MODELS OF ν MASSES FROM OSCILLATIONS WITH LARGE MIXING

F. FERUGLIO (Padova)

in collaboration with G. ALTARELLI

* Phys. Lett. B439 (1998) 112 [hep-ph/9807353]
* JHEP 11 (1998) 21 [hep-ph/9809596]
* hep-ph/9812475 to appear on Phys. Lett. B

PLAN

- starting assumptions
- A mechanism to generate large ν_μ-ν_τ mixing
- A semi-quantitative example in the framework of flavor symmetries
- conclusion

• INPUTS (SHORT SUMMARY)

(0) EVIDENCES FOR ν OSCILLATIONS

	$\Delta m^2 (eV^2)$	$\sin^2 2\vartheta$	
- solar	$(5-9)\cdot 10^{-11}$	• 0.55 – 0.9	$\nu_e \to \nu_{\mu,\tau}$
	$(4-10)\cdot 10^{-6}$	$(2-10)\cdot 10^{-3}$	$\nu_e \to \nu_{\mu,\tau}$
	$(1-10)\cdot 10^{-5}$	• 0.7 – 1.0	$\nu_e \to \nu_s$
- atmospheric	$(1-7)\cdot 10^{-3}$	• 0.75 – 1	$\nu_\mu \to \nu_\tau$ $\nu_\mu \to \nu_s$
- LSND	$(0.1-10)$	$(1-10)\cdot 10^{-3}$	$\bar\nu_\mu \to \bar\nu_e$ $\nu_\mu \to \nu_e$

→ ν-masses with, at least, 1 large mixing angle.

(1) CONSERVATIVE ASSUMPTION

only 3 light ν's (no sterile ν)

Can we accomodate solar + atm + LSND ?

→ 3 independent Δm^2 → 4 light ν's: $\nu_e, \nu_\mu, \nu_\tau + \nu_s$.

→ Assume that some among solar ν experiments is wrong and that solar ν_e deficit is due to E averaged vacuum oscillation.

$$\Delta m^2_{13} \leftrightarrow \text{LSND} \gg \Delta m^2_{12} \leftrightarrow \text{atm}$$

Compute the SK asymmetries $A_{e,\mu}(\vartheta_{12}, \vartheta_{13}, \vartheta_{23})$

→ no match to experimental data
(Barbieri, Hall, Smith, Strumia, Weiner 9807235)

(2) A USEFUL 0^{th}- ORDER APPROXIMATION

$$\underbrace{\Delta m^2_{12}}_{\text{solar}} \ll \underbrace{\Delta m^2_{13} \sim \Delta m^2_{23}}_{\text{atm}}$$

← we keep only these...

→ All experiments for lab and atm distances are sensitive to Δm^2_{23}, ϑ_{23}, ϑ_{13}.

$$P_{ee} = 1 - \sin^2 2\vartheta_{13} \sin^2\left(\frac{\Delta m^2_{23} L}{4E}\right)$$

> 0,9 at Chooz

$$\Delta m^2_{23} > 10^{-3} eV^2 \rightarrow \vartheta_{13} < 13°$$

As a 0^{th}-order approximation we may set $\vartheta_{13} = 0$

$$\begin{cases} \text{solar} \leftrightarrow \Delta m^2_{12}, \vartheta_{12} \\ \text{atm} \leftrightarrow \Delta m^2_{23}, \vartheta_{23} \end{cases}$$

<u>decoupled</u>

ν mixing matrix (omitting CP phases):

$$U_{fi} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} c_{23} & c_{12} c_{23} & -s_{23} \\ s_{12} s_{23} & c_{12} s_{23} & c_{23} \end{pmatrix}$$

$$\sin^2 2\vartheta_{23} \sim 1$$

$$\sin^2 2\vartheta_{12} \begin{cases} 10^{-2} \div 10^{-3} \\ \sim 1 \end{cases}$$

$$U m^2_{diag} U^+ = m_\nu$$

in the basis where the charged lepton mass is <u>diagonal</u>.

(3) SEA-SAW

SMALLNESS OF ν MASSES $\longleftrightarrow$ $\cancel{L}$ at some heavy scale M

$$\frac{\ell \ell h h}{M} \rightarrow$$ M close to the GUT scale

Many GUTs automatically provide a ν^c allowing for the sea-saw mechanism:

$$m_\nu = \frac{m_D^2}{M} + \cdots \qquad \mathcal{L} = \cdots - \nu^c m_D \nu + h.c. - \frac{1}{2} \nu^c M \nu^c + h.c.$$

3 x 3 case

$$m_\nu = - m_D^T M^{-1} m_D$$

We expect m_D hierarchical (one dominant eigenvalue) : cfr quarks, charged leptons.

- How to reconcile a hierarchical m_D with a large $\nu_\mu - \nu_\tau$ mixing without fine-tuning between m_D and M ?

I OUR FAVORITE MECHANISM

[for other possibilities see e.g. A.Yu. Smirnov, hep-ph/9901208]

(Notation $\nu^c m_D \nu \equiv \overline{\nu}_R m_D \nu_L$)

$$m_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A & B \end{bmatrix} v \qquad A, B \sim O(1)$$

<u>hierarchical</u>

eigenvalues : $\left(\sqrt{A^2+B^2}\, v, 0, 0\right)$.

Take any M : $(M^{-1})_{33} \neq 0$

$$m_\nu = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A^2 & AB \\ 0 & AB & B^2 \end{bmatrix} \frac{v^2}{\Lambda}$$

$$m_1 = m_2 = 0 \qquad m_3 = (A^2+B^2)\frac{v^2}{\Lambda}$$

→ good starting point for $\Delta m_{12}^2 \ll \Delta m_{13}^2$

The mixing matrix U_{fi} is precisely what needed with:

- $\tan \vartheta_{23} = -\dfrac{A}{B} \sim O(1)$ $\qquad \sin^2 2\vartheta_{23} \gtrsim 0{,}8$ → $0{,}6 \lesssim \left|\dfrac{A}{B}\right| \lesssim 1{,}6$
- $\vartheta_{13} = 0$
- ϑ_{12} undetermined, at this stage

0^{th}- ORDER APPROXIMATION $\leftrightarrow$ SYMMETRY

Simplest possibility : $U(1)_F$ flavour symmetry

	F- charges		
$\ell \equiv \begin{bmatrix} \nu \\ e \end{bmatrix}$	3	0	0
ν^c	1	-1	0
e^c	3	2	0

$$m_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & O(1) & O(1) \end{bmatrix} v_\mu \to \langle \varphi_u \rangle$$

$$M = \begin{bmatrix} 0 & O(1) & 0 \\ O(1) & 0 & 0 \\ 0 & 0 & O(1) \end{bmatrix} \bar{M} \qquad \det M \simeq O(\bar{M}^3)$$

$$m_e = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & O(1) & O(1) \end{bmatrix} v_d \to \langle \varphi_d \rangle$$

(i) Diagonalize m_e : $(m_e)_{diag} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & O(1) \end{bmatrix} v_d$

This requires a rotation $\begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix} - \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix} \leftarrow m_\tau$

that modifies the $O(1)$ coefficients of m_D.

Barring special choices they remain of $O(1)$.

(ii) After the sea-saw $\quad m_\nu = \begin{bmatrix} 0 & 0 & 0 \\ 0 & O(1) & O(1) \\ 0 & O(1) & O(1) \end{bmatrix} \left(\frac{v_u^2}{\bar{M}} \right)$

$\det = 0$

[Without sea-saw,
→ uncorrelated $O(1)$
coefficients]

III

BEYOND THE 0^{th}-ORDER APPROXIMATION	$\leftrightarrow$	SPONTANEOUSLY BROKEN FLAVOR SYMMETRY

Example:

$\nu_e^c \nu_e \varphi_u$ has $F=+4$ in previous example

$\to$ Add a new scalar field $\bar{\vartheta}$ with $F=-1$.

$$\underbrace{\left(\frac{\bar{\vartheta}}{M_{pe}}\right)^4 \nu_e^c \nu_e \varphi_u}_{\text{allowed by } U(1)} \xrightarrow[\frac{\langle\bar{\vartheta}\rangle}{M_{pe}} = \lambda < 1]{\langle\bar{\vartheta}\rangle \neq 0} (\lambda^4 v_u)\, \nu_e^c \nu_e$$

$\to$ Also ϑ with $F=+1$ $\quad \langle\vartheta\rangle / M_{pe} = \lambda' \sim \lambda$

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & \lambda & \lambda \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix} \bar{M}$$

$$m_e = \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_d \xrightarrow{(i)+(ii)} m_\nu = \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\bar{M}}$$

det $= O(\lambda^2)$

- $m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1$

$\to$ $\lambda \sim$ of the order of the Cabibbo angle

- For ν's: $\quad m_1 : m_2 : m_3 = \lambda^4 : \lambda^2 : 1 \qquad m_3 \sim \frac{v_u^2}{\bar{M}}$

$\to$ $\bar{M} \sim O(10^{15}\,\text{GeV})$ to hit Δm^2_{atm}

$\to$ $\Delta m^2_{12} = O(\lambda^4)\, m_3^2 \sim 10^{-5}\,\text{eV}^2 \leftrightarrow$ MSW range

- ν mixings:
  - * ϑ_{23} large
  - * $\vartheta_{12} \sim (\lambda/2) \to \sin^2 2\vartheta_{12} \sim \lambda^2$ $\nearrow$ MSW small angle
  - * $\vartheta_{13} \sim \lambda^3 \quad \to$ Chooz o.k.

IV $\left\{ \bar{M} \simeq 10^{15}\,\mathrm{GeV} \right.$ $\longrightarrow$ GUT

ν^c

Any additional insight from GUTs ?
[e.g $m_b(M_{GUT}) = m_\tau(M_{GUT})$, ...]

Simplest possibility : (SUSY) SU(5)

matter : 3 copies of Ψ_{10}^a , $\Psi_{\bar{5}}^a$, Ψ_1^a

minimal higgs structure : φ_5 , $\varphi_{\bar{5}}$

	F-charges	
$\Psi_{10}^a \equiv (q, u^c, e^c)$	$(3, 2, 0)$	$\varphi_{5,\bar{5}}$ neutral under $U(1)_F$
$\Psi_{\bar{5}}^a \equiv (l, d^c)$	$(3, 0, 0)$	
$\Psi_1^a \equiv \nu^c$	$(1, -1, 0)$	

In the quark sector :

$$m_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} v_u \quad \rightarrow \quad m_t : m_c : m_u = 1 : \lambda^4 : \lambda^6$$

$$m_d = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} v_d \quad \rightarrow \quad m_b : m_s : m_d = 1 : \lambda^2 : \lambda^6$$

$$U_L^{u,d} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \sim V_{CKM} \equiv (U_L^u)^+ U_L^d$$

→ correct at the order-of-magnitude level.

– lepton sector

In 1st approximation (only φ_5, $\varphi_{\bar 5}$ higgses)

m_e as before. Notice that:

$$m_e = (m_d)^+$$

$\rightarrow$ large μ-τ mixing $\leftrightarrow$ large s^c-b^c (or $\underbrace{s_R - b_R}_{\text{R-handed chiralities}}$) mixing

$\hookrightarrow$ unobservable in weak transitions!

[also:
- Albright, Babu, Barr '98
- Berezhiani, A. Rossi '98]

* In models where $m_D \propto m_u$, approximately diagonal (as expected from SO(10)), the ν_μ-ν_τ mixing could be mostly due to an asymmetric $m_e = (m_d)^+$ mass matrix.

*

* MSW-SA singled out?

Keep only $\bar\vartheta$ (out of $\vartheta, \bar\vartheta$): only the $F > 0$ can be compensated in building $U(1)_F$ invariants.

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & 0 & 0 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} \bar M$$

$$\rightarrow \quad m_3 : m_2 : m_1 = 1 : \lambda^3 : \lambda^3$$

$$\begin{cases} \Delta m^2_{12} \sim O(\lambda^9)\, m_3^2 \\ \operatorname{tg} \vartheta_{12} \sim 1 \end{cases} \leftrightarrow \text{vacuum oscillations}$$

(V) COMMENTS

* Previous example is merely indicative that the mechanism outlined in (I) can work!

* Some issues have not been addressed:
  - $\not{C}\not{P}$ parameters
  - FCNC (depends on the detail of SUSY)

* Some features might be improved:

minimal Higgs content : $m_e \equiv (m_d)^+$

$$\begin{cases} m_b = m_\tau & \text{o.k.} \\ \dfrac{m_e}{m_\mu} = \dfrac{m_d}{m_s} & \text{not good} \end{cases}$$

$$\begin{cases} \bar{\vartheta} & SU(5) \text{ singlet} \\ \bar{\vartheta}_{24} & \text{adjoint} \end{cases}$$

$$\Psi_{10}^a \, \Psi_{\bar{5}}^b \, \frac{(\bar{\vartheta})^p (\bar{\vartheta}_{24})^q}{M_{Pe}^{\,p+q}} \, \varphi_{\bar{5}}$$

$$(24)^q \times \bar{5} = \underbrace{\bar{5} + \overline{45}}_{\text{differentiate e from d}} + \cdots$$

* some other, not satisfactory, aspect e.g. $m_u/m_t \sim O(\lambda^6)$ would require a control of the $O(1)$ coefficients, beyond the scope of $U(1)_F$ flavor symmetries.

* Nevertheless, we find remarkable that 12 masses and 6 mixing angles come out close to the right range, in such a simple scheme.

CONCLUSIONS

Assuming

- 3 light ν's
- ϑ_{13} small
- sea-saw mechanism

a LARGE $\nu_\mu \nu_\tau$ MIXING

- may come as unespected but it is not unnatural
- is perfectly compatible with the usual hierarchy of Dirac masses
- does not require a strong fine-tuning among Dirac and Majorana sectors
- is presumably related, via SU(5) embedding, to a large s^c-b^c mixing, compatible with known weak interactions
- can be supported by a spontaneously broken flavour symmetry, at least at the level of orders-of-magnitude

* SOLAR ν

large or small mixing angles are equally compatible with the present mechanism.