Dirac and Majorana Neutrino Masses

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Neutrino Unbound: http://www.nu.to.infn.it

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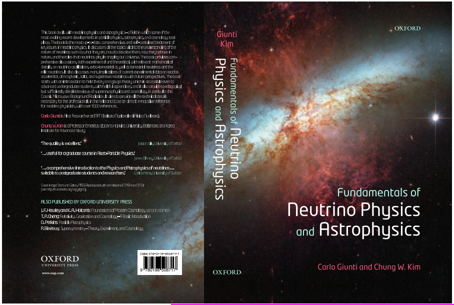
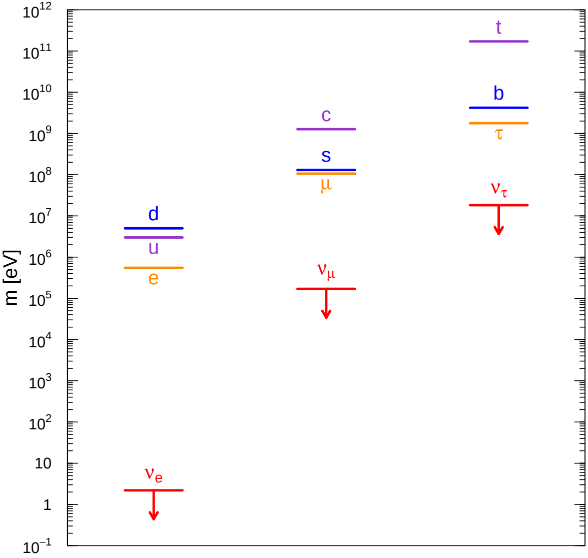


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics
and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Fermion Mass Spectrum



Dirac Lepton Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}^{\text{D}} = -y^{\ell}\,\overline{L_L}\,\Phi\,\ell_R - y^{\nu}\,\overline{L_L}\,\widetilde{\Phi}\,\nu_R + \text{H.c.}$$

Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2\,\Phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}^{\text{D}} = & -\frac{y^{\ell}}{\sqrt{2}}\begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\ell_R \\ & -\frac{y^{\nu}}{\sqrt{2}}\begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix}\begin{pmatrix} v \\ 0 \end{pmatrix}\nu_R + \text{H.c.} \end{aligned}$$

$$\mathscr{L}^{\mathrm{D}} = -y^{\ell} \frac{v}{\sqrt{2}} \overline{\ell_L} \ell_R - y^{\nu} \frac{v}{\sqrt{2}} \overline{\nu_L} \nu_R + \mathrm{H.c.}$$

$$m_{\ell} = y^{\ell} \frac{v}{\sqrt{2}} \qquad\qquad m_{\nu} = y^{\nu} \frac{v}{\sqrt{2}}$$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}^{\text{D}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \text{H.c.}$$

Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\mathscr{L}^{\text{D}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[\frac{v}{\sqrt{2}} Y'^{\ell}_{\alpha\beta} \, \overline{\ell'_{\alpha L}} \, \ell'_{\beta R} + \frac{v}{\sqrt{2}} Y'^{\nu}_{\alpha\beta} \, \overline{\nu'_{\alpha L}} \, \nu'_{\beta R} \right] + \text{H.c.}$$

$$\mathscr{L}^{\text{D}} = - \left[\overline{\boldsymbol{\ell}'_L} \, M'^{\ell} \, \boldsymbol{\ell}'_R + \frac{v}{\sqrt{2}} \overline{\boldsymbol{\nu}'_L} \, M'^{\nu} \, \boldsymbol{\nu}'_R \right] + \text{H.c.}$$

$$\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \boldsymbol{\ell}'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$M'^{\ell} = \frac{v}{\sqrt{2}} \, Y'^{\ell} \qquad M'^{\nu} = \frac{v}{\sqrt{2}} \, Y'^{\nu}$$

$$M'^{\ell} \equiv \begin{pmatrix} M'^{\ell}_{ee} & M'^{\ell}_{e\mu} & M'^{\ell}_{e\tau} \\ M'^{\ell}_{\mu e} & M'^{\ell}_{\mu\mu} & M'^{\ell}_{\mu\tau} \\ M'^{\ell}_{\tau e} & M'^{\ell}_{\tau\mu} & M'^{\ell}_{\tau\tau} \end{pmatrix} \qquad M'^{\nu} \equiv \begin{pmatrix} M'^{\nu}_{ee} & M'^{\nu}_{e\mu} & M'^{\nu}_{e\tau} \\ M'^{\nu}_{\mu e} & M'^{\nu}_{\mu\mu} & M'^{\nu}_{\mu\tau} \\ M'^{\nu}_{\tau e} & M'^{\nu}_{\tau\mu} & M'^{\nu}_{\tau\tau} \end{pmatrix}$$

$$\mathscr{L}^{\mathrm{D}} = -\overline{\boldsymbol{\ell}'_L}\, M'^{\ell}\, \boldsymbol{\ell}'_R - \overline{\boldsymbol{\nu}'_L}\, M'^{\nu}\, \boldsymbol{\nu}'_R + \mathrm{H.c.}$$

Diagonalization of M'^{ℓ} and M'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \mathbf{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \mathbf{n}_R$$

Kinetic terms are invariant under unitary transformations of the fields

$$\mathscr{L}^{\mathrm{D}} = -\overline{\boldsymbol{\ell}_L} V^{\ell\dagger}_L M'^{\ell} V^{\ell}_R \boldsymbol{\ell}_R - \overline{\boldsymbol{\nu}_L} V^{\nu\dagger}_L M'^{\nu} V^{\nu}_R \boldsymbol{\nu}_R + \mathrm{H.c.}$$

$$V^{\ell\dagger}_L \; M'^{\ell} \; V^{\ell}_R = M^{\ell} \qquad M^{\ell}_{\alpha\beta} = m^{\ell}_{\alpha}\, \delta_{\alpha\beta} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$V^{\nu\dagger}_L \; M'^{\nu} \; V^{\nu}_R = M^{\nu} \qquad M^{\nu}_{kj} = m^{\nu}_{k}\, \delta_{kj} \qquad (k, j = 1, 2, 3)$$

Real and Positive m^{ℓ}_{α}, m^{ν}_{k}

Massive Chiral Lepton Fields

$\ell_L = V_L^{\ell\dagger} \ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\ell_R = V_R^{\ell\dagger} \ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger} \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger} \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}^{\mathrm{D}} &= -\,\overline{\ell_L}\, M^{\ell}\, \ell_R - \overline{\mathbf{n}_L}\, M^{\nu}\, n_R + \text{H.c.} \\ &= -\sum_{\alpha=e,\mu,\tau} m_\alpha^\ell\, \overline{\ell_{\alpha L}}\, \ell_{\alpha R} - \sum_{k=1}^{3} m_k^\nu\, \overline{\nu_{kL}}\, \nu_{kR} + \text{H.c.} \end{aligned}$$

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \mathrm{H.c.}$$

Weak Charged Current: $j_W^\rho = j_{W,\mathrm{L}}^\rho + j_{W,\mathrm{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\mathrm{L}}^\rho = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{\alpha L}} \gamma^\rho \ell'_{\alpha L} = 2 \overline{\boldsymbol{\nu}'_L} \gamma^\rho \boldsymbol{\ell}'_L$$

$$\boldsymbol{\ell}'_L = V_L^\ell \boldsymbol{\ell}_L \qquad \boldsymbol{\nu}'_L = V_L^\nu \mathbf{n}_L$$

$$j_{W,\mathrm{L}}^\rho = 2 \overline{\mathbf{n}_L} V_L^{\nu\dagger} \gamma^\rho V_L^\ell \boldsymbol{\ell}_L = 2 \overline{\mathbf{n}_L} V_L^{\nu\dagger} V_L^\ell \gamma^\rho \boldsymbol{\ell}_L = 2 \overline{\mathbf{n}_L} U^\dagger \gamma^\rho \boldsymbol{\ell}_L$$

Mixing Matrix

$$U^\dagger = V_L^{\nu\dagger} V_L^\ell \qquad \boxed{U = V_L^{\ell\dagger} V_L^\nu}$$

- Definition: Left-Handed Flavor Neutrino Fields

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton:

$$j^{\rho}_{W,\mathrm{L}} = 2\left(\overline{\nu_{eL}}\,\gamma^{\rho}\,e_L + \overline{\nu_{\mu L}}\,\gamma^{\rho}\,\mu_L + \overline{\nu_{\tau L}}\,\gamma^{\rho}\,\tau_L\right)$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Standard Model: Lepton numbers are conserved

$$\mathscr{L}^{\mathrm{D}} = - \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

L is conserved: $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Mixing Matrix

- $U = V_L^{\ell\dagger} \, V_L^{\nu} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

- Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \quad \Longrightarrow \quad \begin{array}{ll} \dfrac{N(N-1)}{2} = 3 & \text{Mixing Angles} \\ \dfrac{N(N+1)}{2} = 6 & \text{Phases} \end{array}$$

- Not all phases are physical observables

- Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, U^{*}_{\alpha k} \, \gamma^{\rho} \, \ell_{\alpha L}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)

$$\nu_k \to e^{i\varphi_k} \, \nu_k \quad (k = 1, 2, 3) \, , \qquad \ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

- Performing this transformation, the Charged Current becomes

$$j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, e^{-i\varphi_k} \, U^{*}_{\alpha k} \, e^{i\varphi_\alpha} \, \gamma^{\rho} \, \ell_{\alpha L}$$

$$j^{\rho}_{W,\mathrm{L}} = 2 \, \underbrace{e^{-i(\varphi_1 - \varphi_e)}}_{1} \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, \underbrace{e^{-i(\varphi_k - \varphi_1)}}_{2} \, U^{*}_{\alpha k} \, \underbrace{e^{i(\varphi_\alpha - \varphi_e)}}_{2} \, \gamma^{\rho} \, \ell_{\alpha L}$$

- There are 5 arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant $\Longleftrightarrow$ conservation of Total Lepton Number.

- The mixing matrix contains 1 Physical Phase.
- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Majorana Mass

Dirac Mass Lagrangian

$$\mathscr{L}^{\mathrm{D}} = -m \left(\overline{\nu_R} \, \nu_L + \overline{\nu_L} \, \nu_R \right)$$

$$\nu_R \quad \to \quad \nu_L^C = \mathcal{C} \, \overline{\nu_L}^T$$

$$\frac{1}{2} \mathscr{L}^{\mathrm{D}} \quad \to \quad -\frac{m}{2} \left(-\nu_L^T \, \mathcal{C}^\dagger \, \nu_L + \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T \right)$$

Majorana Mass Lagrangian

$$\mathscr{L}^{\mathrm{M}} = -\frac{m}{2} \left(-\nu_L^T \, \mathcal{C}^\dagger \, \nu_L + \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T \right) = -\frac{m}{2} \left(\overline{\nu_L^C} \, \nu_L + \overline{\nu_L} \, \nu_L^C \right)$$

- Majorana Field: $\nu = \nu_L + \nu_L^C$
- Majorana Condition: $\nu^C = \nu$
- Majorana Lagrangian: $\mathscr{L}^{\text{M}} = \frac{1}{2} m \overline{\nu} \nu$
- The factor $1/2$ distinguishes the Majorana Lagrangian from the Dirac Lagrangian
- Common terminology:

  Majorana neutrino with negative helicity $\equiv$ neutrino
  Majorana neutrino with positive helicity $\equiv$ antineutrino

Lepton Number

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu = \nu^C} \quad \longrightarrow \quad \cancel{L = -1}$$

$$\nu_L \quad \Longrightarrow \quad L = +1 \qquad\qquad \nu_L^C \quad \Longrightarrow \quad L = -1$$

$$\mathscr{L}^{\mathrm{M}} = -\frac{m}{2} \left(\overline{\nu_L^C} \, \nu_L + \overline{\nu_L} \, \nu_L^C \right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta_{0\nu}^-)$$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta_{0\nu}^+)$$

No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T \, \mathcal{C}^\dagger \, \nu_L - \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM (one for each lepton generation)
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed Higgs triplet with $Y = 2$

Mixing of Three Majorana Neutrinos

- $\nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$

$$\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \nu_L'^T \, \mathcal{C}^\dagger \, M^L \, \nu'_L + \text{H.c.}$$

$$= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}'^T \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} + \text{H.c.}$$

- In general, the matrix M^L is a complex symmetric matrix

$$\sum_{\alpha,\beta} \nu_{\alpha L}'^T \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} = - \sum_{\alpha,\beta} \nu_{\beta L}'^T \, M^L_{\alpha\beta} \, (\mathcal{C}^\dagger)^T \, \nu'_{\alpha L}$$

$$= \sum_{\alpha,\beta} \nu_{\beta L}'^T \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu_{\alpha L}'^T \, \mathcal{C}^\dagger \, M^L_{\beta\alpha} \, \nu'_{\beta L}$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \quad \Longleftrightarrow \quad M^L = M^{L\,T}$$

- $\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \boldsymbol{\nu}_L'^{T} \, \mathcal{C}^\dagger \, M^L \, \boldsymbol{\nu}_L' + \text{H.c.}$
- $\boldsymbol{\nu}_L' = V_L^\nu \, \mathbf{n}_L \qquad \Longrightarrow \qquad \mathscr{L}^{\mathrm{M}} = \frac{1}{2} \boldsymbol{\nu}_L'^{T} \, (V_L^\nu)^T \, \mathcal{C}^\dagger \, M^L \, V_L^\nu \, \boldsymbol{\nu}_L' + \text{H.c.}$
- $(V_L^\nu)^T \, M^L \, V_L^\nu = M \,, \qquad M_{kj} = m_k \, \delta_{kj} \qquad (k, j = 1, 2, 3)$
- Left-handed chiral fields with definite mass: $\mathbf{n}_L = V_L^{\nu\dagger} \, \boldsymbol{\nu}_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k \left(\nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu_{kL}^T \right)$$

- Majorana fields of massive neutrinos: $\nu_k = \nu_{kL} + \nu_{kL}^C \qquad \boxed{\nu_k^C = \nu_k}$

$$\mathscr{L}^{\mathrm{M}} = -\frac{1}{2} \sum_{k=1}^{3} m_k \overline{\nu_k} \nu_k$$

Mixing Matrix

- Leptonic Weak Charged Current:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L \qquad \text{with} \qquad U = V^{\ell\dagger}_L\,V^{\nu}_L$$

- Definition of the left-handed flavor neutrino fields:

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V^{\ell\dagger}_L\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- Leptonic Weak Charged Current has the SM form

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- Majorana Mass Term $\mathscr{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k \, \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + \text{H.c.}$ is not invariant under the global U(1) gauge transformations

$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \qquad (k = 1, 2, 3)$$

- Left-handed massive neutrino fields cannot be rephased in order to eliminate two Majorana phases factorized on the right of mixing matrix:

$$\boxed{U = U^{\text{D}} \, D^{\text{M}}} \qquad D^{\text{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- U^{D} is analogous to a Dirac mixing matrix, with one Dirac phase
- Standard parameterization:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

One Generation Dirac-Majorana Mass Term

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}^{\text{D+M}} = \mathscr{L}^{\text{D}} + \mathscr{L}^{L} + \mathscr{L}^{R}$$

$$\mathscr{L}^{\text{D}} = -m_{\text{D}} \, \overline{\nu_R} \, \nu_L + \text{H.c.}$$ Dirac Mass Term

$$\mathscr{L}^{L} = \frac{1}{2} \, m_L \, \nu_L^T \, \mathcal{C}^\dagger \, \nu_L + \text{H.c.}$$ Majorana Mass Term

$$\mathscr{L}^{R} = \frac{1}{2} \, m_R \, \nu_R^T \, \mathcal{C}^\dagger \, \nu_R + \text{H.c.}$$ New Majorana Mass Term!

- Column matrix of left-handed chiral fields: $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\,\overline{\nu_R}^T \end{pmatrix}$

$$\mathscr{L}^{\text{D+M}} = \frac{1}{2}\, N_L^T\, \mathcal{C}^\dagger\, M\, N_L + \text{H.c.} \qquad M = \begin{pmatrix} m_L & m_\text{D} \\ m_\text{D} & m_R \end{pmatrix}$$

- The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- Diagonalization: $n_L = U^\dagger\, N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T\, M\, U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \text{Real } m_k \geq 0$$

- $\mathscr{L}^{\text{D+M}} = \frac{1}{2} \sum_{k=1,2} m_k\, \nu_{kL}^T\, \mathcal{C}^\dagger\, \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k\, \overline{\nu_k}\, \nu_k$

$$\nu_k = \nu_{kL} + \nu_{kL}^C$$

- Massive neutrinos are Majorana! $\qquad \nu_k = \nu_k^C$

Real Mass Matrix

- CP is conserved if the mass matrix is real: $M = M^*$
- $M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$ we consider real and positive m_R and m_{D} and real m_L
- A real symmetric mass matrix can be diagonalized with $U = \mathcal{O}\,\rho$

$$\mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \qquad \rho_k^2 = \pm 1$$

- $\mathcal{O}^T\, M\, \mathcal{O} = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix}$ $\qquad \tan 2\vartheta = \dfrac{2 m_{\text{D}}}{m_R - m_L}$

$$m'_{2,1} = \frac{1}{2}\left[m_L + m_R \pm \sqrt{\left(m_L - m_R\right)^2 + 4\, m_{\text{D}}^2} \right]$$

- m'_1 is negative if $m_L m_R < m_{\text{D}}^2$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m'_1 & 0 \\ 0 & \rho_2^2 m'_2 \end{pmatrix} \implies \boxed{m_k = \rho_k^2\, m'_k}$$

- m_2' is always positive:

$$m_2 = m_2' = \frac{1}{2}\left[m_L + m_R + \sqrt{(m_L - m_R)^2 + 4\,m_{\rm D}^2}\right]$$

- If $m_L m_R \geq m_{\rm D}^2$, then $m_1' \geq 0$ and $\rho_1^2 = 1$

$$m_1 = \frac{1}{2}\left[m_L + m_R - \sqrt{(m_L - m_R)^2 + 4\,m_{\rm D}^2}\right]$$

$$\rho_1 = 1 \text{ and } \rho_2 = 1 \quad \Longrightarrow \quad U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

- If $m_L m_R < m_{\rm D}^2$, then $m_1' < 0$ and $\rho_1^2 = -1$

$$m_1 = \frac{1}{2}\left[\sqrt{(m_L - m_R)^2 + 4\,m_{\rm D}^2} - (m_L + m_R)\right]$$

$$\rho_1 = i \text{ and } \rho_2 = 1 \quad \Longrightarrow \quad U = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix}$$

Special cases:

- $m_L = m_R \implies$ Maximal Mixing
- $m_L = m_R = 0 \implies$ Dirac Limit
- $|m_L|, m_R \ll m_{\text{D}} \implies$ Pseudo-Dirac Neutrinos
- $m_L = 0 \quad m_{\text{D}} \ll m_R \implies$ See-Saw Mechanism

Maximal Mixing

$$\boxed{m_L = m_R}$$

$$\vartheta = \pi/4$$

$$m'_{2,1} = m_L \pm m_\text{D}$$

$$\begin{cases} \rho_1^2 = +1\,, \quad m_1 = m_L - m_\text{D} \quad \text{if} \quad m_L \geq m_\text{D} \\ \rho_1^2 = -1\,, \quad m_1 = m_\text{D} - m_L \quad \text{if} \quad m_L < m_\text{D} \end{cases}$$

$$m_2 = m_L + m_\text{D}$$

$\underline{m_L < m_\text{D}}$

$$\begin{cases} \nu_{1L} = \frac{-i}{\sqrt{2}} \left(\nu_L - \nu_R^C \right) \\ \nu_{2L} = \frac{1}{\sqrt{2}} \left(\nu_L + \nu_R^C \right) \end{cases}$$

$$\begin{cases} \nu_1 = \nu_{1L} + \nu_{1L}^C = \frac{-i}{\sqrt{2}} \left[(\nu_L + \nu_R) - \left(\nu_L^C + \nu_R^C \right) \right] \\ \nu_2 = \nu_{2L} + \nu_{2L}^C = \frac{1}{\sqrt{2}} \left[(\nu_L + \nu_R) + \left(\nu_L^C + \nu_R^C \right) \right] \end{cases}$$

Dirac Limit

$$m_L = m_R = 0$$

- $m'_{2,1} = \pm m_{\mathrm{D}} \implies \begin{cases} \rho_1^2 = -1\,, & m_1 = m_{\mathrm{D}} \\ \rho_2^2 = +1\,, & m_2 = m_{\mathrm{D}} \end{cases}$
- The two Majorana fields ν_1 and ν_2 can be combined to give one Dirac field:

$$\nu = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) = \nu_L + \nu_R$$

- A Dirac field ν can always be split in two Majorana fields:

$$\begin{aligned} \nu &= \frac{1}{2} \left[\left(\nu - \nu^C \right) + \left(\nu + \nu^C \right) \right] \\ &= \frac{i}{\sqrt{2}} \left(-i \, \frac{\nu - \nu^C}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left(\frac{\nu + \nu^C}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) \end{aligned}$$

- A Dirac field is equivalent to two Majorana fields with the same mass

Pseudo-Dirac Neutrinos

$$|m_L|\,,\ m_R \ll m_{\text{D}}$$

- $m'_{2,1} \simeq \dfrac{m_L + m_R}{2} \pm m_{\text{D}}$
- $m'_1 < 0 \quad \Longrightarrow \quad \rho_1^2 = -1 \quad \Longrightarrow \quad m_{2,1} \simeq m_{\text{D}} \pm \dfrac{m_L + m_R}{2}$
- The two massive Majorana neutrinos are almost degenerate in mass
- The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_{\text{D}}\,(m_L + m_R)$$

- The oscillations occur with practically maximal mixing:

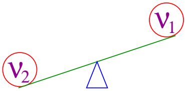
$$\tan 2\vartheta = \frac{2m_{\text{D}}}{m_R - m_L} \gg 1 \quad \Longrightarrow \quad \vartheta \simeq \pi/4$$

See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

$$m_L = 0 \qquad m_{\mathrm{D}} \ll m_R$$

- $\mathscr{L}^L$ is forbidden by SM symmetries $\Longrightarrow$ $m_L = 0$
- $m_{\mathrm{D}} \lesssim v \sim 100\,\mathrm{GeV}$ is generated by SM Higgs Mechanism (protected by SM symmetries)
- m_R is not protected by SM symmetries $\Longrightarrow$ $m_R \sim \mathcal{M}_{\mathrm{GUT}} \gg v$
- $$\left. \begin{array}{l} m'_1 \simeq -\dfrac{m_{\mathrm{D}}^2}{m_R} \\ m'_2 \simeq m_R \end{array} \right\} \Longrightarrow \left\{ \begin{array}{ll} \rho_1^2 = -1\,, & m_1 \simeq \dfrac{m_{\mathrm{D}}^2}{m_R} \\ \rho_2^2 = +1\,, & m_2 \simeq m_R \end{array} \right.$$



- Natural explanation of smallness of neutrino masses
- Mixing angle is very small: $\tan 2\vartheta = 2\,\dfrac{m_{\mathrm{D}}}{m_R} \ll 1$
- ν_1 is composed mainly of ν_L: $\nu_{1L} \simeq -i\,\nu_L$
- ν_2 is composed mainly of ν_R: $\nu_{2L} \simeq \nu_R^C$

Three-Generation Mixing

$$\mathscr{L}^{\text{D+M}} = \mathscr{L}^{\text{D}} + \mathscr{L}^{L} + \mathscr{L}^{R}$$

$$\mathscr{L}^{\text{D}} = -\sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}}\, M^{\text{D}}_{s\alpha}\, \nu'_{\alpha L} + \text{H.c.}$$

$$\mathscr{L}^{L} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L}\, \mathcal{C}^{\dagger}\, M^{L}_{\alpha\beta}\, \nu'_{\beta L} + \text{H.c.}$$

$$\mathscr{L}^{R} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'^{T}_{sR}\, \mathcal{C}^{\dagger}\, M^{R}_{ss'}\, \nu'_{s'R} + \text{H.c.}$$

$$\mathbf{N}'_L \equiv \begin{pmatrix} \boldsymbol{\nu}'_L \\ \boldsymbol{\nu}'^{C}_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'^{C}_R \equiv \begin{pmatrix} \nu'^{C}_{1R} \\ \vdots \\ \nu'^{C}_{N_S R} \end{pmatrix}$$

$$\mathscr{L}^{\text{D+M}} = \frac{1}{2} \mathbf{N}'^{T}_L\, \mathcal{C}^{\dagger}\, M^{\text{D+M}}\, \mathbf{N}'_L + \text{H.c.} \qquad M^{\text{D+M}} = \begin{pmatrix} M^{L} & M^{\text{D}\,T} \\ M^{\text{D}} & M^{R} \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term $\Longrightarrow$ massive Majorana neutrinos
- See-Saw Mechanism $\Longrightarrow$ right-handed neutrinos have large masses and are decoupled from the low-energy phenomenology
- At low energy we have an effective mixing of three Majorana neutrinos