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ν Masses: a Theoretical Perspective

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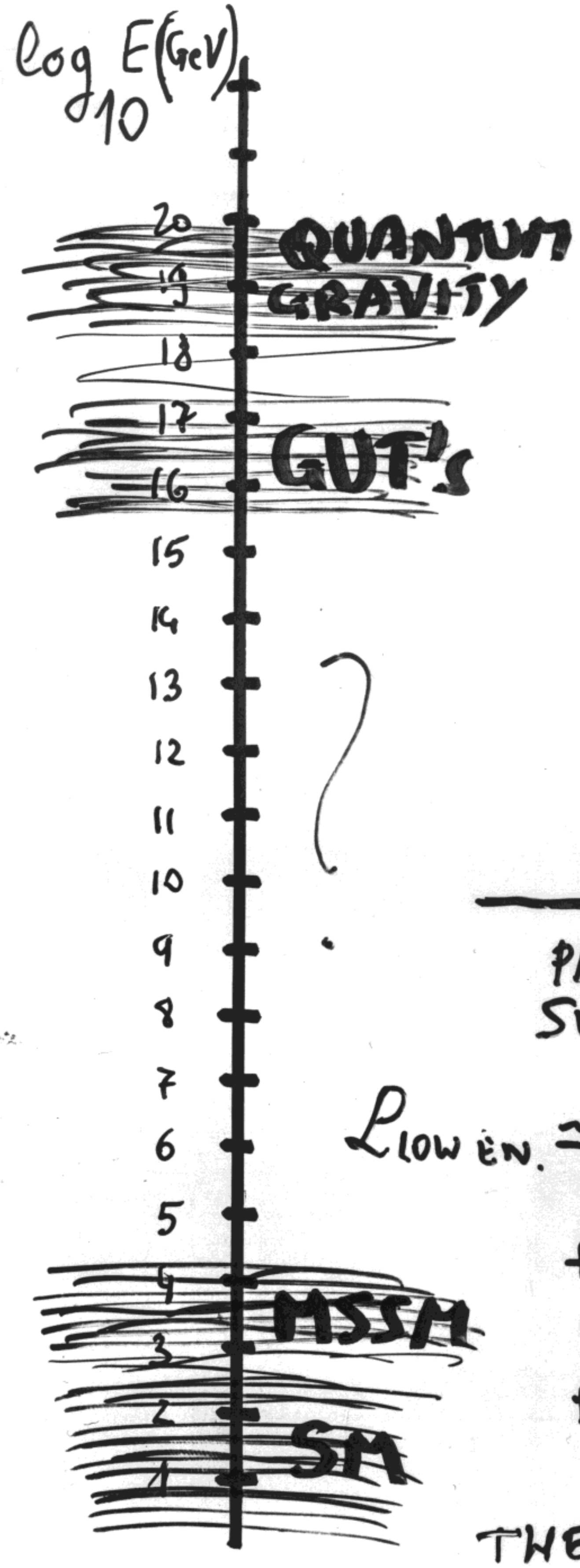
THE CONCEPTUAL IMPACT OF ν MASSES ON OUR PRESENT VIEW OF PARTICLE PHYSICS

A DOUBLE APPROACH TO PARTICLE PHYSICS



FROM ABOVE:

QUANTUM GRAVITY
SUPERSTRING THEORY
GUT's (ν masses, p decay...)

UNDERGROUND EXP'S
COSMIC RAY DETECTORS
SATELLITES (COBE, IRAS...,
.....

FROM BELOW:

SEARCH FOR HIGGS,
FOR SM EXTENSION,

EXPLORING THE FLAVOUR RIDDLE (CP, ...)

PROTECTED BY $SU(2) \otimes U(1)$ AND CHIRAL SYMM. $\sim m \ln \Lambda$

PROTECTED BY SUSY $\sim \Delta M^2_{SUSY}$

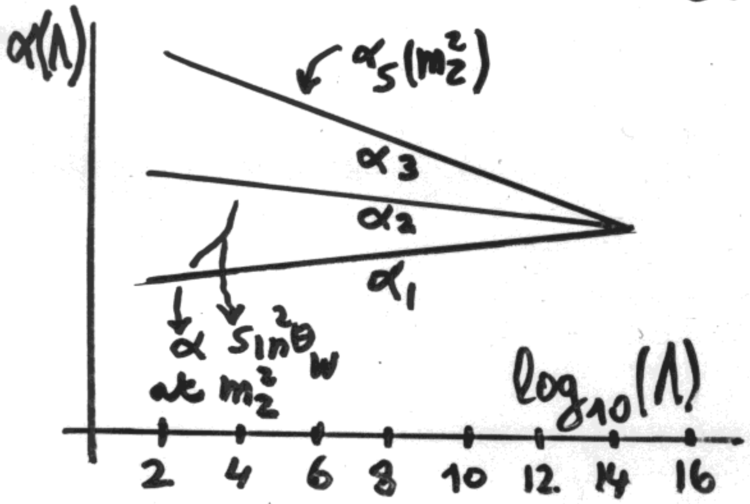
$$\mathcal{L}_{\text{LOW EN.}} \simeq \mathcal{O}(\Lambda^2)\,\mathcal{L}_2 + \mathcal{O}(\Lambda)\,\mathcal{L}_3 + \mathcal{O}(1)\,\mathcal{L}_4 + \mathcal{O}\left(\frac{1}{\Lambda}\right)\mathcal{L}_5 + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\mathcal{L}_6 + \ldots\ldots$$

[$\mathcal{O}(1) \to g^2 \ln \Lambda$]

THE LOW ENERGY THEORY IS AN EFFECTIVE THEORY $[\Lambda \equiv M_{GUT}, M_{Pl}]$

HINTS IN FAVOUR OF SUSY

- UNIFICATION OF COUPLINGS



FROM $\alpha(m_Z^2)$, $\sin^2\theta_W(m_Z^2)$

PREDICT $\alpha_s(m_Z^2)$:

GUT's $\xrightarrow{\text{desert}}$ $\mathcal{L}_{\text{LOW ENERGY}}$

SM : $\alpha_s(m_Z^2) = 0.073 \pm 0.002$

MSSM: $\alpha_s(m_Z^2) = 0.129 \pm 0.010$

$\alpha_s(m_Z^2)_{EXP} \simeq 0.119 \pm 0.004$

- A GOOD COLD DARK MATTER CANDIDATE

A NEUTRALINO : A SUPERPOSITION OF

$\tilde{\gamma}, \tilde{Z}, \underbrace{\tilde{h}_1, \tilde{h}_2}_{\text{higgsimos}}$

- AGREEMENT WITH E-W PRECISION TESTS

FOR NOT TOO LIGHT S-PARTNERS

↳ NOW ESTABLISHED BY DIRECT LIMIT.

SUSY RAD. CORR'S = SM RAD. CORR'S

WITH m_H LIGHT (~ 100 GeV)

↳ INDICATED BY THE DATA

LHC WILL TELL US!

ν OSCILLATIONS $\Longrightarrow$

ν-FLAVOUR MIXING $\Longrightarrow$

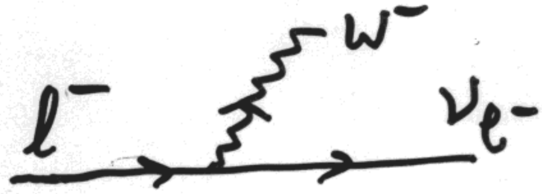
ν MASSES !

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad UU^\dagger = U^\dagger U = 1$$

$\downarrow$ WEAK ISOSPIN EIGENSTATES

$\uparrow$ MASS EIGENSTATES

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$



U MIXING MATRIX OF ν'S IN BASIS WHERE m_{l^-} DIAGONAL

[ANALOGOUS TO V_{CKM} FOR D-QUARKS]

ν MASSES ARE VERY SMALL

DIRECT LIMITS:

$m_{\nu_e} \lesssim \sim 5\, eV$ $\left(m^2_{Best\ Fit} < 0\ ??\right)$

$m_{\nu_\mu} \lesssim 170\ keV$

$m_{\nu_\tau} \lesssim 18\ MeV$

COSMOLOGY:

FOR (SUFFICIENTLY) STABLE ν's

$$\sum_{stable} m_{\nu_i} \lesssim 6\, eV$$

ALSO:

LEP: NO MORE SEQUENTIAL ν's WITH $m_\nu \lesssim 45\, GeV$

[ONLY "STERILE" ν's WITH NO WEAK INT'S (NO $Z \to \nu\nu$) ALLOWED]

ν MASSES AND COSMOLOGY

$$\rho_0 \equiv \Omega \rho_c = \Omega \frac{3H_0^2}{8\pi G} = \Omega h^2 \cdot 11 \frac{keV}{cm^3}$$

$H_0 = 100\, h$ km/s/Mpc

$\Omega = \Omega_m + \Omega_\Lambda = 1$ (FAVOURED BY INFLATION)

EXP: $\Omega_\Lambda \simeq 0.62 \pm 0.16 \pm ?$; $\Omega_m \simeq 0.24 \pm 0.10 \pm ?$

$\Omega \simeq 1 \pm 0.2 \pm ?$ $\quad \Omega_m h^2 \simeq 0.12 \pm 0.05$

$h \simeq 0.7 \pm 0.2$

$\Omega_\nu \lesssim 0.3\, \Omega_m$ $\quad \Omega_\nu h^2 \lesssim 0.04 \div 0.06$

AT DECOUPLING ($T \sim MeV$):

$$\frac{n_\nu}{n_\gamma} \cong \frac{3}{11}$$

(ν: LIGHT NEUTRINO WITH $m_\nu <$ MeV)

$n_\gamma \sim 400/cm^3 \Rightarrow n_\nu \sim 110/cm^3$

$$\rho_\nu = n_\nu m_\nu = 110\, m_\nu/cm^3 = \Omega_\nu h^2\, 11 \frac{keV}{cm^3}$$

$$\boxed{\sum m_\nu \approx \Omega_\nu h^2\, 100\, eV \lesssim 6\, eV}$$

$m_\nu \sim 0.05\, eV \Rightarrow \Omega_\nu \sim 10^{-3}$ $\quad [\Omega_{stars} \simeq 5 \cdot 10^{-3}]$

SOME EVIDENCE FOR $\Omega_\Lambda \neq 0$

- WITH $\Omega_\Lambda \neq 0$ IT IS NO MORE CLEAR THAT HOT DM IS NEEDED
- Ω_ν COULD BE SMALL.

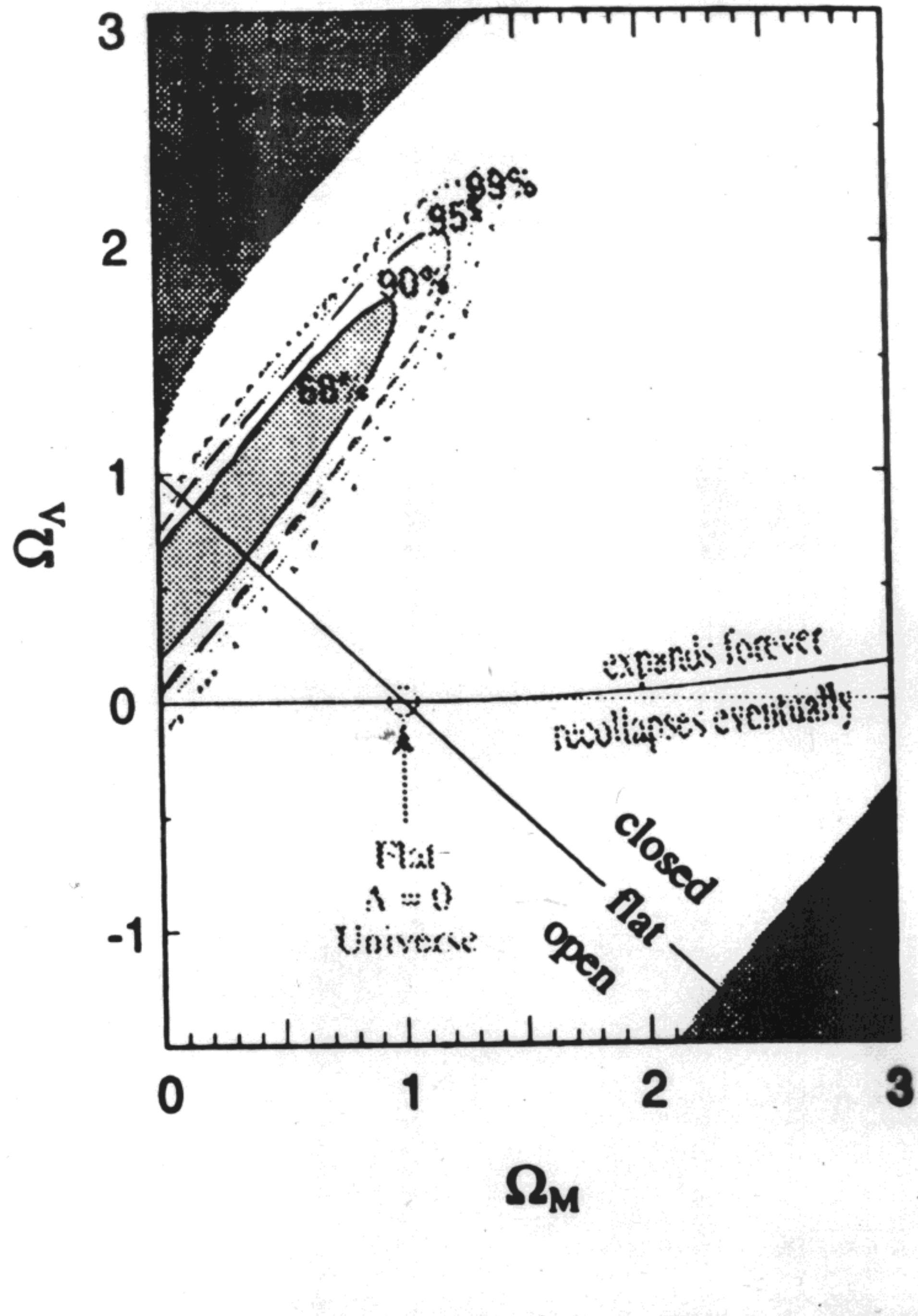
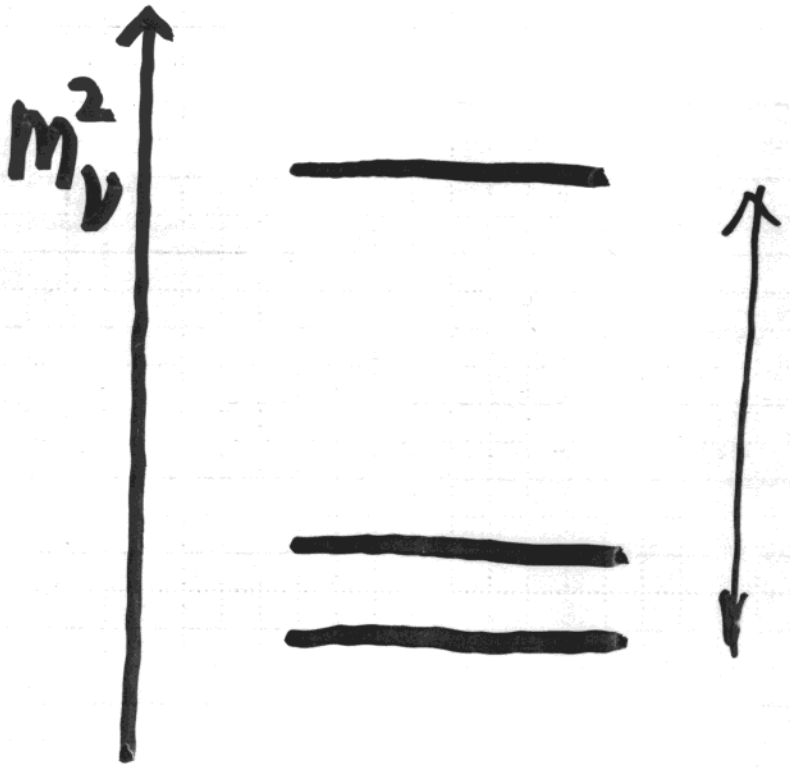


Figure 2: Supernova Cosmology Project results.

$\Omega_\Lambda + \Omega_M \simeq 1 \pm 0.2 \pm ?$ IS INDICATED BY 1st ACOUSTIC PEAK IN CMB $[\ell(\ell+1)C_\ell$ vs $\ell]$



$\Delta m^2_{atm} \sim 10^{-3}\,eV^2$

$\Delta m^2_{\odot} \lesssim 10^{-5}\,eV^2$

$+ \quad m_{\nu_e} \lesssim$ a few eV

$\sum_i m_{\nu_i} \lesssim 6\,eV$

$\Downarrow$

ALL $m_{\nu_i} \lesssim 2\,eV$

ν MASSES $\Rightarrow$ ν_R OR $\not{L}$ OR BOTH

MOST LIKELY BOTH

L CONSERVED ?

DIRAC ν MASS : $\bar{\nu}_L m \nu_R + h.c.$

BUT:
WITHIN EACH GENERATION FERMION MASS SPREAD IS NOT THAT LARGE:

u ~5 MeV	c ~1 GeV	t 175 GeV
d ~8	s ~0.1	b ~5
e^- ~0.5	μ^- ~0.1	τ^- ~2

WHILE $m_{\nu_i} \lesssim o(\mathrm{eV})$

IF L CONSERVED $\Rightarrow$ ADDITIONAL ENORMOUS SPREAD OF YUKAWA's

MORE APPEALING ALTERNATIVE:

ν'S ARE LIGHT BECAUSE THEY ARE THE ONLY NEUTRAL FERMIONS:

L VIOLATED $\Rightarrow$ ν MAJORANA

IN S.M. THERE IS NO LORENTZ INV., DIM ≤ 4 OPERATOR (COMPATIBLE WITH $SU(3)\times SU(2)\times U(1)$) THAT VIOLATES B AND/OR L EXCEPT $\nu_R^T \nu_R$

FOR EXAMPLE :

$$u + u \to e^+ + \bar{d}$$

IS Ok (e.g. COLOUR : $3 + 3 = 6 + \bar{3}$)

BUT : $\frac{\lambda}{M^2} \bar{d}^c u \; \bar{e}^c u \Rightarrow$ DIM 6

$SU(5)$ $\quad p \to e^+ \pi^0$

$M \equiv M_{GUT}$

[SAME IN SUSY WITH R-PARITY CONSERV'N]

BUT IF ν_R IS INTRODUCED THEN $\nu_R^T \nu_R$ IS $SU(3)\times SU(2)\times U(1)$ INVARIANT AND $\Delta L = 2$!

WITH ν_R L CONSERV. NOT AUTOMATIC!!

GUT's MAKE ν MASSES VERY PLAUSIBLE

- L (AND B) VIOLATION IS A GENERAL PROPERTY OF GUT's

- LARGE M_{GUT} OFFERS A NATURAL MECHANISM FOR m_ν VERY SMALL : $m_\nu \sim \frac{m^2}{M}$ ← LARGE SCALE OF $\not{L}$

NO ν_R ?

NON-RENORM. OPERATORS:

$$O_5 \simeq \nu_{Li}^T \frac{\lambda^2_{ij}}{M} \nu_{Lj} H H$$

← ORDINARY HIGGS

↳ $\Delta L = 2$

$$\Rightarrow m_\nu \sim \frac{\lambda^2 v^2}{M}$$

$v = \langle 0|H|0\rangle$ ← 175 GeV

$M \sim M_{GUT}, M_P$

- ν_R PRESENT IN SO(10), E6 ....

SEE-SAW MECHANISM

$$\Rightarrow m_\nu \sim \frac{m_D^2}{M}$$

m_D : DIRAC ν-mass

$M \sim M_{GUT}, M_{Pl}$

SO(10) IS VERY IMPRESSIVE:

ONE WHOLE FAMILY IN A SINGLE SO(10) REPRESANTATION

$$\underset{SO(10)}{16} \supset \underbrace{\bar{5} + 10 + 1}_{SU(5)}$$

TOO STRIKING NOT TO BE A SIGN.

SO(10) MUST BE RELEVANT AT LEAST AS A CLASSIFICATION GROUP

FOR EXAMPLE WE COULD HAVE

$$SO(10) \xrightarrow[16]{} SU(5) \times U(1) \longrightarrow$$

AT M_{Pl} (B-L BROKEN) AT M_{GUT}

$$\xrightarrow[45]{} SU(3)_c \otimes SU(2) \otimes U(1)$$

ONE-SCALE BREAKING FROM M_{GUT} TO M_W (GOOD PREDICTION OF COUPLING UNIF. KEPT)

$$SO(10) \xrightarrow[54]{} SU(4) \otimes SU(2)_L \otimes SU(2)_R \rightarrow$$

$$\xrightarrow[45]{} SU(3)_c \otimes U(1)_{B-L} \otimes SU(2)_L \otimes SU(2)_R$$

$$\xrightarrow[16]{} SM$$

SEE-SAW MECHANISM

Yanagida; Gell-Mann, Ramond, Slansky

$\nu_R^T \nu_R$ ALLOWED BY $SU(2) \otimes U(1)$

↳ LARGE MAJORANA MASS M

DIRAC MASS m BY HIGGS DOUBLET(S)

$$\begin{array}{c|cc} & \nu_L & \nu_R \\ \nu_L & 0 & m_D \\ \nu_R & m_D & M \end{array}$$

$$\begin{matrix} & \nu_L & \nu_R \\ \nu_L & \multicolumn{2}{c}{\multirow{2}{*}{}} \end{matrix}$$

$\underline{M \gg m}$

EIGENVALUES

$$\nu_{light} \simeq -\frac{m_D^2}{M}, \quad \nu_{heavy} \simeq M$$

SIGN IRRELEVANT FOR FERMIONS

ALSO, THIS ZERO COULD BE FILLED UP $\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$

NOT BY TRIPLET HIGGS $\nu_L^T \nu_L \phi_{TRIPLET}$ (SPONTANEOUS $\not{L}$ $\Rightarrow$ TRIPLET MAJORON EXCLUDED BY Γ_{inv} AT LEP)

PERHAPS BY DOUBLETS WITH NON RENORM. COUPLINGS:

$$\frac{\lambda^2}{M} \underbrace{\nu_L^T \nu_L H H}_{\text{dim } 5}$$

$$m_\nu \sim \frac{\lambda^2 v^2}{M}$$

AGAIN OF $O\left(\frac{m_D^2}{M}\right)$

IN GENERAL BOTH $O_5 \sim \nu_L^T \frac{\lambda^2}{M} \nu_L HH$

AND THE SEE-SAW MECHANISM

ARE OPERATIVE :

$$\nu_L^T m_\nu \nu_L = \nu_L^T m_D^T M^{-1} m_D \nu_L + \nu_L^T \frac{\lambda^2 v^2}{M} \nu_L$$

THE 2 TERMS HAVE THE SAME FORM, THE SAME TRANSF. PROPERTIES UNDER $\nu_L' = U \nu_L$, BUT DIFFERENT ORIGINS [e.g. in GUT'S m_D RELATED TO q & l DIRAC MASSES]

THEY CAN BE OF COMPARABLE OR OF VERY DIFFENT SIZE [e.g. $1/M_{GUT}$ VS $1/M_{Pl}$]

MORE FLAVOURS (e.g. ν_e, ν_μ, ν_τ)

$$\mathcal{L} = -\overline{\Psi}_R m_D \Psi_L + \frac{1}{2} \overline{\Psi}_R M \overline{\Psi}_R^T + h.c$$

(m_D ↑ DIRAC; M ↑ MAJORANA)

m : NOT HERMITIAN, NOT SYMM. $M = M^T$ SYMM.

EQ.S OF MOTION FOR $\overline{\Psi}_R$

(IN STATIC LIMIT : $\partial \Psi = 0$)

$$-\frac{\partial \mathcal{L}}{\partial \overline{\Psi}_R} = m_D \Psi_L - M \overline{\Psi}_R^T = 0$$

$$\overline{\Psi}_R^T = M^{-1} m_D \Psi_L$$

$$\overline{\Psi}_R = \Psi_L^T m_D^T M^{-1}$$

BY ELIMINATION OF $\overline{\Psi}_R$:

$$\overline{\Psi}_R m_D \Psi_L \to \Psi_L^T m_D^T M^{-1} m_D \Psi_L$$

$$L^T L \rightsquigarrow m_\nu = + m_D^T M^{-1} m_D$$

(m_D ↑ $\overline{R}L$)

↙

EFFECTIVE LIGHT ν MASS MATRIX

ν MASSES:

L CONSERVED

$\downarrow$

ν_R

$\downarrow$

$$m_\nu \simeq \bar{\nu}_R m_D \nu_L$$

BUT:

EXTREME SMALLNESS OF m_ν NOT EXPLAINED

WHY L NOT VIOLATED BY $\nu_R^T \nu_R$??

VERY UNLIKELY

L VIOLATED

$\downarrow$

NO ν_R → NON REN INTERACTION

AND ν_R → NON REN INTERACTION; SEE-SAW

$$\nu_L^T \frac{\lambda}{M} \nu_L \varphi\varphi$$

$$\begin{pmatrix} ? & m_D \\ m_D & M \end{pmatrix}$$

$\frac{m_D^2}{M}$; M: $\nu_R^T \nu_R$

$\downarrow$

$$m_\nu \simeq \nu_L^T \frac{\lambda v^2}{M} \nu_L \overset{?}{+} \nu_L^T m_D M^{-1} m_D^T \nu_L$$

LARGE M → SMALL m_ν

M: M_{GUT}, M_{Planck}

ν - OSCILLATION DATA

ν'98
WIN 99

ATMOSPHERIC ν'S

SK, MACRO

$\nu_\mu \to \nu_\tau \qquad \Delta m^2_{atm} \sim 3.5 \cdot 10^{-3} eV$

NOT MUCH $\nu_e \to$ ANY (CHOOZ)

MAXIMAL MIXING : $\sin^2 2\theta_{atm} \gtrsim 0.9$!

SOLAR ν'S

$\nu_e \leftrightarrow \nu_{\mu,\tau}$

WARNING :
EXISTENCE OF OSCILL'NS : SOLID
DETAILED VALUES OF $\Delta m^2, \theta$: MAY CHANG

TWO POSSIBILITIES : (Bahcall et al, Barbieri et

- VACUUM OSCILL. $\Delta m^2_{sun} \sim 0.65 \cdot 10^{-10} eV$

  MAXIMAL MIXING : $\sin^2 2\theta_{sun} \sim 0.75$!

- MSW (SMALL θ_{sun}) $\Delta m^2_{sun} \sim 5 \cdot 10^{-6} eV$

  $\sin^2 2\theta_{sun} = 5.5 \cdot 10^{-3}$

LNSD

NOT CONFIRMED. DISREGARDED HERE

↓ ALSO CALLED "BIMIXING"

DOUBLE MAXIMAL MIXING

[e.g. VACUUM OSC'S FOR ☉ ν'S]

IF $m_3 \gg m_2 \gtrsim m_1$:

$$\Delta m^2_{atm} \sim m_3^2 \sim 3.5 \cdot 10^{-3}\ eV^2 \simeq (0.06\ eV)$$

$$m_3 \sim \frac{m_t^2 \text{ or } v^2}{M} \sim 0.06\ eV \quad (v \sim m_t \sim \sim 200\ GeV$$

$$\Rightarrow M \simeq 0.5\ 10^{15}\ GeV \approx M_{GUT}$$

SUPPORTS RELATION WITH GUT'S!

$$\Delta m^2_{sun} \sim m_2^2 \sim 0.65\ 10^{-10}\ eV^2$$

$$m_2 \sim 0.8 \cdot 10^{-5}\ eV$$

$$\Rightarrow \frac{m_2}{m_3} = (1.3 \cdot 10^{-2})^2 \sim \left(\frac{m_c}{m_t}\right)^2$$

VERY ATTRACTIVE!

SMALL ANGLE MSW

$$\Delta m^2_{sun} \simeq m_2^2 - m_1^2 \sim m_2^2 \sim 5 \cdot 10^{-6}\, eV^2$$

$$m_2 \sim 2.2\ 10^{-3}\ eV$$

$$\Rightarrow \frac{m_2}{m_3} \sim \left[\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right]^{1/2} \sim 0.04$$

$$\sim (0.2)^2$$

RELATION WITH $\frac{m_c}{m_t}$ NO MORE INDICATED

RATHER, POSSIBLY WITH

$$\lambda \simeq \sin\theta_c \simeq 0.22$$

OR $\sqrt{\frac{m_\mu}{m_\tau}} \simeq 0.24$

THE U MATRIX

- 3 FLAVOURS
- 2 FREQUENCIES
- NO $e \to$ ANY FOR $\Delta \nu_{atm}$ (CHOOZ) } CAN BE RELAXED IN 2nd APPROX
- MAXIMAL MIXING FOR $\Delta \nu_{atm}$

$\sin\theta$ → $-s$; $U_{e3} = 0$ CHOOZ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

(the $-1/\sqrt{2}$ and $1/\sqrt{2}$ entries: MAXIMAL $\Delta \nu_{atm}$ MIXING)

<u>NOTE</u>:
- ~~CP~~ NEGLECTED (U REAL)
- SOME SIGNS ARE CONVENTIONAL

$$\begin{cases} P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = \frac{1}{2}\sin^2 2\theta \sin^2 \Delta_{sun} \\ P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - \frac{1}{4}\sin^2 2\theta \sin^2 \Delta_{sun} \end{cases}$$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \qquad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

Maki, Nakagawa, Sakata: 1962

U IS ANALOGOUS TO V_{CKM}

$\Rightarrow$ SAME GENERAL FORM

e.g. MAIANI

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta} \\ \cdots & \cdots & s_{23}c_{23} \\ \cdots & \cdots & c_{23}c_{13} \end{bmatrix}$$

FOR: $s_{13} \sim 0$, $s_{23} = s_\gamma \sim \frac{1}{\sqrt{2}}$, $s_{12} = s$, $\delta \sim 0$
$c_{23} = c_\gamma \sim \frac{1}{\sqrt{2}}$, $c_{12} = c$

$$U = \begin{bmatrix} c & -s & 0 \\ s c_\gamma & c c_\gamma & -s_\gamma \\ s s_\gamma & c s_\gamma & c_\gamma \end{bmatrix}$$

FOR $s_\gamma \sim c_\gamma = \frac{1}{\sqrt{2}}$ SAME AS ABOVE APART FROM SIGNS (← CONVENTION)

MOST GENERAL m_ν

$$m_\nu \sim m_D^T M^{-1} m_D \sim U \begin{bmatrix} e^{i\varphi_1} m_1 & & \\ & e^{i\varphi_2} m_2 & \\ & & m_3 \end{bmatrix} U^T$$

(m_ν ↑ $L^T m_\nu L$; m_D ↑ $\bar{R} m_D L$)

9 NEW PARAM'S ADDED TO SM:
3 MASSES
3 MIXINGS
3 PHASES

GIVEN $|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle$

AND $m_\nu^{diag} = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$

THEN $m_\nu = U \, m_\nu^{diag} \, U^T$

$$m_\nu = \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2)\frac{cs}{\sqrt{2}} & (m_1 - m_2)\frac{cs}{\sqrt{2}} \\ \cdots & \frac{m_3}{2} + \frac{m_1 s^2}{2} + \frac{m_2 c^2}{2} & \cdots \\ \cdots & -\frac{m_3}{2} + \frac{m_1 s^2}{2} + \frac{m_2 c^2}{2} & \frac{m_3}{2} + \frac{m_1 s^2}{2} + \frac{m_2 c^2}{2} \end{bmatrix}$$

THIS IS IN BASIS WHERE m_ℓ DIAGONAL (ℓ ← CHARGED LEPTONS)

NOTE: m_ν IS SYMMETRIC

m_ν IS THE EFFECTIVE LIGHT ν MATRIX

$$L^T m_\nu L \xrightarrow{\text{SEE-SAW}} m_\nu = m_D M^{-1} m_D^T$$

(M ← MAJORANA; m_D → $\bar{L}R$ (DIRAC))

FOR EXAMPLE : ASSUME $|m_3| \gg |m_{1,2}|$

BY NEGLECTING SMALL TERMS OF ORDER $m_{1,2}/m_3$:

$$m_\nu^{diag} \simeq \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3 \end{pmatrix}$$

$$m_\nu = U\, m_\nu^{diag}\, U^T = $$ IN BASIS WHERE m_ℓ^D - DIAGONAL

$$= \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

↳ $\det[23] = 0$

<u>NOTE</u>: THIS IS INDEPENDENT OF s

(BIMIXING : $s \cong \frac{1}{\sqrt{2}}$, MSW : s SMALL)

SIGN CONVENTIONAL : $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ ALSO OK!

ZEROTH ORDER APPROXIMATION

$$U = \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

BASIS OF m_e^D DIAGONAL

$\rightarrow$ $m_\nu = U \, \mathrm{Diag} \, U^T$

hierarchy		m_{diag}	double maximal mixing $s = \frac{1}{\sqrt{2}}$	single maximal mixing $s = 0$
$\vert m_3\vert \gg \vert m_{1,2}\vert$	A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
$\vert m_1\vert \sim \vert m_2\vert \gg \vert m_3\vert$	B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
	B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
$\vert m_1\vert \approx \vert m_2\vert \approx \vert m_3\vert$	C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
	C3	Diag[1,1,-1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

G.A., F. FERUGLIO II

Table I : Zeroth order form of the neutrino mass matrix for double and single max xing, according to the different possible hierarchies given in eq. (6).

G.A., F. Feruglio I hep-ph/9807353 PL B439 (1998) 112
II /9809596 JHEP 11 (1998) 21
III /9812475

A HOT PROBLEM:

FIND A NATURAL EXPLANATION OF LARGE ν MIXINGS

MANY MATRICES INVOLVED:

e.g.

$$m_\nu = m_D^\nu M^{-1} m_D^{\nu T}$$

↳ IN BASIS WHERE m_D^e DIAGONAL

$$m_D^\nu, m_D^e \xrightarrow{\text{GUTS}} m_D^u, m_D^d$$

(m_D^u, m_D^d: SMALL MIXINGS)

GUT'S ⇒ q AND ℓ RELATED

BUT q MIXINGS SMALL

WHILE ν MIXINGS LARGE

3 DEGENERATE ν'S ?

ATTRACTIVE BECAUSE OF HOT D.M.

$$|m_i| = m \sim 1-3\ eV \Rightarrow \Omega_\nu \sim 0.1-0.3$$

ν-LESS DOUBLE β-DECAY $\Rightarrow m_\nu^{ee} < 0.46\,eV$

$$m_\nu^{ee} = m_1 c^2 + m_2 s^2 \lesssim 0.46\ eV$$

$$\hookrightarrow m_1 = -m_2 \qquad c^2 \sim s^2$$

DM IMPLIES BIMIXING !! Georgi Glashow Vissani

ARE SMALL SPLITTINGS NATURAL ?

$$\Delta m^2_{atm} = m_2^2 - m_1^2 \cong 2m(m_2 - m_1) = 2m\,\Delta m$$

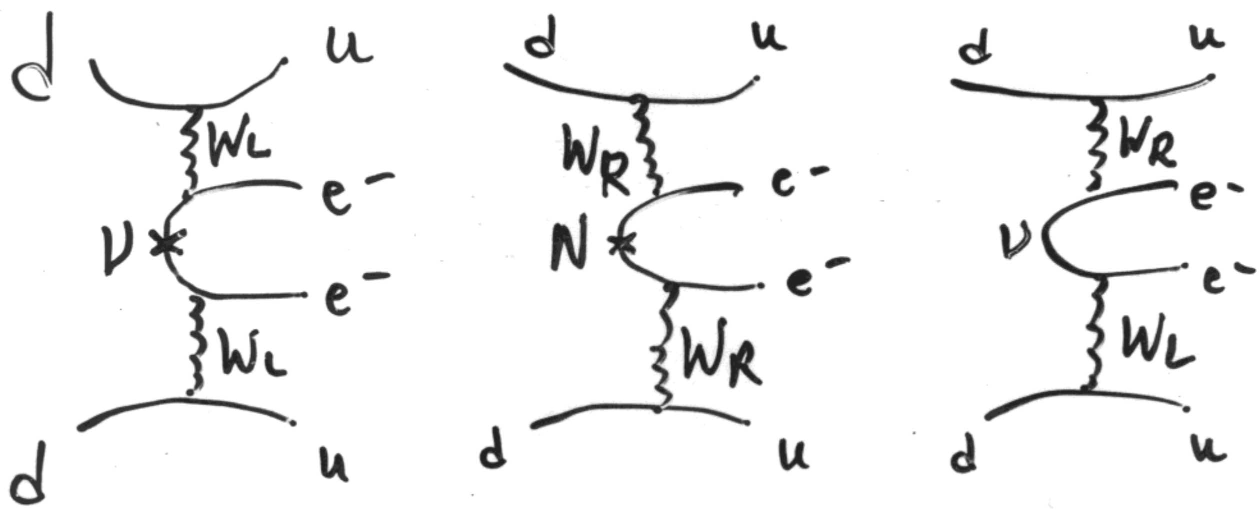
$$\frac{\Delta m}{m} = \frac{\Delta m^2_{atm}}{2m^2} \simeq 10^{-3} - 10^{-4}$$

EVEN MUCH SMALLER FOR $\Delta\nu_\odot$

$$\frac{\Delta m}{m} = \frac{\Delta m^2_{sun}}{2m^2} \simeq 10^{-10} - 10^{-11} \ !!$$

LIMIT FROM $\beta\beta0\nu$: $(\Delta L = -2)$

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$



PROBABLY W_R, N HEAVY ENOUGH TO BE DISREGARDED

$$m_\nu^{ee} \equiv \sum_\alpha U_{e\alpha}^2 \, m_\alpha \lesssim 0.46 \text{ eV}$$

A STRONG CONSTRAINT ON THE MASS OF LIGHT NEUTRINOS FROM THE SEE-SAW MECHANISM.

FOR $|m_1| \sim |m_2| \sim |m_3| \gtrsim 1$ eV

ONLY ONE POSSIBILITY

BIMIXING (C1 EQUIVALENT C2)

	m_{diag}	double maximal mixing	single maximal mixing
A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
C3	Diag[1,1,-1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Table I : Zeroth order form of the neutrino mass matrix for double and single max ixing, according to the different possible hierarchies given in eq. (6).

A MODEL WHICH IS SIMPLE TO STATE BUT DIFFICULT TO REALIZE

Fritzsch, Zhing

ASSUME THAN IN 1st APPROX.

$$m_{q,\ell^-} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[\text{DIAG.}]{U} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 3 \end{pmatrix}$$

"DEMOCRATIC"

IN SAME BASIS FOR ν'S

$$|m_\nu| = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

i.e. ν'S ARE DIAGONAL IN BASIS WHERE q, ℓ DEMOCRATIC

THEN IN BASIS WHERE ℓ DIAGONAL

$$m_\nu' = U m_\nu U^T$$

APPROX. BIMIXING

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

$\sin^2 2\vartheta_{atm} =$
$= 4 s^2 c^2 = 4 \frac{4}{6} \frac{1}{3}$
$= 8/9$

NATURAL MODELS WITH $|m_1| \simeq |m_2| \simeq |m_3| \sim 2eV$ ARE DIFFICULT TO CONSTRUCT.

FOR THIS THE MOST PROMISING WAY IS THAT THE TERM

$$O_5 \simeq L^T \frac{\lambda^2}{M} L\ H\ H$$

IS DOMINANT [NO DIRECT CONNECTION WITH m^{ν}_{Dirac} (WHICH IS PRESUMABLY HIERARCHICAL AND MORE CONSTRAINED BY GUT'S)]

A SYMMETRY IS NEEDED TO GUARANTEE EQUAL MASSES FOR ν'S IN SYMM. LIMIT

DISCRETE	NON ABELIAN
Mohapatra, Nussinov	e.g. SO(3)
....	Ma
	Wetterich
	Barbieri, Hall, Kane, Ross
	X.-L. Wu
	.....

NOTE: $\frac{v^2}{1\,eV} \sim M \sim 10^{13}\,GeV$: TOO LOW ?

SIMPLEST STRATEGY [G.A., F.Feruglio]

- ONLY 3 LIGHT ν'S
- $SO(10) \to \nu_R$ + ASSUME SEE-SAW DOMINANT
- FOR u, d, ℓ DIRAC MATRICES: THE 3rd GENERATION EIGENVALUE DOMINANT:

$$d^u \equiv \mathrm{diag}[m_D^u]$$

$$d^u \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_t \end{pmatrix}, \quad d^d \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_b \end{pmatrix}, \quad d^\ell \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix}$$

- NATURAL TO ASSUME THIS IS ALSO TRUE FOR d^ν [SIMPLEST $SO(10) \to m_D^\nu \sim m_D^u$]

$$d^\nu \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_{\nu_3} \end{pmatrix}$$

- THEN, AFTER SEE-SAW: $m_\nu = m_D^{\nu T} M^{-1} m_D^\nu$

FINE TUNED NEAR DEGENERACIES IN m_ν ARE UNPLAUSIBLE

e.g. FOR DARK MATTER (WITH BIMIXING) $\quad \frac{\Delta m_{23}}{m} \sim 10^{-4}$

$\Rightarrow m_\nu$ IS HIERARCHICAL TOO $\quad \frac{\Delta m_{12}}{m} \sim 10^{-11}$

$$\mathrm{diag}[m_\nu] \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3 \end{pmatrix}$$

- IF NO FINE TUNED DEGENERACIES, THEN WE NEED **BOTH** **LARGE $m_3 - m_2$ SPLITTING** AND **LARGE MIXING** IN 2-3 SECTOR

$$\begin{cases} m_3 \simeq \sqrt{\Delta m^2_{atm}} \simeq 6 \cdot 10^{-2}\, eV \\ m_2 \simeq \sqrt{\Delta m^2_{sun}} \simeq 2 \cdot 10^{-3}\, eV \text{ (MSW) or } \sim 10^{-5}\, eV \text{ (VO)} \end{cases}$$

- THE "THEOREM" THAT LARGE Δm_{32} IMPLIES SMALL 23 MIXING IS IN GENERAL FALSE

ALL ONE NEEDS IS $\text{Sub Det}[23] \approx 0$

EXAMPLE:

$$M_{23} = \begin{pmatrix} x^2 & x \\ x & 1 \end{pmatrix}$$

$\mathrm{Det}\, M_{23} = 0$

Eigenvalues: $0,\ 1+x^2$

Mixing: $\sin^2 2\theta = \frac{4x^2}{(1+x^2)^2}$

FOR $x \sim 1$
LARGE SPLITTING AND
LARGE MIXING.

SO WE NEED A MECHANISM
FOR $\mathrm{Det}[23] = 0$ AUTOMATIC

ONE POSSIBLE MECHANISM G.A., F. Feruglio

WE FAVOUR $|m_3| \gg |m_{1,2}|$ LIKE FOR q & ℓ

ASSUME THAT IN BASIS WHERE m_D^ℓ DIAG.:

$$m_D^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{pmatrix} = V d U^T = V \begin{pmatrix} 0 & & \\ & 0 & \\ & & y \end{pmatrix} U^T$$

$\hookrightarrow \bar{R} L$ (V: RIGHT, U: LEFT)

WITH $V = 1$

THEN FOR A GENERIC M^{-1}

$$m_\nu = m_D^{\nu T} M^{-1} m_D^\nu = \text{const} \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix}$$

IN FACT $m_\nu = U (d M^{-1} d) U^T$

d ACTS AS A PROJECTOR $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k \end{pmatrix}$

$k = (M^{-1})_{33}$

ONE FINDS: $\sin^2 2\vartheta_{atm} = \dfrac{4x^2}{(1+x^2)^2}$

EXP. $\sin^2 2\vartheta_{atm} \gtrsim 0.8 \longrightarrow 0.6 \lesssim |x| \lesssim 1.6$

$$U_L = \begin{bmatrix} c & -s & 0 \\ sc_\gamma & cc_\gamma & -s_\gamma \\ ss_\gamma & cs_\gamma & c_\gamma \end{bmatrix}$$

$s_\gamma = -x/\sqrt{1+x^2}$

$c_\gamma = 1/\sqrt{1+x^2}$

INTERESTING OBSERVATION:

MIXING COULD BE ENHANCED BY RUNNING DOWN FROM GUT SCALE.

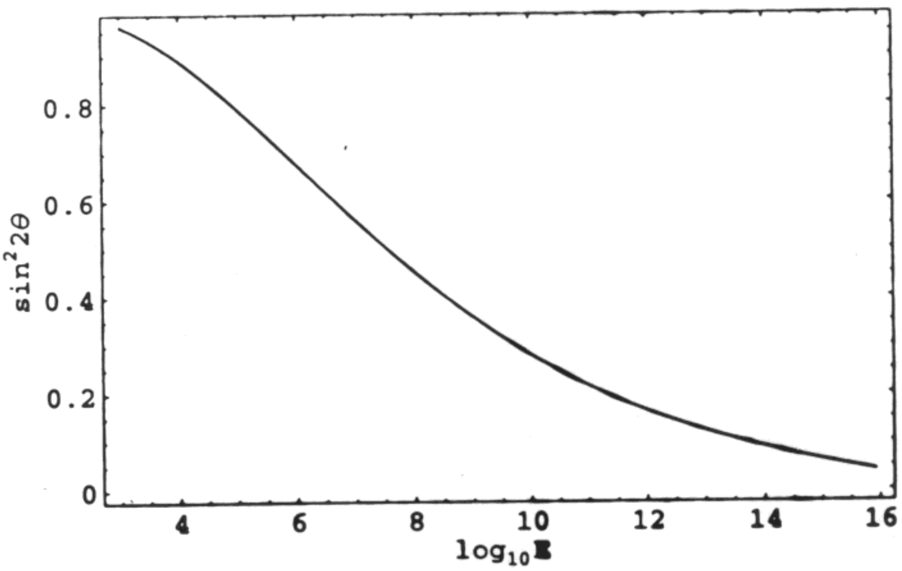


Figure 5: *An example of the renormalisation-group enhancement of the 2 × 2 light-neutrino mixing angle, starting from a small value at the GUT scale. We assume initial Yukawa couplings $h_t = h_b = h_\tau = 2.0$, corresponding to a large value of $\tan\beta$.*

J. Ellis, Leontaris, Lola, Nanopoulos

$$8\pi^2 \frac{d}{dt} S_{23} = -S_{23}(1-S_{23})\left(|Y_\tau|^2-|Y_\mu|^2\right)\frac{m_{33}-m_{22}}{m_{33}-m_{22}}$$

↑ $\sin^2 2\theta_{23}$

τ, μ Yukawa Couplings

$m_{ij} \equiv (m^T M^{-1} m)_{ij}$

Chankowski, Pluciennik '93

Leung, Babu, Pantaleone '93

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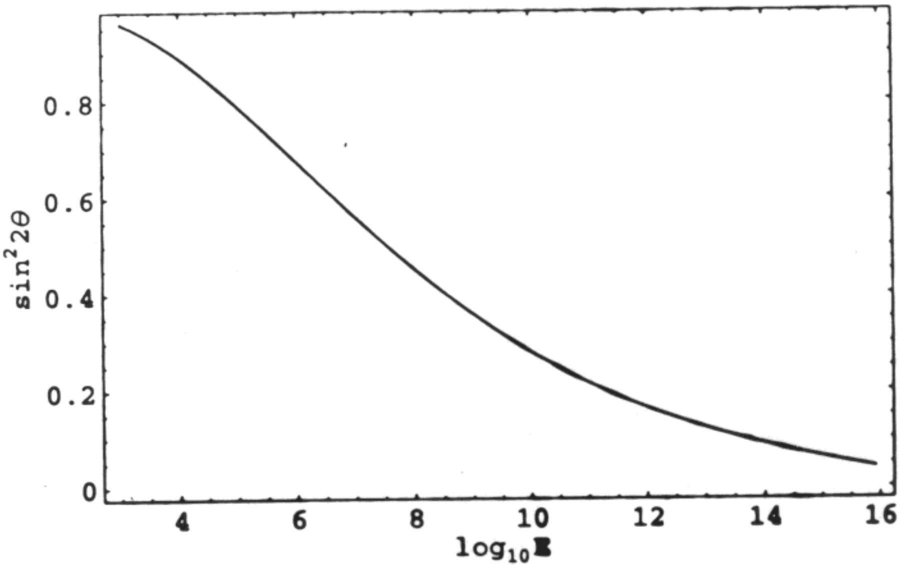

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S_{23} ↑ $\sin^2 2\theta_{23}$

τ, μ Yukawa Couplings

$m_{ij} \equiv (m^T M^{-1} m)_{ij}$

Chankowski, Pluciennik '93

Leung, Babu, Pantaleone '93

NOTE : WE WORKED IN BASIS m_ℓ^D DIAGONAL

BUT THE SAME TEXTURE COULD BE TAKEN FOR BOTH m_ℓ AND m_ν (AT LEAST ONE A_ℓ OR A_ν LARGE)

$$m_\ell^D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A_\ell & B_\ell \end{bmatrix}; \qquad m_\nu^D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A_\nu & B_\nu \end{bmatrix}$$

AFTER DIAGONALISATION OF m_e^D:

$$m_e^D = V_e \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix} U_e^T$$

WITH:

$$V_e = 1 \quad ; \quad U_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_e & -s_e \\ 0 & s_e & c_e \end{pmatrix}$$

$s_e = \frac{-A_\ell}{\sqrt{A_\ell^2+B_\ell^2}}$

$c_e = \frac{1}{\sqrt{A_\ell^2+B_\ell^2}}$

$$m_\nu^D \to \text{constant} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & X & 1 \end{pmatrix}$$

$$|X| = \frac{|A_\ell B_\nu - A_\nu B_e|}{|A_\nu A_\ell + B_\nu B_e|}$$

THUS: NO INCREASE OF THE NUMBER OF PARAMETERS TO FIX BY HAND.

IMPORTANT REMARK

LEFT-HANDED QUARKS : SMALL MIXINGS

$$V_{CKM} = U_u^\dagger U_d \simeq \begin{bmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

$\lambda \simeq \sin\theta_c \approx 0.22$

$A \simeq 0.8 \, , \sqrt{\rho^2+\eta^2} \sim 0.4$

BUT RIGHT-HANDED QUARKS CAN HAVE LARGE MIXINGS (UNKNOWN)

IN SU(5) THE OPPOSITE IS NATURALLY TRUE FOR LEPTONS

$$\bar{5} : (\underbrace{\bar{d}\,\bar{d}\,\bar{d}}_{R}\ \underbrace{\nu\, e^-}_{L})$$

$$m_D^d \sim \underset{\bar{5}}{\bar{d}_R}\, \overset{10}{d_L}$$

$$m_D^\ell \sim \underset{10}{\bar{\ell}_R}\, \overset{5}{\ell_L} \rightsquigarrow (m_D^d)^T \quad !!$$

REMARK :

IN SU(5) : $\bar{5}_i$, 10_i, 1_i i: family 1,2,3

$m_U \to 10_i\, 10_j\, H$ CAN BE DIAGONALISED BY ROTATION OF 10_i

$m_{d,e} \to \bar{5}_i\, 10_j\, H$

$m_{D\nu} \to \bar{5}_i\, 1_j\, H$ (DIRAC ν)

$M_{RR} \to 1_i\, 1_j$ CAN BE DIAG. BY ROT. OF 1_i

$M_{LL} \to \bar{5}_i\, \bar{5}_j$ CAN BE DIAG. BY ROT. OF $\bar{5}_i$

↑

m_ν Effective Light ν matrix.

IN THIS BASIS IF $m_d \sim m_e^T$

$$m_d^T m_d = V_{CKM}^T \,\mathrm{diag}\, m_d^2 \; V_{CKM}$$

$$m_d m_d^T = m_e^T m_e = U^T \,\mathrm{diag}\, m_e^2 \; U$$

↑ ν MIXING MATRIX

or G.A., F. Feruglio

WE FAVOUR A CLASS OF MODELS WHERE: Hagiwara, Okamura; Berezhiani, Rossi; Albright, Babu, Barr

- m_U, m_ν ARE NEAR DIAGONAL
- m_d HAS LARGE <u>RIGHT</u>-HANDED MIXINGS
- THE APPROXIMATE SU(5) RELATION

  $$\boxed{m_d = m_e^T}$$ (i.e. LEFT ⟺ RIGHT)

  INDUCES LARGE <u>LEFT</u>-HANDED MIXINGS IN m_e
- SubDet [23] ≈ 0 NATURALLY

<u>NOTE</u>: BY R-H MIXING WE MEAN OFF DIAGONAL TERMS THAT CAN ONLY BE DIAGONALISED BY A R-H ROTATION V

ABOVE STATEMENTS ARE VALID IN SU(5) BASIS [A GIVEN $\bar{5}_i$, 10_i, CONTAINS BOTH R-H AND L-H FERMIONS]

CERN-TH/98-410
DFPD-98/TH/52

A Simple Grand Unification View of Neutrino Mixing and Fermion Mass Matrices

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Abstract

Assuming three light neutrinos and the see-saw mechanism we present a semiquantitative model of fermion masses based on (SUSY) SU(5) and abelian horizontal charges. A good description of the observed pattern of quark and lepton masses is obtained. For neutrinos we naturally obtain widely split masses and large atmospheric neutrino mixing as a consequence of SU(5)-related asymmetric mass matrices for *d* quarks and charged leptons.

CERN-TH/98-410
DFPD-98/TH/52
December 1998

A MORE MINIMALISTIC VIEW

ALL MATRICES ARE SYMMETRIC AND NEARLY DIAGONAL (LEFT-RIGHT SYMMETRY?)

LARGE $\mu\tau$ MIXING IS OBTAINED BY A FLUCTUATION OF NOT SO SMALL NUMBERS:

e.g. $\frac{m_\mu}{m_\tau} \simeq \frac{0.1}{1.8} \simeq 0.055$

$$\Rightarrow 2\sqrt{\frac{m_\mu}{m_\tau}} \sim 0.5 \sim \left(\frac{m_\mu}{m_\tau}\right)^{1/4}$$

ALSO $\theta^{\ell}_{\mu\tau}$ AND $\theta^{\nu}_{\mu\tau}$ FROM CHARGED LEPTON DIAG. AND FROM m_ν COULD ADD

$$\theta_{atm} \sim \theta^{\ell}_{\mu\tau} + \theta^{\nu}_{\mu\tau} \qquad \theta_{ij} \sim \left(\frac{m_i}{m_j}\right)^{q}$$

FOR MODERATE HIERARCHIES (e.g. MSW BETTER THAN VO) COULD WORK

Babu, Pati, Wilczek

EXAMPLE (23 SUBMATRIX: FOR SIMPLICITY)

AT ORDER ZERO $m_{D\nu} \sim \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

IF M IS GENERIC $\rightarrow m_\nu \sim \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

ASSUME FOR CHARGED LEPTONS:

$$m_e \sim \begin{pmatrix} a^2 & ra \\ ra & 1 \end{pmatrix} \qquad r \sim o(1), \quad a \sim \sqrt{\frac{m_\mu}{m_\tau}} \sim 0.25$$

EIGENVALUES

$\lambda_1 \sim 1, \; \lambda_2 \sim a^2$

THE SYMM. MATRIX m_e DIAGON'D BY

$V_R^+ m_e U_L = \text{diag}\, m_e$ WITH $V_R = V_L = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$

IN BASIS WHERE m_e DIAGONAL:

$$m_\nu \sim \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} s^2 & sc \\ sc & c^2 \end{pmatrix}$$

$\Rightarrow \det[23] = 0, \quad \sin 2\theta \sim 2sc \sim 2ra$

LARGE FOR $r \sim 1\text{-}2$

Leontaris, Lola, Ross '95

Lola, Ross '99

SUMMARY AND CONCLUSION

- ν OSCILLATIONS ? PROBABLY YES!
  $\Rightarrow$ ν MASSES
- DATA STILL CRUDE
  VALUES OF Δm^2_{atm}, Δm^2_{sun}, ... ?
  LNSD ?
  MSW VO
- ν MASSES INDICATE $\not{L}$ (MAJORANA ν's)
  $m_\nu \sim \frac{m^2}{M}$ M: SCALE OF $\not{L}$
  $M \sim M_{GUT}$
  A WINDOW ON GUT'S
  BARYOGENESIS VIA LEPTOGENESIS.
- IF ~~LNSD~~ $\Rightarrow$ 3 LIGHT ν's ENOUGH
  (NO NEED OF STERILE ν's)
- ν AS HOT DM ? $\Rightarrow$ $|m_3| \cong |m_2| \cong |m_1| \sim 2eV$
  MODELS INVOLVE $m_\nu \sim L^T \frac{\lambda}{M} L\, HH$
  + DISCRETE OR NON ABELIAN FAMILY SYMM.
  νLESS-2β DECAY : IF $|U_{e3}| = 0$ $\Rightarrow$ BIMIXING

↙ MORE LIKELY ?

- HIERARCHICAL ν MASSES : e.g. $m_3 \gg |m_{2,1}|$

$$m_3 \sim \sqrt{\Delta m^2_{atm}} \sim 0.06 \text{ eV}$$

SMALL COSMOLOGICAL RELEVANCE

MODELS : ν_R + SEE-SAW

m^{ν}_{DIRAC} HIERARCHICAL LIKE FOR q, ℓ

$$\text{diag } m^{\nu}_{DIRAC} \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3 \end{pmatrix}$$

$m_\nu \sim m_D^T M^{-1} m_D$ ALSO HIERARCHICAL

(LARGE SPLITTINGS AND MIXINGS)

↳ subdet[23] = 0

EITHER : ALL q, ℓ MIXINGS SMALL

$$\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}$$

LARGE θ_{atm} IS FLUCTUATION

(SOLAR : MSW)

OR : RH q MIXINGS LARGE

$RH_d \xrightarrow{SU(5)?} LH_e$

(SOLAR : MSW OR VO)