Introduzione alle masse e ai mescolamenti dei neutrini (II)

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Outline:

Pedagogical Introduction
Neutrino masses and spinor fields
Neutrinoless double beta decay
2ν & 3ν oscillations in vacuum
[Homework]

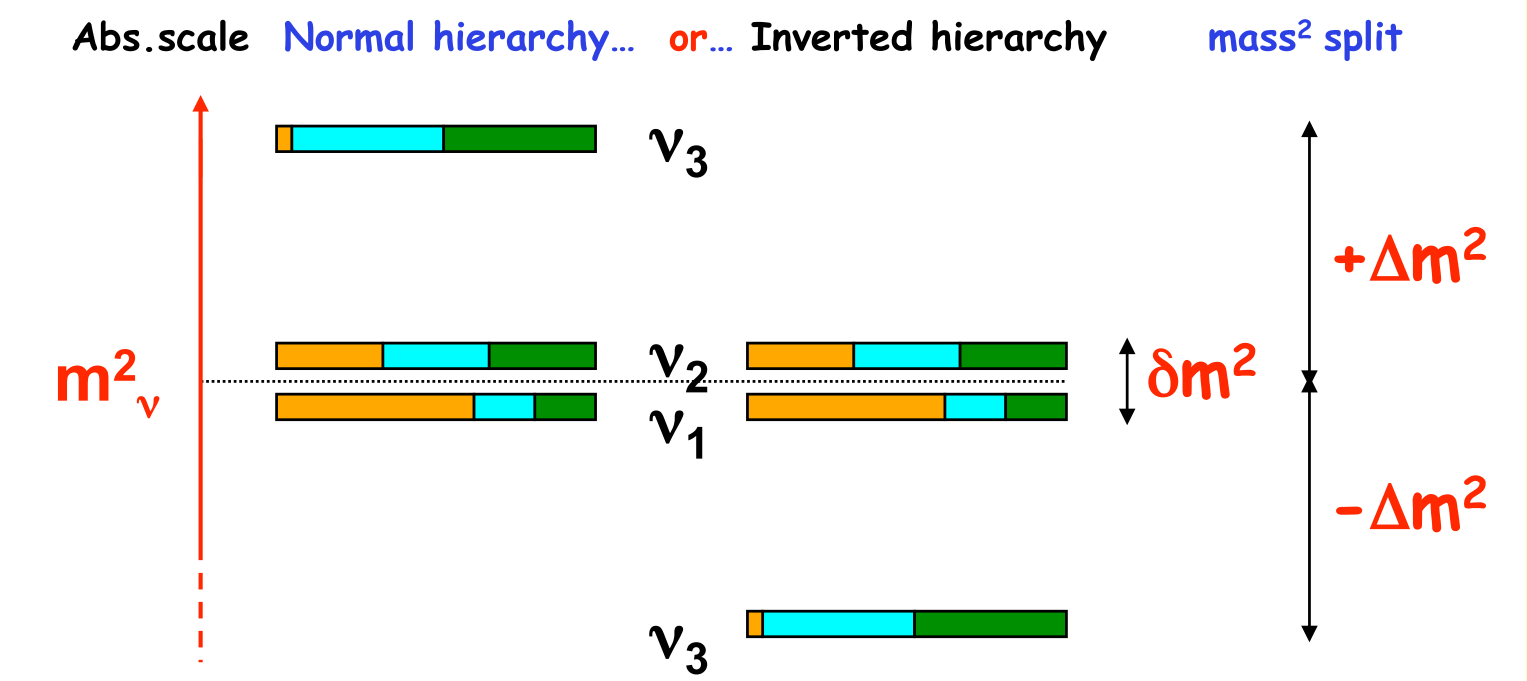
I

Recap
2ν oscillations in matter
Solar and KamLAND oscillations
Absolute neutrino masses
[Homework]

II

3ν mass-mixing overview in just one slide

(here, with 1 digit accuracy). Flavors = e μ τ



$\delta m^2 \sim 8 \times 10^{-5}\ \mathrm{eV}^2$ $\quad$ $\sin^2\theta_{12} \sim 0.3$

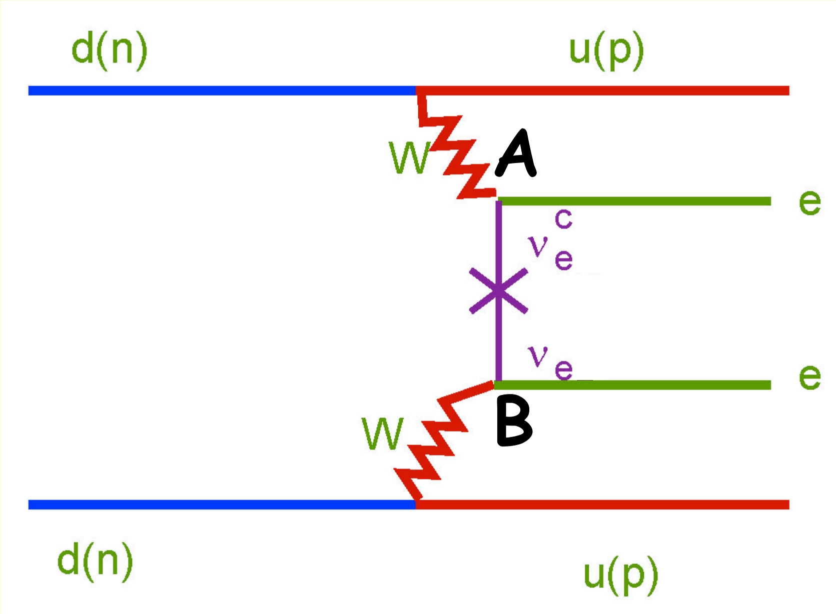
$\Delta m^2 \sim 3 \times 10^{-3}\ \mathrm{eV}^2$ $\quad$ $\sin^2\theta_{23} \sim 0.5$

$m_\nu < O(1)\ \mathrm{eV}$ $\quad$ $\sin^2\theta_{13} < \mathrm{few}\%$

$\mathrm{sign}(\pm\Delta m^2)$ unknown $\quad$ δ (CP) unknown

Implications of 3ν mixing for 0νββ decay

For each mass state ν_i, 0νββ amplitude proportional to:



... mixing of ν_e with ν_i

... mass of ν_i

... mixing of ν_i with ν_e

(times an unknown ν_i phase)

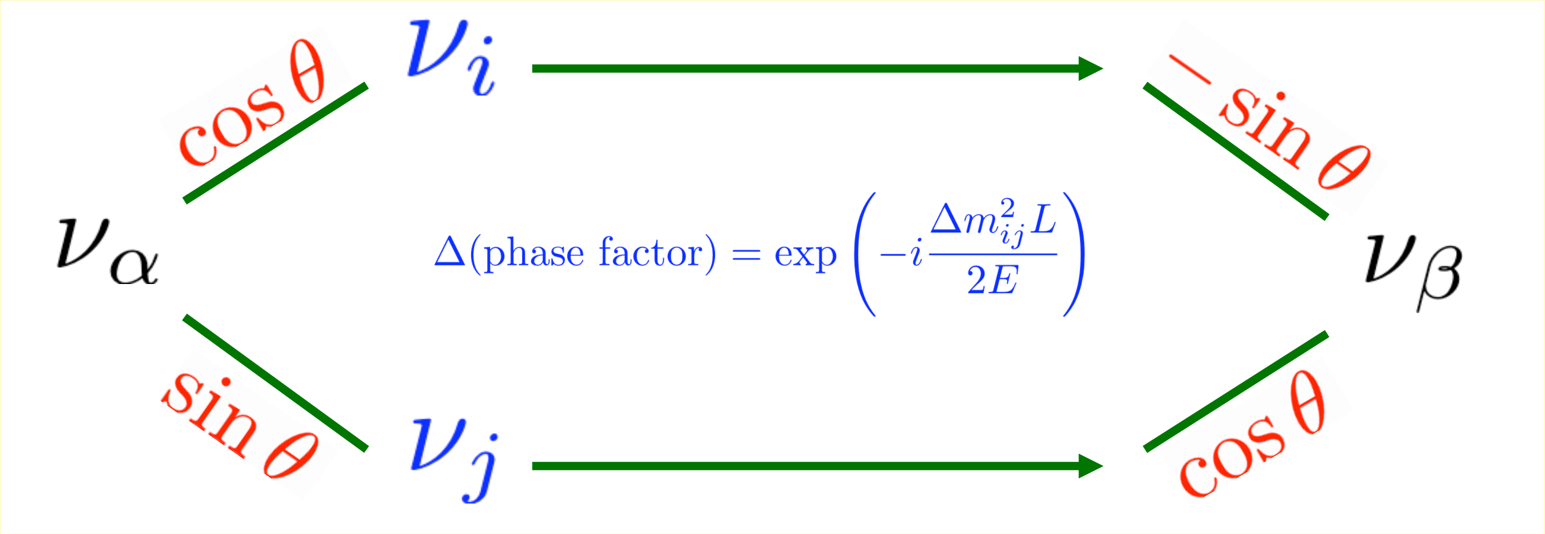
Summing up for three massive neutrinos:

Amplitude ~ "effective Majorana mass"

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

[complex linear combination of masses; c_{ij} = cos θ_{ij} etc.]

Oscillations in vacuum: analogy with a two-slit experiment

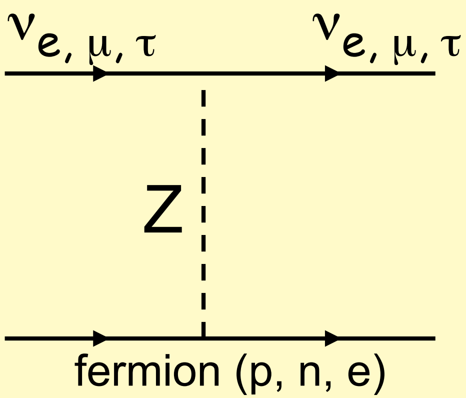


This is the simplest case (only 2 neutrinos involved, no interactions with matter). It shows that, if neutrinos are massive and mixed (like quarks), then flavor is not a good quantum number during propagation. Indeed, it changes ("oscillates") significantly over a distance L (≈Δt) dictated by the uncertainty relation:

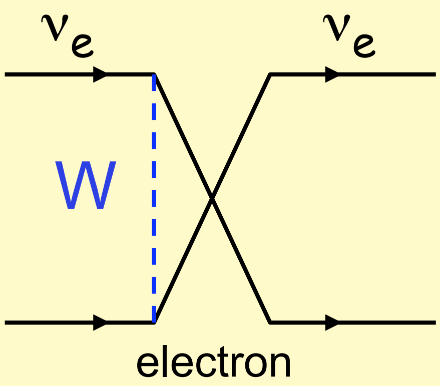
$$P(\nu_\alpha \rightarrow \nu_\beta) = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$

Neutrino flavor oscillations in matter

Neutrinos of all flavors ($\nu_{e,\ \mu,\ \tau}$) have the same amplitude for coherent forward scattering in matter via NC. However, only ν_e can further scatter via CC, since ordinary matter contains e, not μ or τ. This fact implies a difference in the relative propagation of ν_e versus $\nu_{\mu,\ \tau}$, (but not between ν_μ and ν_τ): the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

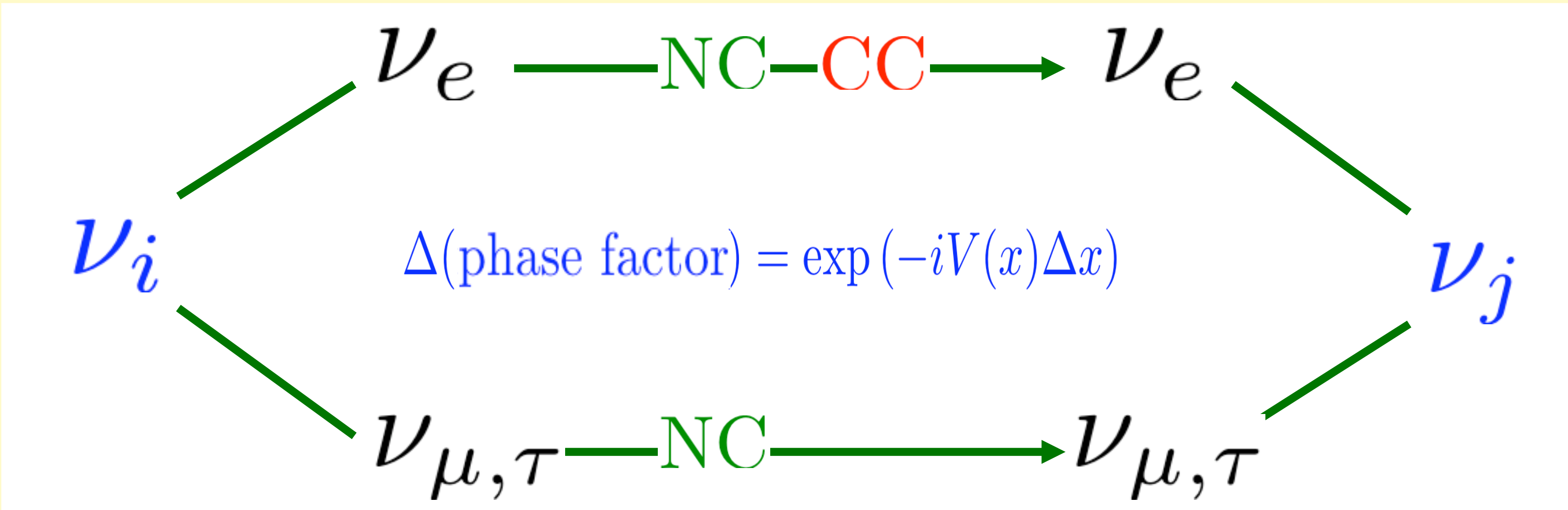


ν_μ & ν_τ (e.g., atmospheric) feel background fermions in the same way (through NC); no relative phase change while propagating (~ vacuum-like propagation, as anticipated)



But ν_e , in addition to NC, have CC interac. with background electrons (density N_e). Energy difference: $V = +\sqrt{2}\ G_F\ N_e$ leads to a phase difference in matter

Again, analogy with the two-slit experiment:
one "arm" (flavor) feels a different "refraction index"



governed by the local (electron) density:

$$V(x) = V_e - V_{\mu,\tau} = \sqrt{2}\, G_F\, N_e(x) \quad [N_e = \text{electron density}]$$

(-V for antineutrinos)

Exercise 5. Prove that oscillations between ν_e and ν_x (=ν_μ,ν_τ) in matter with **constant** density lead to Pontecorvo's formula

$$P(\nu_e \to \nu_x) = \sin^2 2\tilde{\theta} \sin^2 \left(\frac{\Delta \tilde{m}^2 L}{4E} \right)$$

with effective (tilde) parameters defined as

$$\frac{\Delta \tilde{m}^2}{\Delta m^2} = \frac{\sin 2\theta}{\sin 2\tilde{\theta}} = \sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2} \right)^2 + (\sin 2\theta)^2}$$

where $A = 2VE = 2\sqrt{2} G_F N_e E$

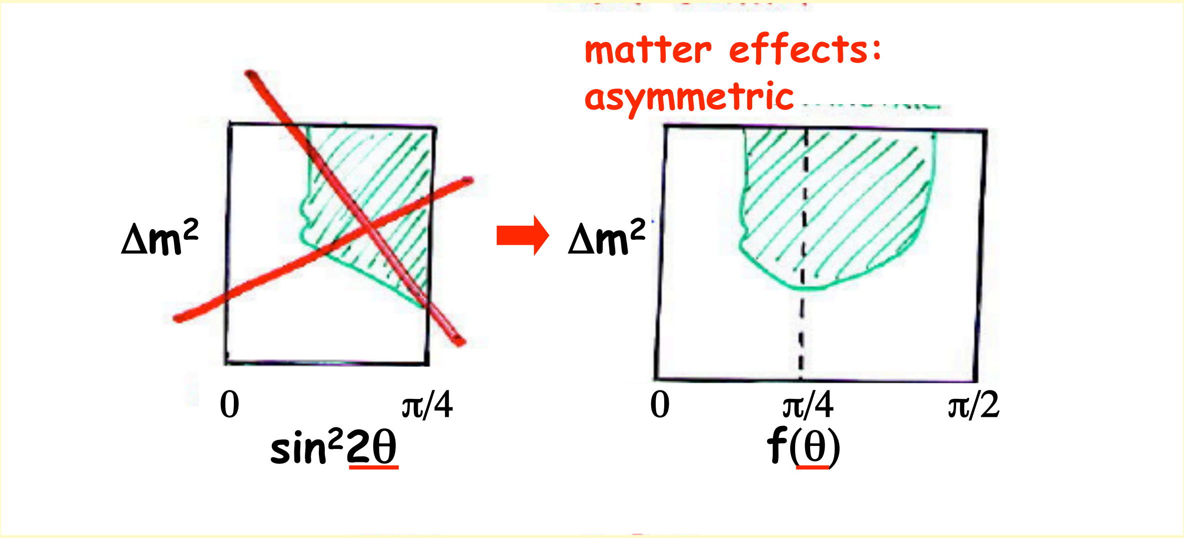
Exercise 6 (Conversion factors). Prove that

$$\frac{A}{\Delta m^2} = 1.526 \times 10^{-7} \left(\frac{N_e}{\text{mol/cm}^3} \right) \left(\frac{E}{\text{MeV}} \right) \left(\frac{\text{eV}^2}{\Delta m^2} \right)$$

Rule of thumb (**~valid also for non-constant density**):

Expect strong matter effects when **$A/\Delta m^2 \sim O(1)$.**

Note: matter effects are octant-asymmetric; need to unfold second octant.

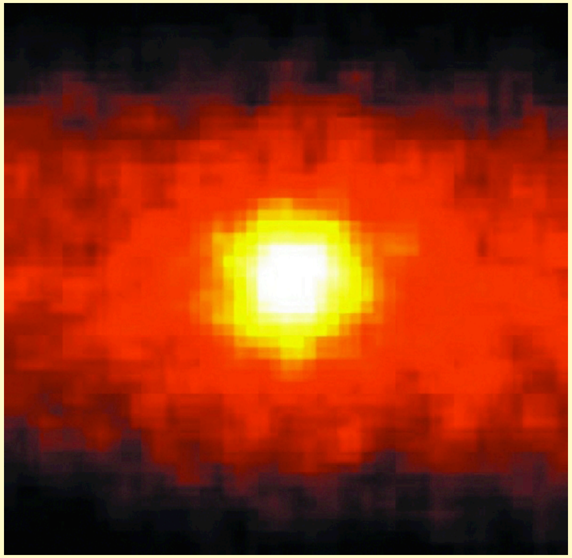


Asymmetry is particular pronounced for solar neutrinos, with mass-mixing parameters (**δm^2**, **θ_{12}**)

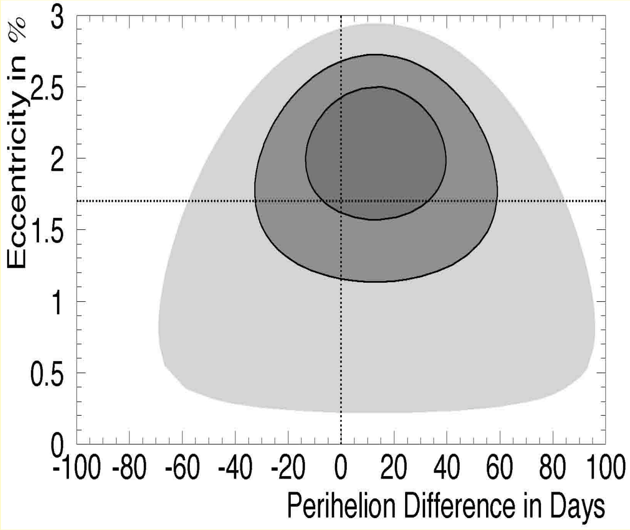
[N.B.: Effects also depend on sign of squared mass difference: Handle to hierarchy discrimination.]

Experiments sensitive to the "small" δm^2:

Solar neutrinos [see talk by E. Bellotti]



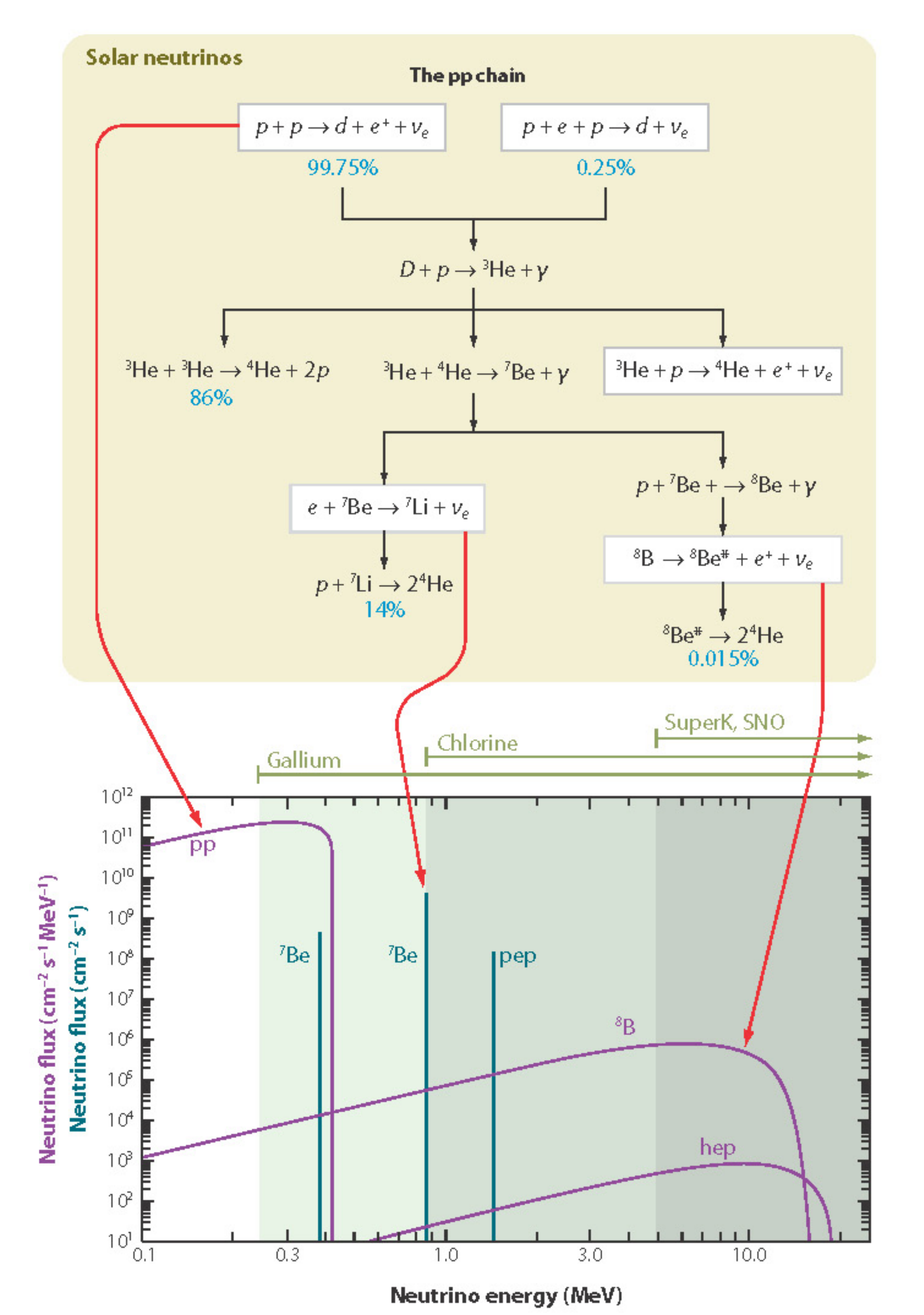
The Sun seen with neutrinos (SK)

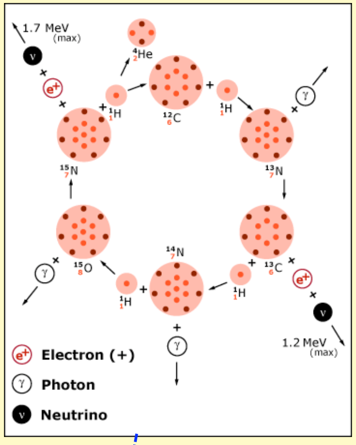


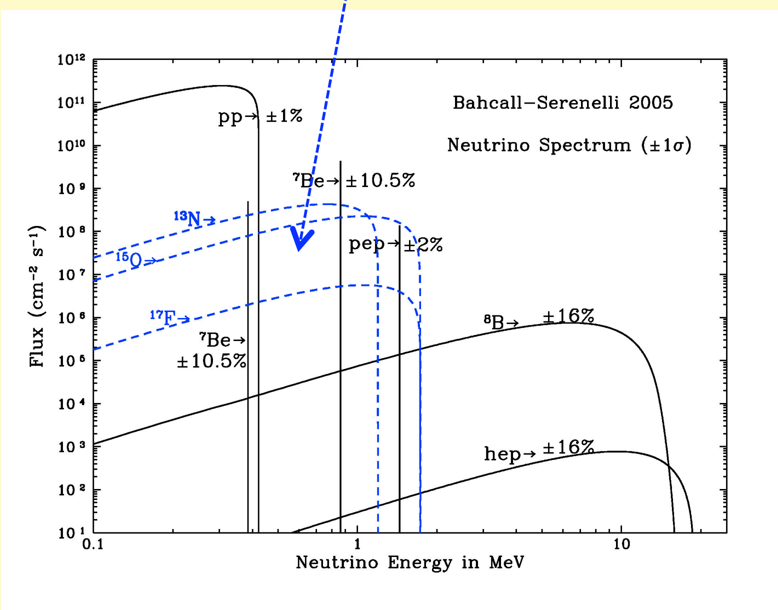
Earth orbit from solar ν (SK)

Production
pp (+CNO) cycle

[See talk by A. Caciolli]

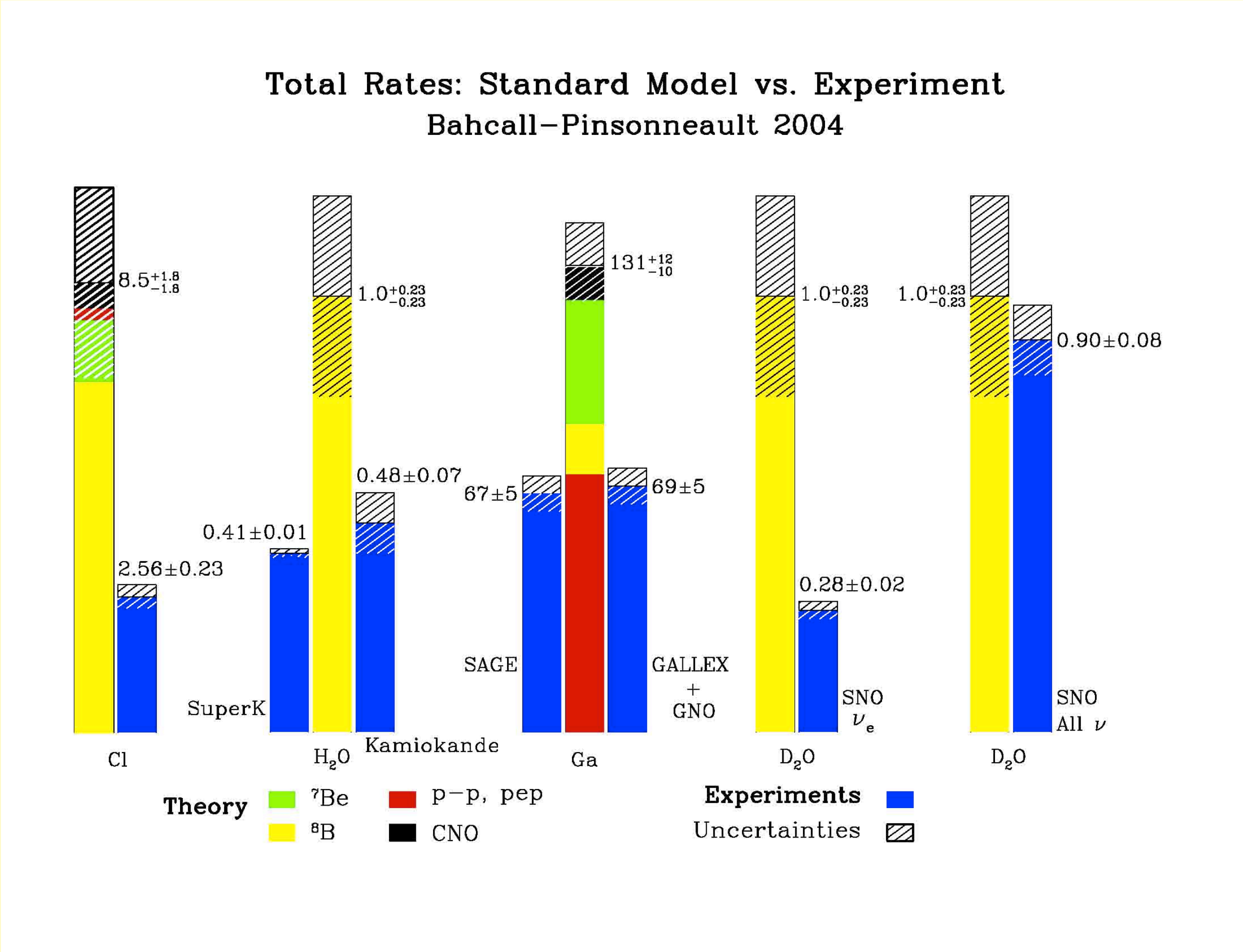






Experimental Results

All results in CC mode indicated a ν_e deficit…

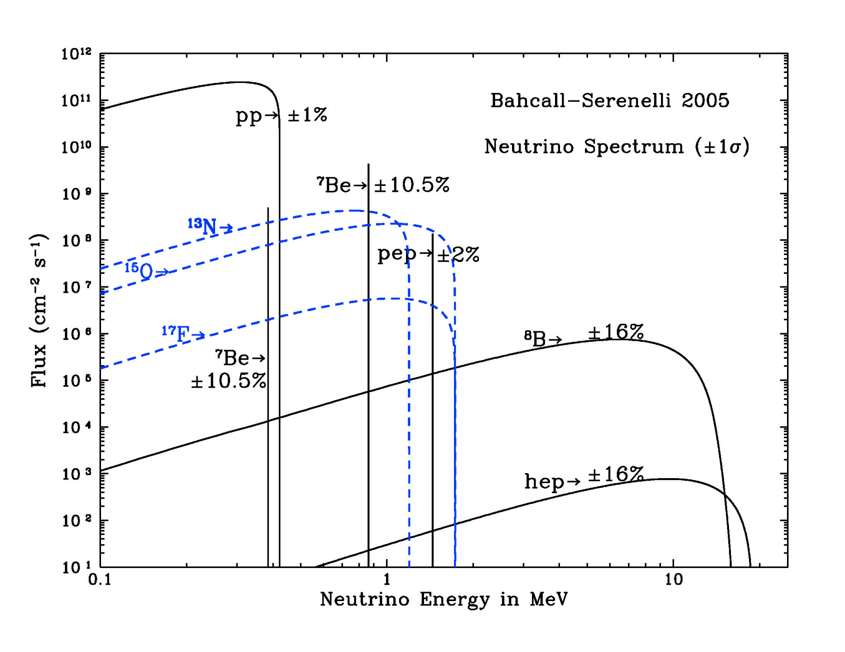


…as compared to solar model expectations

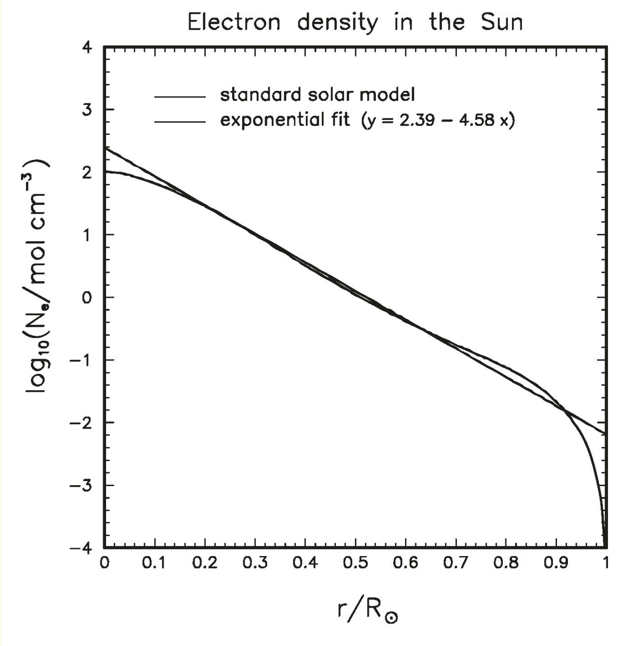
Latest confirmation: BOREXINO [see talk by L. Ludhova]

Interpretation

The Sun is an intense source of ν_e with $E \sim O(10^{\pm 1})$ MeV …



… and its electron density range is $\sim O(10^{\pm 2})$ mol/cm^3

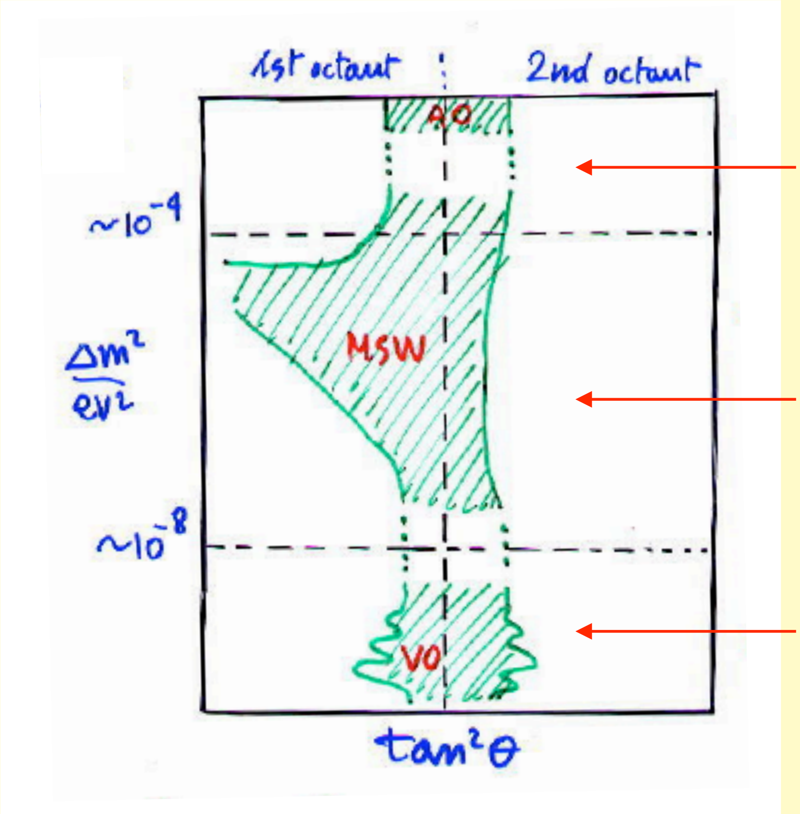


$$\ldots\text{therefore, } A/\delta m^2 \sim O(1) \text{ if } \delta m^2 \sim O(10^{-10} - 10^{-3})\ \text{eV}^2$$

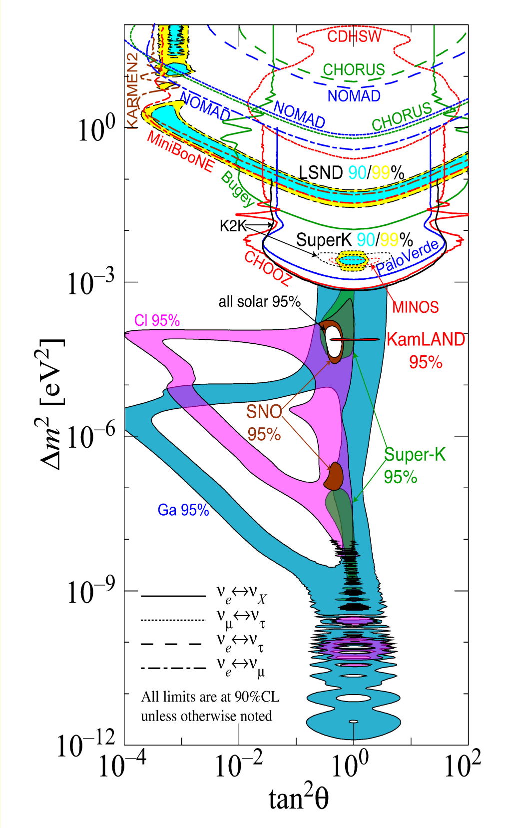
The Sun is an ideal place to look for oscillations in matter, driven the "**small" squared mass difference** **δm^2** (not the "large" Δm^2), and Nature has been kind enough to fulfill these expectations! The corresponding (solar) mixing angle is **θ_{12}**

Complications… (until a few years ago)

Large parameter space

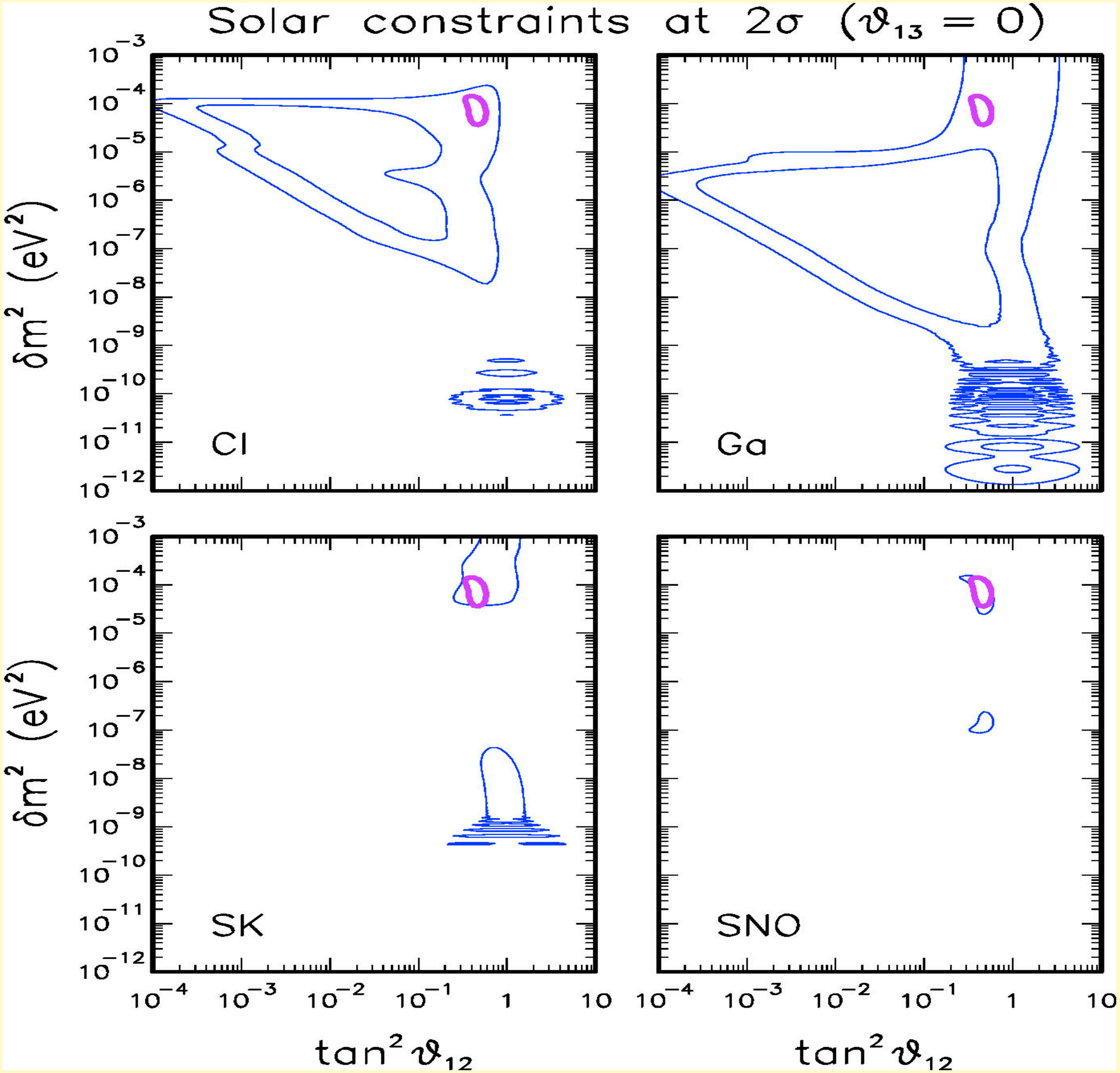


- Averaged (vacuum) oscillations
  Octant symmetric
- MSW transitions
  Not octant symmetric
- Vacuum oscillations
  Octant symmetric



Large literature on (semi)analytic or numerical solutions: constant density approximation generally not applicable

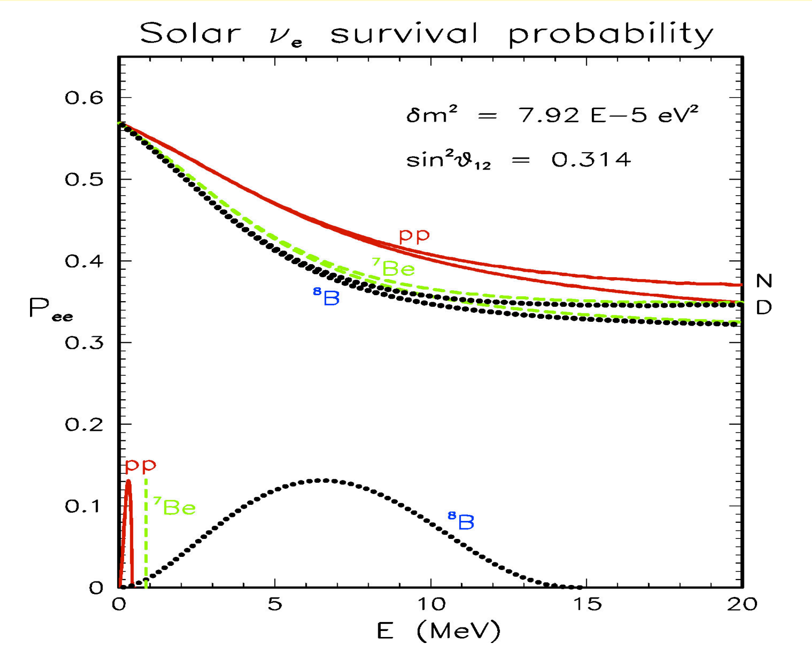
But, in 2002 (“annus mirabilis”), one global solution was finally singled out by combination of data (“large mixing angle” or LMA). [See talk by E. Bellotti]



For the parameters (δm^2, θ_{12}) in the LMA region, one can use the next approximation to "constant density," namely, the approximation of "slowly varying density" (with respect to oscillation frequency):

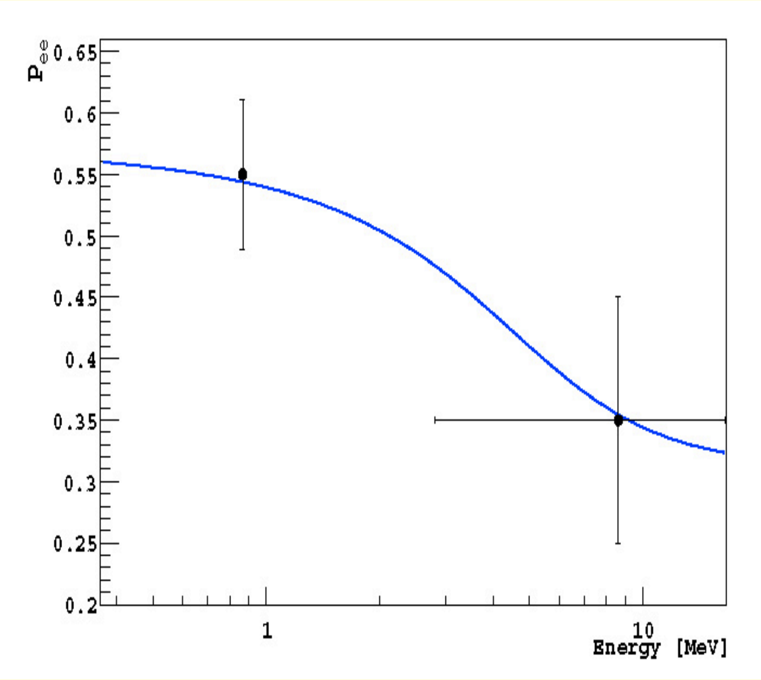
adiabatic approximation **(see Appendix)**

Expected probability profile



In the Earth: small day/night (D/N) effects, not yet seen.

Test with recent Borexino data



[See talk by L. Ludhova]

Also in 2002… KamLAND: 1000 ton mineral oil detector, "surrounded" by nuclear reactors producing anti-ν_e. Characteristics:

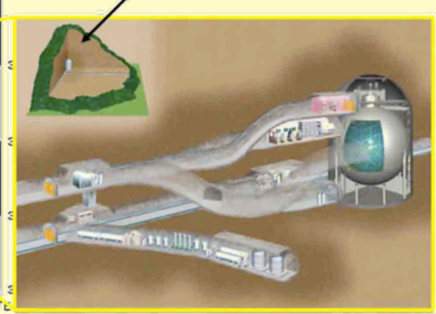
$A/\delta m^2 \ll 1$ in Earth crust (vacuum approxim. OK)
L~100-200 km
E_ν~ few MeV

⇨

With previous $(\delta m^2, \theta_{12})$ parameters it is $(\delta m^2 L/4E) \sim O(1)$ and reactor neutrinos should oscillate with large amplitude (large θ_{12})

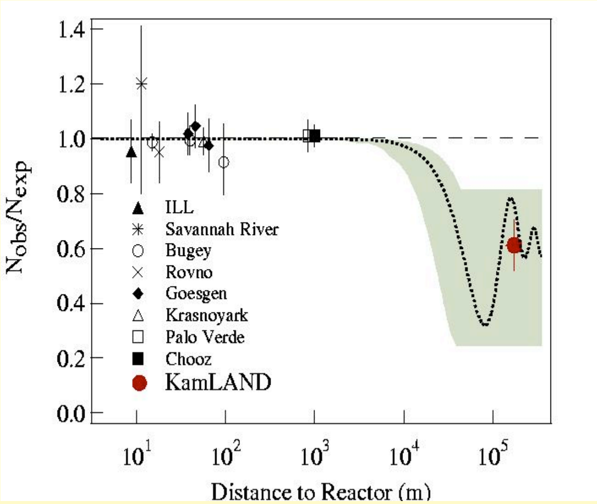
Long-baseline reactor expt

~1 km high Mt Ikenoyama

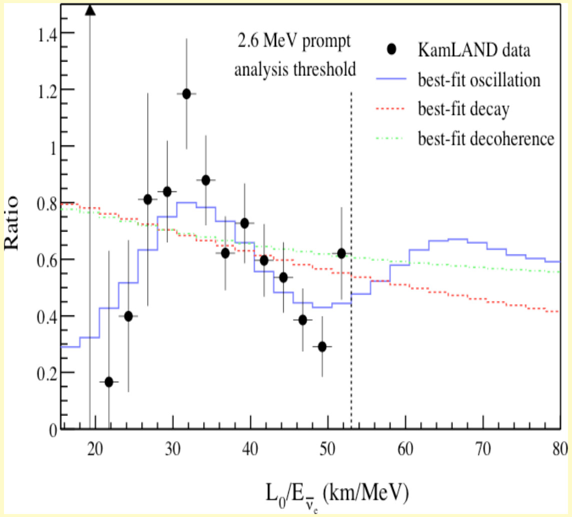


KamLAND results

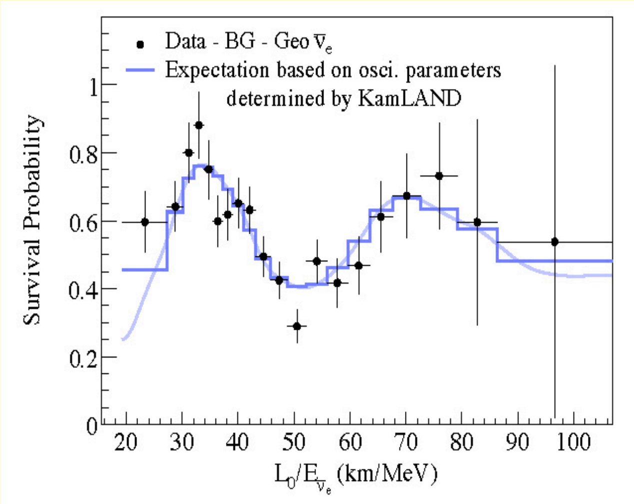
2002: electron flavor disappearance observed



2004: half-period of oscillation observed

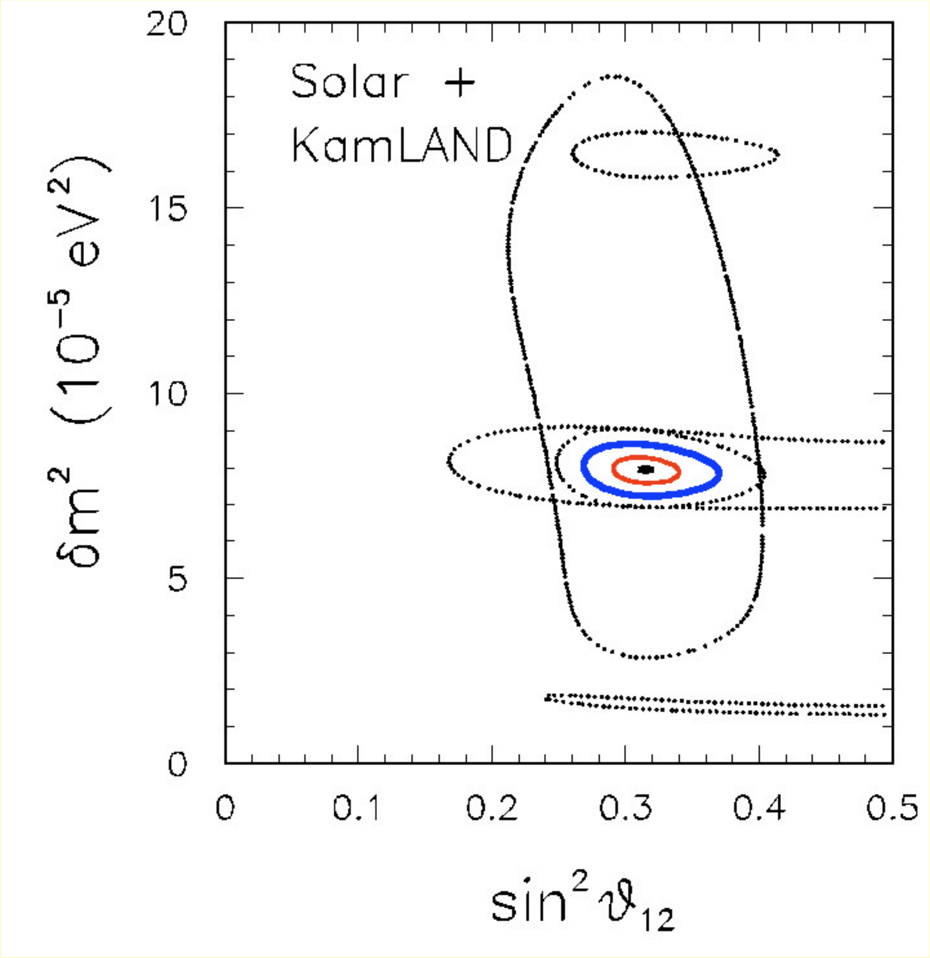


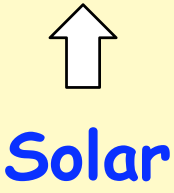
2007: one period of oscillation observed



Direct observation of δm^2 oscillations

(δm^2, θ_{12}) - complementarity of solar/reactor neutrinos

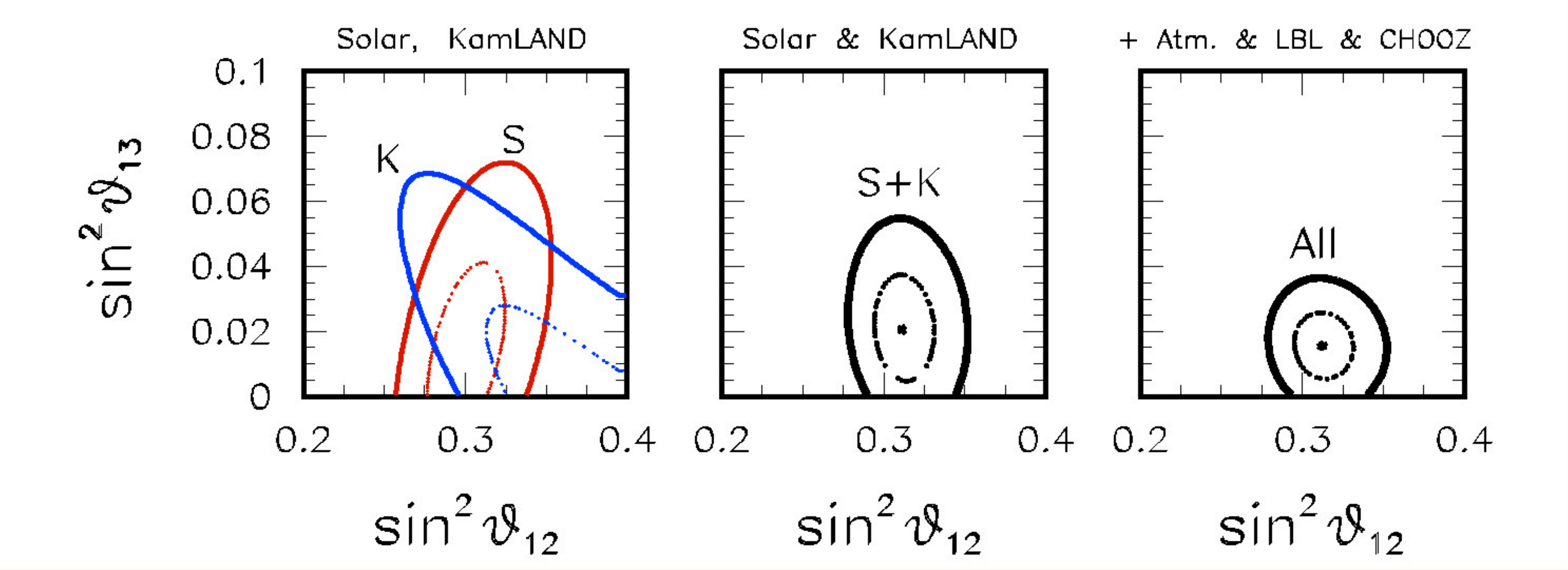




More refined (3ν) interpretation

Go beyond dominant 3ν oscillations. Include subleading effects due to θ_{13} and averaged Δm^2 oscillations in vacuum/matter.

Interesting (small) effects emerge. [See arXiv:0806.2649].



Hint of $\theta_{13} > 0$? Time will tell.

Synopsis of neutrino mass^2 and mixing parameters:
central values and n-σ ranges from global 3ν analysis

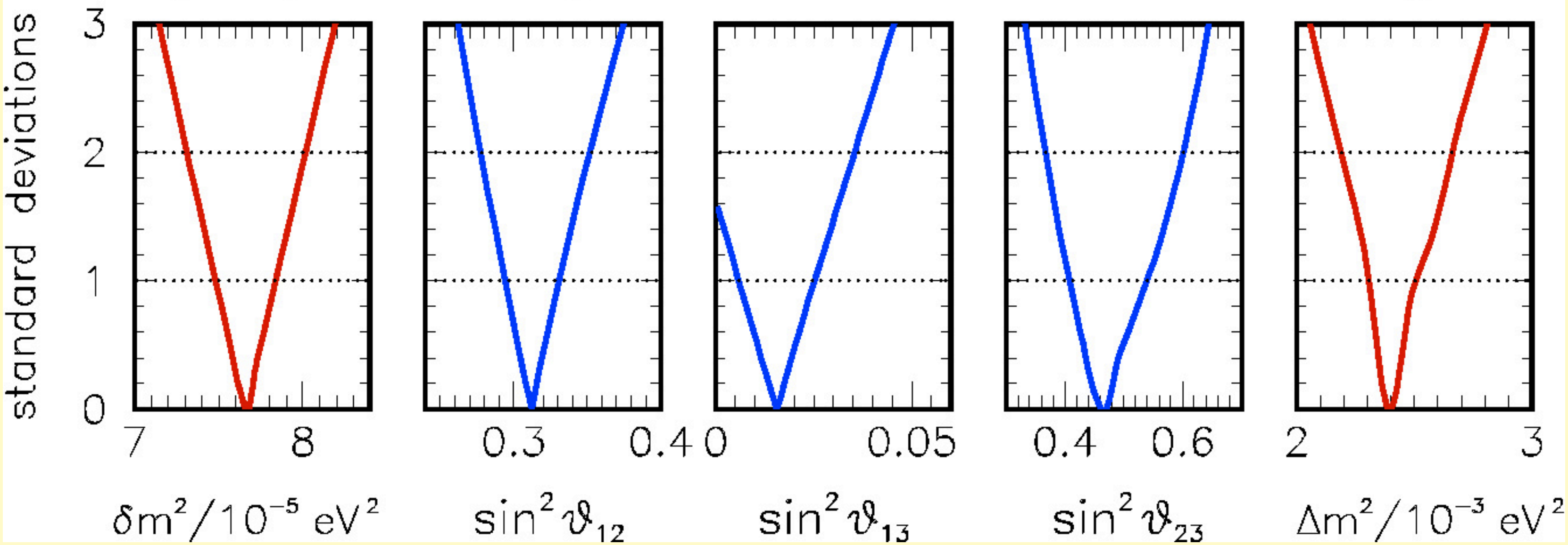


TABLE I: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges for the mass-mixing parameters.

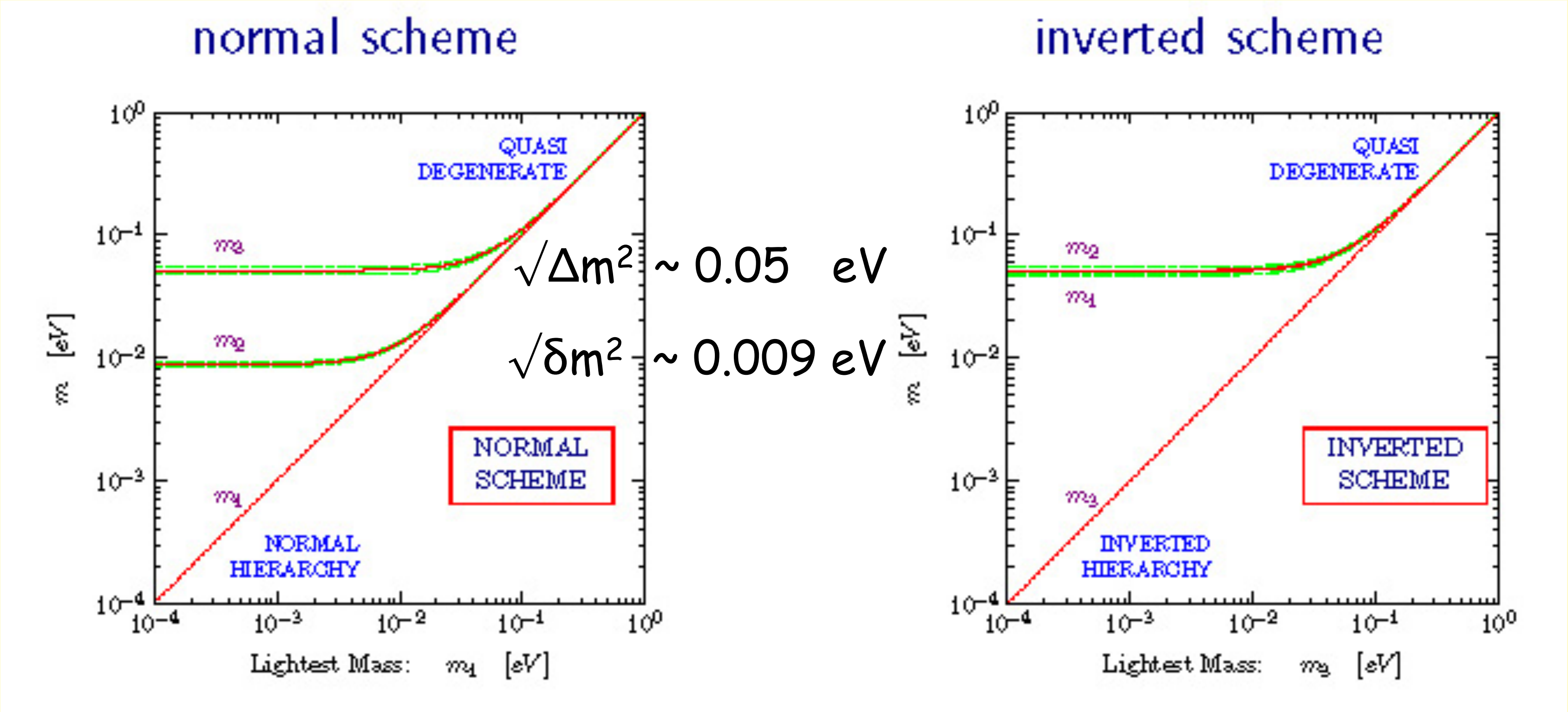
Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

arXiv:0805.2517

Absolute neutrino masses: Current phenomenology

Oscillations constrain neutrino mixings and mass splittings
but not the absolute mass scale.

E.g., can take the lightest neutrino mass as free parameter:



$\sqrt{\Delta m^2} \sim 0.05$ eV

$\sqrt{\delta m^2} \sim 0.009$ eV

However, the lightest neutrino mass is not really an “observable”

We know three realistic observables to attack ν masses →

(m_β, $m_{\beta\beta}$, Σ)

1) β decay: $m^2_i \neq 0$ can affect spectrum endpoint. Sensitive to the "effective electron neutrino mass":

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

2) $0\nu\beta\beta$ decay: Can occur if $m^2_i \neq 0$ and $\overline{\nu}=\nu$ (Majorana, not Dirac) Sensitive to the "effective Majorana mass" (and phases):

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

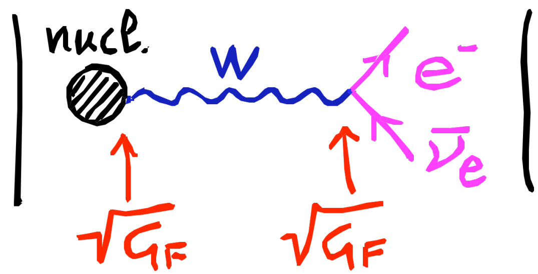
3) Cosmology: $m^2_i \neq 0$ can affect large scale structures in (standard) cosmology constrained by CMB + other data. Sensitive to:

$$\Sigma = m_1 + m_2 + m_3$$

Classic kinematic search for neutrino mass: look at high-energy endpoint Q of spectrum.

β-decay

rate: $d\Gamma \propto G_F^2 \times (\text{phase sp.})$

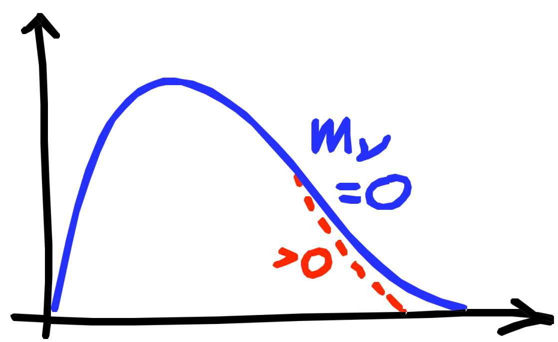


$\propto G_F^2$

energy spectrum:

$$\frac{d\Gamma}{dE_e} \propto G_F^2\, p_e E_e (Q - E_e)^2 \qquad (m_\nu = 0)$$

$$G_F^2\, p_e E_e (Q - E_e)\sqrt{(Q - E_e)^2 + m_\nu^2} \qquad (>0)$$



μ-decay

$$\Gamma_\mu = \frac{1}{\tau_\mu} \propto G_F^2 m_\mu^5$$

"defines" G_F

For just one (electron) neutrino family: sensitivity to $m^2(\nu_e)$ (obsolete)

For three neutrino families ν_i, and individual masses experimentally unresolved in beta decay: sensitivity to the sum of $m^2(\nu_i)$, weighted by squared mixings $|U_{ei}|^2$ with the electron neutrino. Observable:

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

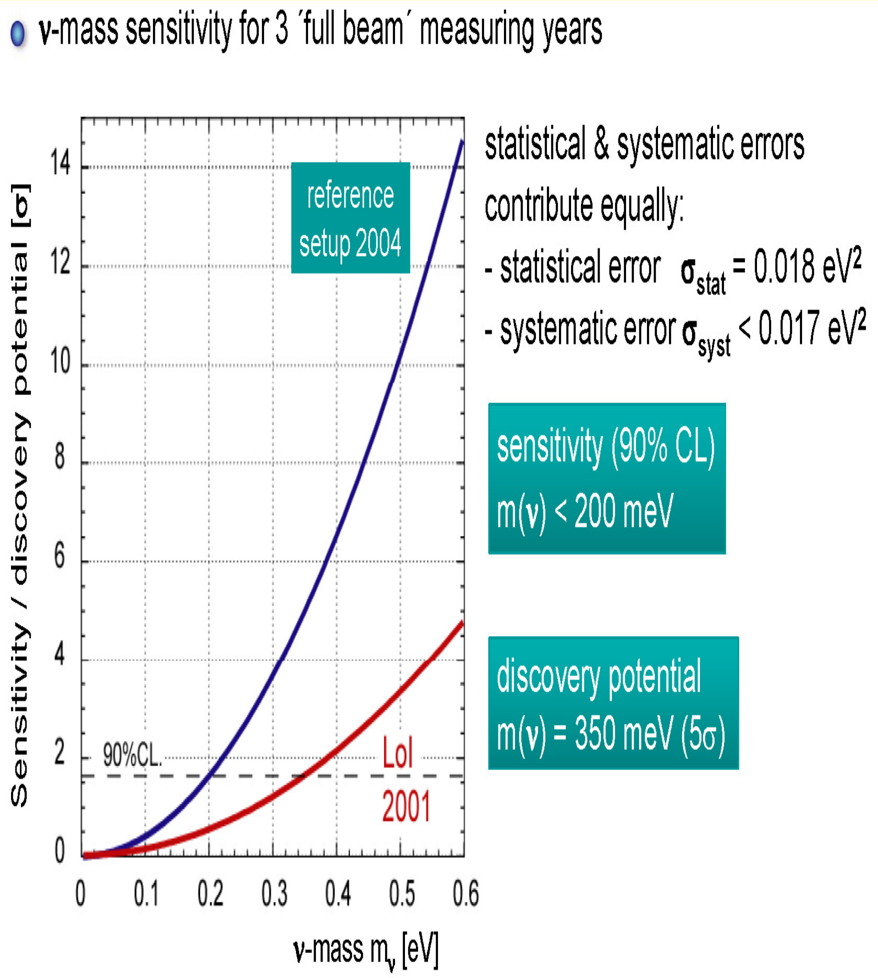
(so-called "effective electron neutrino mass")

Note: mass state with largest electron flavor component is ν_1:

$$|U_{e1}|^2 \approx \cos^2\theta_{12} \approx 0.7$$

… and we can't exclude that ν_1 is ~massless in normal hierarchy.

In construction: KATRIN. Sensitivity:



Mainz + Troitsk: $m_\beta < 2$ eV

KATRIN: O(10) improvement

Examples of prospective results at KATRIN ($\pm 1\sigma$, [eV]):

$m_\beta = 0.35 \pm 0.07$ (5σ discovery)

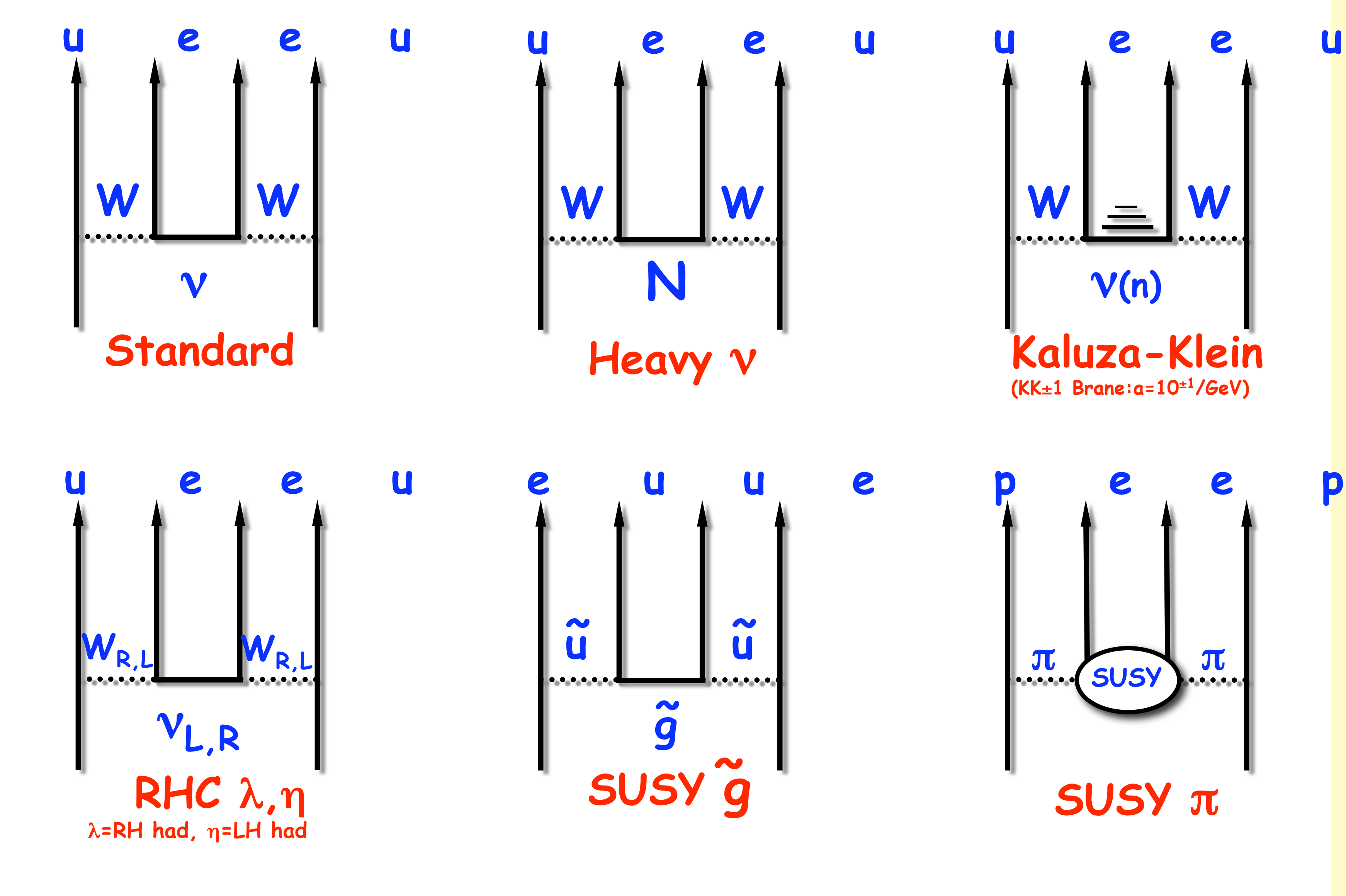
$m_\beta = 0.30 \pm 0.10$ (3σ evidence)

$m_\beta = 0 \pm 0.12$ (<0.2 at 90% CL)

Need new ideas [MARE ?] to go below ~0.2 eV…

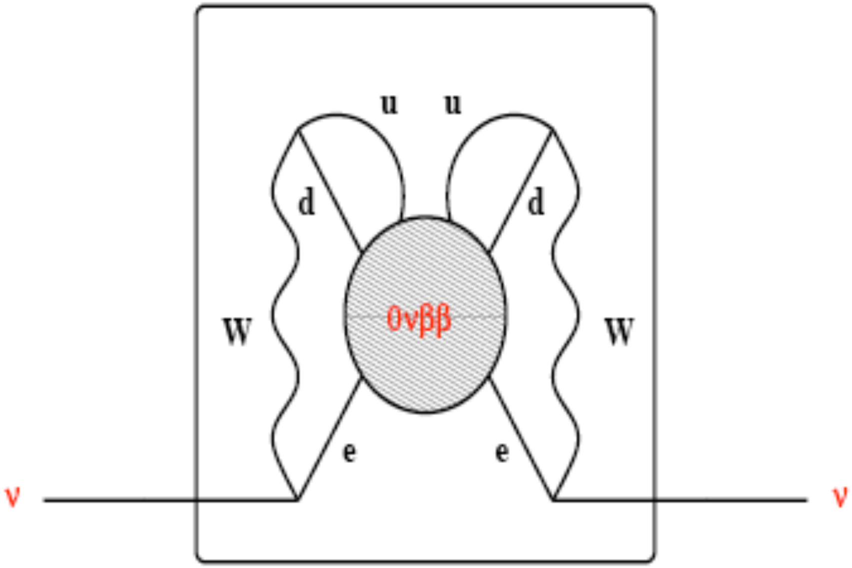
...Probably the "ultimate" spectrometer of this kind!

$0\nu\beta\beta$ decay: already discussed. Warning: might also arise from new physics!



[See talk by F. Feruglio]

However: whatever the mechanism…



Schechter & Valle, 1982

Independent of mechanism of $0\nu\beta\beta$ decay Majorana neutrino mass will appear in higher order!

Thus:

Observe $0\nu\beta\beta$ decay

$\equiv$

Neutrinos are Majorana particles

Cosmology

(a "modern" probe)

Standard big bang cosmology predicts a relic neutrino background with total number density $336/cm^3$ and temper. $T_\nu \sim 2$ K $\sim 1.7 \times 10^{-4}$ eV $\ll \sqrt{\delta m^2}, \sqrt{\Delta m^2}$.

→At least two relic neutrino species are nonrelativistic today (we can't exclude the lightest to be ~ massless)

→Their total mass contributes to the normalized energy density as $\Omega_\nu \approx \Sigma/50$ eV, where

$$\Sigma = m_1 + m_2 + m_3$$

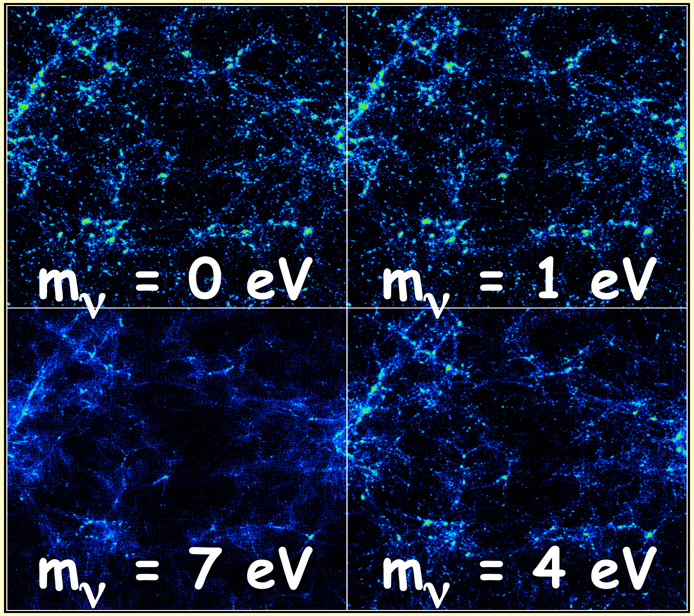
→So, if we just impose that neutrinos do not saturate the total matter density, $\Omega_\nu < \Omega_m \approx 0.25$, we get

$m_i < 4$ eV – not bad!

Much better bounds can be derived from neutrino effects on structure formation.

Massive neutrinos are difficult to cluster because of their relatively high velocities: they suppress matter fluctuations on scales smaller than their mass-dependent free-streaming scale.

→ Get mass-dependent suppression of small-scale structures



(E..g., Ma 1996)

[See talk by A. Melchiorri]

Hunting absolute masses... with a trident

ν oscillations

β decay

$0\nu2\beta$ decay

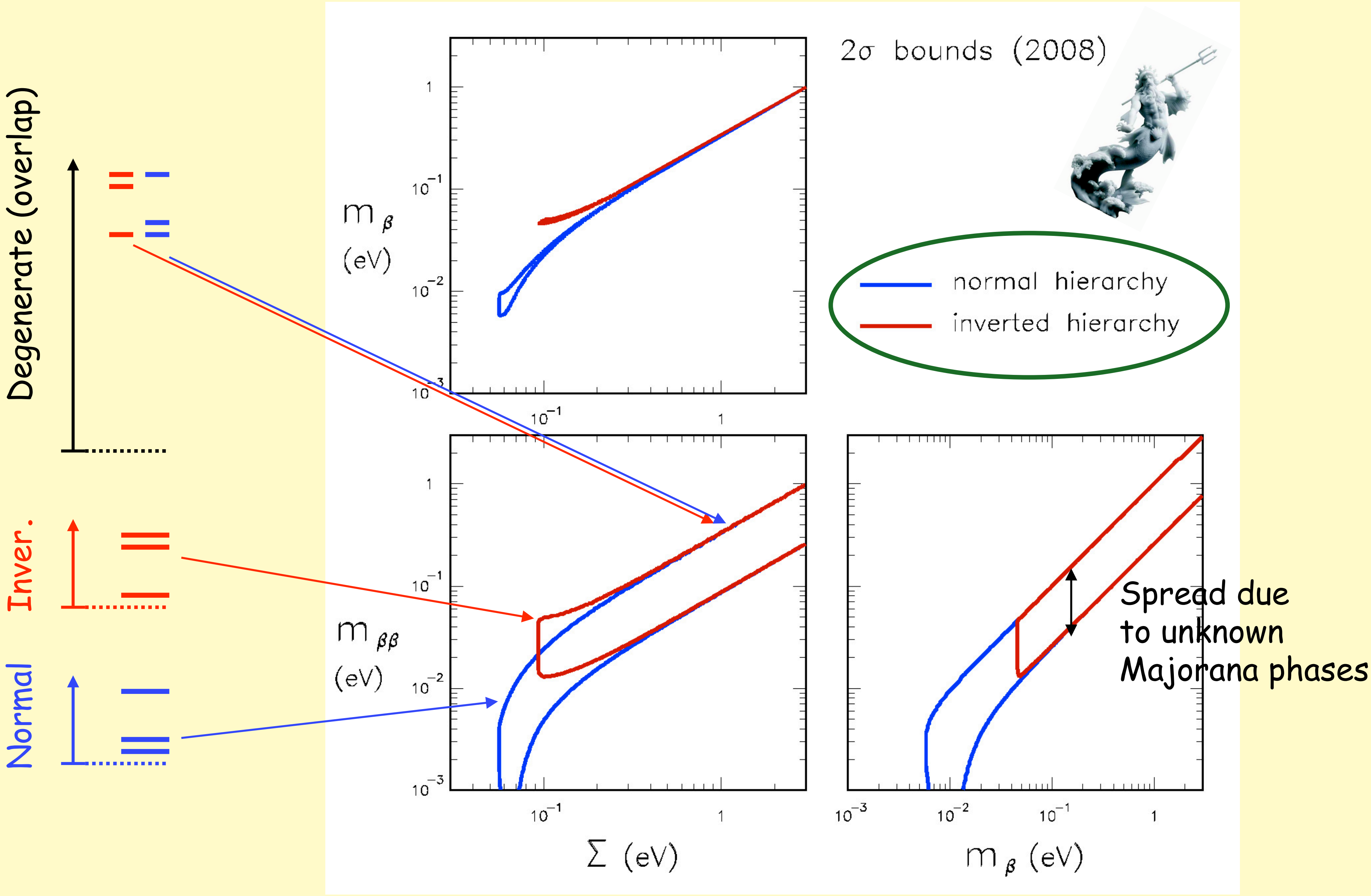
cosmology

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

$$\Sigma = m_1 + m_2 + m_3$$

The “spear” (oscill. data) sets the “hunting direction” in the **$(m_\beta,\ m_{\beta\beta},\ \Sigma)$** parameter space:



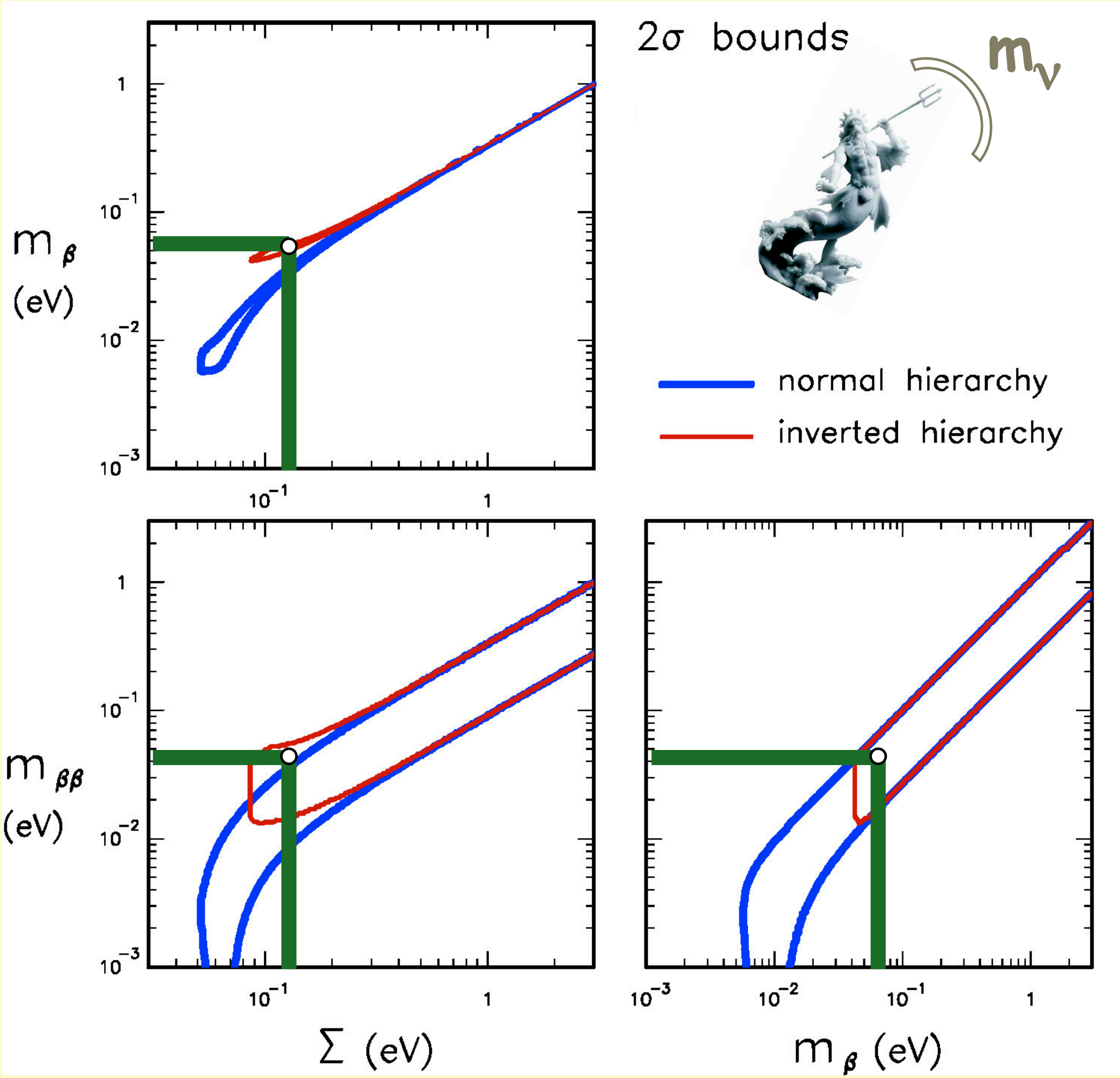
With "dreamlike" nonoscillation data one could, e.g.

Determine the mass scale…

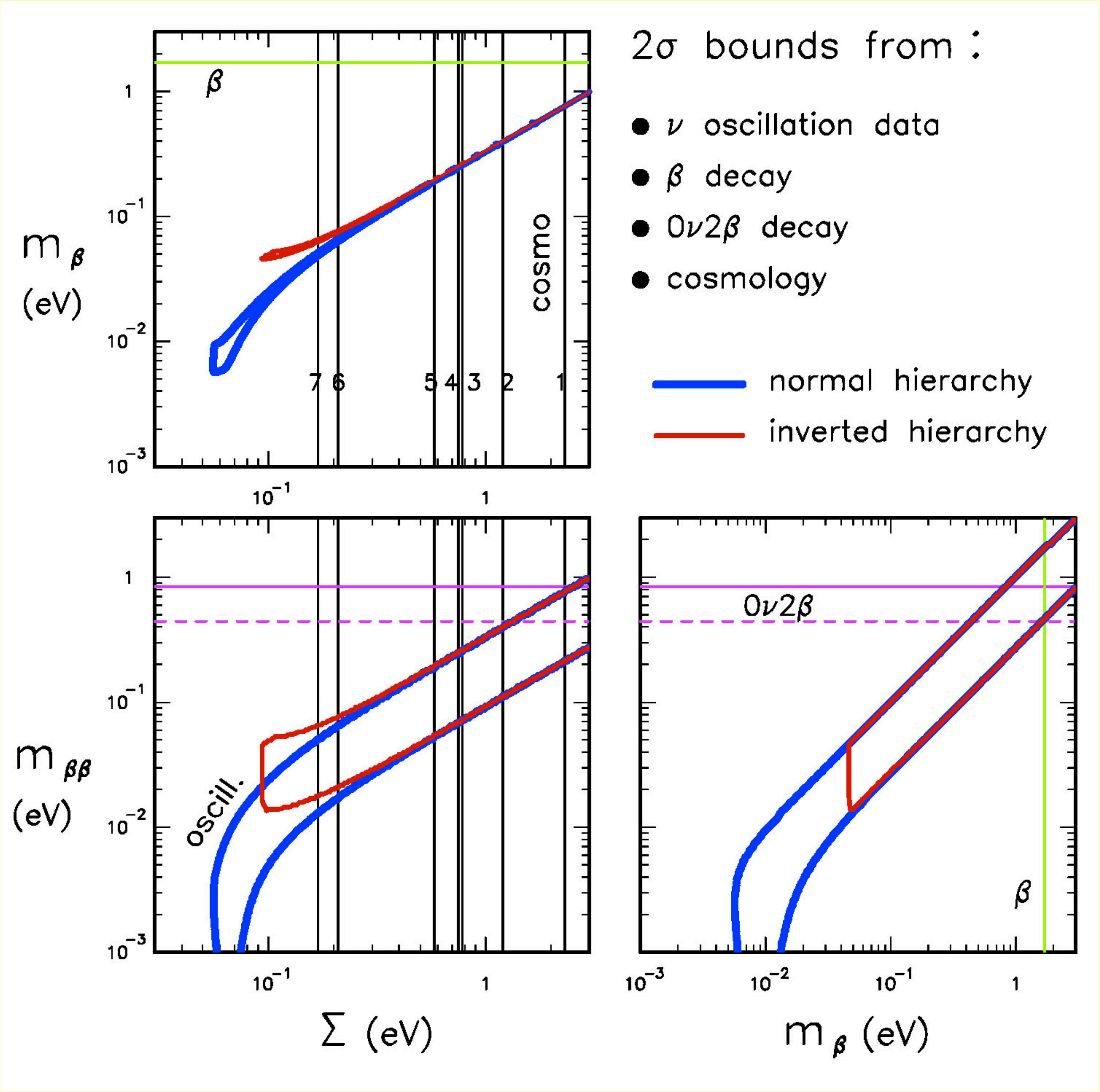
Check 3ν consistency …

Identify the hierarchy …

Probe the Majorana phase(s) …



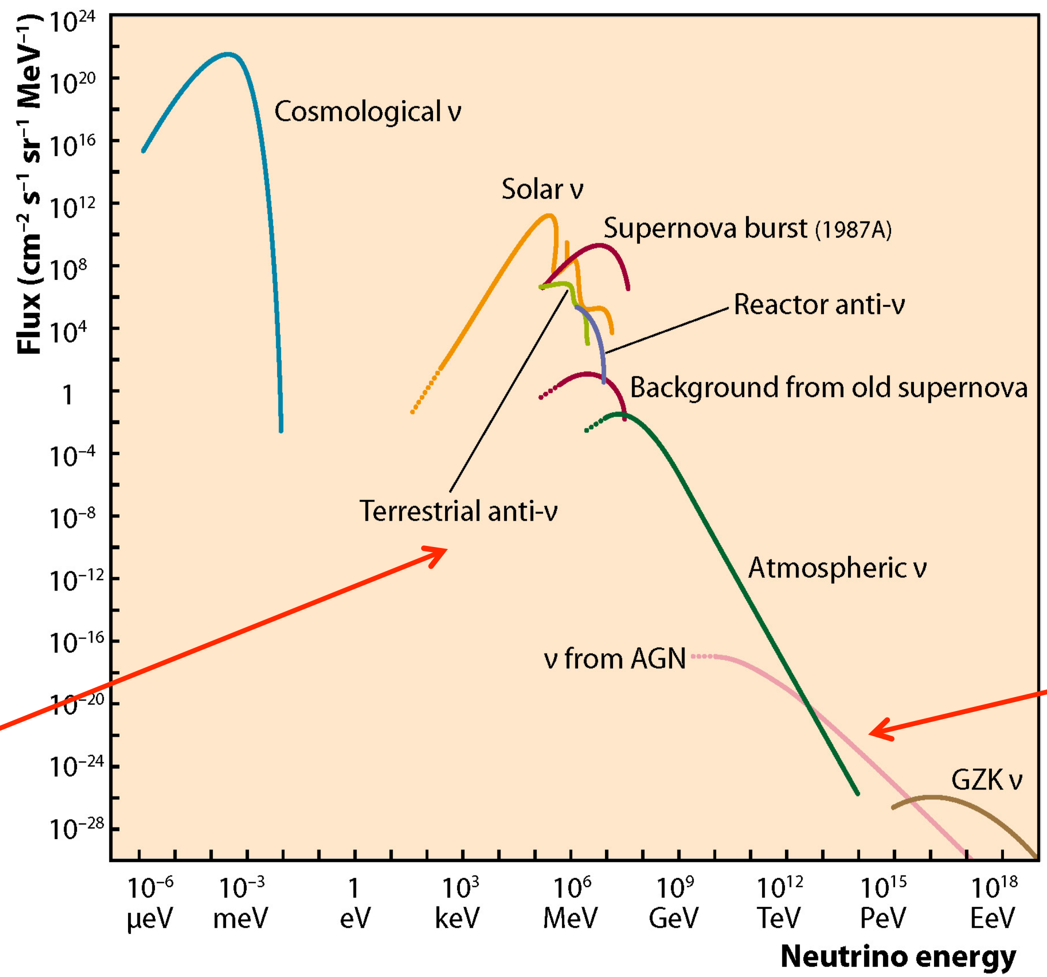
We are still far from this situation (an example with ~2006 data):



Different choices ⇒ Different possible combinations (and implications)

Progress in Neutrino Physics is not just limited to cornering neutrino mass and mixing parameters… there is much more!

Vast lands to be explored …



A synoptic view of neutrino fluxes. **(from ASPERA roadmap)**

[See talk by L. Ludhova]

[See talk by V. Flaminio]

Conclusions and Open Problems

Great progress in recent years …

Neutrino mass & mixing: established fact
Determination of ($\delta m^2,\theta_{12}$) and ($\Delta m^2,\theta_{23}$)
Upper bounds on θ_{13}
Observation of (half)-period of oscillations
Direct evidence for solar ν flavor change
Evidence for matter effects in the Sun
Upper bounds on ν masses in (sub)eV range
…………

Determination of θ_{13}
Appearance of ν_e, ν_τ
Leptonic CP violation
Absolute m_ν from β-decay and cosmology
Test of $0\nu2\beta$ claim and of Dirac/Majorana ν
Matter effects in the Earth, Supernovae…
Normal vs inverted hierarchy
(Dis)confirmation of standard 3ν scenario
Deeper theoretical understanding
Neutrino geo- and astro-physics
…………

… and great challenges for the future!

NOW 2010 Poster: www.ba.infn.it/now

The neutrino tree continues to grow.

Many opportunities open for your research activity!

Thank you for your attention.

HOMEWORK

Solution 5

- Mass basis: $\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$, $\Delta m^2 = m_2^2 - m_1^2$
- Flavor basis: $\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$; $\nu_x = \nu_{\mu,\tau}$; $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

- Hamiltonian in vacuum, flavor basis (see Exercise 2):

$$H = U \begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & +\frac{\Delta m^2}{4E} \end{pmatrix} U^T$$

- Hamiltonian in matter, flavor basis:

$H \rightarrow \tilde{H} = H + \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}$ with $V = \sqrt{2} G_F N_e$ (extra ν_e energy in matter)

- It is convenient to put $\tilde{H}$ in traceless form (extract $\mathrm{tr}(\tilde{H}) \cdot \mathbb{1}$):

$$\tilde{H} = \frac{1}{4E} \begin{bmatrix} A - \cos 2\theta\, \Delta m^2 & \sin 2\theta\, \Delta m^2 \\ \sin 2\theta\, \Delta m^2 & -A + \cos 2\theta\, \Delta m^2 \end{bmatrix} , \quad A = 2VE$$

(diagonalization becomes easier)

Solution 5 (ctd)

- Eigenvalues of $\tilde{H}$: $\pm \frac{\Delta\tilde{m}^2}{4E}$ with $\Delta\tilde{m}^2 = \Delta m^2 \sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}$

- Diagonalizing rotation:

$$\tilde{H} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} -\frac{\Delta\tilde{m}^2}{4E} & 0 \\ 0 & +\frac{\Delta\tilde{m}^2}{4E} \end{pmatrix} \begin{pmatrix} \cos\tilde{\theta} & -\sin\tilde{\theta} \\ \sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}$$

with $\sin 2\tilde{\theta} = \frac{\sin 2\theta}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$; $\cos 2\tilde{\theta} = \frac{\cos 2\theta - \frac{A}{\Delta m^2}}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$

This is analogous to the vacuum case, with the replacement $\theta \to \tilde{\theta}$ and $\Delta m^2 \to \Delta\tilde{m}^2$. [Note that $\Delta\tilde{m}^2 \sin 2\tilde{\theta} = \Delta m^2 \sin 2\theta$].

- If $A = \text{const}$ (i.e., $\tilde{\theta}$ is constant), then the evolution operator can be obtained by exponentiation as in Exercise 2. Then one gets in a similar way:

$$P(\nu_e \to \nu_x) = \sin^2 2\tilde{\theta} \, \sin^2\left(\frac{\Delta\tilde{m}^2 L}{4E}\right).$$

Solution 6

Let's prove first that $1\ \frac{\text{mol}}{\text{cm}^3} = 4.267 \times 10^{-9}\ \text{MeV}^3$

with $1\ \text{mol} = 6.022 \times 10^{23}$ particles (Avogadro number):

$$1\ \frac{\text{mol}}{\text{cm}^3} = \frac{6.022 \times 10^{23}}{10^{-6}\ \text{m}^3}\left(\frac{\text{MeV}^3}{\text{MeV}^3}\right) = 6.022 \times 10^{29}\ \frac{1}{(\text{m}\cdot\text{MeV})^3}\ \text{MeV}^3 = \frac{6.022 \times 10^{29}}{(5.0677 \times 10^{12})^3}\ \text{MeV}^3$$

$$= 4.627 \times 10^{-9}\ \text{MeV}^3$$

↑ see Exercise 4

Then: $A = 2\sqrt{2}\, G_F N_e E$ with $G_F = 1.16637 \times 10^{-5}\ \text{GeV}^{-2} = 1.16637 \times 10^{-11}\ \text{MeV}^{-2}$ (Fermi const.)

$$\frac{A}{\Delta m^2} = \frac{2\sqrt{2}\, G_F N_e E}{\Delta m^2} = 2\sqrt{2}\left(1.16637 \times 10^{-11}\ \text{MeV}^{-2}\right)\left(\frac{N_e}{\text{mol/cm}^3}\cdot \text{mol/cm}^3\right)\left(\frac{E}{\text{MeV}}\cdot \text{MeV}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\cdot\frac{1}{\text{eV}^2}\right)$$

$$= 3.299 \times 10^{-11}\ \frac{\text{MeV}^{-2}\ \text{MeV}}{\text{eV}^2}\ \frac{\text{mol}}{\text{cm}^3}\left(\frac{N_e}{\text{mol/cm}^3}\right)\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\right)$$

$$3.299 \times 10^{-11}\ \frac{\text{MeV}^{-2}\ \text{MeV}}{\text{eV}^2}\ \frac{\text{mol}}{\text{cm}^3} = 3.299 \times 10^{-11}\ \frac{10^{12}}{\cancel{\text{MeV}^3}} \times 4.627 \times 10^{-9}\ \cancel{\text{MeV}^3} = 1.526 \times 10^{-7}$$

$$\frac{A}{\Delta m^2} = 1.526 \times 10^{-7}\left(\frac{N_e}{\text{mol/cm}^3}\right)\left(\frac{E}{\text{MeV}}\right)\left(\frac{\text{eV}^2}{\Delta m^2}\right)$$

Appendix: Adiabatic approximation

Adiabatic 2ν case

Prove that, if $N_e(x)$ changes "slowly" from $x = x_i$ to $x = x_f$, then the AVERAGE survival probability $P(\nu_e \to \nu_e)$ is given by:

$$P_{ee}^{(2\nu)}(\text{adiab.}) = \cos^2\tilde{\theta}_i \cos^2\tilde{\theta}_f + \sin^2\tilde{\theta}_i \sin^2\tilde{\theta}_f$$

where $\tilde{\theta}_i$ and $\tilde{\theta}_f$ are the effective mixing angles in matter at $x = x_i$ and $x = x_f$.

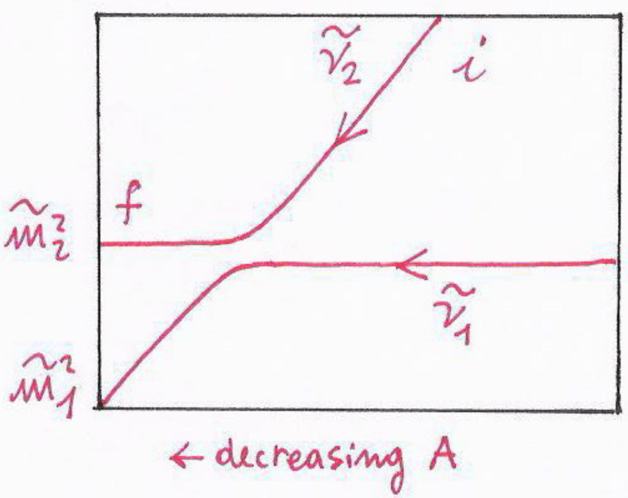
This is a good approximation for solar ν_e's, for the $(\delta m^2, \theta_{12})$ parameters chosen by nature!

Solution:

For a quasi-constant hamiltonian, one can solve the evolution equation at "one x" at a time, and then patch the solutions from x_i to x_f. This means that, given the initial state

$$|\nu_e^i\rangle = \cos\tilde{\theta}_i\, |\tilde{\nu}_1^i\rangle + \sin\tilde{\theta}_i\, |\tilde{\nu}_2^i\rangle \quad ,$$

the effective eigenstates of the hamiltonian at $x = x_i$ ($|\tilde{\nu}_{1,2}^i\rangle$) slowly transform into $|\tilde{\nu}_{1,2}^f\rangle$ at $x = x_f$, respectively:



$$|\tilde{\nu}_1^i\rangle \to |\tilde{\nu}_1^f\rangle \quad \text{with} \quad |\langle \nu_1^f | \nu_1^i \rangle| = 1$$

$$|\tilde{\nu}_2^i\rangle \to |\tilde{\nu}_2^f\rangle \quad \text{with} \quad |\langle \nu_2^f | \nu_2^i \rangle| = 1$$

$$\text{and} \quad |\langle \nu_{2,1}^f | \nu_{1,2}^i \rangle| = 0$$

(no "level crossing")

$$|\nu_e^f\rangle = \cos\tilde{\theta}_f\, |\nu_1^f\rangle + \sin\tilde{\theta}_f\, |\nu_2^f\rangle$$

We have then:

$$P_{ee}^{2\nu} = |\langle \nu_e^f | \nu_e^i \rangle|^2 =$$

$$= |\cos\hat{\theta}_i \cos\hat{\theta}_f \langle \nu_1^f | \nu_1^i \rangle + \sin\hat{\theta}_i \sin\hat{\theta}_f \langle \nu_2^f | \nu_2^i \rangle|^2 .$$

If we average out interference terms and phases (ok for many/fast oscillations along the ν trajectory):

$$P_{ee}^{2\nu} \simeq \cos^2\hat{\theta}_i \cos^2\hat{\theta}_f \, |\langle \nu_1^f | \nu_1^i \rangle|^2 + \sin^2\hat{\theta}_i \sin^2\hat{\theta}_f \, |\langle \nu_2^f | \nu_2^i \rangle|^2$$

$$= \cos^2\hat{\theta}_i \cos^2\hat{\theta}_f + \sin^2\hat{\theta}_i \sin^2\hat{\theta}_f$$

[Solar ν's oscillate many times from Sun to Earth for $\delta m^2 \sim 7.7 \times 10^{-5}$ eV^2.]

Equivalent form: $P_{ee}^{2\nu} = \frac{1}{2} + \frac{1}{2} \cos 2\hat{\theta}_i \cos 2\hat{\theta}_f$

Application to solar ν_e :

$\theta = \theta_{12}$; $\tilde{\theta}_{12}(x_f) = \theta_{12}$ (vacuum value at exit from the Sun)

$$P_{ee}^{2\nu}(\text{solar}) \simeq \cos^2\theta_{12}\cos^2\tilde{\theta}_{12}(x_0) + \sin^2\theta_{12}\sin^2\tilde{\theta}_{12}(x_0)$$

where $\tilde{\theta}(x_0)$ is the effective mixing angle at production point x_0.

limiting cases :

- **E ≲ few MeV** (vacuum-dominated) : $A/\delta m^2 \lesssim 1$

  → $\tilde{\theta}_{12}(x_0) \simeq \theta_{12}$ and $P_{ee} \simeq c_{12}^4 + s_{12}^4 = 1 - \frac{1}{2}\sin^2 2\theta_{12}$

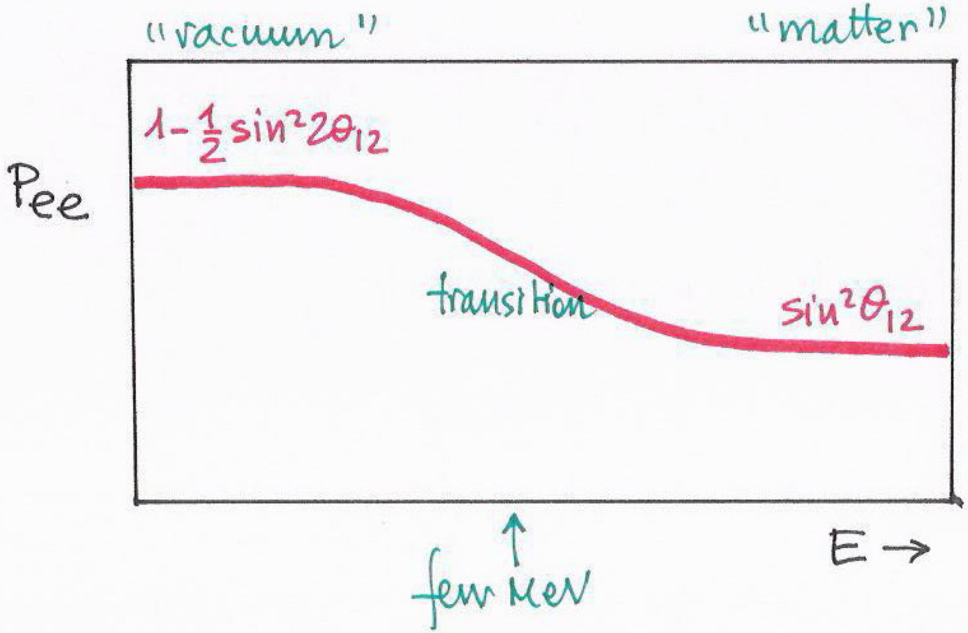
  → Pee equals the average vacuum probability

- **E ≳ few MeV** (matter dominated) : $A/\delta m^2 \gtrsim 1$

  → $\tilde{\theta}_{12}(x_0) \sim \pi/2$ and $P_{ee} \simeq \sin^2\theta_{12}$

  → Pee is octant asymmetric

Energy profile of P_{ee}:



The P_{ee} transition from "low" to "high" energy ν_e's is a signature of matter effects.

Thanks to matter effects we can determine the octant of the mixing angle θ_{12}.