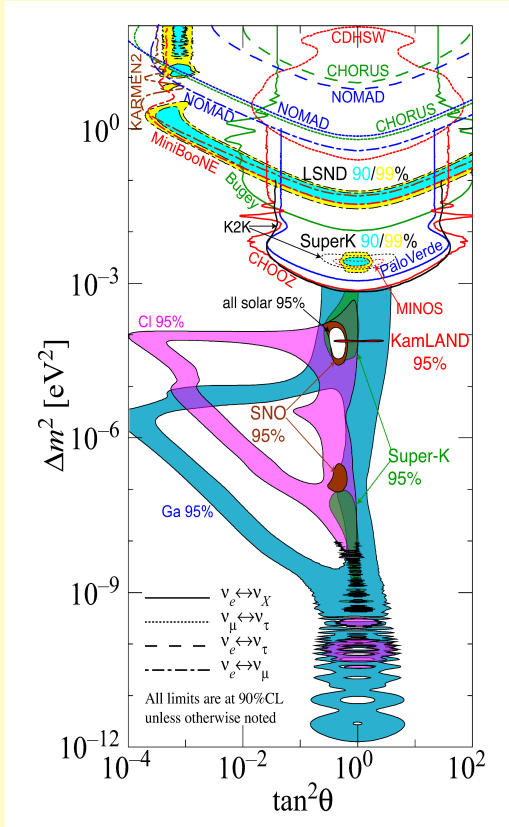
Complications… (until ~9 years ago)

Large parameter space



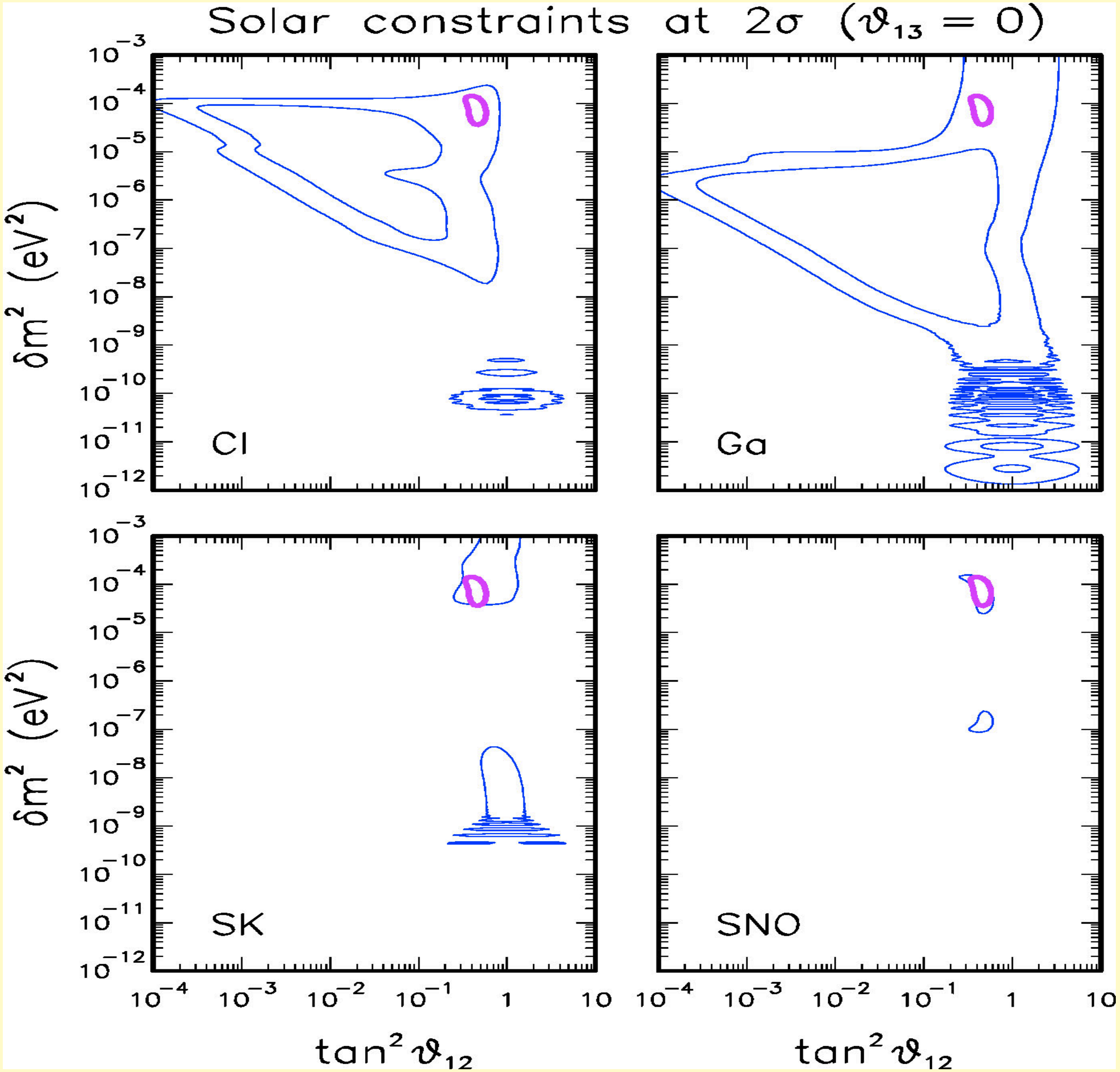
"small" / "large" mixing

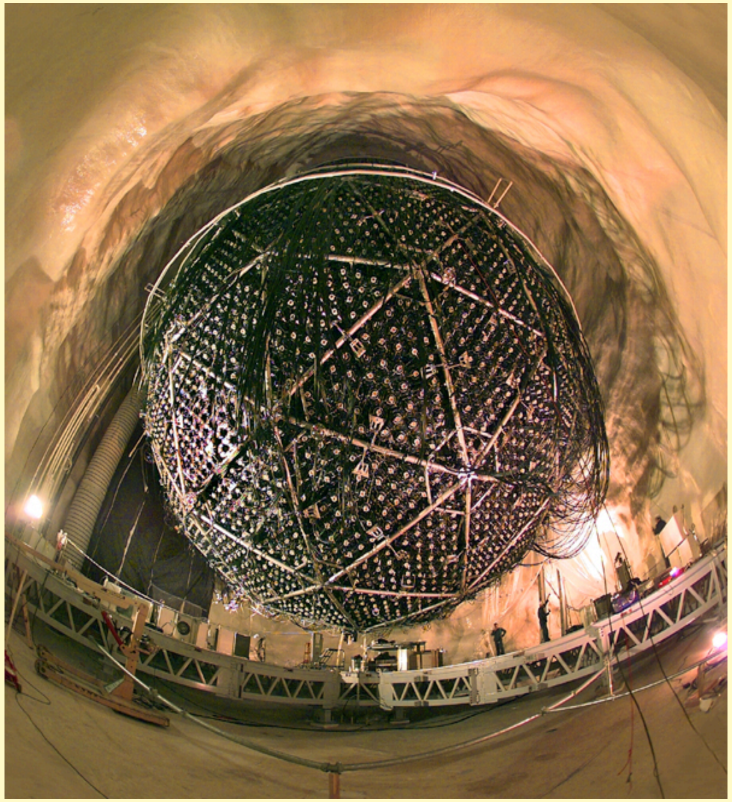
- Averaged (vacuum) oscillations — Octant symmetric
- MSW transitions — Not octant symmetric
- Vacuum oscillations — Octant symmetric



Vast literature on (semi)analytic or numerical solutions: constant density approximation generally not applicable

But, in 2002 ("annus mirabilis"), one global solution was finally singled out by combination of data ("large mixing angle" or **LMA**).





Crucial role played by

Sudbury Neutrino Observatory:

The **breakthrough**: in deuterium one can separate CC events (induced by ν_e only) from NC events (induced by ν_e, ν_μ, ν_τ), and double check via Elastic Scattering events (due to both NC and CC)

$$\begin{aligned} \mathrm{CC}: &\quad \nu_e + d \rightarrow p + p + e \\ \mathrm{NC}: &\quad \nu_{e,\mu,\tau} + d \rightarrow p + n + \nu_{e,\mu,\tau} \\ \mathrm{ES}: &\quad \nu_{e,\mu,\tau} + e \rightarrow e + \nu_{e,\mu,\tau} \end{aligned}$$

$$\frac{\mathrm{CC}}{\mathrm{NC}} \sim \frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})}$$

thus:

$$\frac{\mathrm{CC}}{\mathrm{NC}} < 1 \Rightarrow \phi(\nu_{\mu,\tau}) > 0 \Rightarrow \nu_e \rightarrow \nu_{\mu,\tau}$$

CC/NC ~ 1/3 < 1

"Smoking gun" proof of flavor change. Solar model OK! Also:

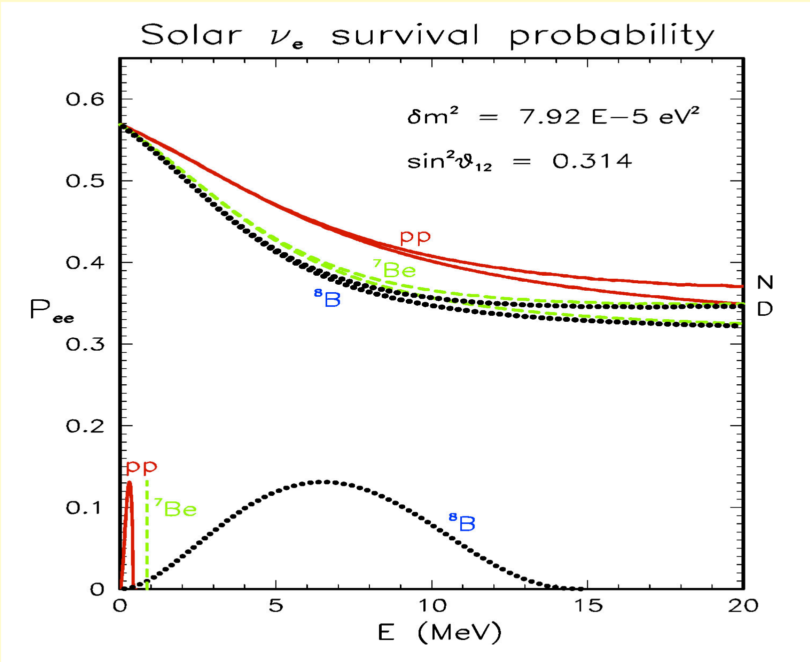
CC/NC ~ Pee ~ $\sin^2\theta_{12}$ (LMA) ~1/3 < ½

Evidence of: mixing in first octant + matter effects

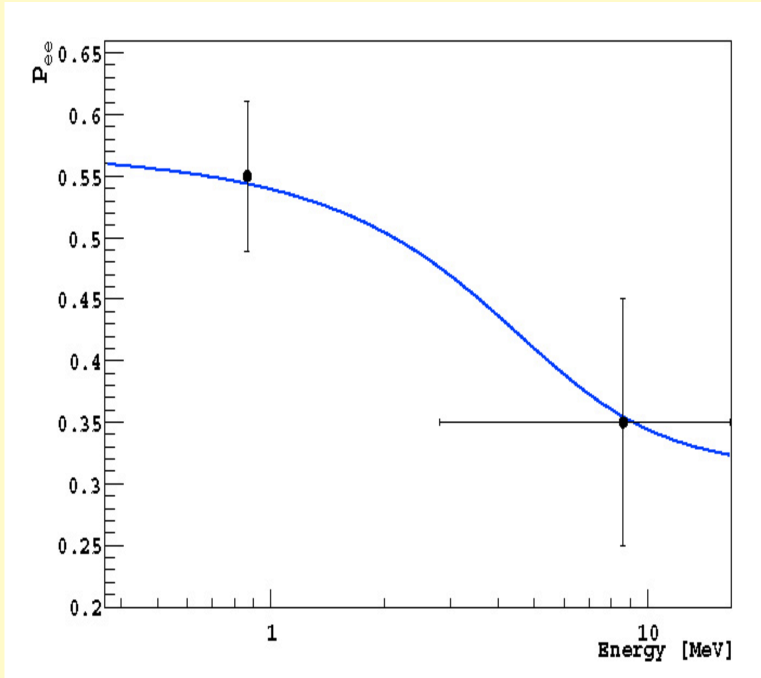
For the parameters (δm^2, θ_{12}) in the LMA region, one can use the next approximation to "constant density," namely, the approximation of "slowly varying density" (with respect to oscillation frequency):

adiabatic approximation **(see Appendix)**

Expected probability profile



Test with recent Borexino data



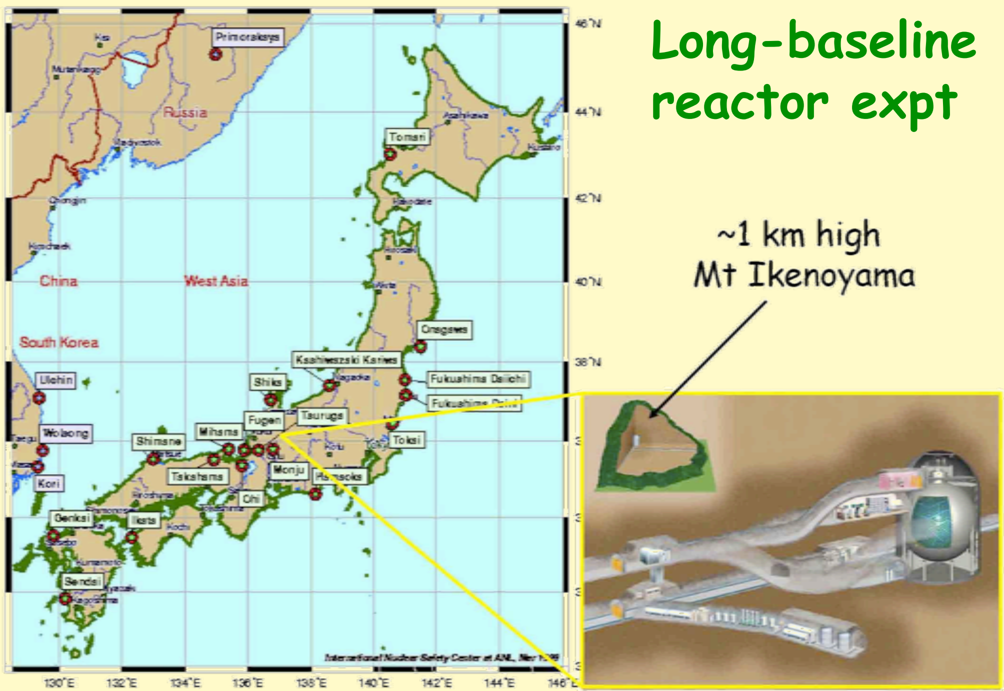
In the Earth: small day/night (D/N) effects, not yet seen.

Also in 2002… KamLAND: 1000 ton mineral oil detector, "surrounded" by nuclear reactors producing anti-ν_e. Characteristics:

$A/\delta m^2 \ll 1$ in Earth crust
(vacuum approxim. OK)
L~100-200 km
E_ν~ few MeV

⇨

With previous (δm^2,θ_{12}) parameters it is ($\delta m^2 L/4E$)~O(1) and reactor neutrinos should oscillate with large amplitude (large θ_{12})

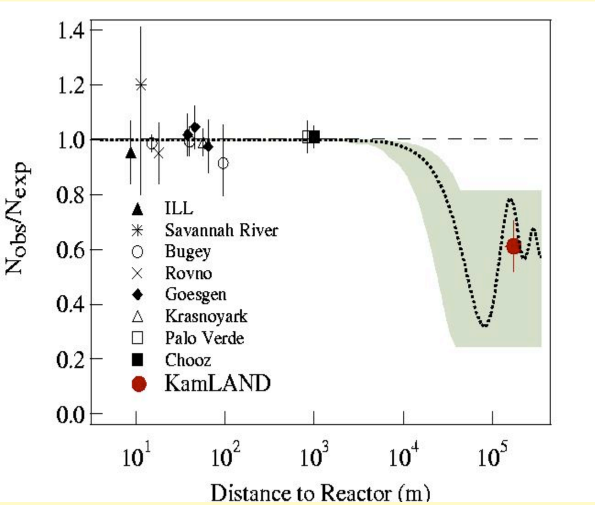


Long-baseline reactor expt

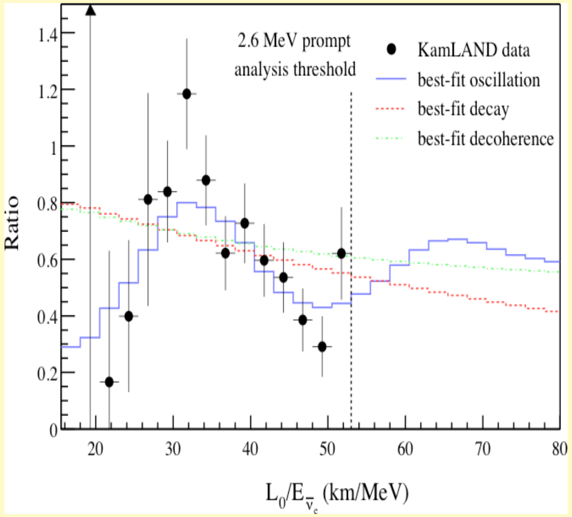
~1 km high
Mt Ikenoyama

KamLAND results

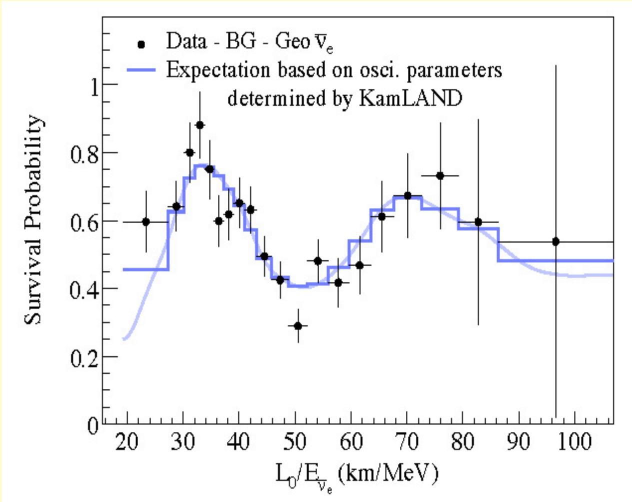
2002: electron flavor disappearance observed



2004: half-period of oscillation observed

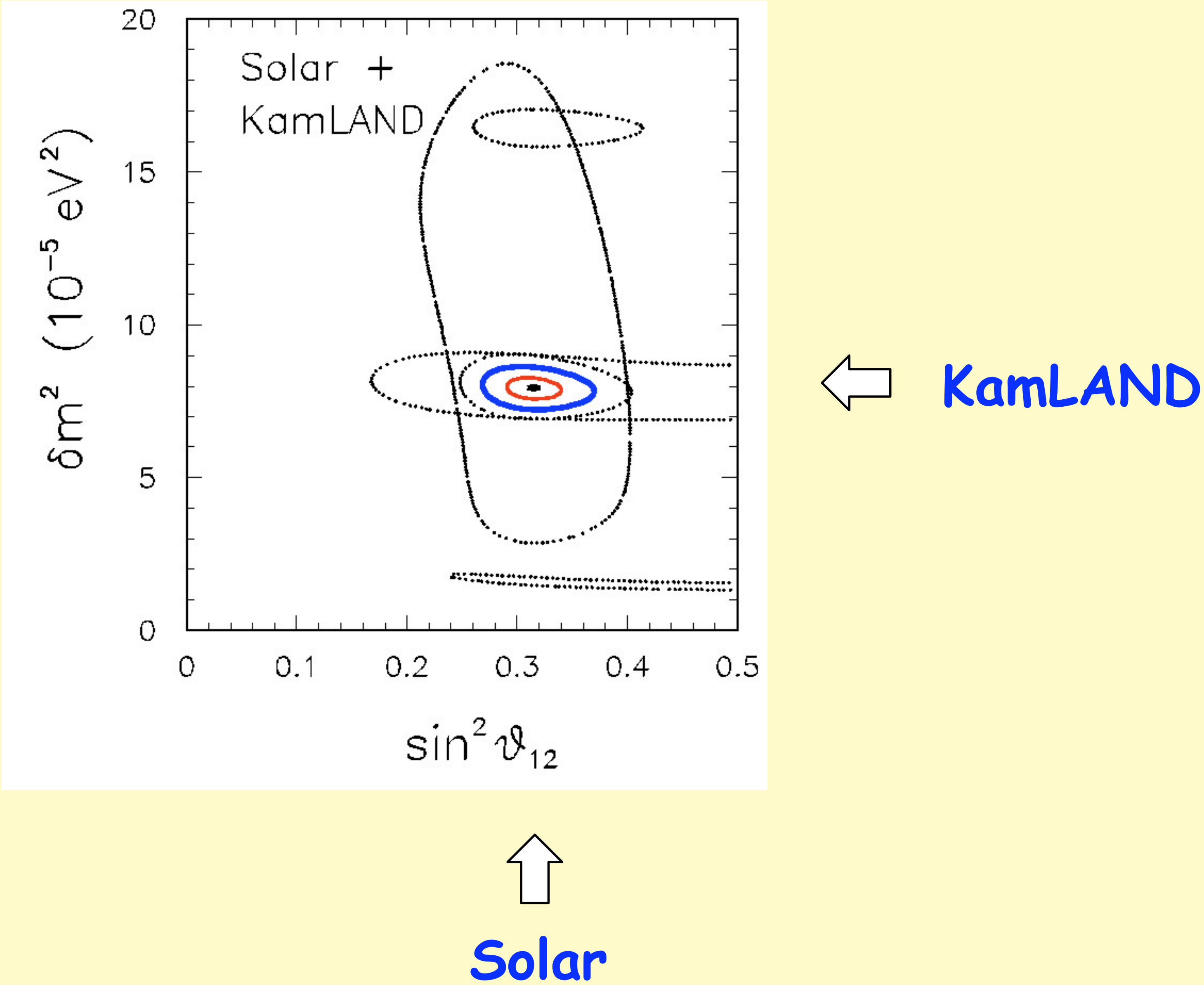


2007: one period of oscillation observed



Direct observation of δm^2 oscillations

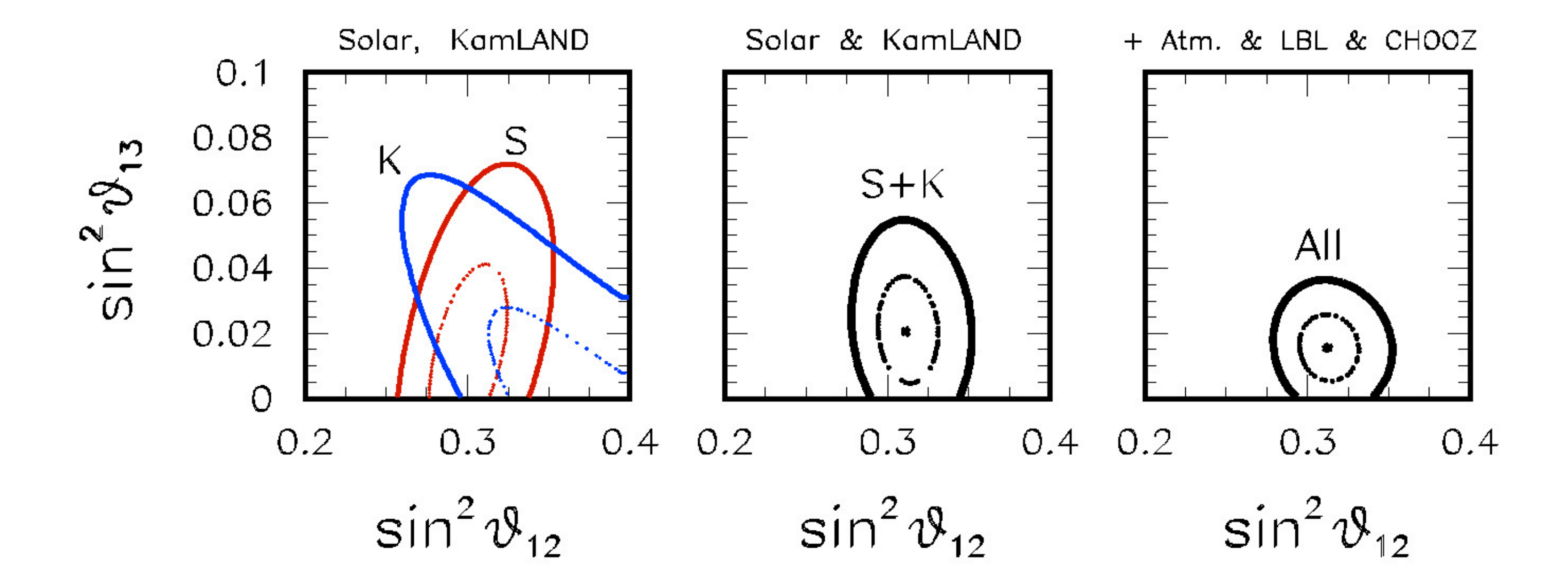
(δm^2, θ_{12}) - complementarity of solar/reactor neutrinos



More refined (3ν) interpretation

Go beyond dominant 3ν oscillations. Include subleading effects due to θ_{13} and averaged Δm^2 oscillations in vacuum/matter.

Interesting (small) effects emerge. [See arXiv:0806.2649].



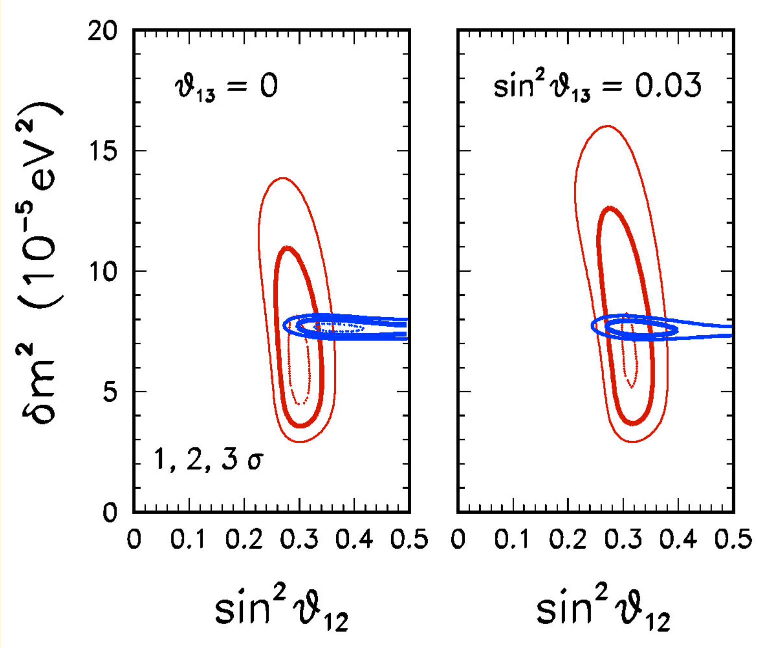
Preference of θ_{13}>0 arises from slight tension on θ_{12} (solar vs KamLAND) and from different correlation bewteen mixing angles, related to different relative signs in P_{ee} (survival probability) of solar vs KamLAND:

Solar, high energy (LMA MSW):

$$P_{ee} \simeq \underbrace{(1-2s^2_{13})}_{-}\underbrace{(+s^2_{12})}_{+}$$

Reactor (~vacuum): KamLAND

$$P_{ee} \simeq \underbrace{(1-2s^2_{13})}_{-}\underbrace{(1-4s^2_{12}c^2_{12}\sin^2(\delta m^2 L/4E))}_{-}$$



Better agreement on a common θ_{12} value for θ_{13}>0

Hint of θ_{13} >0 ? Time will tell.

Synopsis of neutrino mass^2 and mixing parameters:
central values and n-σ ranges from global 3ν analysis

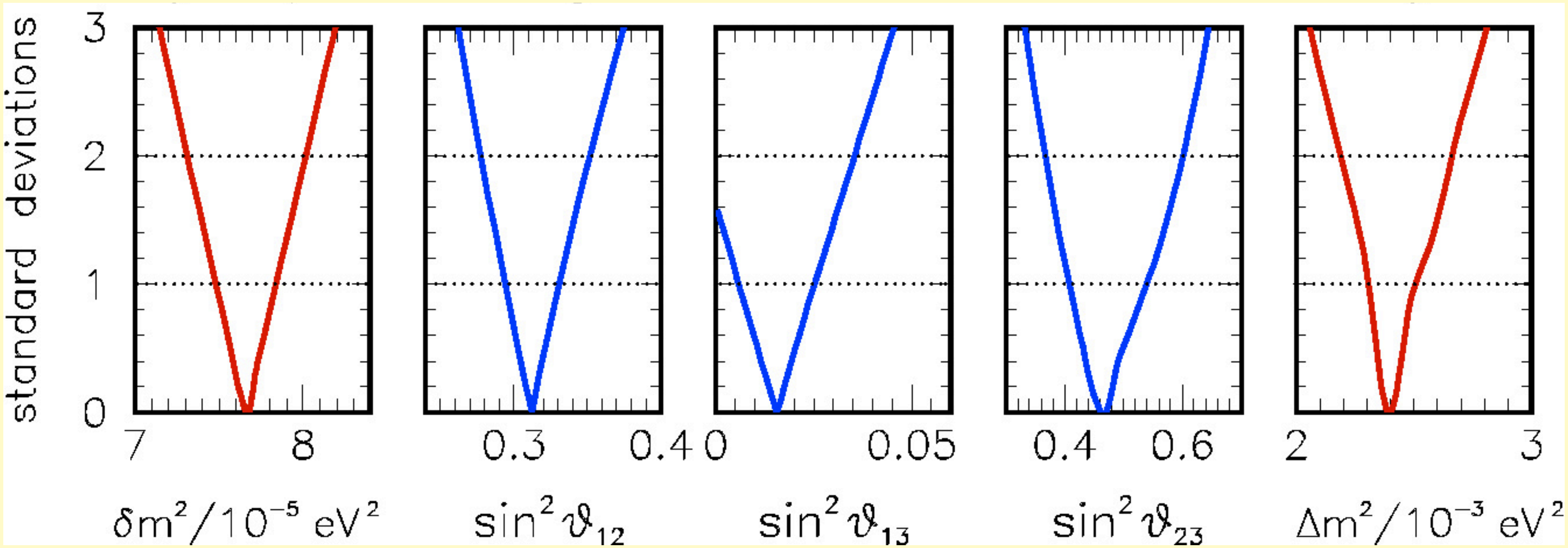


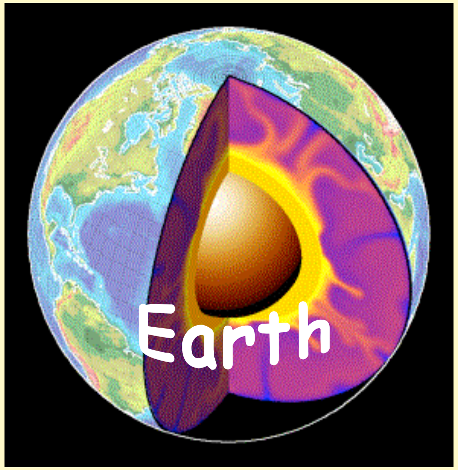
TABLE I: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges for the mass-mixing parameters.

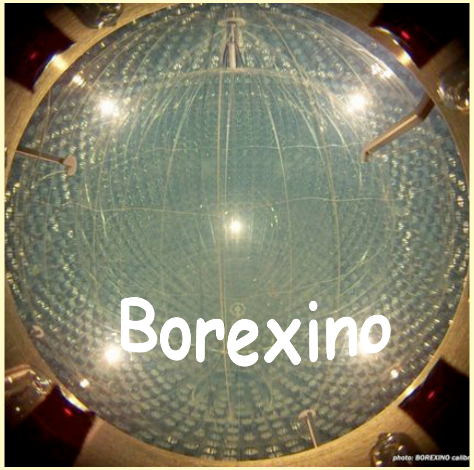
Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

arXiv:0805.2517 (no big change since)

Detour #1… geoneutrinos!

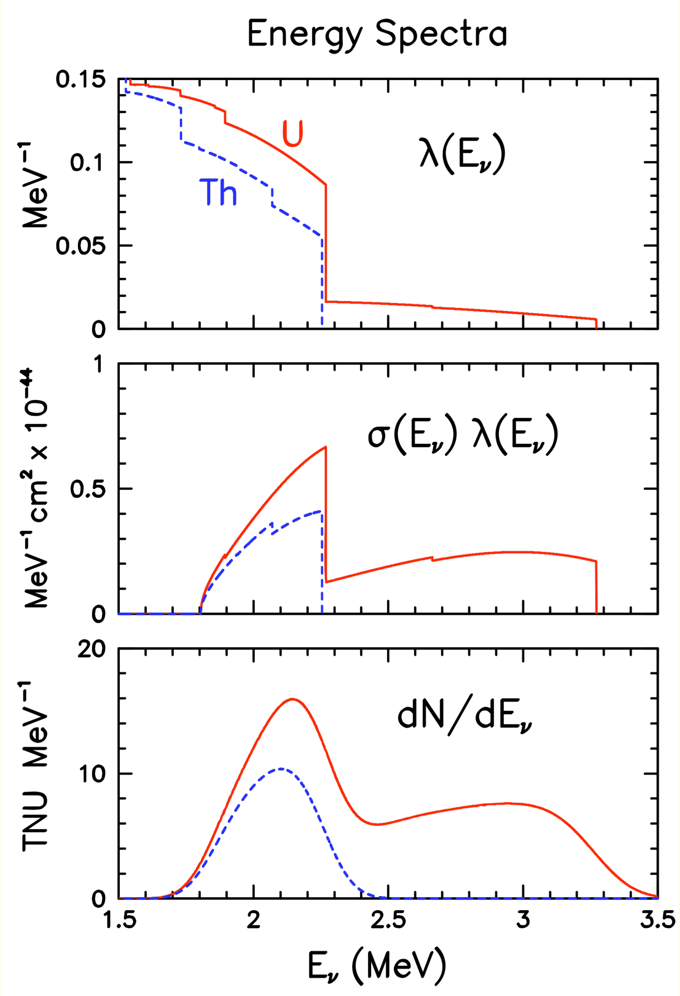
In a cosmo/astro-oriented School, it is worth reminding that the **Earth is an "antineutrino star"**, and that a precious by-product of the KamLAND+Borexino data analysis is the study of **geoneutrinos** (i.e., electron antineutrinos emitted in the decay chains of Th and U present in trace amount in the crust and mantle):





Expect: Th/U ratio ~ 3.9 and significant contribution to Earth's heat

Four geo-ν observables: rates (R) of U, Th events in KL and BX



Energy spectrum of U and Th geo-ν
(for a flux of $10^6/cm^2/s$)

... times absorption cross section
(for 10^{32} target protons)

... plus energy resolution effects
(note Th+U peak and U tail)

Areas under the curves (after ~energy-independent suppression due to neutrino flavor oscillations): R(U), R(Th) for KL and BX

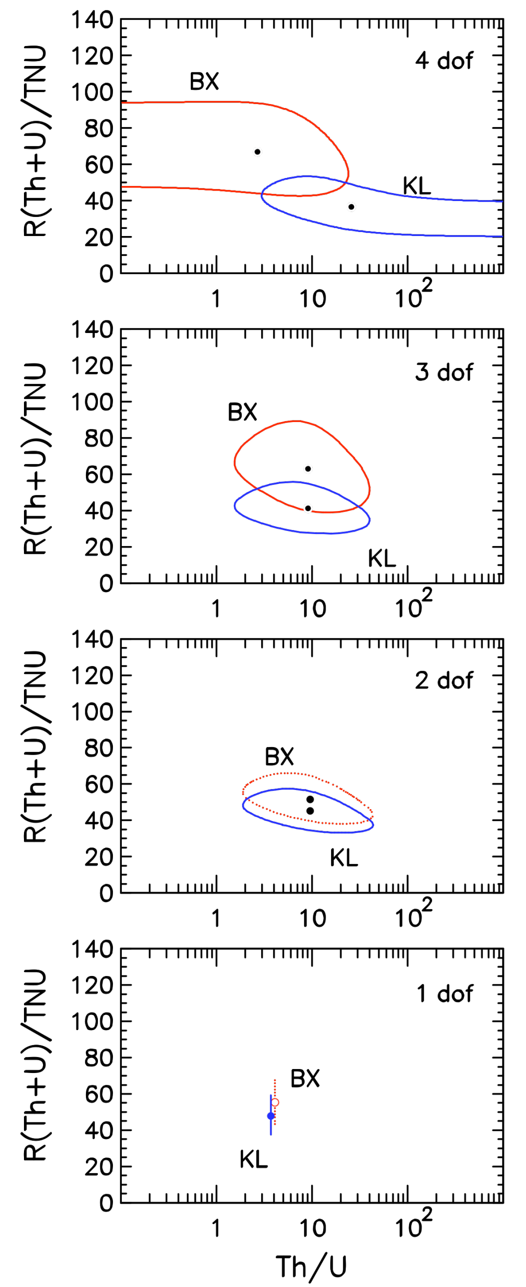
Results: 1σ contours in the plane charted by total rate R vs Th/U

4 dof

3 dof

2 dof

1 dof



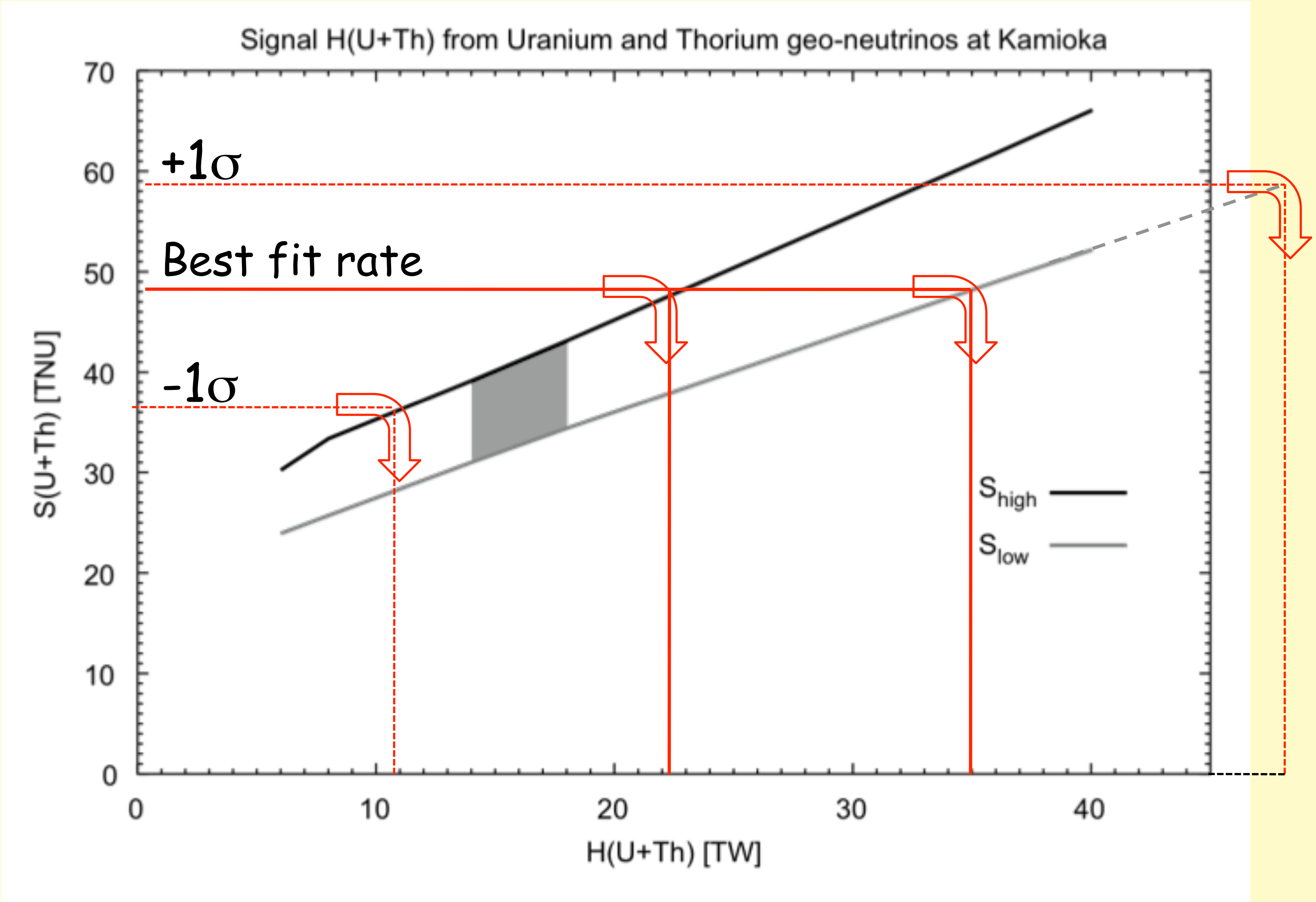
Rates: BX>KL as expected
KL: Th>0 favored, U=0 allowed
BX: U>0 favored, Th=0 allowed
Overlap of Th/U ranges

Assume same Th/U in both expts.
Th/U: 1σ bounds emerge.
Allowed range includes chondritic value

Assume also BX~ 1.15 KL (contin. crust).
Total rate: reduced uncertainty

Assume also Th/U=3.9 (chondrites).
Total rate: further (slight) error reduction

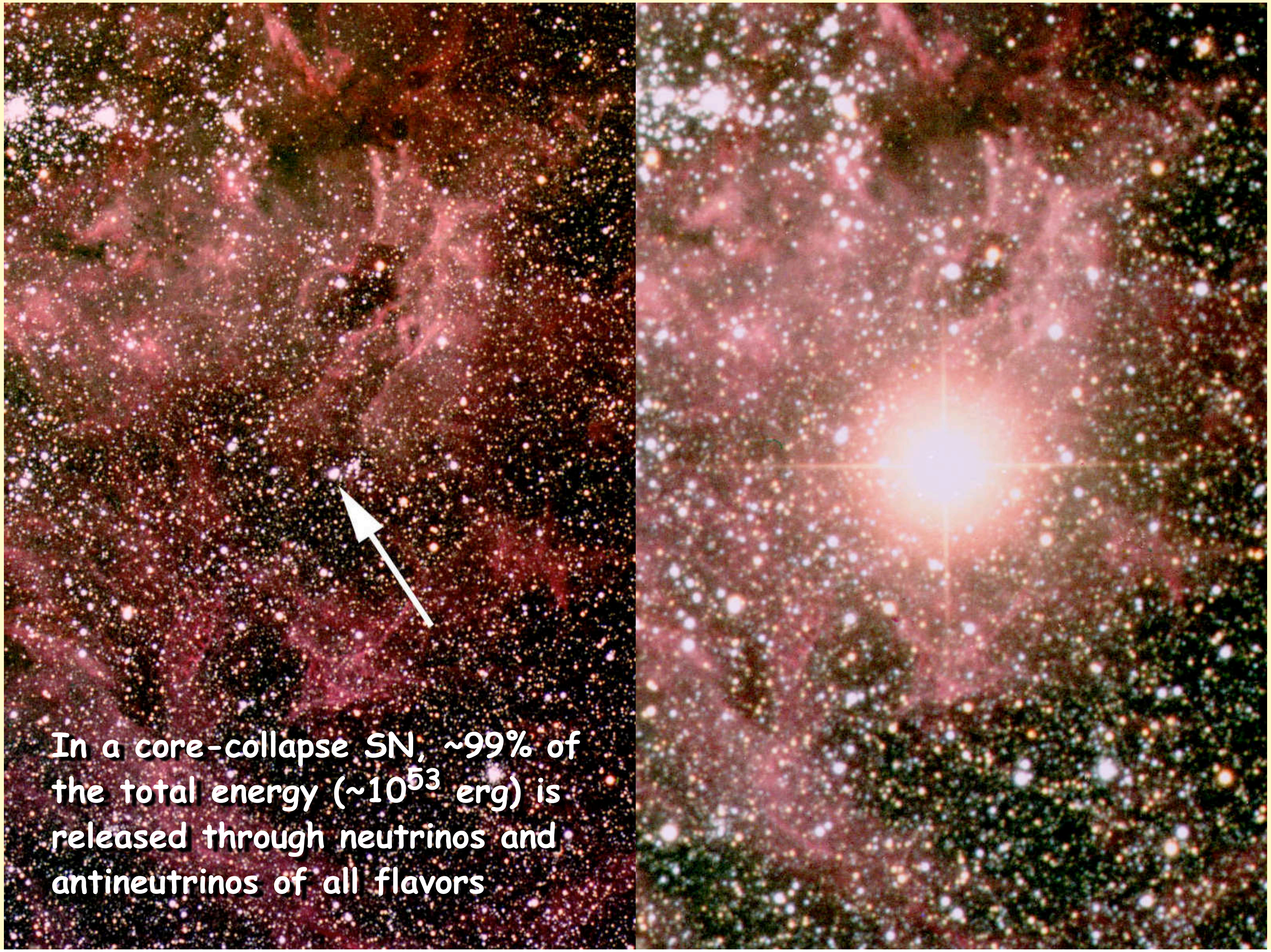
Implications from Heat/Rate correlation …



…relatively high H, in excess of "crust only" minimum, favored at 1σ

Detour #2… supernovae!

In a cosmo/astro-oriented School, it is also worth reminding that the only two known sources in ν astronomy are the Sun and the SN 1987A.

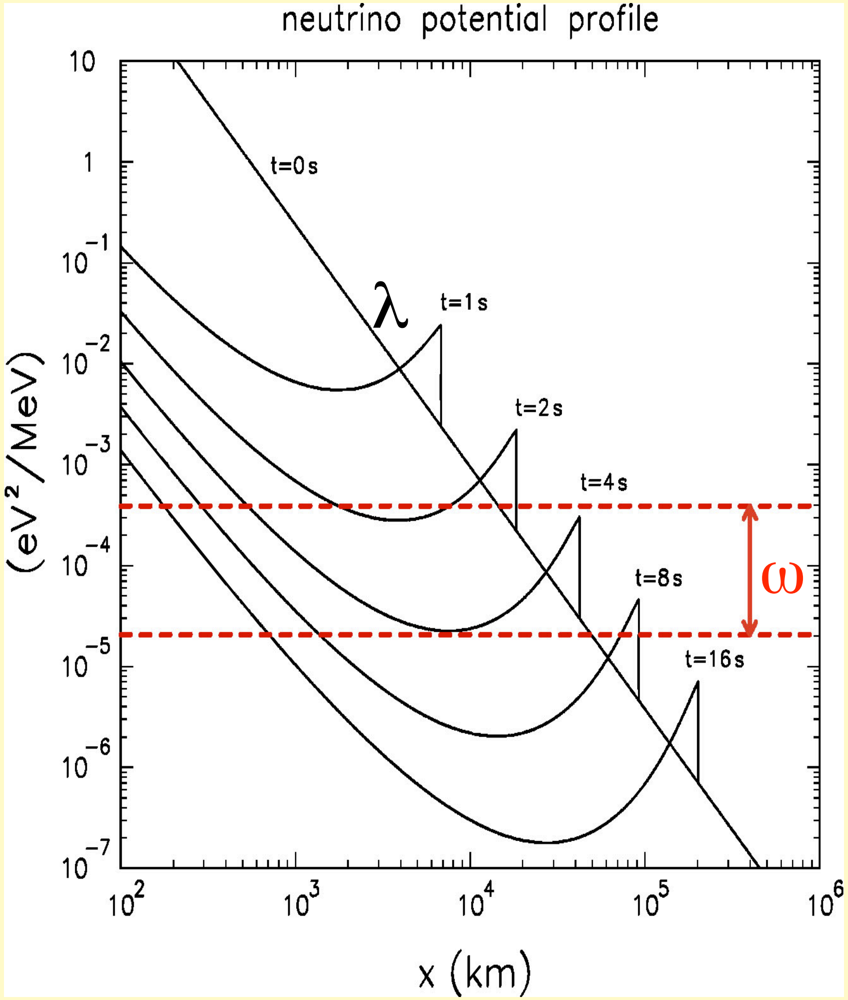


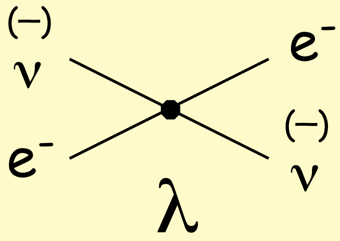
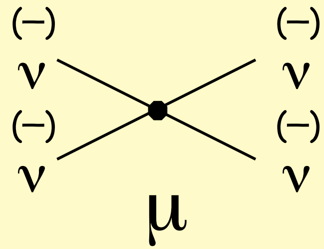
Flavor changes induced by "usual MSW" effects: studied for ~20 y.

Well-known MSW effects can occur in a SN envelope when the ν potential $\lambda=\sqrt{2}\, G_F N_e$ is close to osc. frequency $\omega=\Delta m^2/2E$ ($\Delta m^2=|m^2_3-m^2_{1,2}|$, $\theta_{13}\neq 0$).

For t~few sec after bounce,
$\lambda\sim\omega$ at $x \gg 10^2$ km (large radii).

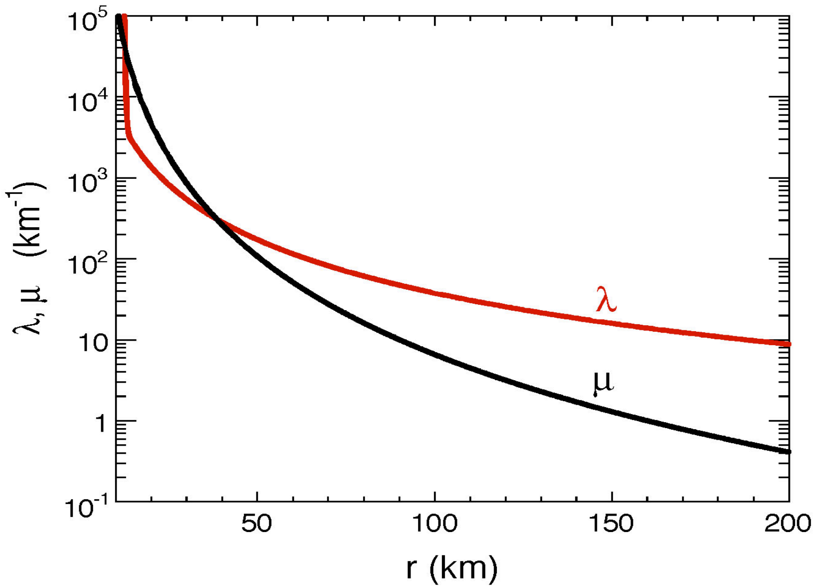
What about small radii?
Popular wisdom:
$\lambda \gg \omega$ at $x < O(10^2)$ km,
thus flavor transitions suppressed.
Incorrect!



Besides  (CC) one has to include  (NC)

At small r, neutrino and antineutrino density (n and n) high enough to make self-interactions important. Strength:

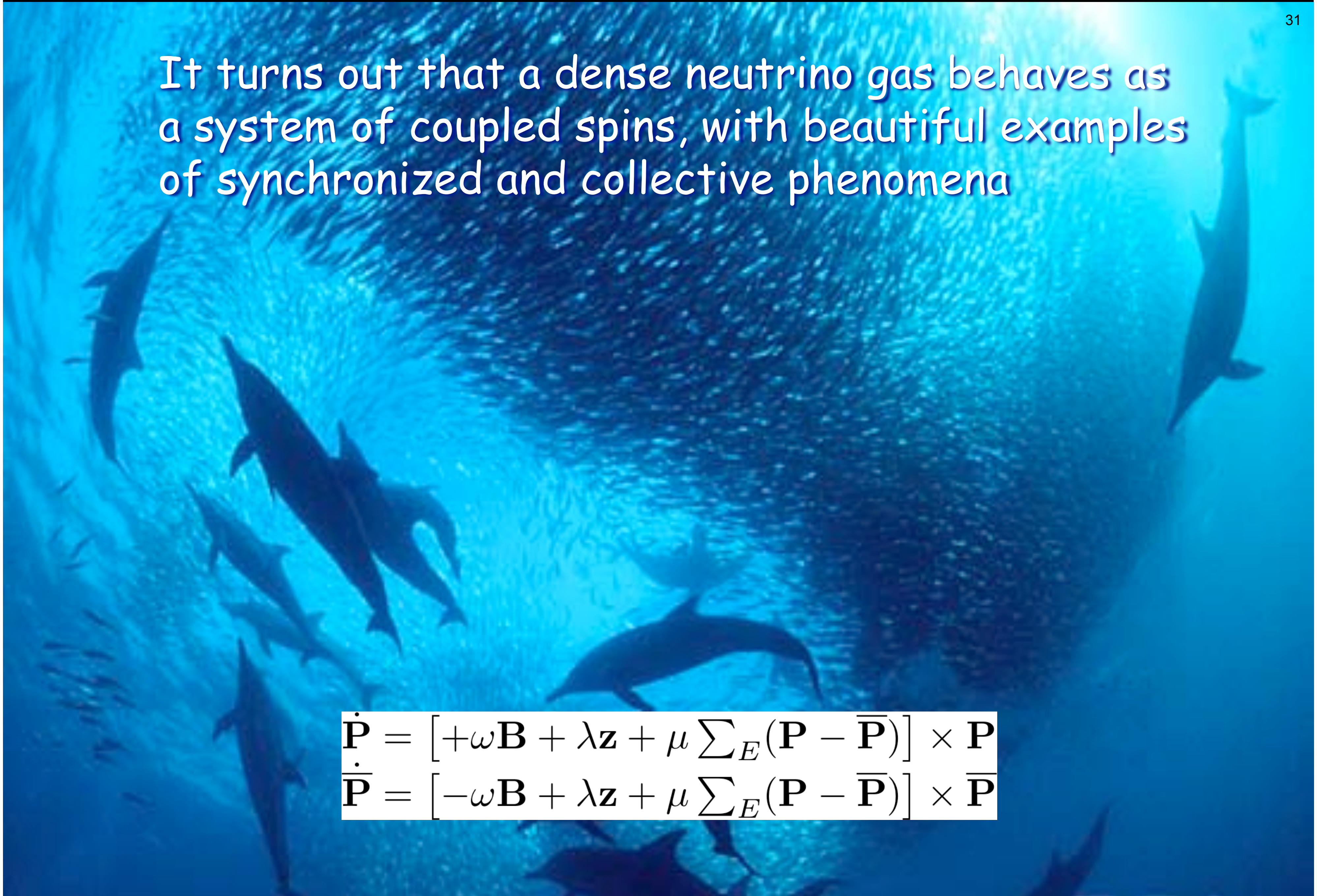
$\mu=\sqrt{2}\, G_F\, (n+\bar{n})$



Angular modulation factor: $(1-\cos\Theta_{ij})$
If averaged: "single-angle" approxim.
Otherwise : "multi-angle" (difficult)

Self-interaction effects known for ~20 y in SN. But, recent boost of interest after new crucial results, first obtained numerically and then analitically.

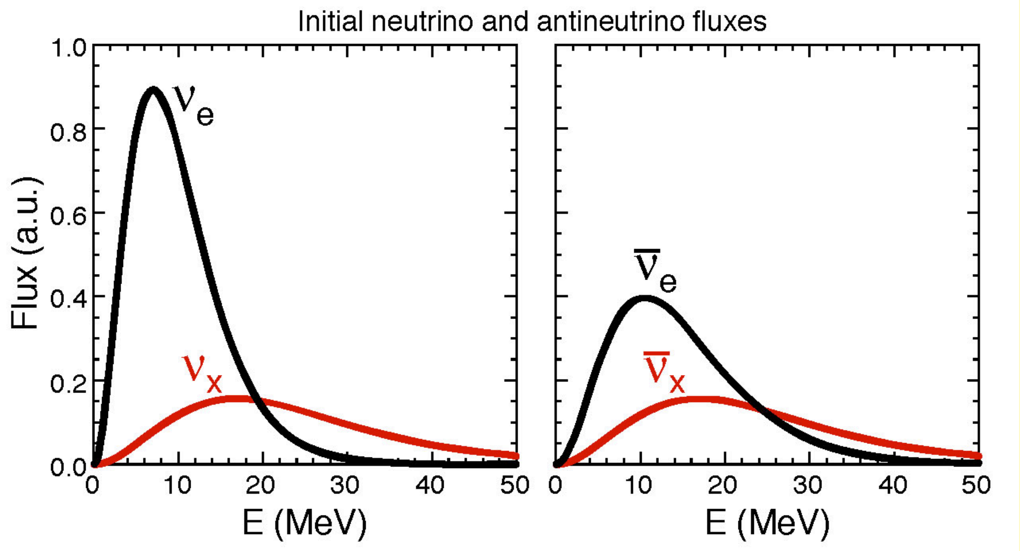
Lesson: self-interactions (μ) can induce large, non-MSW flavor change at small radii, despite large matter density λ



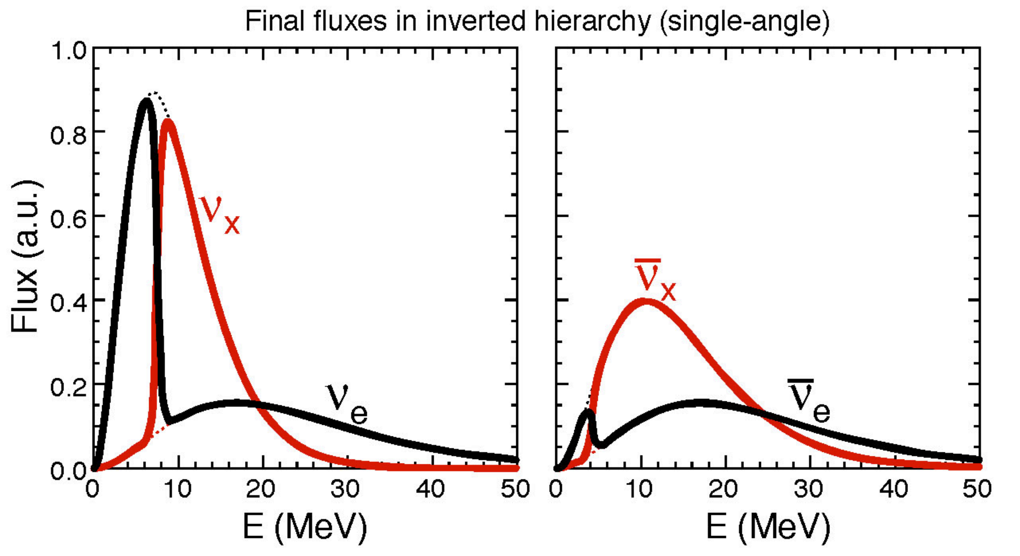
It turns out that a dense neutrino gas behaves as a system of coupled spins, with beautiful examples of synchronized and collective phenomena

$$\dot{\mathbf{P}} = \left[+\omega\mathbf{B} + \lambda\mathbf{z} + \mu \textstyle\sum_E (\mathbf{P} - \overline{\mathbf{P}})\right] \times \mathbf{P}$$
$$\dot{\overline{\mathbf{P}}} = \left[-\omega\mathbf{B} + \lambda\mathbf{z} + \mu \textstyle\sum_E (\mathbf{P} - \overline{\mathbf{P}})\right] \times \overline{\mathbf{P}}$$

E.g., flavor may be swapped abruptly in certain energy ranges for inverted hierarchy ("spectral split")



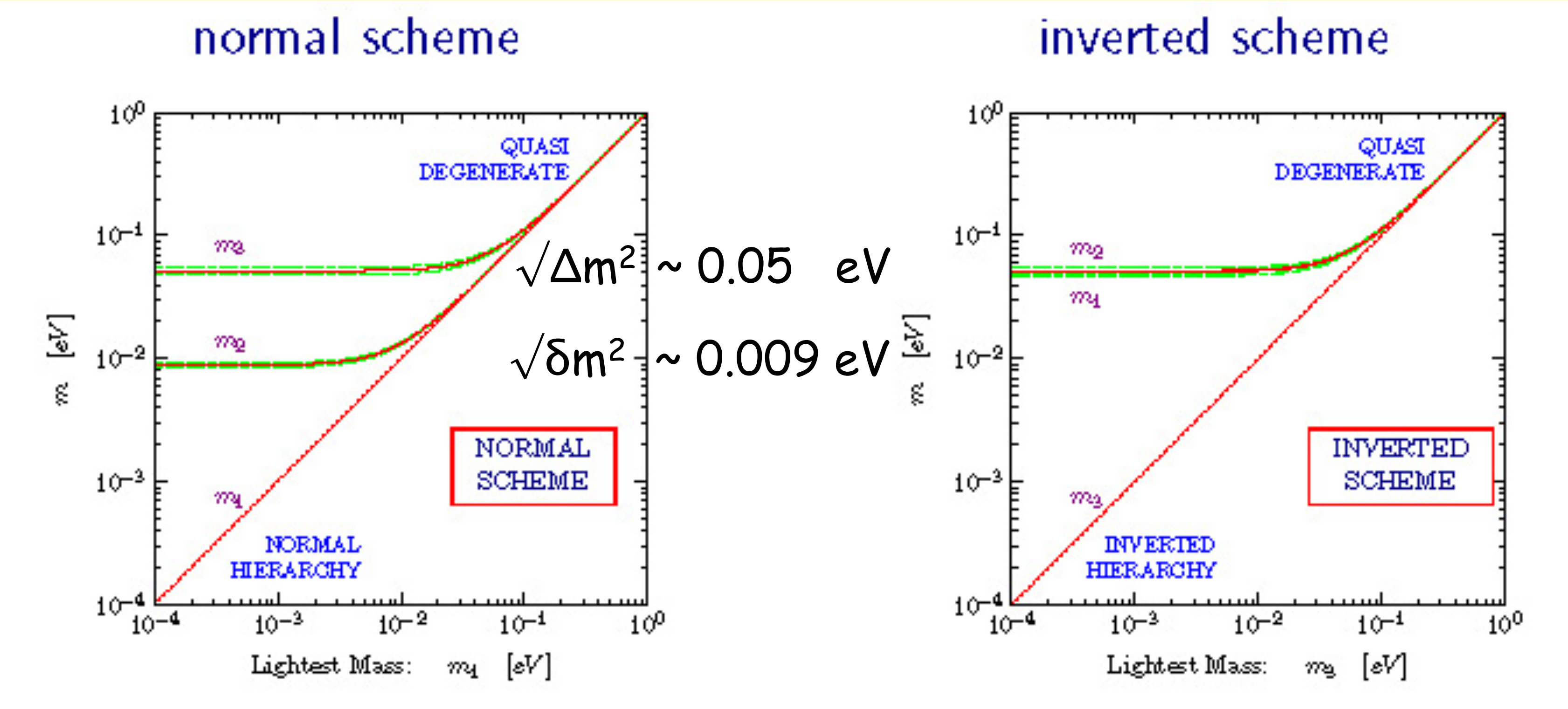
Initial fluxes at the neutrinosphere (r~10 km)



Final fluxes at the end of collective effects (r~200 km)

Absolute neutrino masses: Current phenomenology

RECAP: Oscillations constrain neutrino mixings and mass splittings but not the absolute mass scale.



Three realistic observables to attack ν masses →

The "weapon":

One spear: | **Three prongs:**

The three prongs of the "trident": (m_β, $m_{\beta\beta}$, Σ)

1) β decay: $m^2_i \neq 0$ can affect spectrum endpoint. Sensitive to the "effective electron neutrino mass":

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

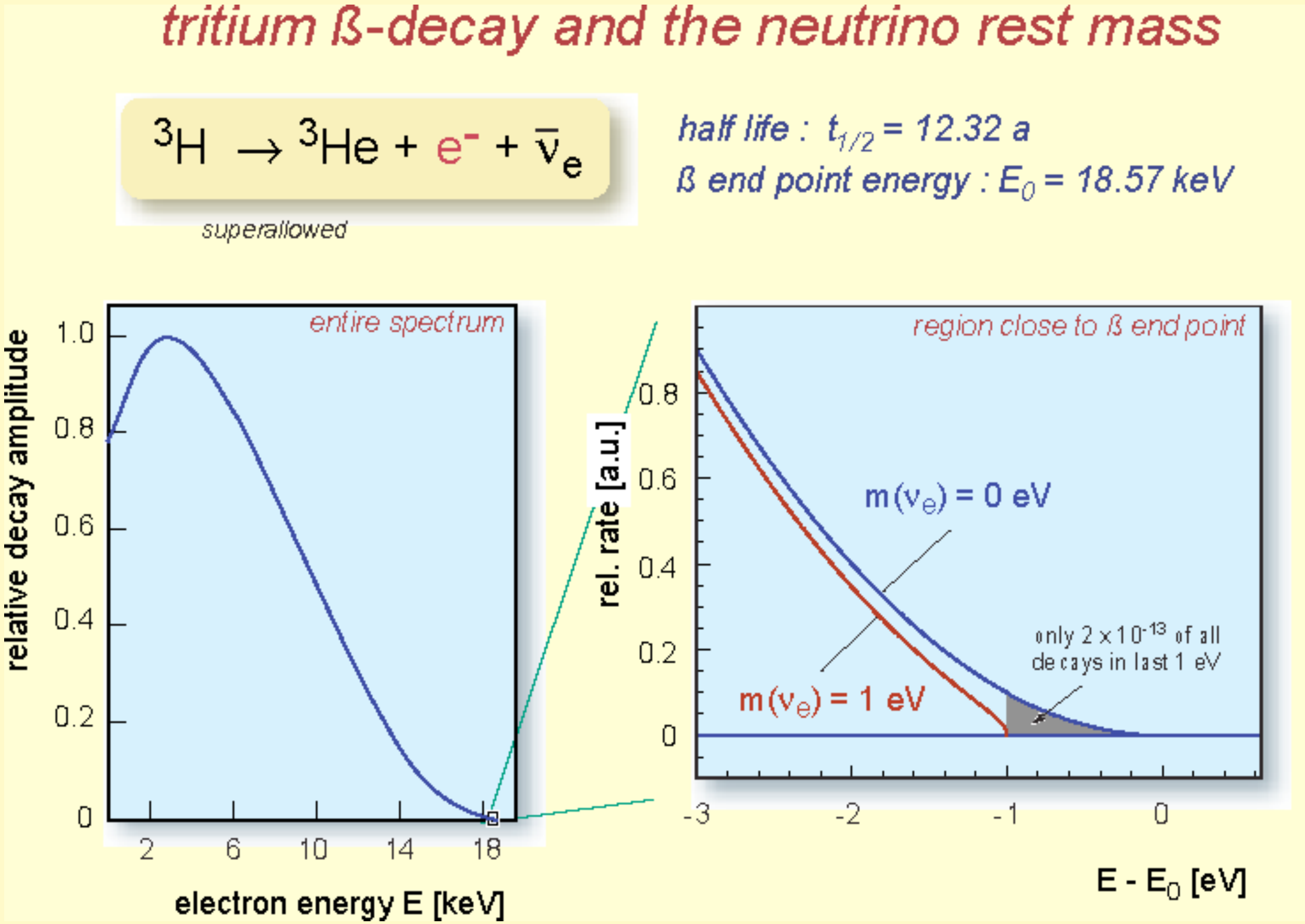
2) $0\nu\beta\beta$ decay: Can occur if $m^2_i \neq 0$ and $\overline{\nu}=\nu$ (Majorana, not Dirac) Sensitive to the "effective Majorana mass" (and phases):

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

3) Cosmology: $m^2_i \neq 0$ can affect large scale structures in (standard) cosmology constrained by CMB + other data. Sensitive to:

$$\Sigma = m_1 + m_2 + m_3$$

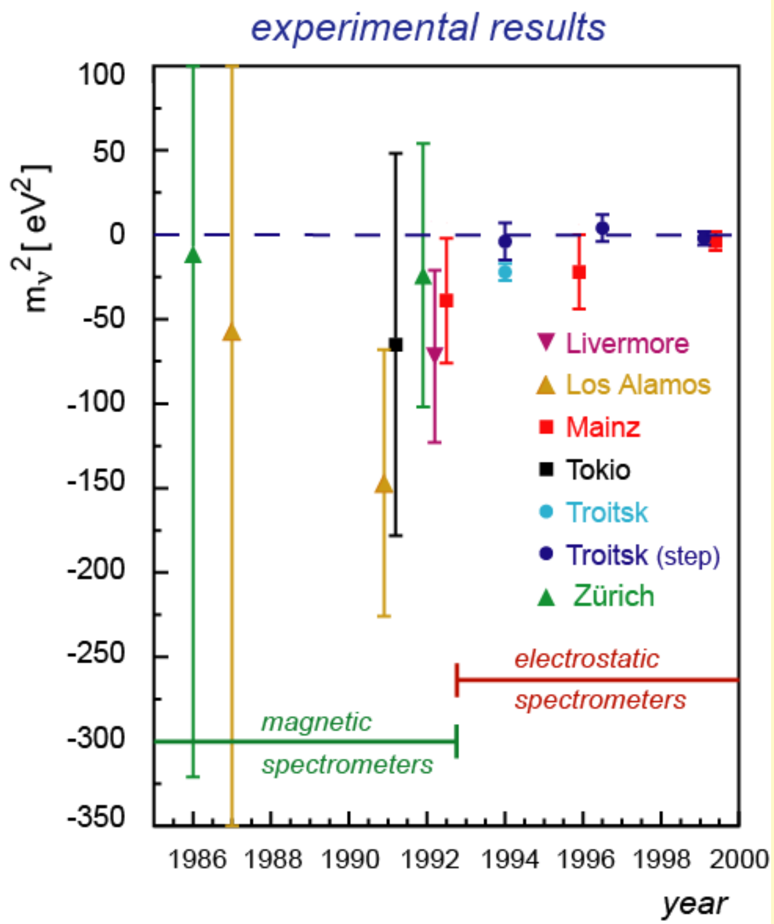
BETA DECAY with Tritium: low-Q, fast decays



Need good energy resolution

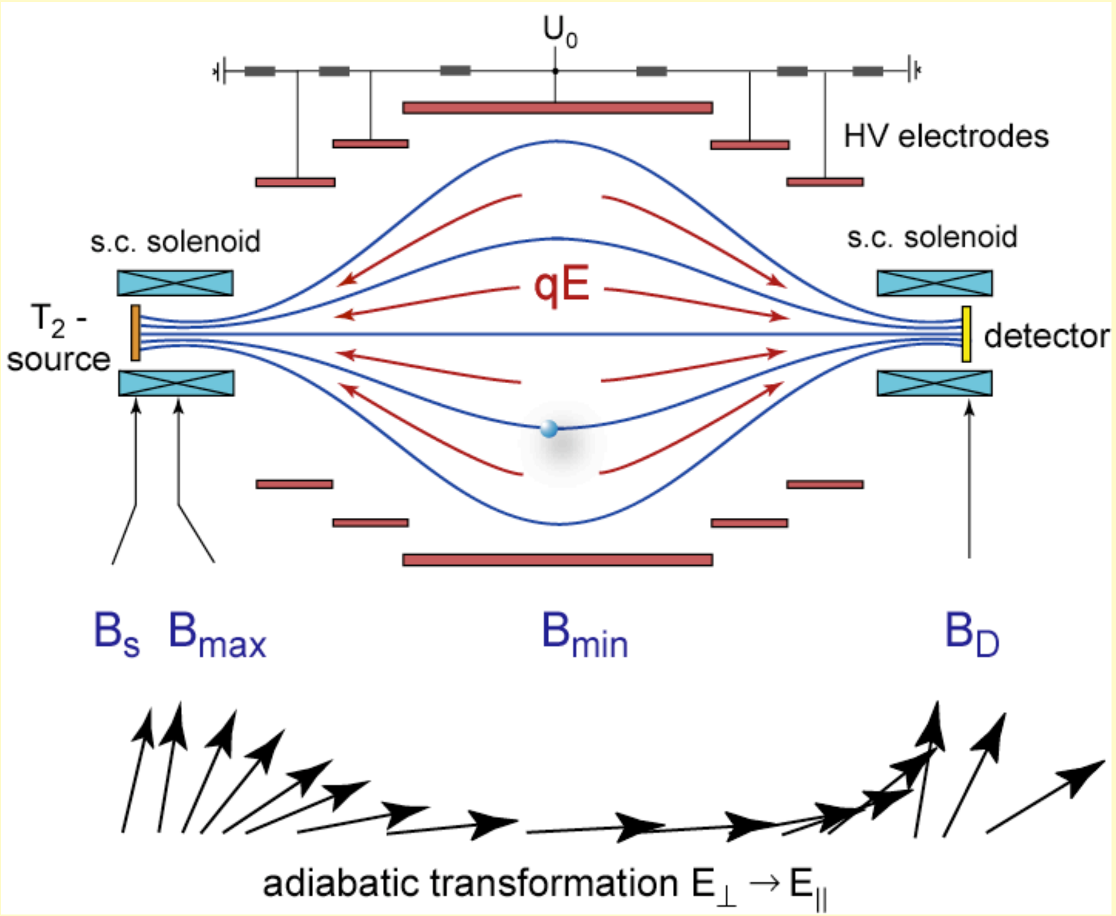
History plot for tritium

Experiment	m_ν
ITEP *T_2 in complex molecule* *magn. spectrometer (Tret'yakov)*	17-40 eV
Los Alamos *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 9.3 eV
Tokio *T - source* *magn. spectrometer (Tret'yakov)*	< 13.1 eV
Livermore *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 7.0 eV
Zürich *T_2 - source impl. on carrier* *magn. spectrometer (Tret'yakov)*	< 11.7 eV
Troitsk (1994-today) *gaseous T_2 - source* *electrostat. spectrometer*	< 2.2 eV
Mainz (1994-today) *frozen T_2 - source* *electrostat. spectrometer*	< 2.3 eV



Latest bounds at the level of ~2 eV

In construction: KATRIN experiment

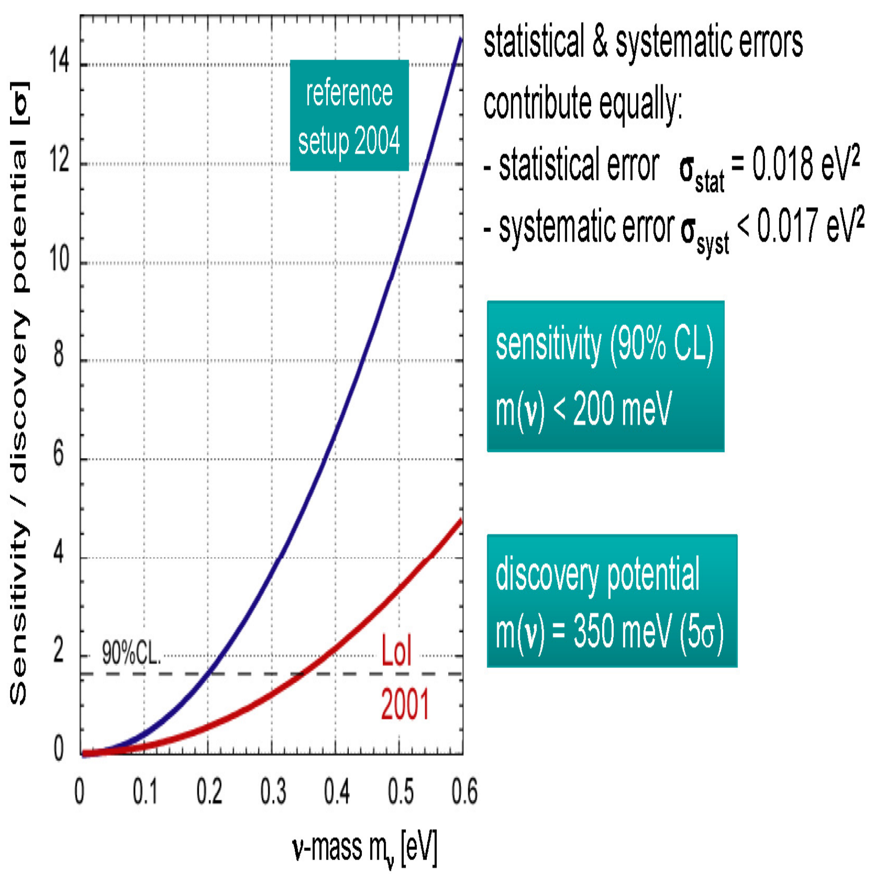


Magnetic Adiabatic Collimation with an Electrostatic Filter

Probably the "ultimate" spectrometer of this kind....

KATRIN sensitivity

ν-mass sensitivity for 3 ´full beam´ measuring years



Mainz + Troitsk: m_β < 2 eV

KATRIN: O(10) improvement

Examples of prospective results at KATRIN (±1σ, [eV]):

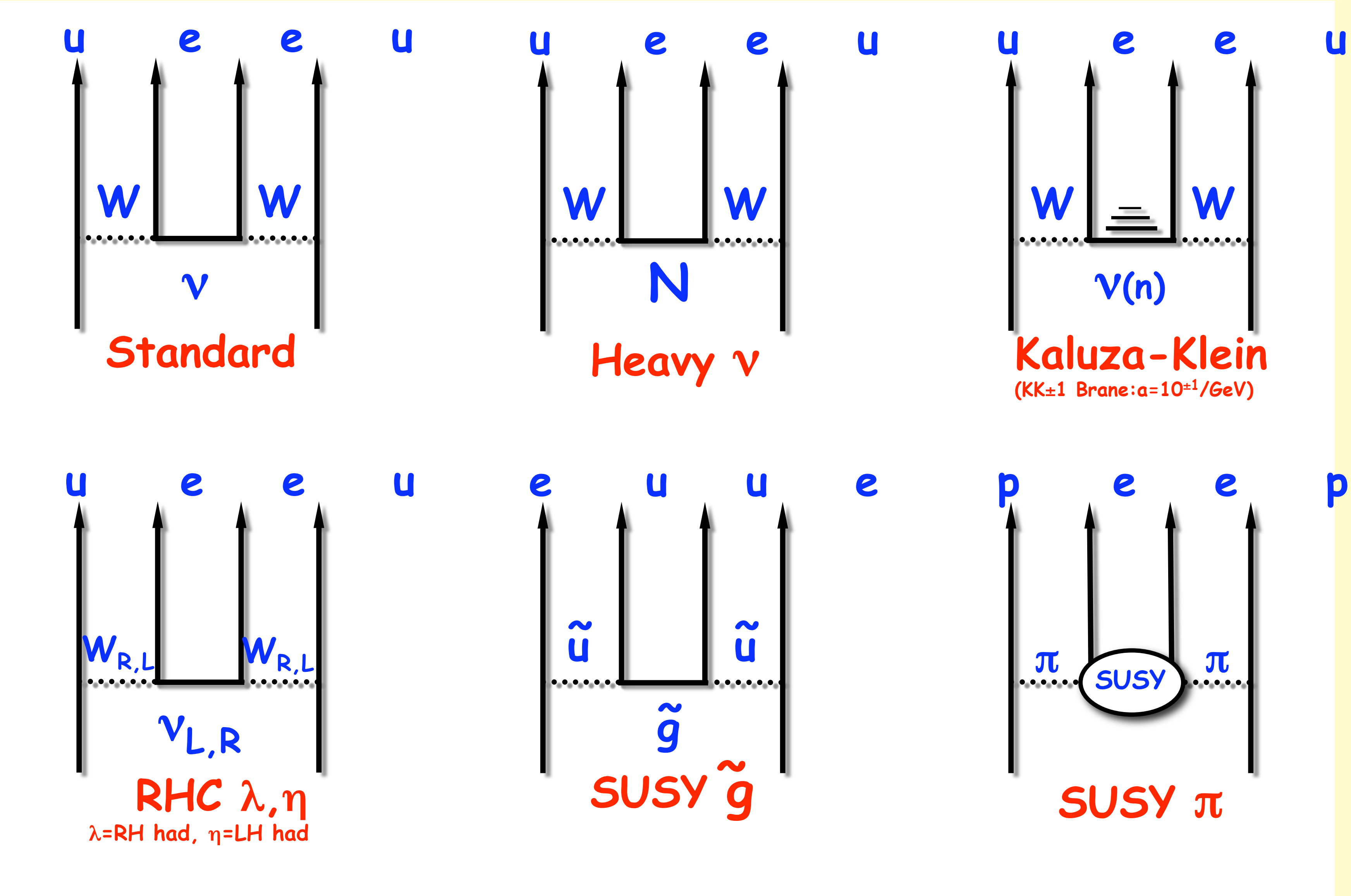
$m_\beta = 0.35 \pm 0.07$ (5σ discovery)

$m_\beta = 0.30 \pm 0.10$ (3σ evidence)

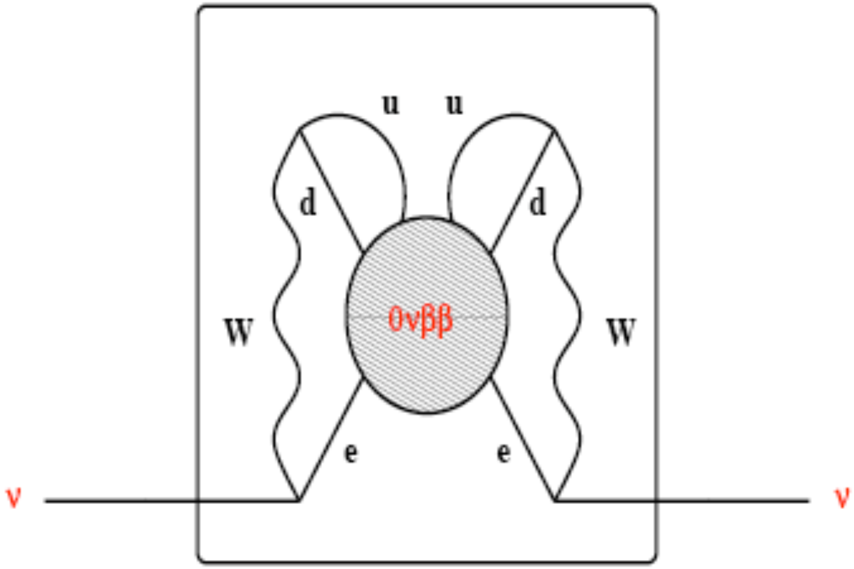
$m_\beta = 0 \pm 0.12$ (<0.2 at 90% CL)

[Need new ideas to go below ~0.2 eV]

0νββ decay: already discussed. Warning: might also arise from new physics!



However: whatever the mechanism…



Schechter & Valle, 1982

Independent of mechanism of $0\nu\beta\beta$ decay Majorana neutrino mass will appear in higher order!

Thus:

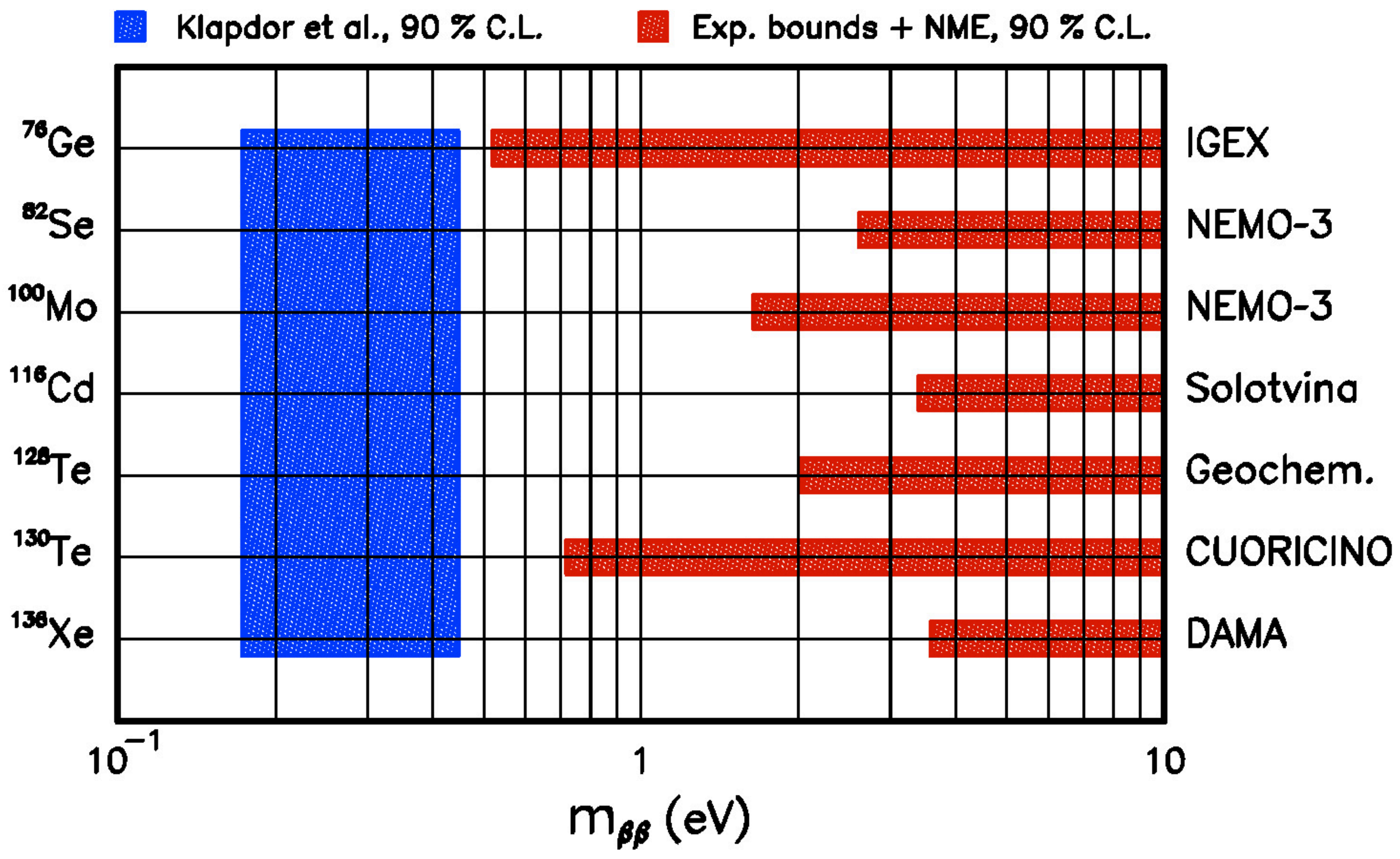
Observe $0\nu\beta\beta$ decay

$\equiv$

Neutrinos are Majorana particles

Assuming standard mechanism and QRPA matrix elements+errors:

Claim versus current limits (in terms of Majorana mass)

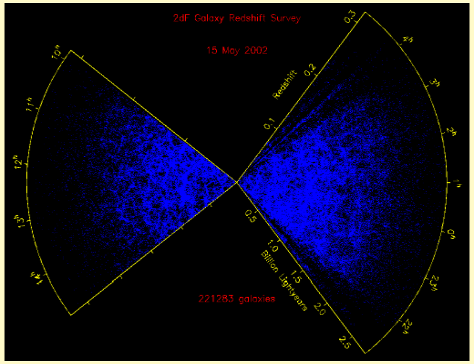


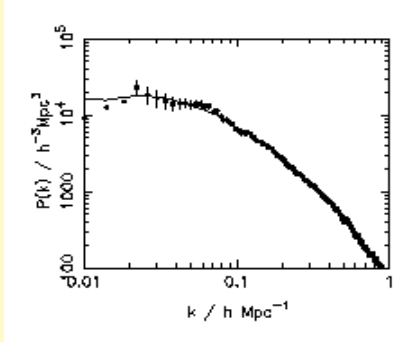
Neutrinos in cosmology: detailed discussions here!

Observations:

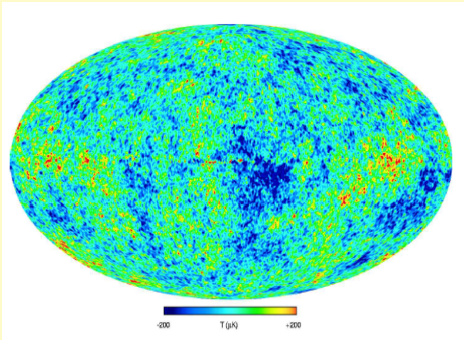
Spectra:

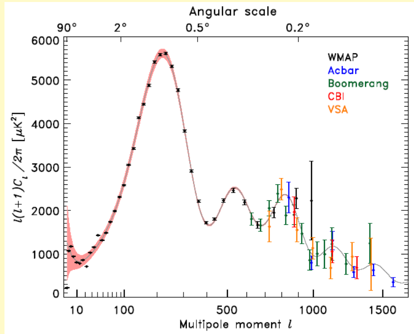
LSS





CMB





Constraints:

TABLE II: Representative cosmological data sets and corresponding 2σ (95% C.L.) constraints on the sum of ν masses Σ.

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

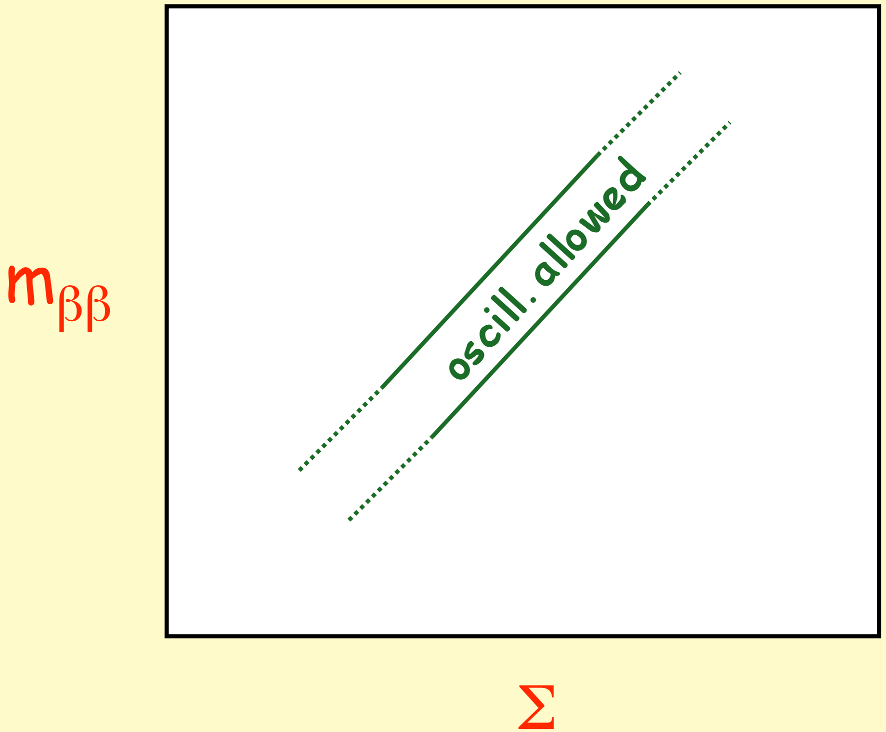
The trident… in action

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

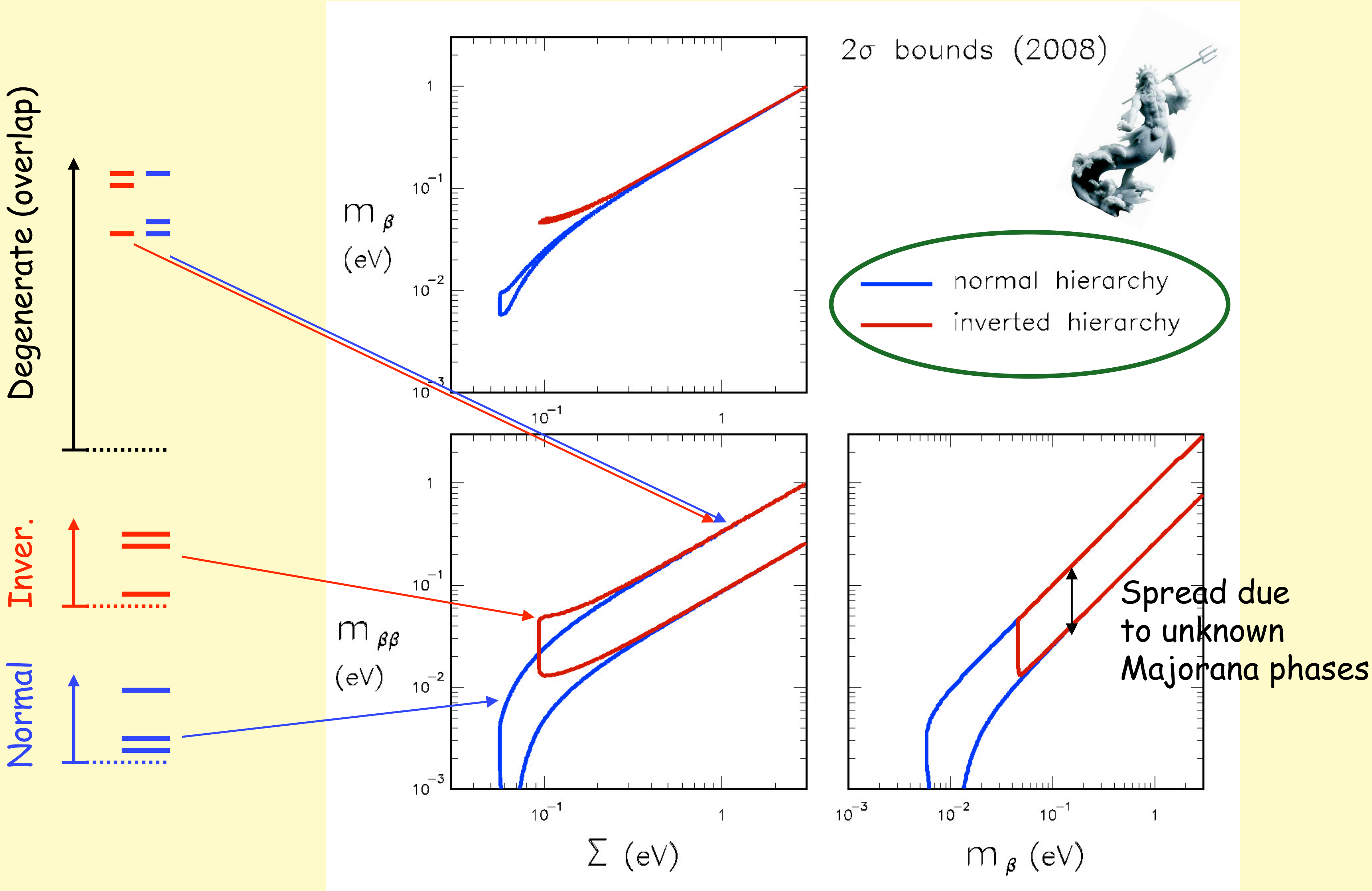
$$\Sigma = m_1 + m_2 + m_3$$

Interplay: Oscillations fix the mass^2 splittings, and thus induce positive correlations between any pair of the three observables (m_β, $m_{\beta\beta}$, Σ), e.g.:

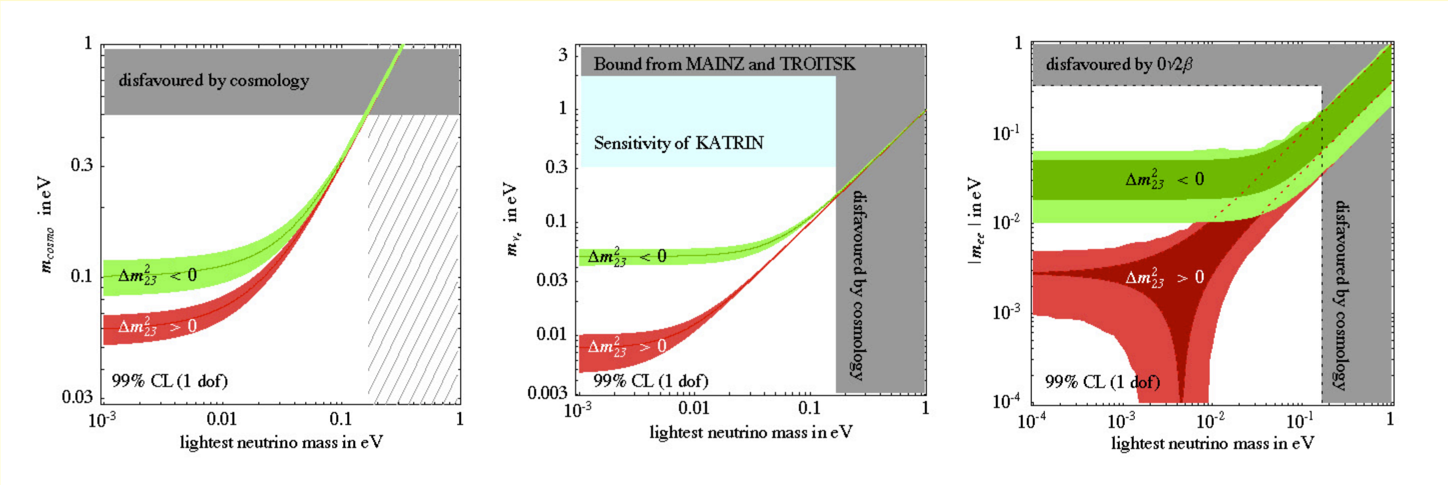


i.e., if one observable increases, the other one (typically) must increase to match mass splitting

The "spear" (oscill. data) sets the "hunting direction" in the $(m_\beta, m_{\beta\beta}, \Sigma)$ parameter space:

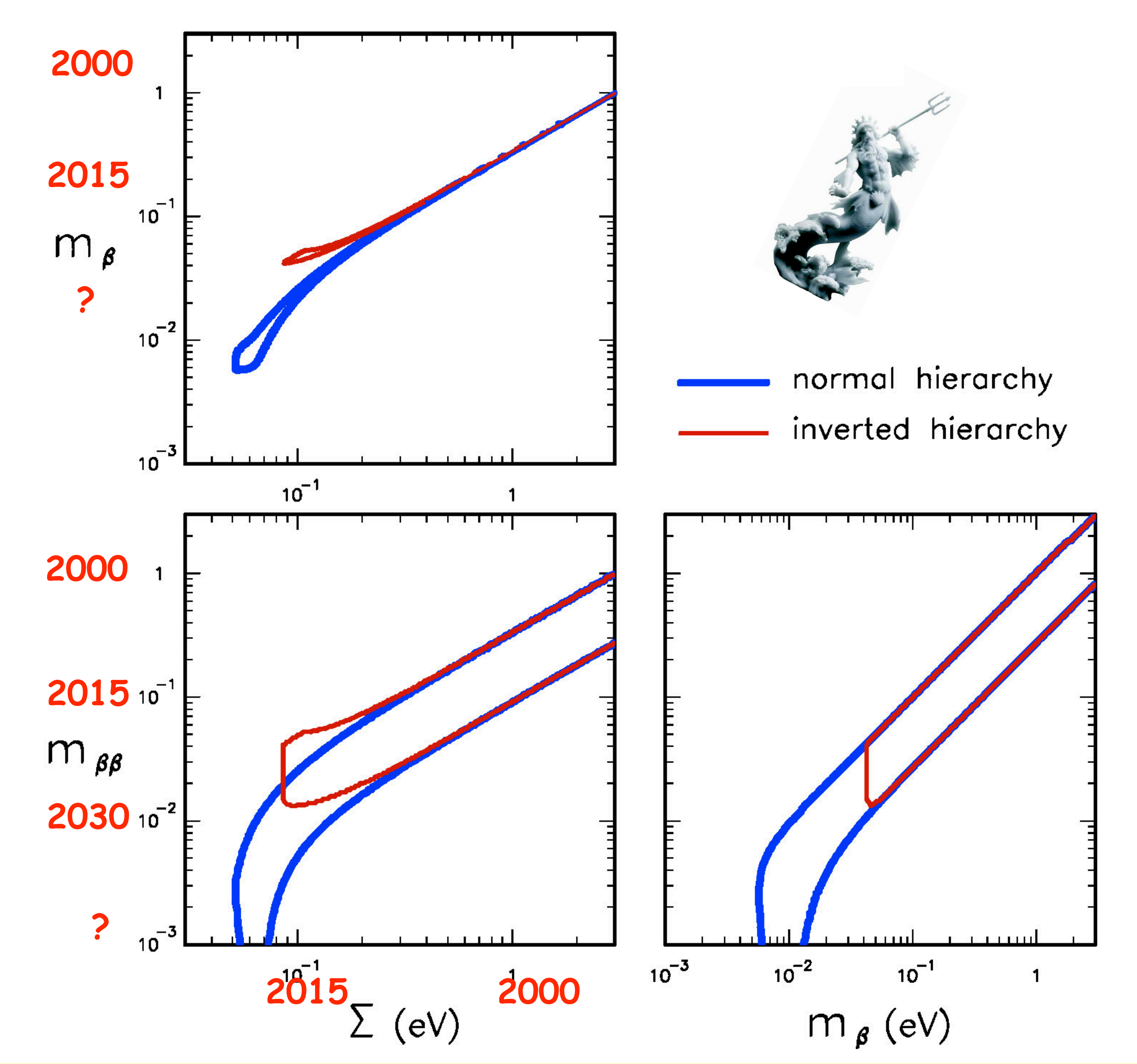


Footnote - Previous plots project away the "unobservable" lightest neutrino mass from graphs like:



Taken from Strumia and Vissani, 2006

History plots → "Moore's law": factor of ~10 improvement every ~15 years



Such "logarithmic progress" seems to be:

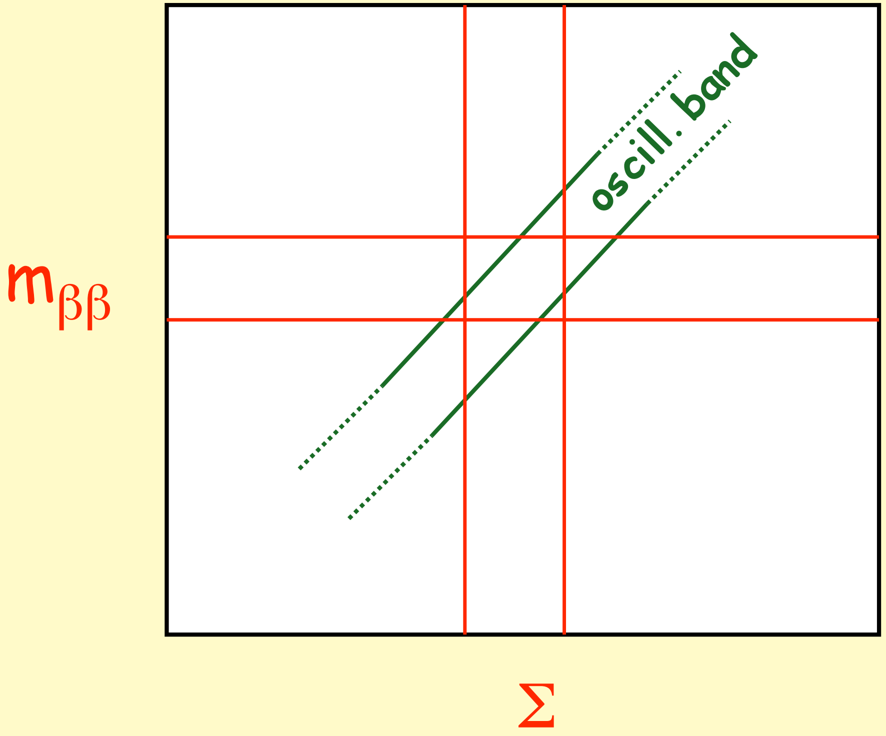
- maybe slowing for β decay (after KATRIN)

- continuing for $0\nu2\beta$ decay

- "accelerating" for cosmology: the only probe where the ultimate goal ($\Sigma_{min} = \sqrt{\Delta m^2} \approx 0.05$ eV) is claimed to be reachable

You have good chances to see first successful results within your career!

Generic expectations: In the absence of new physics (beyond 3ν masses and mixing), any two data among (m_β, $m_{\beta\beta}$, Σ) are expected to cross the oscillation band



This requirement provides either an important consistency check or, if not realized, an indication for new physics (barring expt mistakes)

$\Rightarrow$ Data accuracy/reliability/redundance are crucial

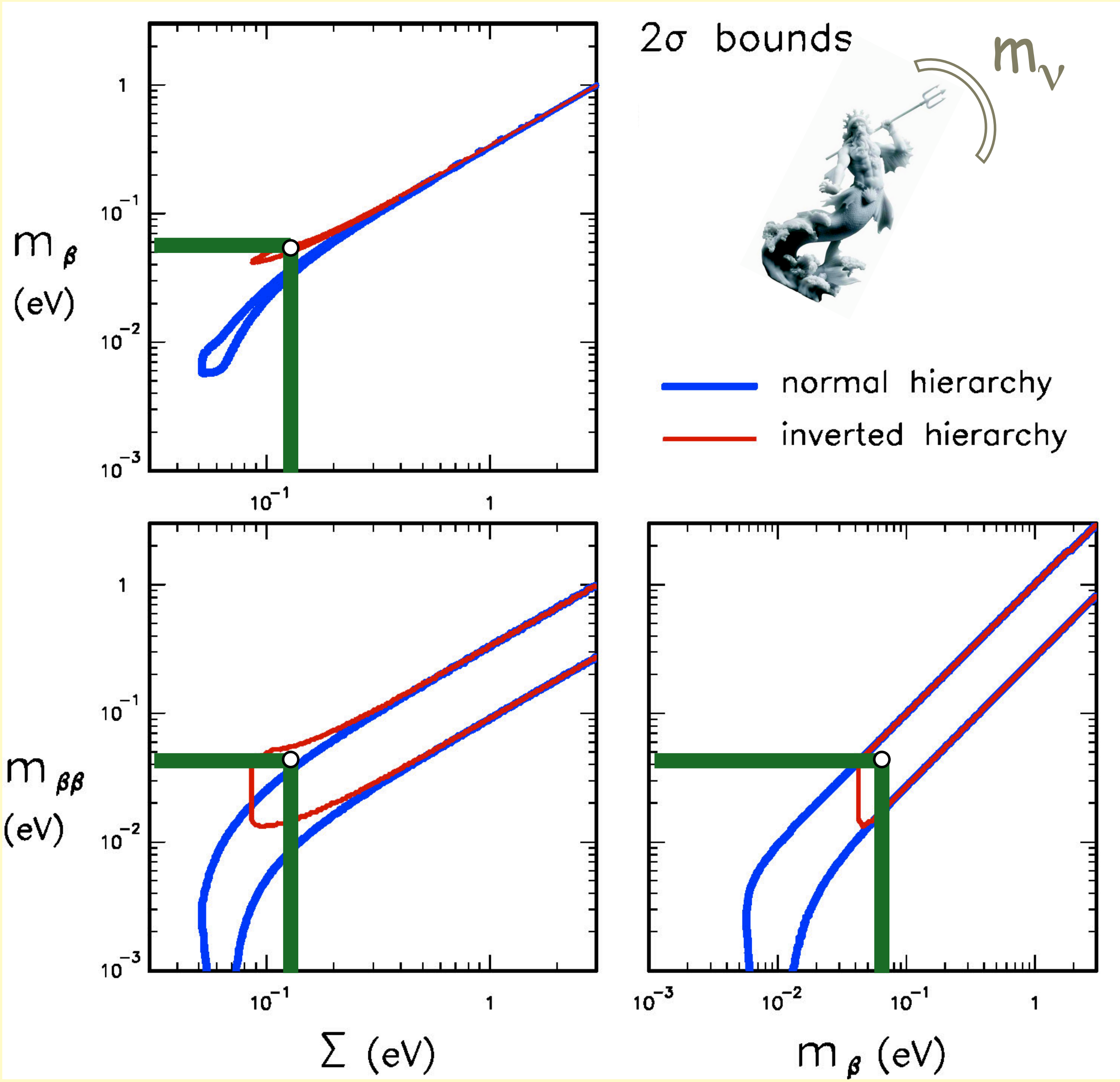
With "dreamlike" data one could, e.g.

Determine the mass scale…

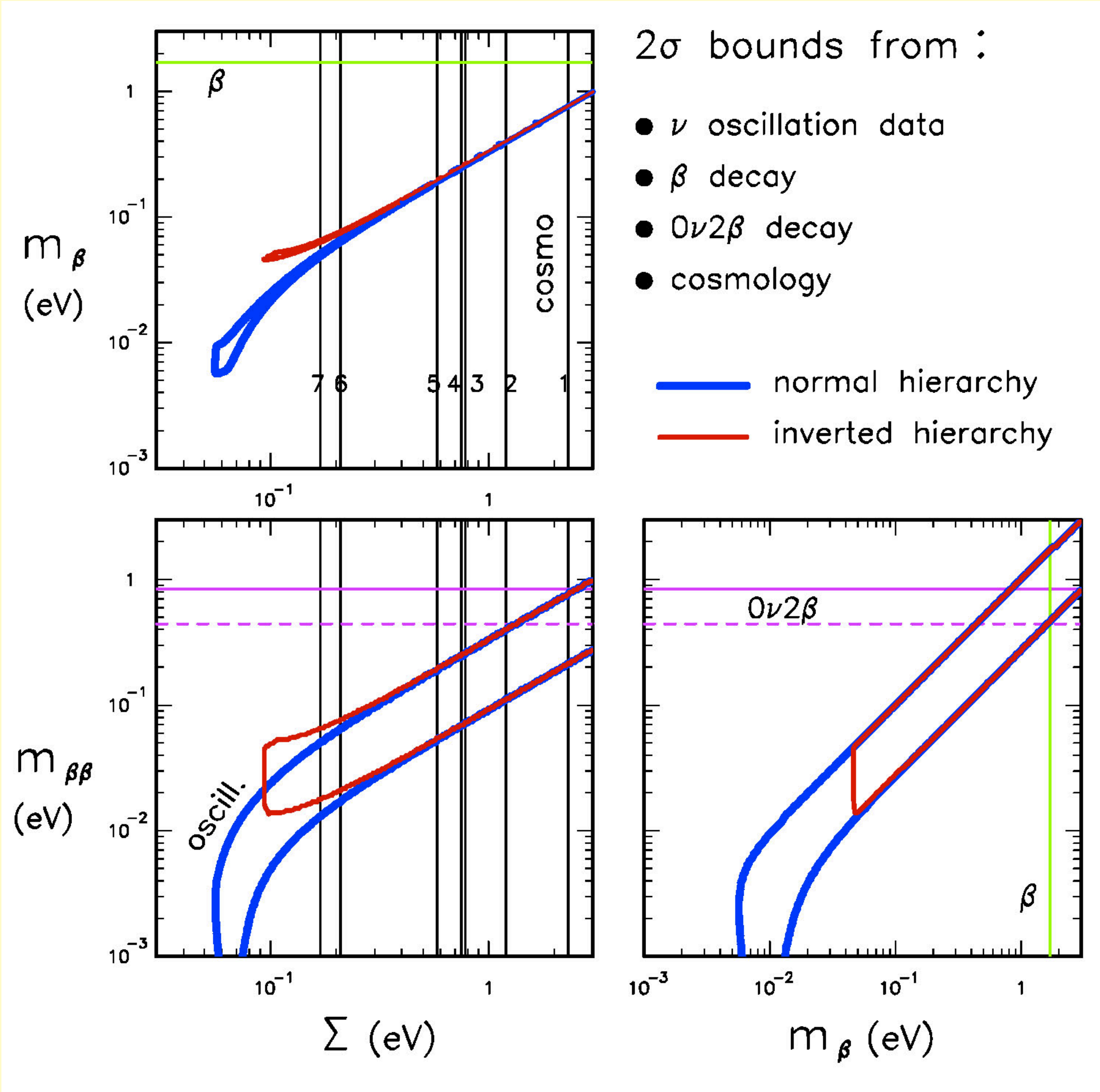
Check 3ν consistency …

Identify the hierarchy …

Probe the Majorana phase(s) …



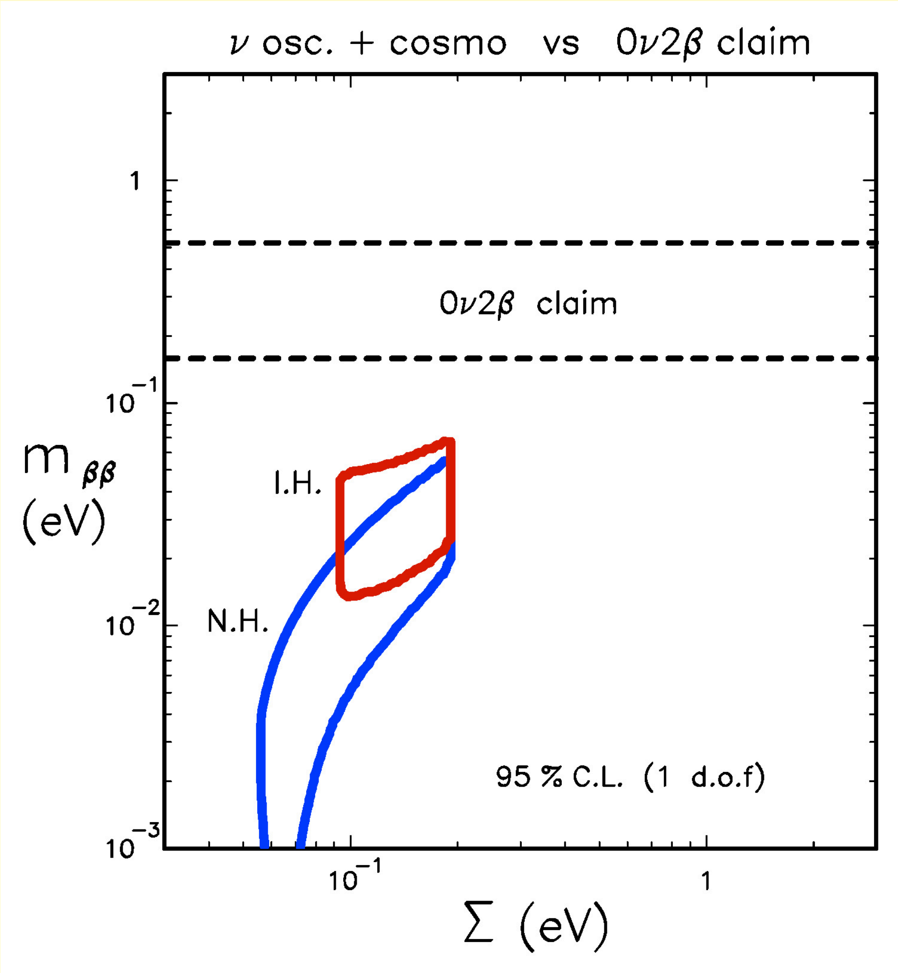
We are still far from this situation (an example with ~2006 data):



Different choices ⇒ Different possible combinations (and implications)

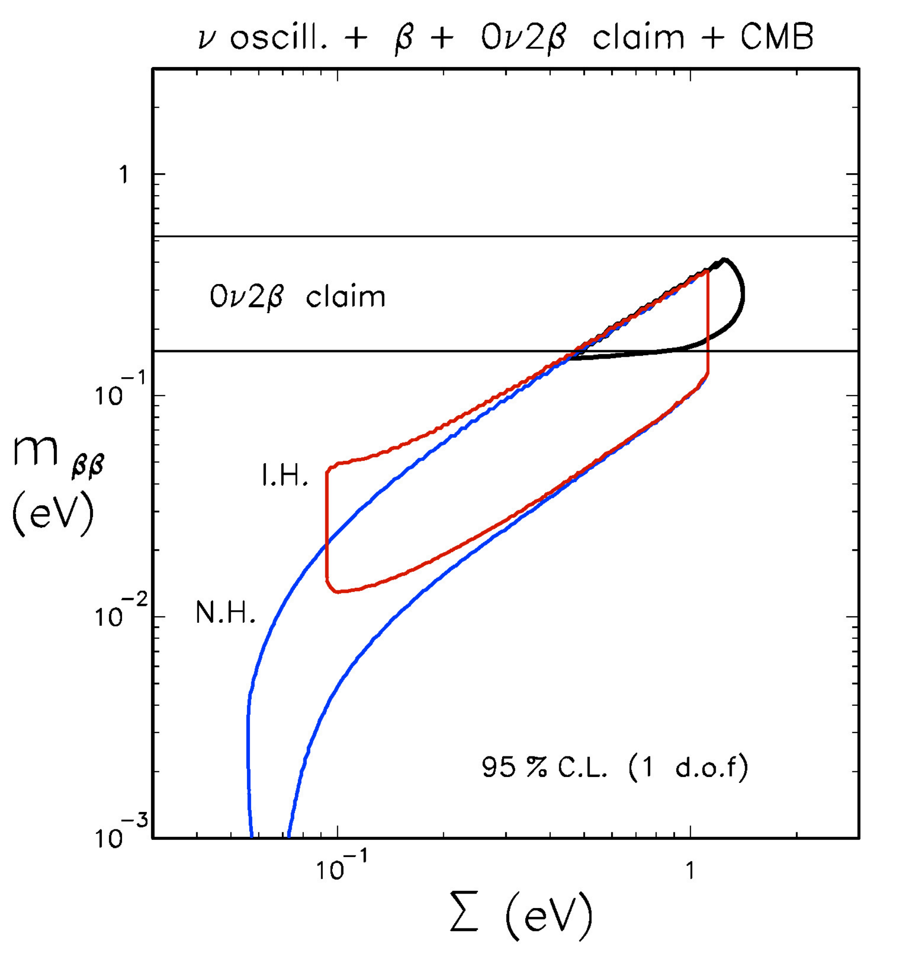
Also the most recent data do not yet lead to definite conclusions. Beta decay: no yet very constraining. Double beta vs cosmology: different possibilities. E.g.,

Cosmo-“aggressive”



The tighest cosmo bounds are not compatible with Klapdor’s claim. Then, either one of the two is wrong, or there is new physics beyond the standard model (of particle physics and/or of cosmology)

Cosmo-"conservative"



Very conservative cosmo bounds can be made compatible with Klapdor's claim, with no new physics required. Then, the combination of data (black wedge) would prefer degenerate neutrino masses, ~few x 10^{-1} eV

Let's entertain the possibility that the "true" answer is just around the corner... For instance, that neutrinos are Majorana, with nearly degenerate and relatively large masses:

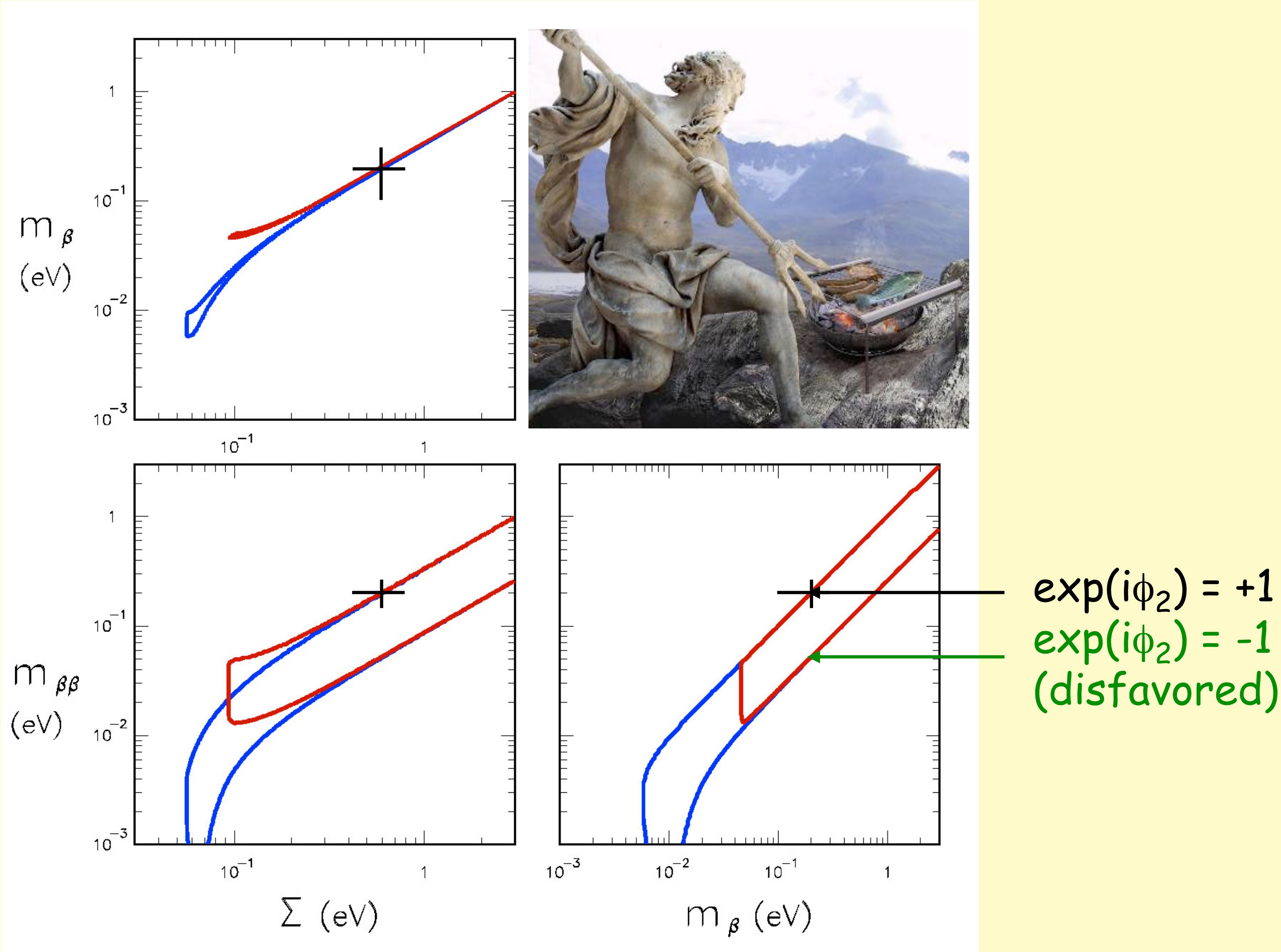
$$m_1 \sim m_2 \sim m_3 \sim 0.2 \text{ eV} .$$

Then we might reasonably hope to observe soon all three nonoscillation signals in next-generation experiments, e.g.,

$$\begin{aligned} m_{\beta\beta} &\simeq 0.2(1 \pm 0.3) \text{ eV} \\ \Sigma &\simeq 0.6(1 \pm 0.3) \text{ eV} \\ m_\beta &\simeq 0.2(1 \pm 0.5) \text{ eV} \end{aligned}$$

in which case...

…The absolute neutrino mass would be established within ~25% uncertainty, and one Majorana phase (ϕ_2) would be constrained…



Absolute masses <u>and</u> mixings crucial for model building

Mixing angles seem to have some "special" values:

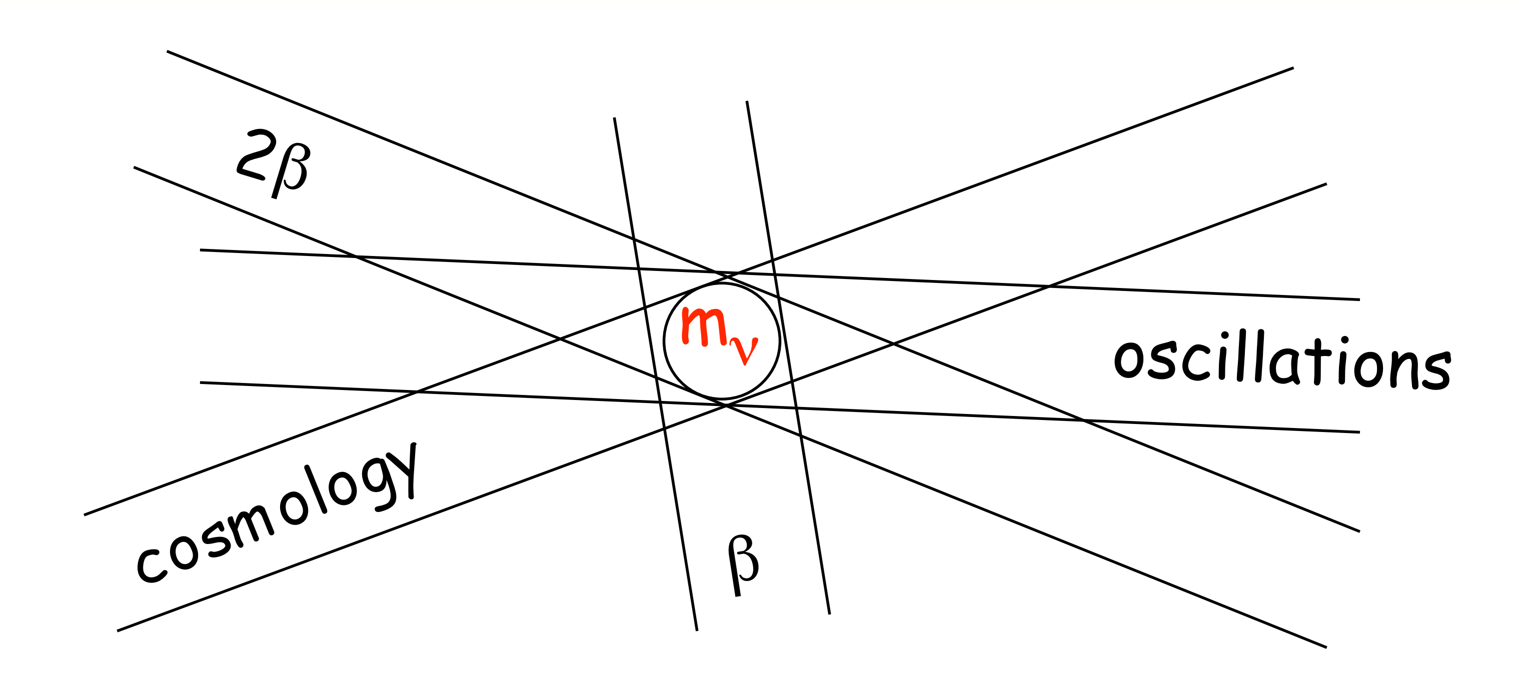
$\sin^2\theta_{23} \approx 1/2$
$\sin^2\theta_{12} \approx 1/3$ "tri-bimaximal mixing"
$\sin^2\theta_{13} \approx 0$
<u>A signal of discrete symmetries in the neutrino sector?</u>

$\theta_{12}+\theta_C \approx \pi/4$ "quark-lepton complementarity"
$[\theta_{23}+\theta_{23,q} \approx \pi/4]$
<u>A possible link between neutrino and quark mixing?</u>

Model diagnostic: also dependent on the above "≈"

RECAP

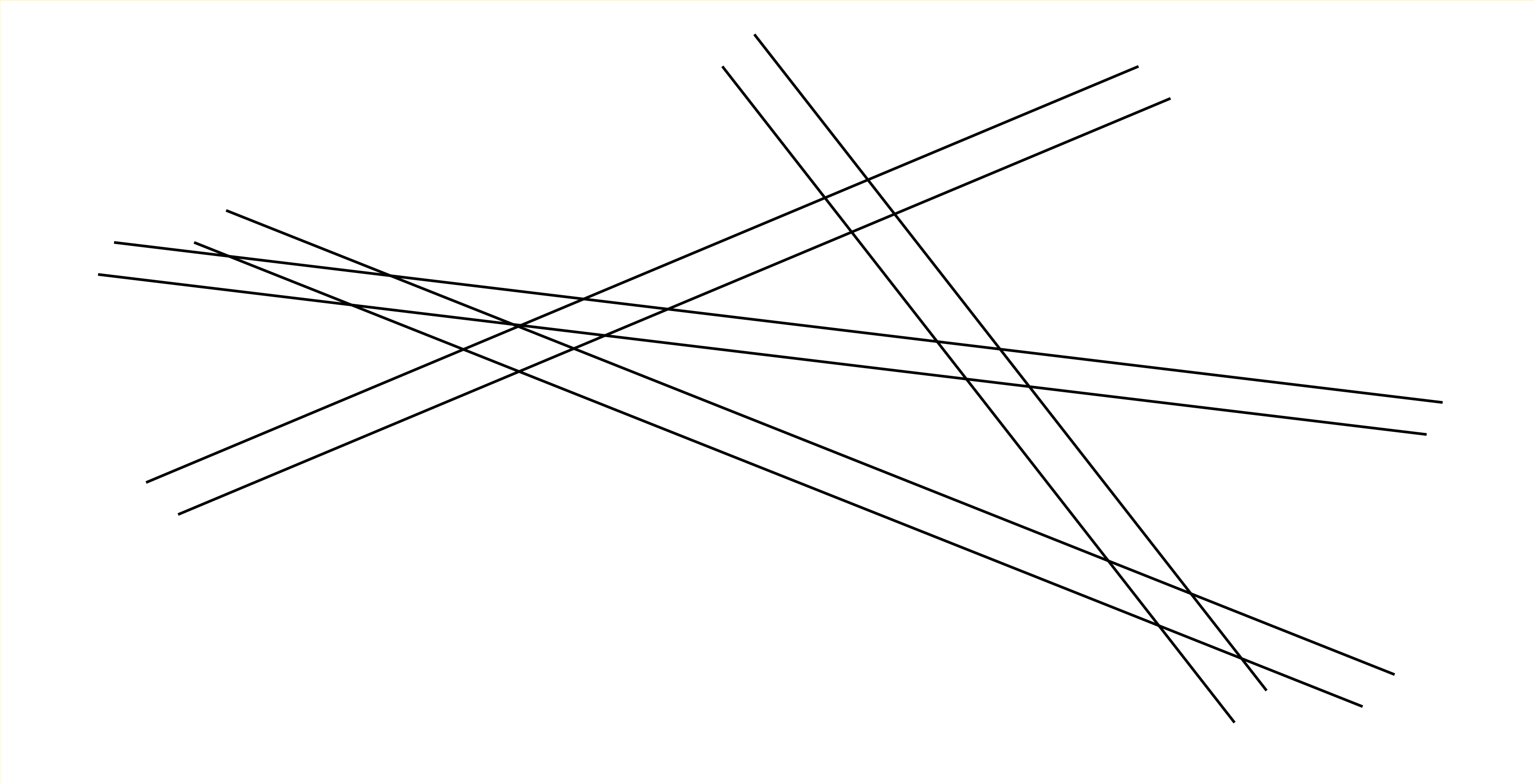
In the (long) process of cornering the neutrino mass …



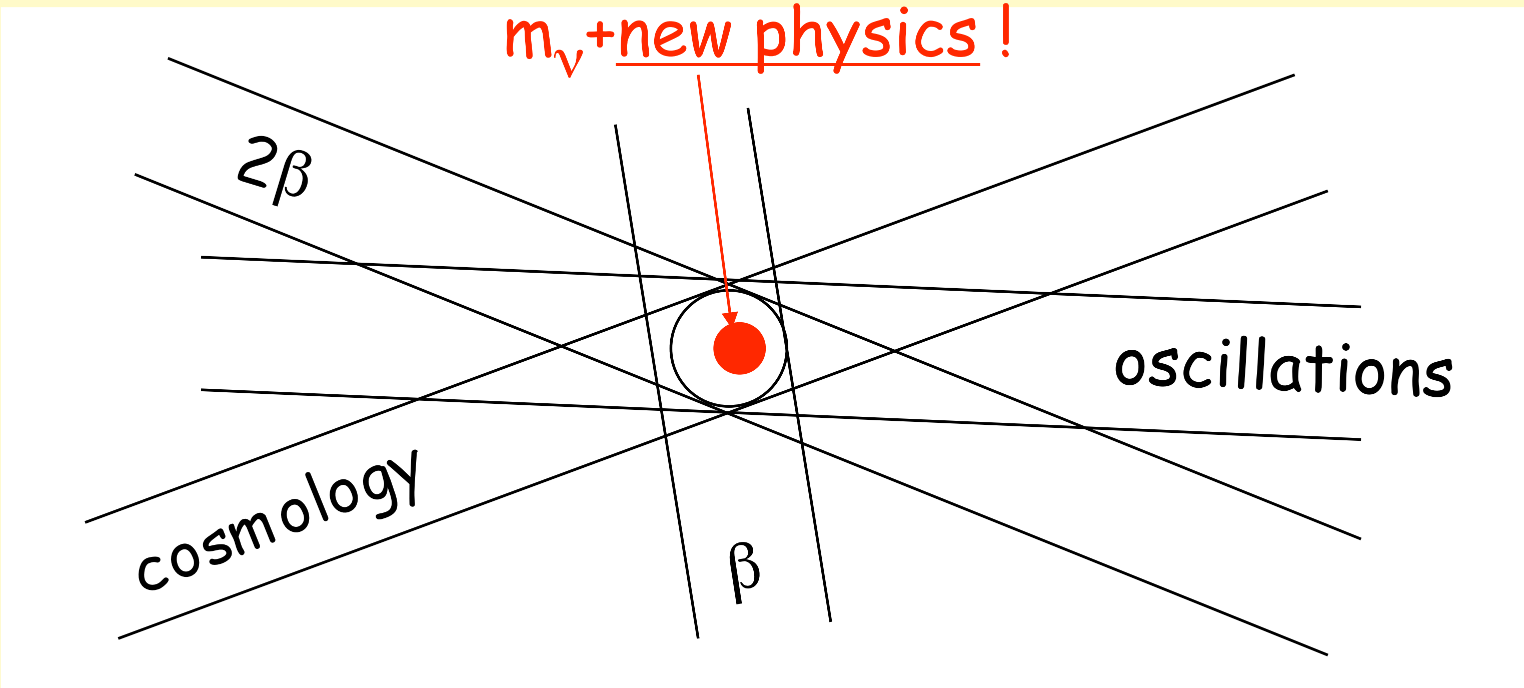
… neutrino oscillations currently provide rather stable and reliable constraints, which will be followed by progress on non-oscillation searches in the next years.

We hope in overall convergence!

Future nightmares, which can't be excluded, might include situations like this (partly realized now?)…

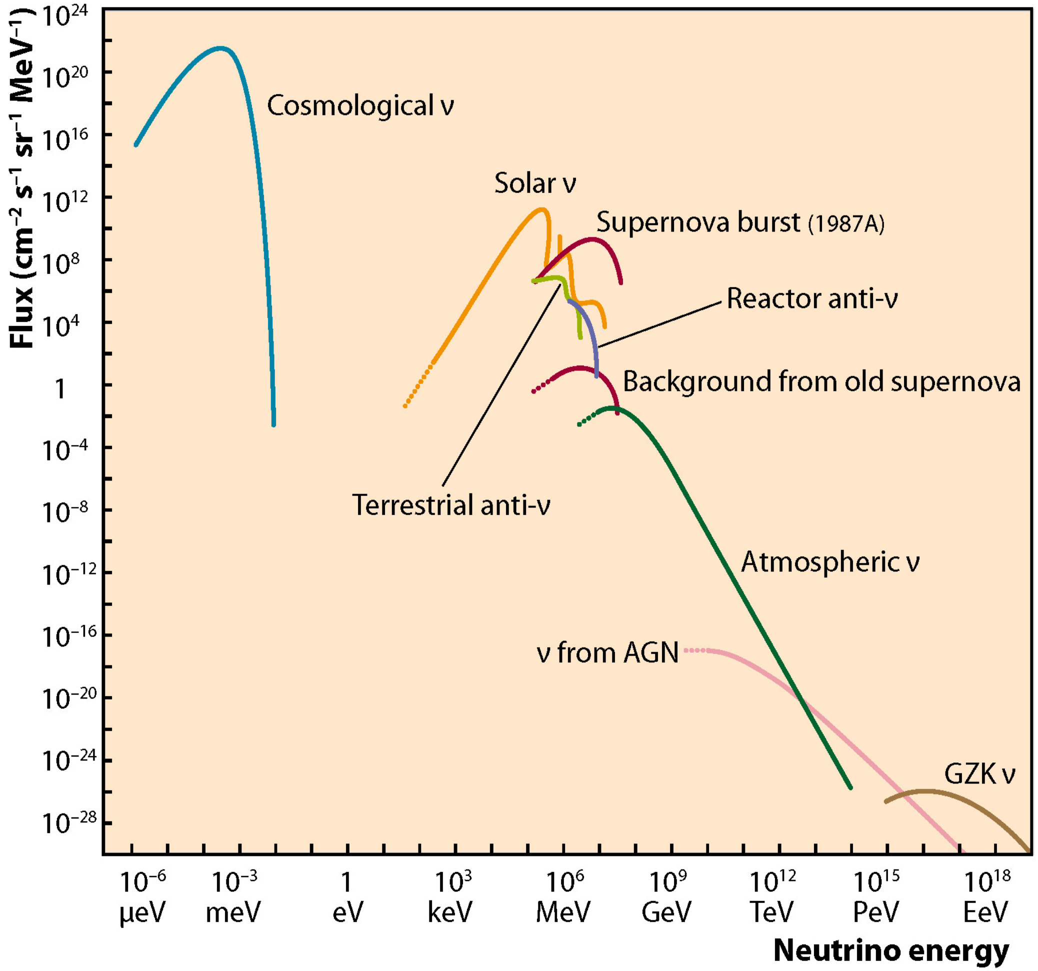


… but we should never forget that such situations might still "converge" if something more exciting happens:


m_ν+new physics !
2β
oscillations
cosmology
β

Progress in Neutrino Physics is not just limited to cornering neutrino mass and mixing parameters… there is much more!

Vast lands to be explored … [Talk by P.Lipari]



A synoptic view of neutrino fluxes. **(from ASPERA roadmap)**

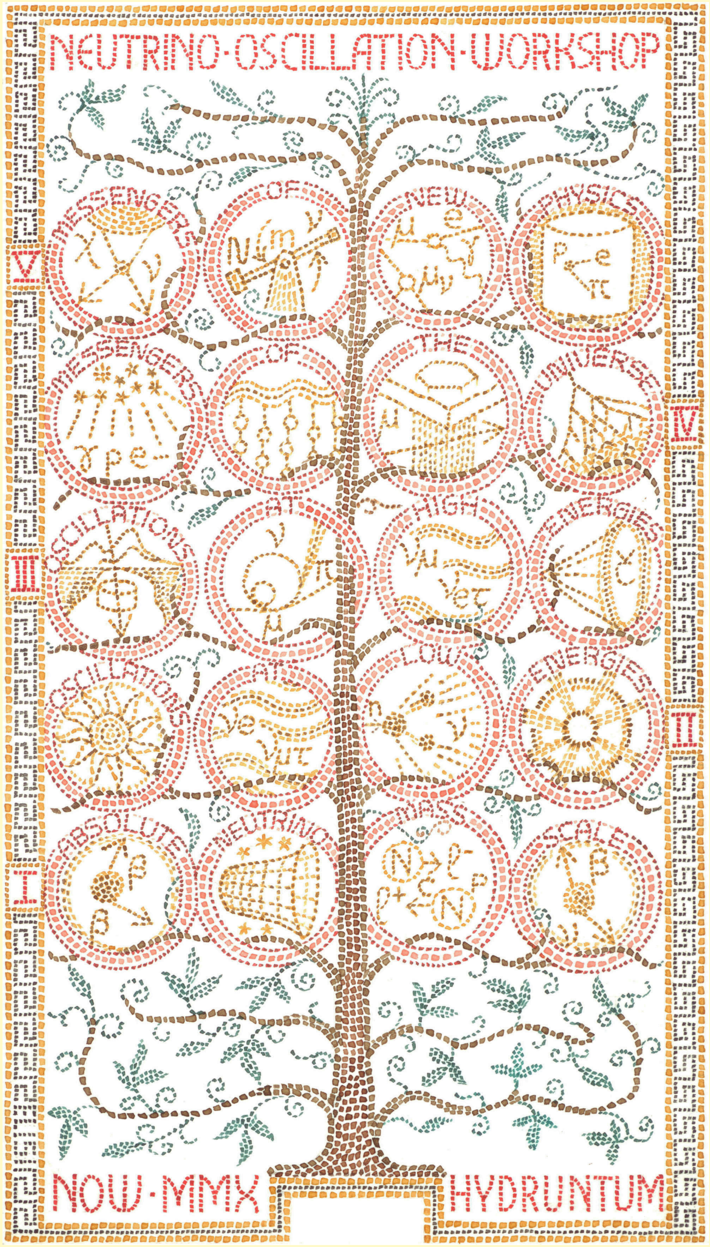
Conclusions and Open Problems

Great progress in recent years ...

Neutrino mass & mixing: established fact
Determination of $(\delta m^2, \theta_{12})$ and $(\Delta m^2, \theta_{23})$
Upper bounds on θ_{13}
Observation of (half)-period of oscillations
Direct evidence for solar ν flavor change
Evidence for matter effects in the Sun
Upper bounds on ν masses in (sub)eV range
....

Determination of θ_{13}
Appearance of ν_e, ν_τ
Leptonic CP violation
Absolute m_ν from β-decay and cosmology
Test of $0\nu2\beta$ claim and of Dirac/Majorana ν
Matter effects in the Earth, Supernovae...
Normal vs inverted hierarchy
Beyond standard 3ν scenario (sterile...)?
Deeper theoretical understanding
Neutrino geo- and astro-physics
....

... and great challenges for the future!



NOW 2010 Poster: www.ba.infn.it/now

The neutrino tree continues to grow.

Many opportunities open for your research activity!

Thank you for your attention.

HOMEWORK

Solution 5

- Mass basis: $\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$, $\Delta m^2 = m_2^2 - m_1^2$
- Flavor basis: $\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$; $\nu_x = \nu_{\mu,\tau}$; $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

- Hamiltonian in vacuum, flavor basis (see Exercise 2):

$$H = U \begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & +\frac{\Delta m^2}{4E} \end{pmatrix} U^T$$

- Hamiltonian in matter, flavor basis:

$H \to \tilde{H} = H + \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}$ with $V = \sqrt{2}\, G_F N_e$ (extra ν_e energy in matter)

- It is convenient to put $\tilde{H}$ in traceless form (extract $\mathrm{tr}(\tilde{H}) \cdot \mathbb{1}$):

$$\tilde{H} = \frac{1}{4E} \begin{bmatrix} A - \cos 2\theta\, \Delta m^2 & \sin 2\theta\, \Delta m^2 \\ \sin 2\theta\, \Delta m^2 & -A + \cos 2\theta\, \Delta m^2 \end{bmatrix} \quad , A = 2VE$$

(diagonalization becomes easier)

Solution 5 (ctd)

- Eigenvalues of $\tilde{H}$: $\pm \frac{\Delta\tilde{m}^2}{4E}$ with $\Delta\tilde{m}^2 = \Delta m^2 \sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}$

- Diagonalizing rotation:

$$\tilde{H} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} -\frac{\Delta\tilde{m}^2}{4E} & 0 \\ 0 & +\frac{\Delta\tilde{m}^2}{4E} \end{pmatrix} \begin{pmatrix} \cos\tilde{\theta} & -\sin\tilde{\theta} \\ \sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}$$

with $\sin 2\tilde{\theta} = \dfrac{\sin 2\theta}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$; $\cos 2\tilde{\theta} = \dfrac{\cos 2\theta - \frac{A}{\Delta m^2}}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$

This ~~a~~ is analogous to the vacuum case, with the replacement $\theta \to \tilde{\theta}$ and $\Delta m^2 \to \Delta\tilde{m}^2$. [Note that $\Delta\tilde{m}^2 \sin 2\tilde{\theta} = \Delta m^2 \sin 2\theta$].

- If $A = \text{const}$ (i.e., $\tilde{\theta}$ is constant), then the evolution operator can be obtained by exponentiation as in Exercise 2. Then one gets in a similar way:

$$P(\nu_e \to \nu_x) = \sin^2 2\tilde{\theta} \sin^2\left(\frac{\Delta\tilde{m}^2 L}{4E}\right).$$

Solution 6

Let's prove first that $1\ \frac{\text{mol}}{\text{cm}^3} = 4.267 \times 10^{-9}\ \text{MeV}^3$

with $1\ \text{mol} = 6.022 \times 10^{23}$ particles (Avogadro number):

$$1\ \frac{\text{mol}}{\text{cm}^3} = \frac{6.022\times 10^{23}}{10^{-6}\,\text{m}^3}\left(\frac{\text{MeV}^3}{\text{MeV}^3}\right) = 6.022\times 10^{29}\ \frac{1}{(\text{m}\cdot\text{MeV})^3}\ \text{MeV}^3 = \frac{6.022\times 10^{29}}{(5.0677\times 10^{12})^3}\ \text{MeV}^3$$

$$= 4.627 \times 10^{-9}\ \text{MeV}^3$$

see Exercise 4

Then: $A = 2\sqrt{2}\, G_F N_e E$ with $G_F = 1.16637\times 10^{-5}\ \text{GeV}^{-2} = 1.16637\times 10^{-11}\ \text{MeV}^{-2}$ (Fermi const.)

$$\frac{A}{\Delta m^2} = \frac{2\sqrt{2}\, G_F N_e E}{\Delta m^2} = 2\sqrt{2}\left(1.16637\times 10^{-11}\ \text{MeV}^{-2}\right)\left(\frac{N_e}{\text{mol/cm}^3}\cdot \text{mol/cm}^3\right)\left(\frac{E}{\text{MeV}}\cdot\text{MeV}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\cdot\frac{1}{\text{eV}^2}\right)$$

$$= 3.299\times 10^{-11}\ \frac{\text{MeV}^{-2}\,\text{MeV}}{\text{eV}^2}\ \frac{\text{mol}}{\text{cm}^3}\left(\frac{N_e}{\text{mol/cm}^3}\right)\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\right)$$

$$3.299\times 10^{-11}\ \frac{\text{MeV}^{-2}\,\text{MeV}}{\text{eV}^2}\ \frac{\text{mol}}{\text{cm}^3} = 3.299\times 10^{-11}\ \frac{10^{12}}{\cancel{\text{MeV}^3}} \times 4.627\times 10^{-9}\ \cancel{\text{MeV}^3} = 1.526\times 10^{-7}$$

$$\frac{A}{\Delta m^2} = 1.526\times 10^{-7}\left(\frac{N_e}{\text{mol/cm}^3}\right)\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\right)$$

Appendix: Adiabatic approximation

Adiabatic 2ν case

Prove that, if $N_e(x)$ changes "slowly" from $x = x_i$ to $x = x_f$, then the AVERAGE survival probability $P(\nu_e \to \nu_e)$ is given by:

$$\boxed{P_{ee}^{(2\nu)}(\text{adiab.}) = \cos^2\tilde{\theta}_i \cos^2\tilde{\theta}_f + \sin^2\tilde{\theta}_i \sin^2\tilde{\theta}_f}$$

where $\tilde{\theta}_i$ and $\hat{\theta}_f$ are the effective mixing angles in matter at $x = x_i$ and $x = x_f$.

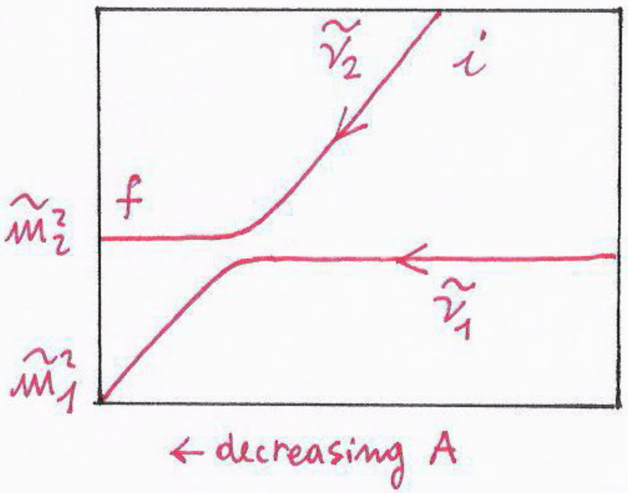
This is a good approximation for solar ν_e's, for the $(\delta m^2, \theta_{12})$ parameters chosen by nature!

Solution:

For a quasi-constant hamiltonian, one can solve the evolution equation at "one x" at a time, and then patch the solutions from x_i to x_f. This means that, given the initial state

$$|\nu_e^i\rangle = \cos\tilde{\theta}_i \, |\tilde{\nu}_1^i\rangle + \sin\tilde{\theta}_i \, |\tilde{\nu}_2^i\rangle \quad ,$$

the effective eigenstates of the hamiltonian at $x = x_i$ ($|\tilde{\nu}_{1,2}^i\rangle$) slowly transform into $|\tilde{\nu}_{1,2}^f\rangle$ at $x = x_f$, respectively:



$$|\tilde{\nu}_1^i\rangle \rightarrow |\tilde{\nu}_1^f\rangle \quad \text{with} \quad |\langle\nu_1^f|\nu_1^i\rangle| = 1$$

$$|\tilde{\nu}_2^i\rangle \rightarrow |\tilde{\nu}_2^f\rangle \quad \text{with} \quad |\langle\nu_2^f|\nu_2^i\rangle| = 1$$

$$\text{and} \quad |\langle\nu_{2,1}^f|\nu_{1,2}^i\rangle| = 0$$

(no "level crossing")

$$|\nu_e^f\rangle = \cos\tilde{\theta}_f \, |\nu_1^f\rangle + \sin\tilde{\theta}_f \, |\nu_2^f\rangle$$

We have then :

$$P_{ee}^{2\nu} = |\langle \nu_e^f | \nu_e^i \rangle|^2 =$$
$$= |\cos\hat{\theta}_i \cos\hat{\theta}_f \langle \nu_1^f | \nu_1^i \rangle + \sin\hat{\theta}_i \sin\hat{\theta}_f \langle \nu_2^f | \nu_2^i \rangle|^2 .$$

If we average out interference terms and phases (ok for many/fast oscillations along the ν trajectory) :

$$P_{ee}^{2\nu} \simeq \cos^2\hat{\theta}_i \cos^2\hat{\theta}_f \, |\langle \nu_1^f | \nu_1^i \rangle|^2 + \sin^2\hat{\theta}_i \sin^2\hat{\theta}_f \, |\langle \nu_2^f | \nu_2^i \rangle|^2$$
$$= \cos^2\hat{\theta}_i \cos^2\hat{\theta}_f + \sin^2\hat{\theta}_i \sin^2\hat{\theta}_f$$

[Solar ν's oscillate many times from Sun to Earth for $\delta m^2 \sim 7.7 \times 10^{-5}$ eV^2.]

Equivalent form: $P_{ee}^{2\nu} = \frac{1}{2} + \frac{1}{2} \cos 2\hat{\theta}_i \cos 2\hat{\theta}_f$

Application to solar ν_e :

$\theta = \theta_{12}$; $\tilde{\theta}_{12}(x_f) = \theta_{12}$ (vacuum value at exit from the Sun)

$$\boxed{P_{ee}^{2\nu}(\text{solar}) \simeq \cos^2\theta_{12}\cos^2\tilde{\theta}_{12}(x_0) + \sin^2\theta_{12}\sin^2\tilde{\theta}_{12}(x_0)}$$

where $\tilde{\theta}(x_0)$ is the effective mixing angle at production point x_0.

limiting cases:

- **E $\lesssim$ few MeV (vacuum-dominated)** : $A/\delta m^2 \lesssim 1$

$\rightarrow$ $\tilde{\theta}_{12}(x_0) \simeq \theta_{12}$ and $P_{ee} \simeq c_{12}^4 + s_{12}^4 = 1 - \frac{1}{2}\sin^2 2\theta_{12}$

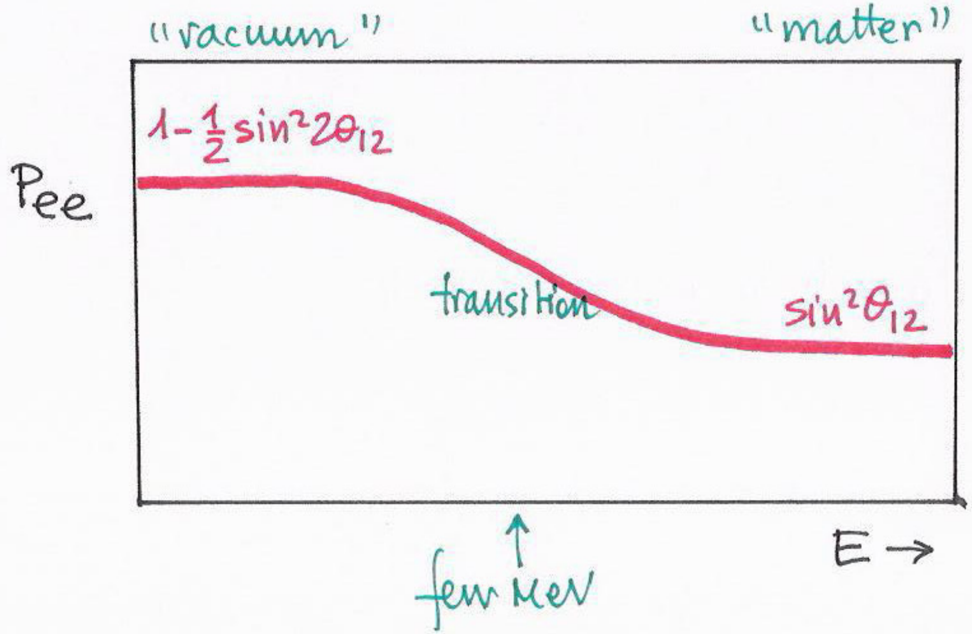
$\rightarrow$ Pee equals the average vacuum probability

- **E $\gtrsim$ few MeV (matter dominated)** : $A/\delta m^2 \gtrsim 1$

$\rightarrow$ $\tilde{\theta}_{12}(x_0) \sim \pi/2$ and $P_{ee} \simeq \sin^2\theta_{12}$

$\rightarrow$ Pee is octant asymmetric

Energy profile of P_{ee}:



The P_{ee} transition from "low" to "high" energy ν_e's is a signature of matter effects.

Thanks to matter effects we can determine the octant of the mixing angle θ_{12}.