PARAMETRIC RESONANCE IN OSCILLATIONS OF SOLAR AND ATMOSPHERIC NEUTRINOS IN THE EARTH

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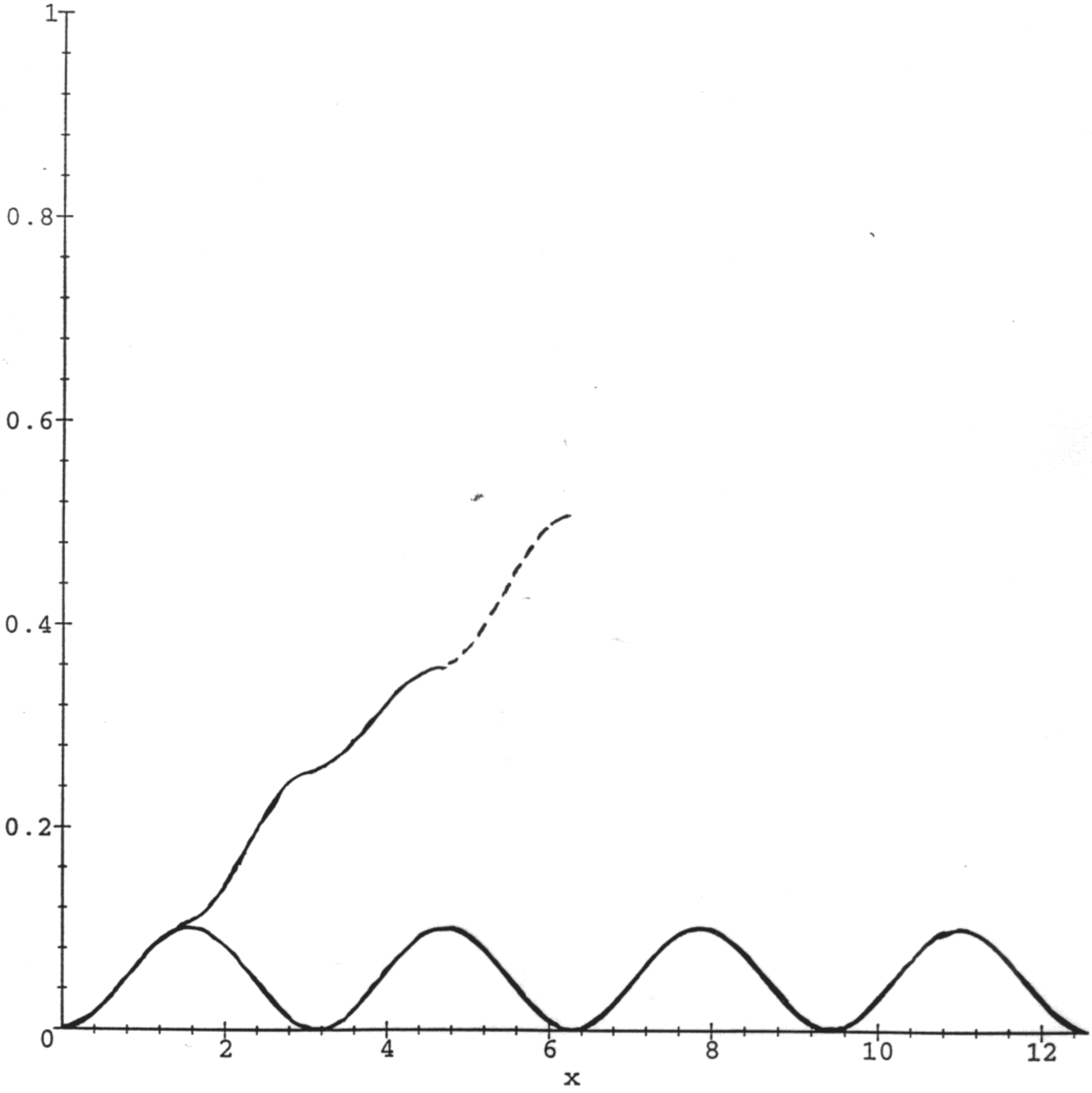
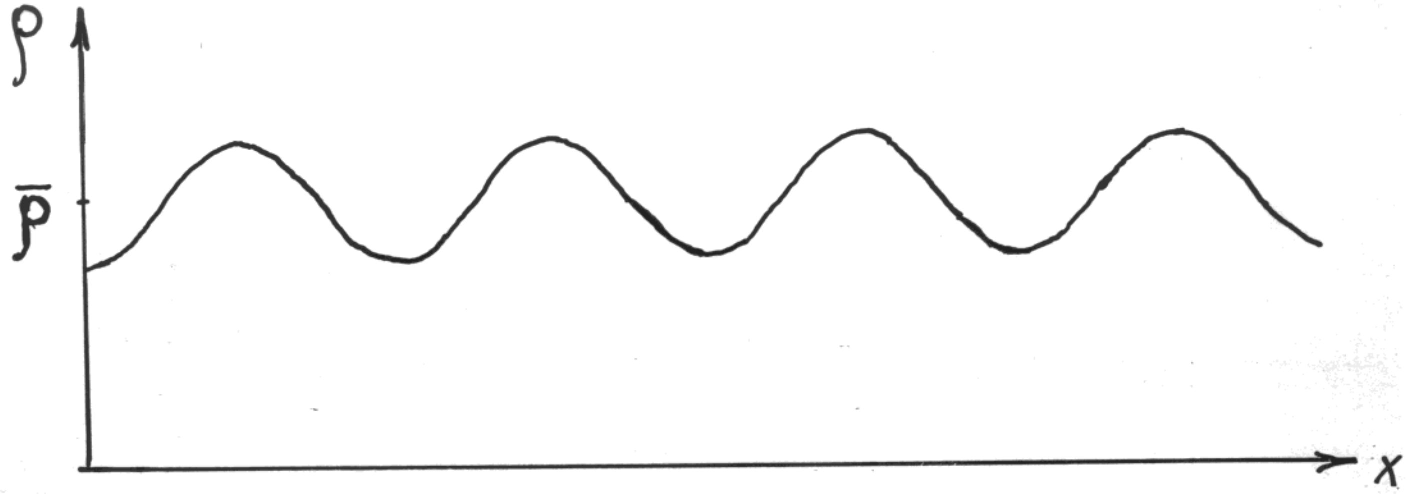
1. Physical nature of parametric resonance
2. Param. resonance of ν oscillations in the earth
3. Stability of param. res. conditions
4. Implications for solar and atmospheric ν's
5. Prospects of experimental observation

Matter effects on neutrino oscillations can be important – MSW is not the sole possibility!

MSW: oscillation amplitude is enhanced.

Parametric resonance: oscillation phase in matter is modified in a special way.





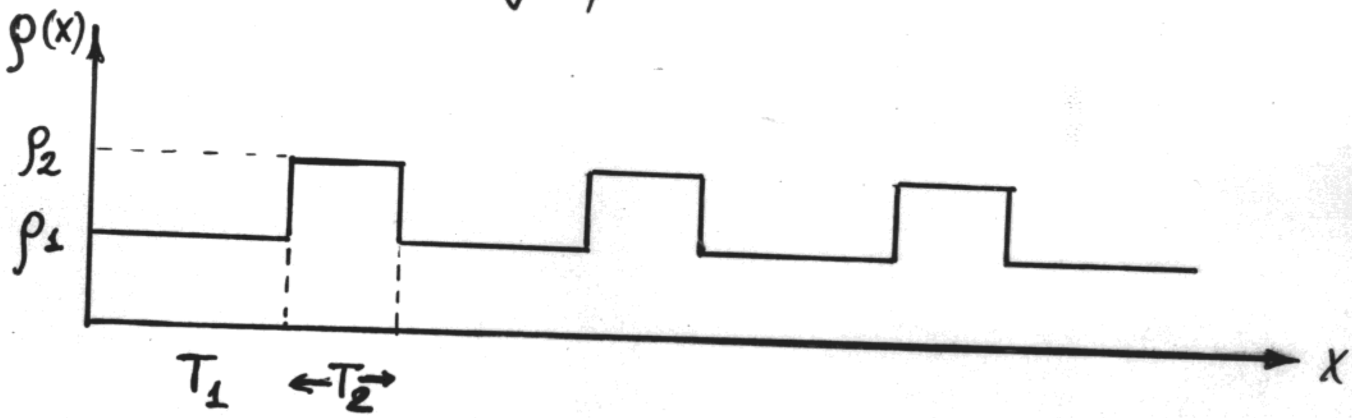
Mathieu equation

Approx. solutions:

Ermilova, Tzarev, Chechin, 1986

E.A., 1987

Periodic step function ("castle wall") density profile:



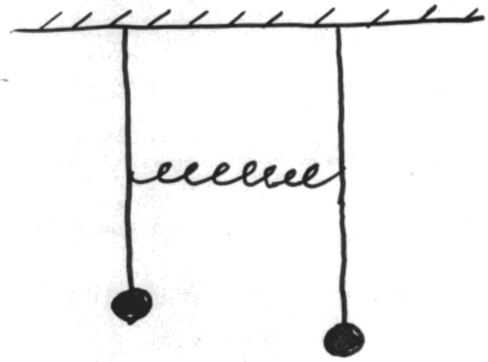
$T = T_1 + T_2$

Allows exact solution! E.A., 1987

Sov. J. Nucl. Phys. 47 (88) 301

Parametric resonance of ν oscillations

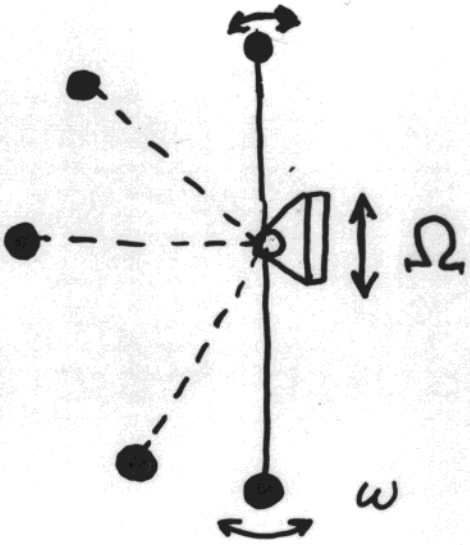
A mechanical model of the MSW effect:



coupled pendulum

Are there any other resonance phenomena in mechanics which can have interesting analogs in neutrino physics?

Parametric resonance: can occur in dynamical systems whose parameters vary periodically with time.



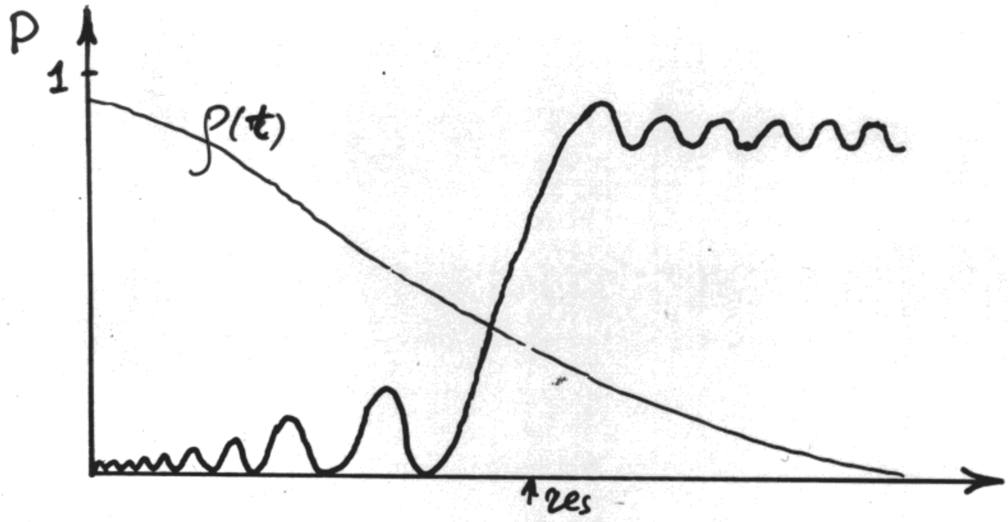
Instability of small oscillations: $\Omega = \frac{2\omega}{n}$ $(n = 1, 2, \ldots$

- In matter of constant density:

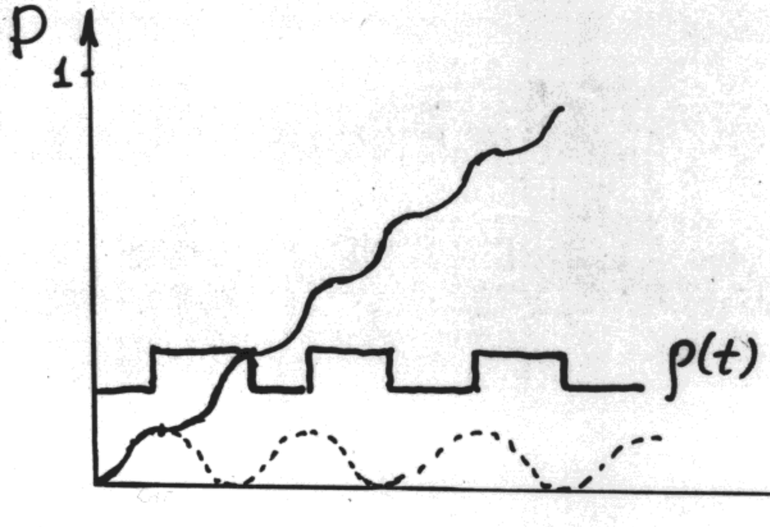
  transition probability cannot exceed $\sin^2 2\theta_m$ no matter how long ν's travel

- In matter with periodic density profile:

  $P \cong 1$ is possible if ν's travel long enough. ⇒ Not necessarily over many periods!



MSW



Parametric resonance

"Castle wall" density profile

Very different from the MSW effect!

$$P(\nu_a \to \nu_b \,;\, t = nT) = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2\left(\Phi \frac{t}{T}\right)$$

- Similar to ν osc. in matter of constant density. But: amplitude and phase are different from those in matter with ρ equal to either ρ_1 or ρ_2!

Resonance: $X_3 = 0 \implies$

$$\phi_1 \equiv \omega_1 T_1 = \frac{\pi}{2}(2k+1) \,;\quad \phi_2 \equiv \omega_2 T_2 = \frac{\pi}{2}(2k'+1)$$

$$k, k' = 0, 1, 2, \ldots$$

At the resonance:

$$P(\nu_a \to \nu_b \,;\, t = nT) = \sin^2[2n(\theta_2 - \theta_1)]$$

θ_1, θ_2 – mixing angles in matter with densities ρ_1 and $\rho_2 \implies$ can be quite small (no MSW-enhancement assumed)

But: for large enough n ($n \sim \frac{\pi}{4(\theta_1 - \theta_2)}$) the transition probability can be close to 1.

Krastev & Smirnov, Phys. Lett. B226 (89) 341:

If the parametric resonance energy is not fa
from the MSW resonance energy,

$$\left(\delta \equiv \frac{\Delta m^2}{4E} \sim V \equiv \frac{G_F}{\sqrt{2}} N_e \right)$$

strong parametric enhancement of the transition probability can occur even for small number (1-2-3) of periods!

Where can the conditions for the parametric resonance be realized?

In the Earth!

Liu, Smirnov, Nucl. Phys. B 524 (98) 505
hep-ph/9712493

Liu, Mikheyev & Smirnov, Phys. Lett. B440 (98) 319
hep-ph/9803415

Atmospheric ν oscillations in the Earth

$\nu_\mu \to \nu_s$ channel, large mixing angle.

Parametric resonance conditions:

$$\Phi_{m1} \approx \Phi_c \approx \Phi_{m2} \approx \pi$$

(principal resonance – $K = K' = 0$)

LS 97

- $$P_{res} = \sin^2(2\theta_c - 4\theta_m)$$

$\frac{\Delta m^2}{2E} < V_{m,c}$ – not the most interesting region.

Petcov, Phys. Lett. B 434 (98) 321
hep-ph/9805262

$\nu_2 \to \nu_e$ oscillations of solar ν's in the earth. Parametric resonance conditions are satisfied to a much better accuracy! Strong enhancement for $V_m \lesssim \frac{\Delta m^2}{2E} \lesssim V_c$.

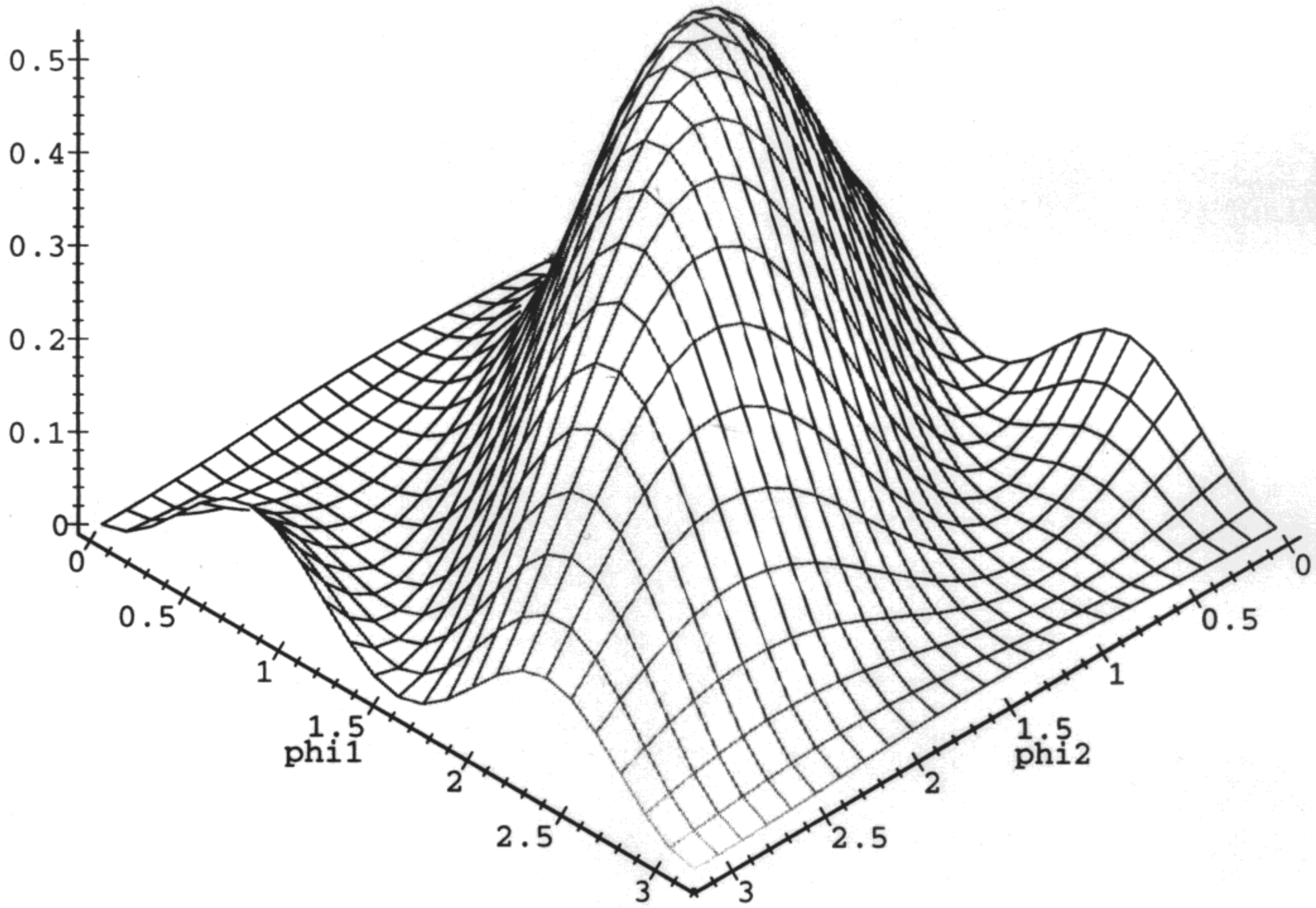
- $$(P_{2e})_{res} = \sin^2(2\theta_c - 4\theta_m + \theta_0)$$

E. A., Nucl. Phys. B 538 (99) 25 (hep-ph/9805272)
E. A., Dighe, Lipari, Smirnov, hep-ph/9808270
Chizhov, Maris, Petcov, hep-ph/9810501

8''

$P(\nu_a \leftrightarrow \nu_b)$ as a function of ϕ_1, ϕ_2

$\sin^2 2\theta_0 = 0.01, \quad \delta = 1.2 \times 10^{-13}$ eV

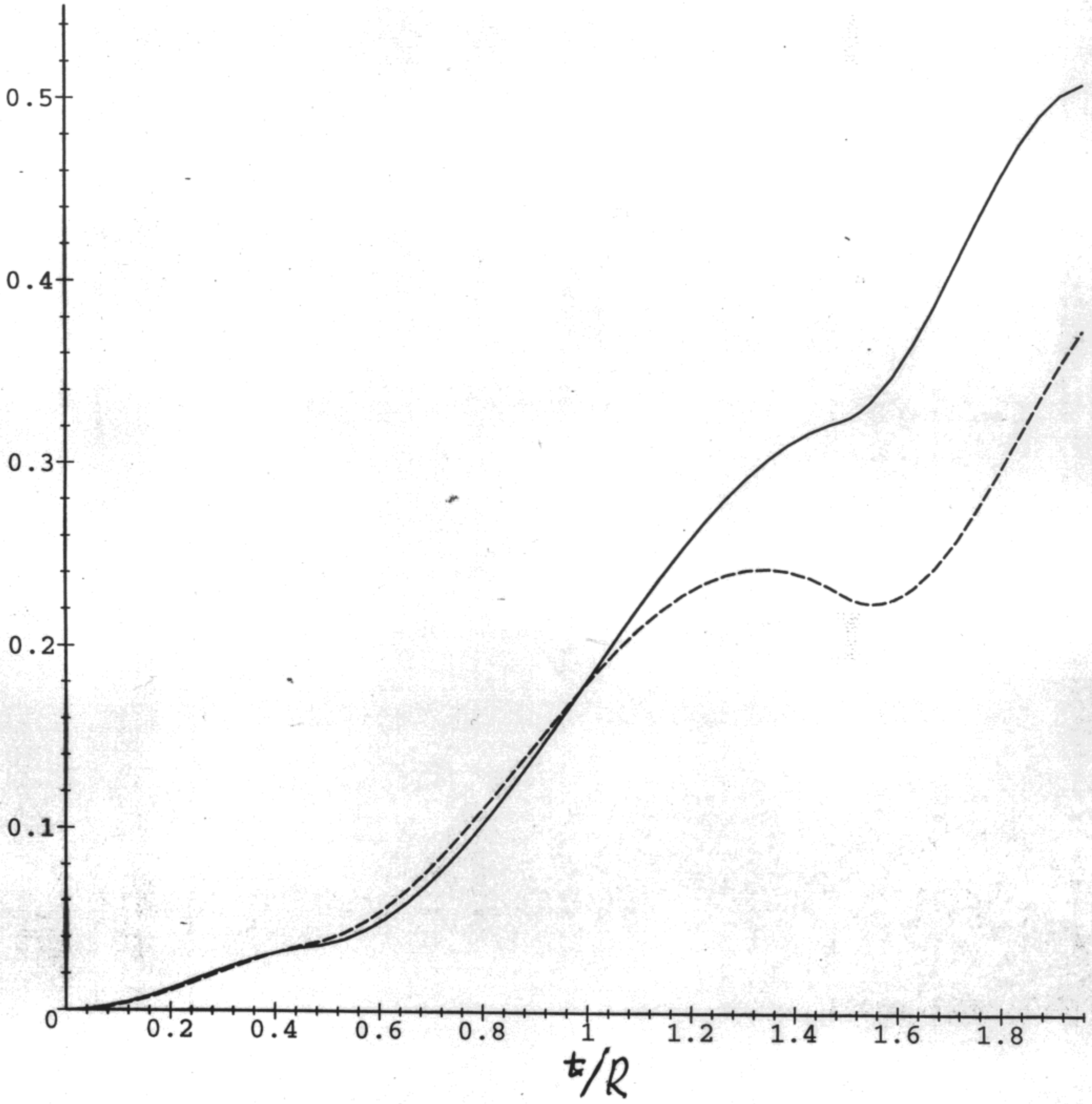


$\sin^2 2\theta_0 = 0.01$

$\Theta = 168.5^\circ$

—— $\frac{\Delta m^2}{4E} = 1.8\times10^{-13}$ eV

- - - $\frac{\Delta m^2}{4E} = 1.7\times10^{-13}$ eV



Why is the parametric resonance in ν oscillations in the earth possible?

- Remarcable coincidence:

$$V_m = \frac{G_F}{\sqrt{2}} (N_e)_{mantle}, \; V_c = \frac{G_F}{\sqrt{2}} (N_e)_{core}, \quad R_\oplus^{-1}$$

and $\left(\frac{\Delta m^2}{4E}\right)_{solar}$, $\left(\frac{\Delta m^2}{4E}\right)_{atm}$

are all of the same order of magnitude

$$\underline{3 \times 10^{-14} \div 3 \times 10^{-13} \ \ eV}$$

- $R_\oplus^{-1} = 3.1 \times 10^{-14}$ eV; $\qquad \rho_m \cong (3 \div 5.5) \frac{g}{cm^3}$ $\Rightarrow$
  $\rho_c \cong (10 \div 13) \frac{g}{cm^3}$

- $V_m = V_{mantle} \cong (5.7 \times 10^{-14} \div 1 \times 10^{-13})$ eV

- $V_c = V_{core} \cong (1.9 \div 2.5) \times 10^{-13}$ eV

Solar ν's: small θ_0 MSW solution $\Rightarrow \Delta m^2 \sim 10^{-6} \div 10^{-}$
eV

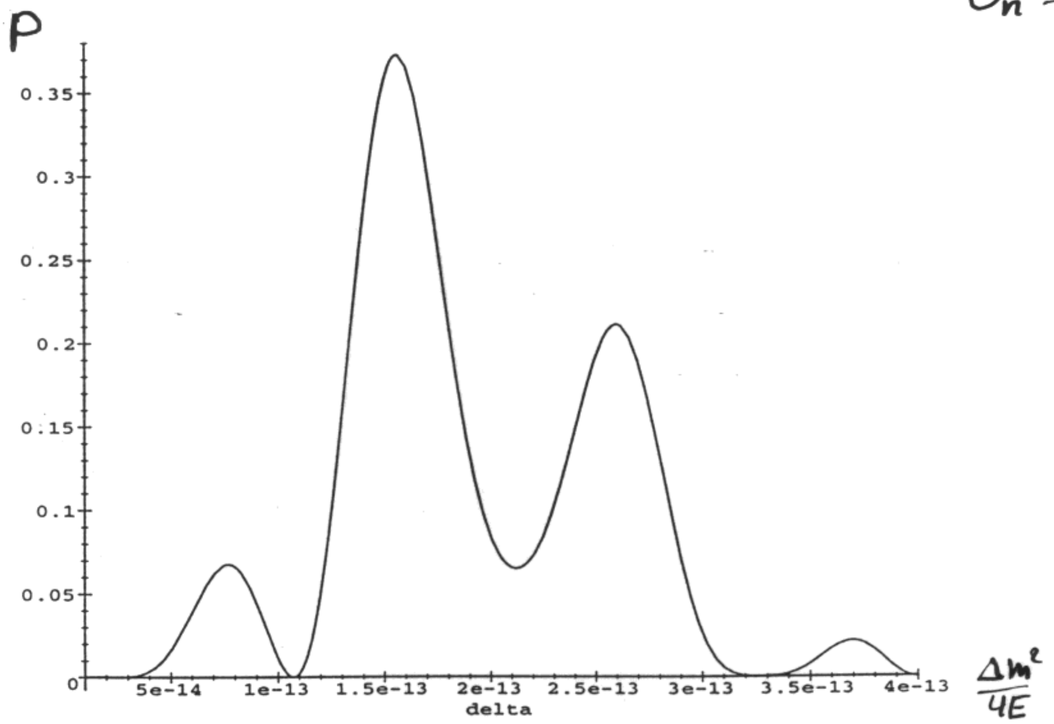
- $\left(\frac{\Delta m^2}{4E}\right)_{solar} \sim (3 \times 10^{-14} \div 3 \cdot 10^{-13})$ eV (for $E \simeq 8$ Me

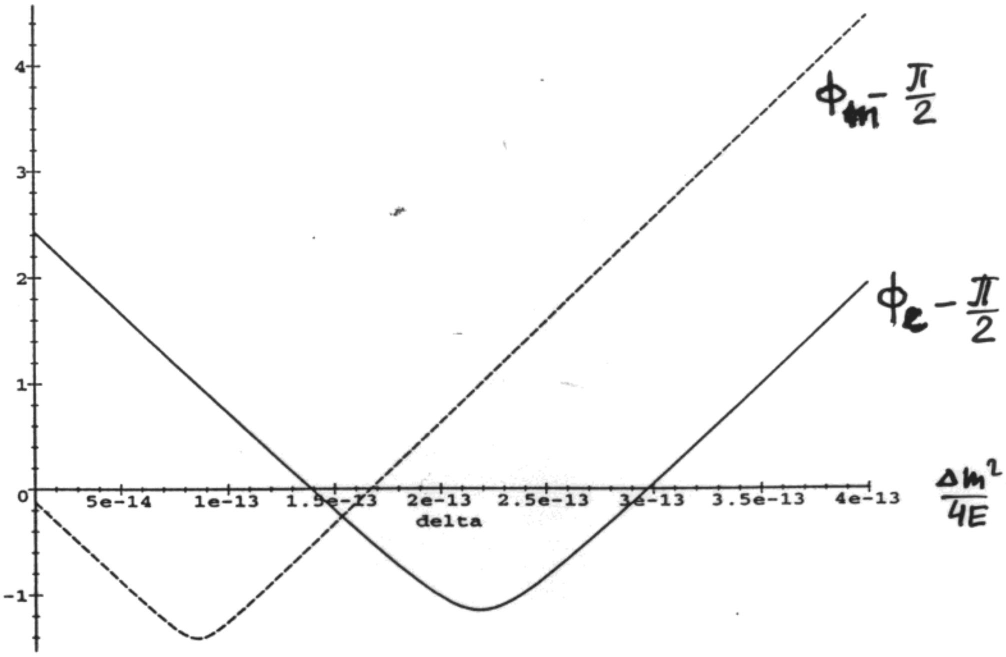
Atm. ν's: for $\Delta m^2 \simeq 10^{-3}$ eV^2

- $\left(\frac{\Delta m^2}{4E}\right)_{atm} \cong \begin{cases} 5 \times 10^{-13} \text{ eV} & (E \sim 0.5 \text{ GeV, sub-Ge} \\ 1 \times 10^{-13} \text{ eV} & (E \sim 2.5 \text{ GeV, multi-G} \end{cases}$

$\sin^2 2\theta_0 = 0.01$

$\theta_n = 27.3^\circ$





$\phi_m = \omega_m R_m$

$\phi_c = \omega_c R_c$

Day - night effect for solar neutrinos. During night a fraction of ν_μ can be re-converted into ν_e's due to the MSW-enhanced oscillations in the earth (regeneration effect).

- Parametric effects can greatly enhance the night-time regeneration for ν's that cross the core of the earth!

  Up to a factor of 6 enhancement ⇒ for small-Θ_0 MSW solution this may be the only possibility to see the day-night effect.

For atmospheric neutrinos:

- Parametric enhancement of the subdominant $\nu_\mu \leftrightarrow \nu_e$ oscillation mode
- Partial explanation of the observed excess of e-like events
- Specific distortion of the zenith-angle distribution and enhanced up-down asymmetry of e-like events.

Very small θ_{13} can be probed!

Probability of finding a solar ν_e at a detector after it traverses the Earth:

◆

$$P_{SE} = \bar{P}_S + \frac{1-2\bar{P}_S}{\cos 2\theta_0}\left(P_{2e} - \sin^2\theta_0\right)$$

$\bar{P}_S$ - averaged survival probability of ν_e in the Sun.

For $\bar{P}_S \approx \frac{1}{2}$ effects of the Earth's matter on ν oscillations strongly suppressed even if P_{2e} is resonantly enhanced!

Possibility of observing effects of ν propagation in the matter of the Earth (including possible parametric enhancement) depends on how close the true values of Δm^2 and $\sin^2 2\theta_0$ are to the line in the parameter space where $\bar{P}_S = \frac{1}{2}$.

12''

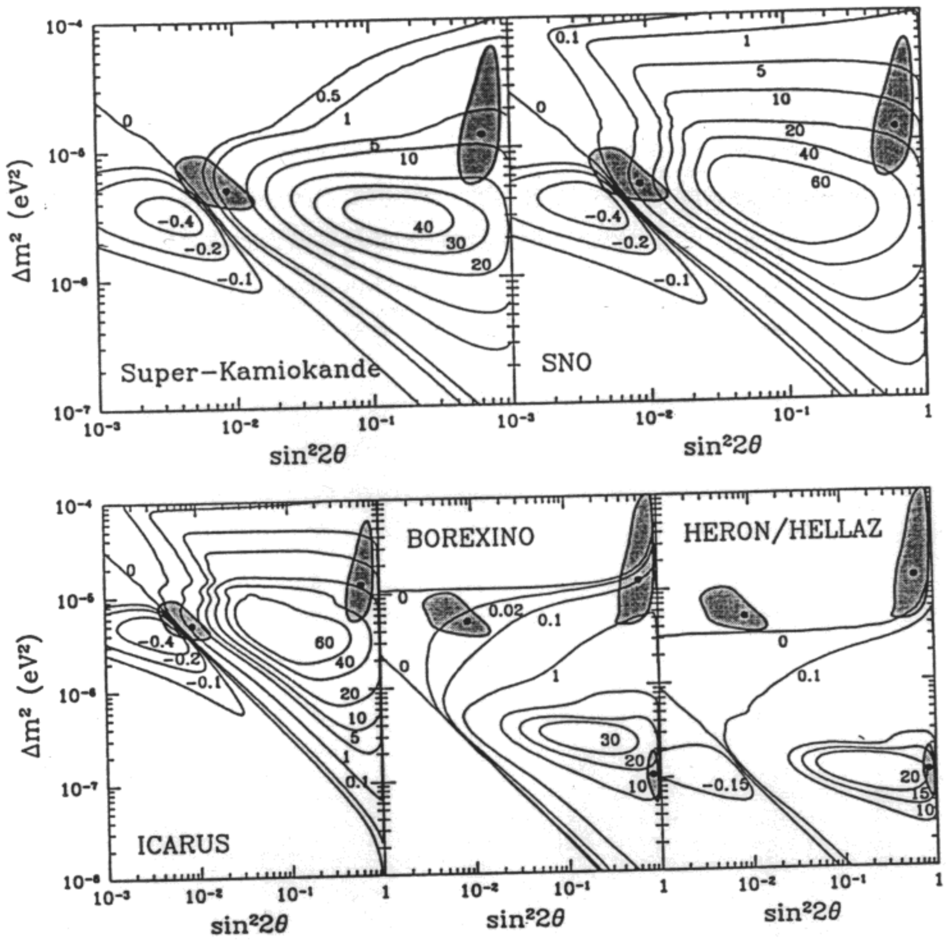


Figure 10

Bahcall, Krastev (1997)

SK 504 days

hep-ex/9812009

FIGURES

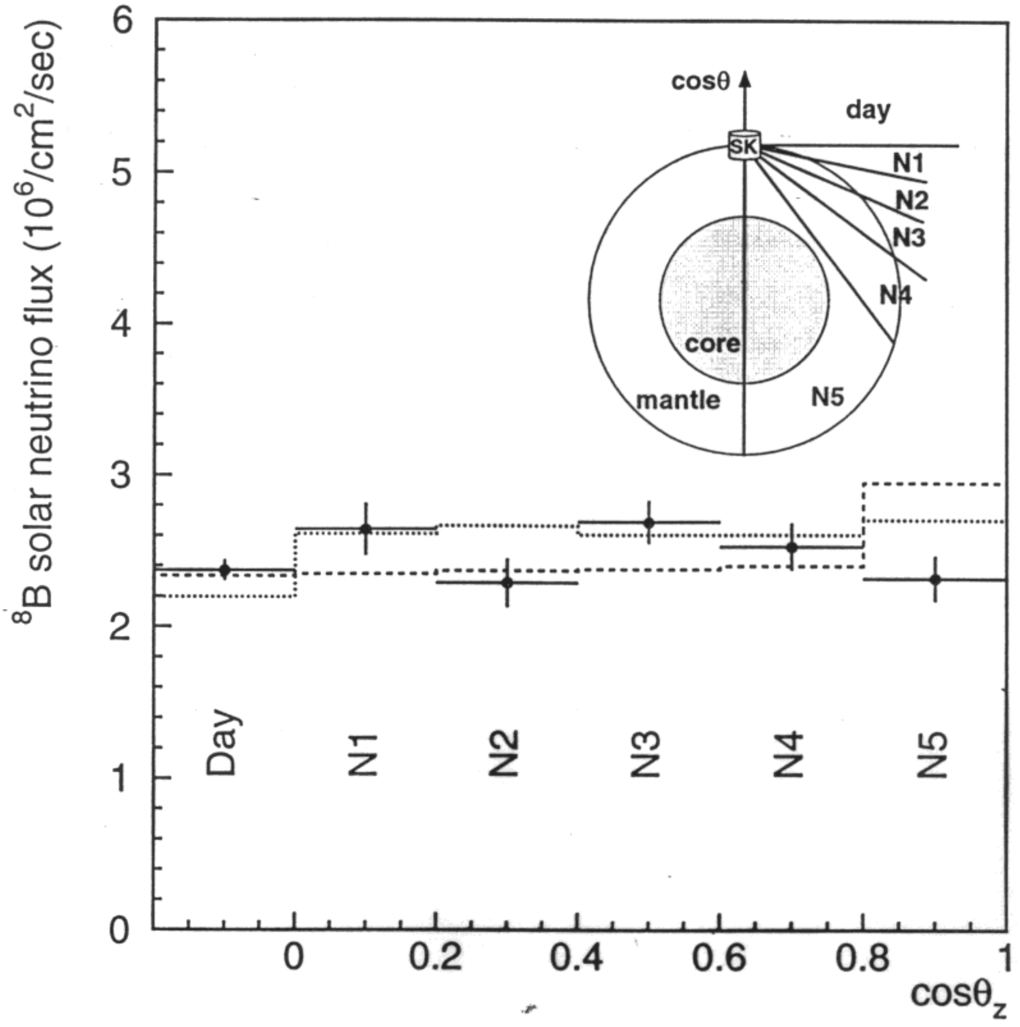
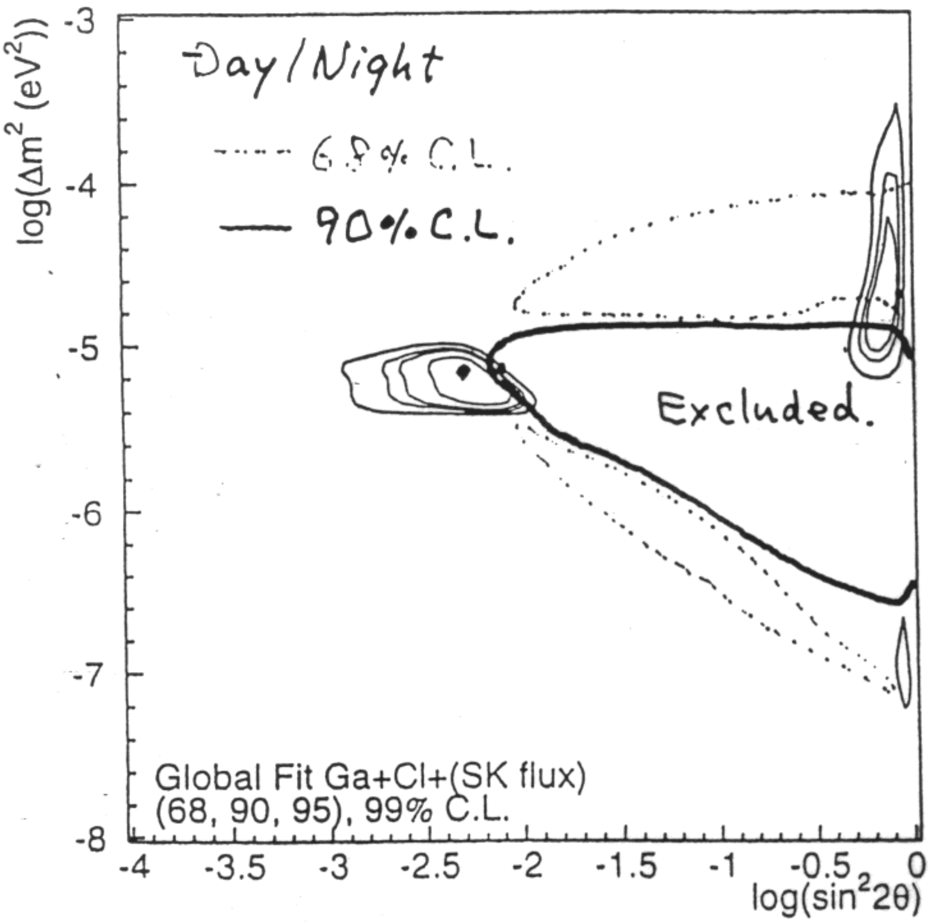


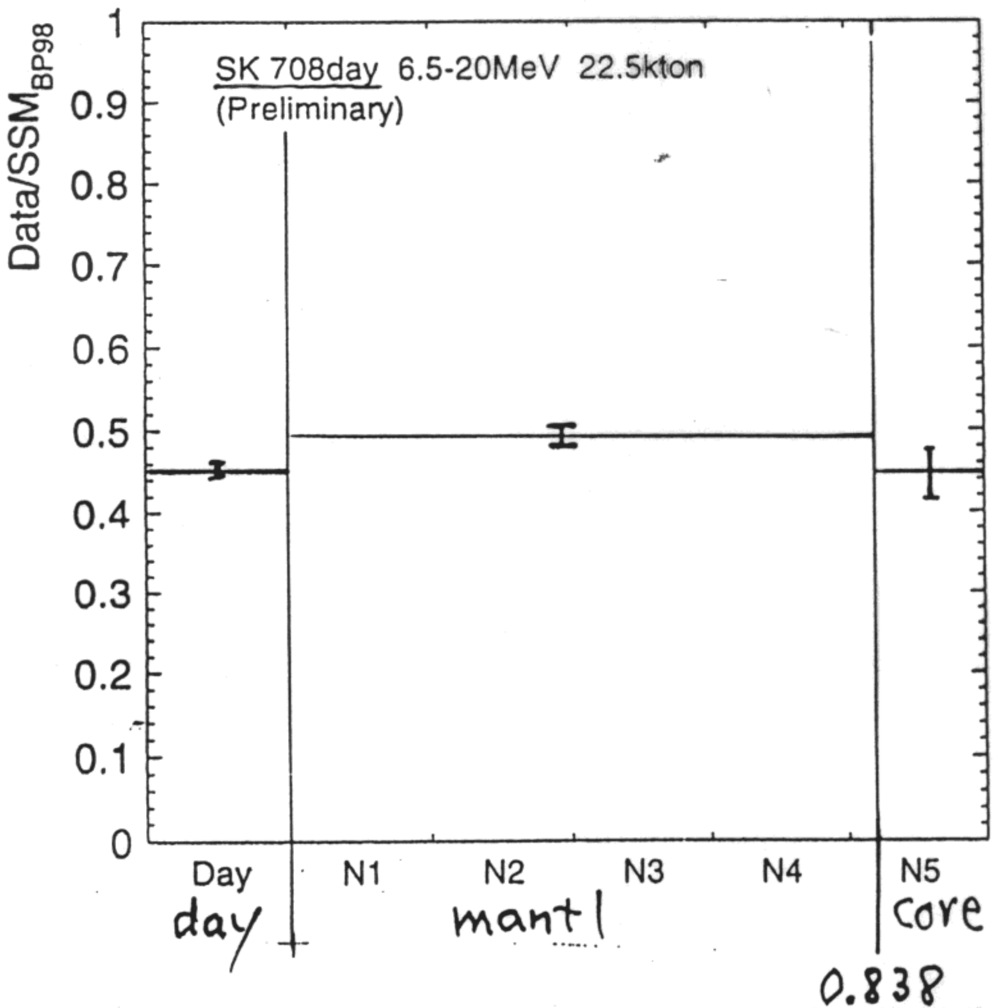
FIG. 1. Measured day/night solar neutrino fluxes as a function of the nadir of the Sun. Error bars represent statistical errors only. Night data is divided into 5 bins. Dotted histogram is the expected variation of a typical large angle solution and dashed histogram is that of a typical small angle solution.

·········· $\sin^2 2\theta_0 = 0.56$, $\Delta m^2 = 1.2\times10^{-5}$ eV^2

- - - - - - $\sin^2 2\theta_0 = 0.01$, $\Delta m^2 = 6.3\times10^{-6}$ eV^2

Y. Suzuki
WIN99





Core enhancement

day	353 days
mantl	310 days
core (outeR)	45 days

No effect.

—— no oscillations

- - - $\sin^2 2\theta_{13} = 0.10$

$\sin^2 \theta_{23} = 0.75$

$\Delta m^2_{32} = 1.7 \times 10^{-3}\ eV^2$

┼ - SK 535 days data

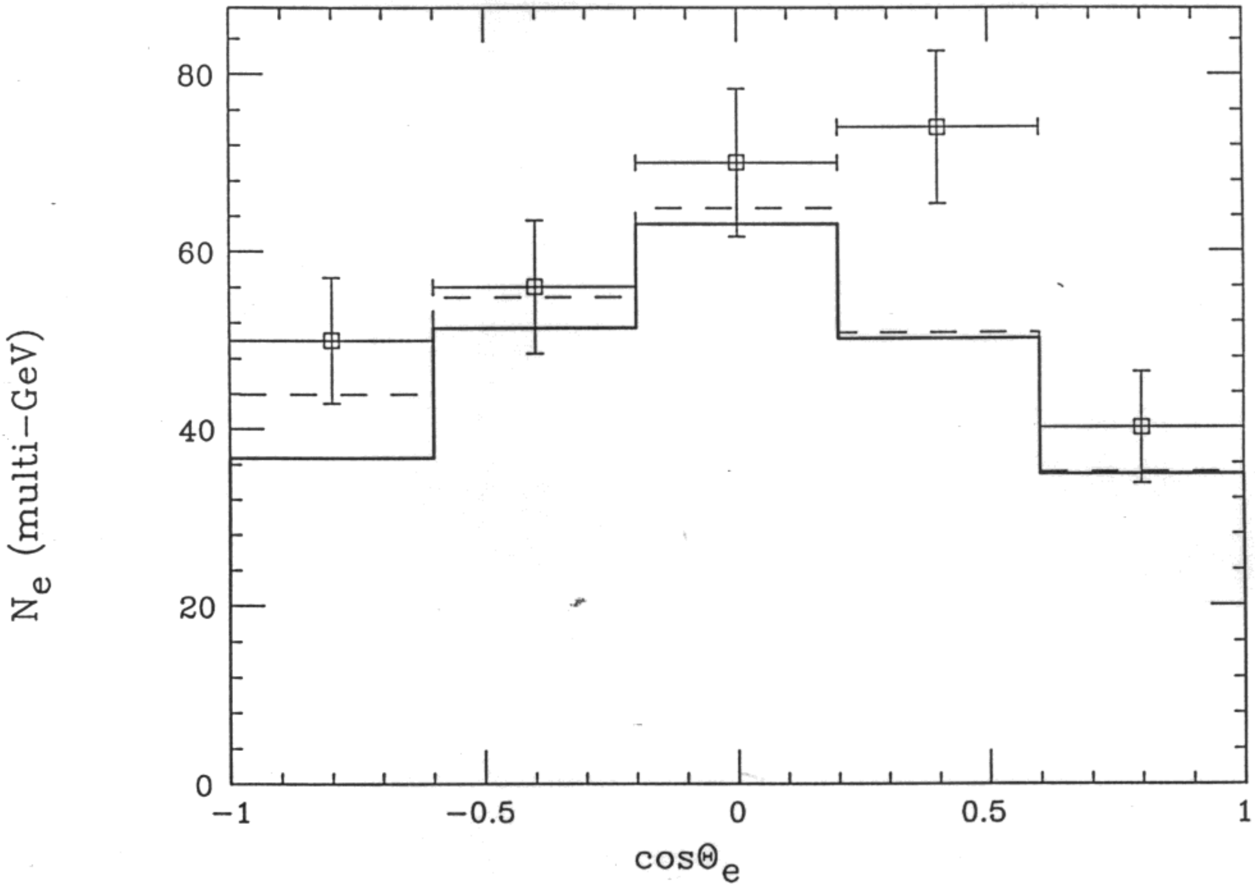


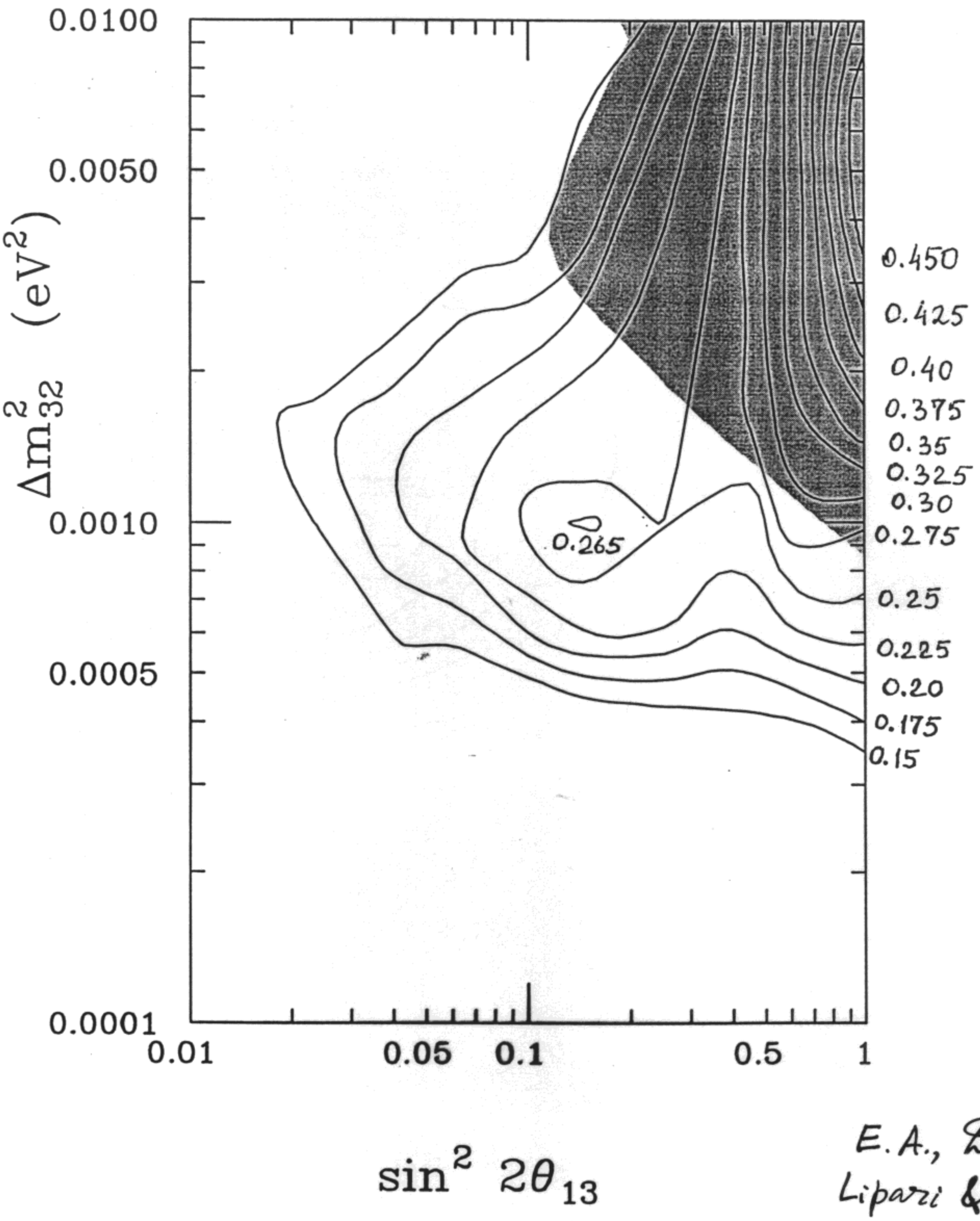
Figure 6

E. A., Dighe, Lipari & Smirnov (1998)

Asymmetry $A_e^{U/D}(|\cos\Theta_e| > 0.6) = 2\,\frac{U-D}{U+D}$

for multi-GeV e-like events

$\sin^2\theta_{23} = 0.75$



E.A., Dighe,
Lipari & Smirn
(1998)

Shaded area – excluded by CHOOZ

Difficulties of experimental observation

1. Atmospheric ν's.

- 1. The trajectory of a detected ν is not known to a good precision: only charged leptons produced by ν's are detected.

  $\langle \theta_{l\nu} \rangle \simeq 60^\circ$ (sub-GeV)

  $\langle \theta_{l\nu} \rangle \simeq 17^\circ$ (multi-GeV)

- 2. Both ν's and $\bar{\nu}$'s are detected whereas oscillations of only ν's (or only $\bar{\nu}$'s) is parametrically enhanced (applies to the MSW effect as well).
- 3. For each group of events (sub-GeV, multi-GeV, upward-going muons) the neutrinos over rather wide range of energies are detected.
- 4. In principle, the parametric enhancement can manifest itself only for a rather narrow range of Δm^2_{31} ($\sim (1 \div 3) 10^{-3}\,\mathrm{eV}^2$ for multi-GeV ν's).

2. Solar ν's.

- 1. Contribution of P_{e2} into D/N effect may be strongly suppressed if $\bar{P}_S$ is close to $\frac{1}{2}$
- 2. Existing detectors have rather small core coverage times (7% of the year for SK; $\Theta_n > 13^\circ$).

Advantages

Solar ν's

- The direction is precisely known
- For a detector near equator the core coverage time can be significantly increased
- With good enough statistics, characteristic ν spectrum distortions can be observed

Atmospheric ν's

- Eearth core fully covered
- Measuring the momentum of recoil nucleon one can reconstruct the ν trajectory
- Small (high density) detectors can have magn. field to discriminate ν and $\bar{\nu}$
- Properly choozing energy cuts and zenith angle binning one can enhance the parametric effects.
- Extremely sensitive probe of the subdominant θ_{13} mixing!
- No additional suppression factors due to compositi

Is it possible to study parametric resonance of neutrino oscillation in laboratory?

Param. resonance: $\underline{l_{osc.} \approx L}$ (density modulation scale)

- $$l_{osc} = \frac{\pi}{\omega_i} \simeq \min\left\{\frac{\pi}{\delta}, \frac{\pi}{V_i}\right\}$$

Require baseline $\lesssim 1$ km

1. $V_i \gtrsim \delta$ (matter domination) $\Rightarrow l_{osc} \simeq \frac{\pi}{V_i}$

   $V_i = \frac{G_F}{\sqrt{2}} N_i$; $l_{osc} < 1\ \text{km} \Rightarrow \underline{\rho_i > 3\times 10^4\ g/cm^3\ !}$

   Conversely, for $\rho_i < 10\ \frac{g}{cm^3}$, $\underline{l_{osc} > 3.3\times 10^3\ \text{km}}$

2. $V_i \ll \delta$; $l_{osc} \simeq l_{vac} = \frac{\pi}{\delta} = \frac{4\pi E}{\Delta m^2}$

   $l_{osc} < 1\ \text{km} \Rightarrow \delta > 2.5\times 10^{-10}$ eV.

   In principle OK. But:

   For $\rho_i < 10\ g/cm^3 \Rightarrow \frac{V_i}{\delta} \lesssim 10^{-3}$ - very small

   $\Rightarrow \theta_i \simeq \theta_0 \left(1 + \frac{V_i}{\delta}\right)$; $\underline{\Delta\theta} = \theta_2 - \theta_1 \cong \frac{\Delta V}{\delta}\theta_0 \lesssim \underline{10^{-3}\theta_0}$

   For small $\Delta\theta$, ν's must travel over many periods ($n \gtrsim \pi/4\Delta\theta \gg 1$). The baseline:

   $\sim \pi^2/4\Delta V\theta_0 \gtrsim 3\times 10^3$ km – again too large!

⇒ Earth may be the only place where the parametric resonance of ν oscillations can be realized !

Liu & Smirnov, 1997.

Conclusions

- A very interesting resonance enhancement effect for ν oscillations in matter, different from MSW. $\Rightarrow$ A source of additional information on ν parameters
- Earth may be the unique place where it can occur!
- Can take place for both solar and atmospheric ν's for the ranges of ν parameters that are preferred by the current exp. data
- For solar ν's – can significantly enhance the D/N effect for small θ_0 MSW solution if the core crossing ν's are selected for the night sample. May be the only hope to see the D/N effect for small θ_0 MSW!
- For atmospheric ν's – a sensitive probe of the subdominant $\nu_\mu \leftrightarrow \nu_e$ oscillations. Improves agreement of predictions with data (excess of e-like events).
  A sensitive probe of a very important parameter θ_{13} – values as small as $\underline{(1-2)\%}$ can be probed!