

NEW ENHANCEMENT MECHANISM OF THE TRANSITIONS
OF SOLAR AND ATMOSPHERIC NEUTRINOS
CROSSING THE EARTH

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M. CHIZHOV, M. MARIS, S.T.P.
SISSA 53/98/EP, 31.7.98
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See also: M. MARIS, S.T.P., PR D56 (197) 7444
PR D58 (98)
(D-N EFFECT) Q.Y. Liu, M. MARIS, S.T.P., PR D56 (197) 599;

ν - Oscillations in Vacuum

(Pontecorvo 1958; 1967 -
relevance to ν -experiments)

ν - Oscillations in vacuum are possible if ν 's with $M(\nu_i) \neq 0$ ($m_i \neq m_j, i \neq j$) and nontrivial lepton (ν -) mixing exist in vacuum. (idea of ν -osc Pontecorvo, '51 Maki et al, '62)

Consider the simplest case:

$$\begin{cases} |\nu_e\rangle = |\nu_1\rangle \cos \theta + |\nu_2\rangle \sin \theta \\ |\nu_x\rangle = -|\nu_1\rangle \sin \theta + |\nu_2\rangle \cos \theta \end{cases} \quad \text{in vacuum}$$

$$\vec{p}, E_1(m_1)$$

$$\vec{p}, E_2(m_2)$$

$$\begin{aligned} m_2 &\neq m_1 \\ \theta &\neq \frac{\pi}{2} k, \\ k &= 0, 1, 2 \end{aligned}$$

$$\nu_x = \nu_\mu \text{ or } \nu_\tau - \text{active, or}$$

$$\nu_x = \nu_\Omega - \text{sterile};$$

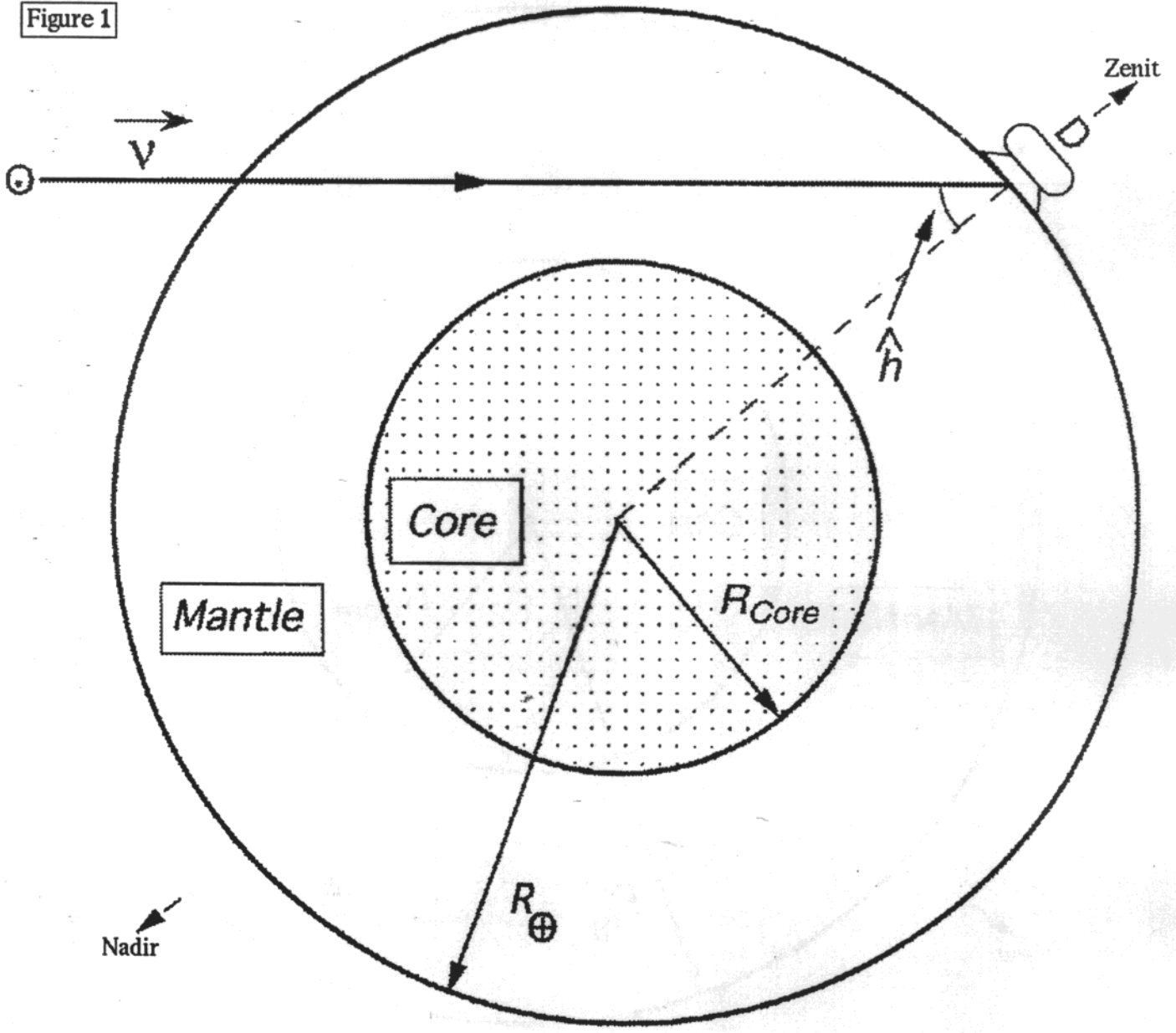
$$\boxed{\nu_e \rightleftharpoons \nu_\mu}_{x=\mu}$$

possible in vacuum.

θ - neutrino mixing angle in vacuum,

ν_1, ν_2 - neutrinos with definite mass in vacuum (vacuum mass-eigenstate ν 's)

Figure 1



$\rho_c \approx (10 - 13) \text{ g/cm}^3$ over a distance of $(R_c = 3486 \text{ km})$

$\rho_m \approx (3.3 - 5.5) \text{ g/cm}^3$ over a distance of $\sim 2885 \text{ km}$

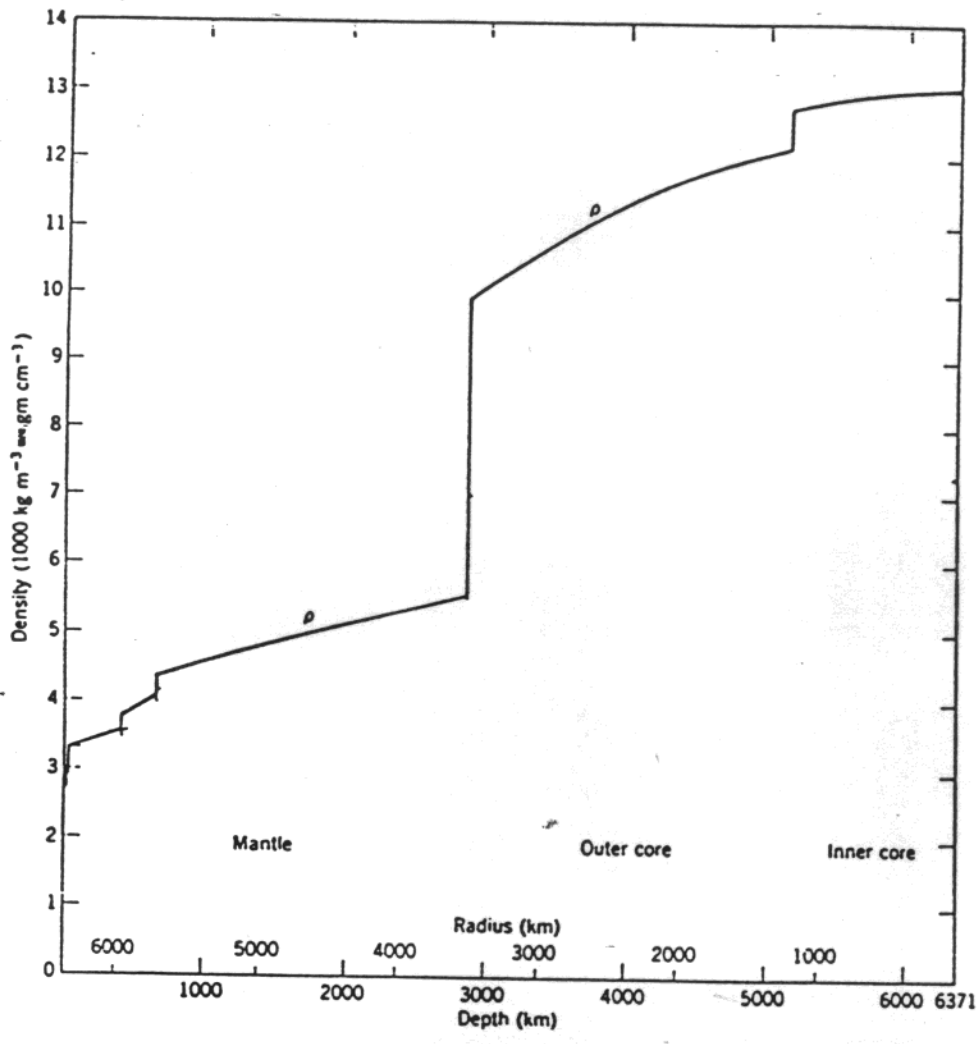


Figura 5.1: Distribuzione di densità della Terra (Stacey, 1977).

$$P(\nu_{\mu}(e) \rightarrow \nu_e(\mu; z))$$

"New Type" of **RESONANCE IN $P_{e2} \equiv P(\nu_2 \rightarrow \nu_e)$** :

(P_{e2} : accounts for the Earth Effect in $P_{\odot}(\nu_e \rightarrow \nu_e)$ in the case of 2- ν mixing MSW solution of the ν_{\odot} -problem)

- **Not the MSW resonance**

S.T.P. : PL B434
Rep-ph/9809587
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- Takes place when

$L_{osc}^{core}, L_{osc}^{man}$ obey certain constraints

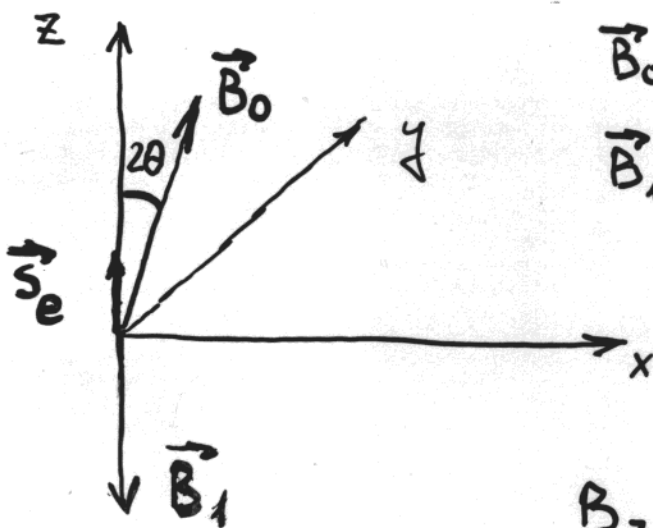
hence, it is a

"neutrino oscillation length resonance"

- Similar to the

electron paramagnetic resonance

L. WOLFENSTEIN

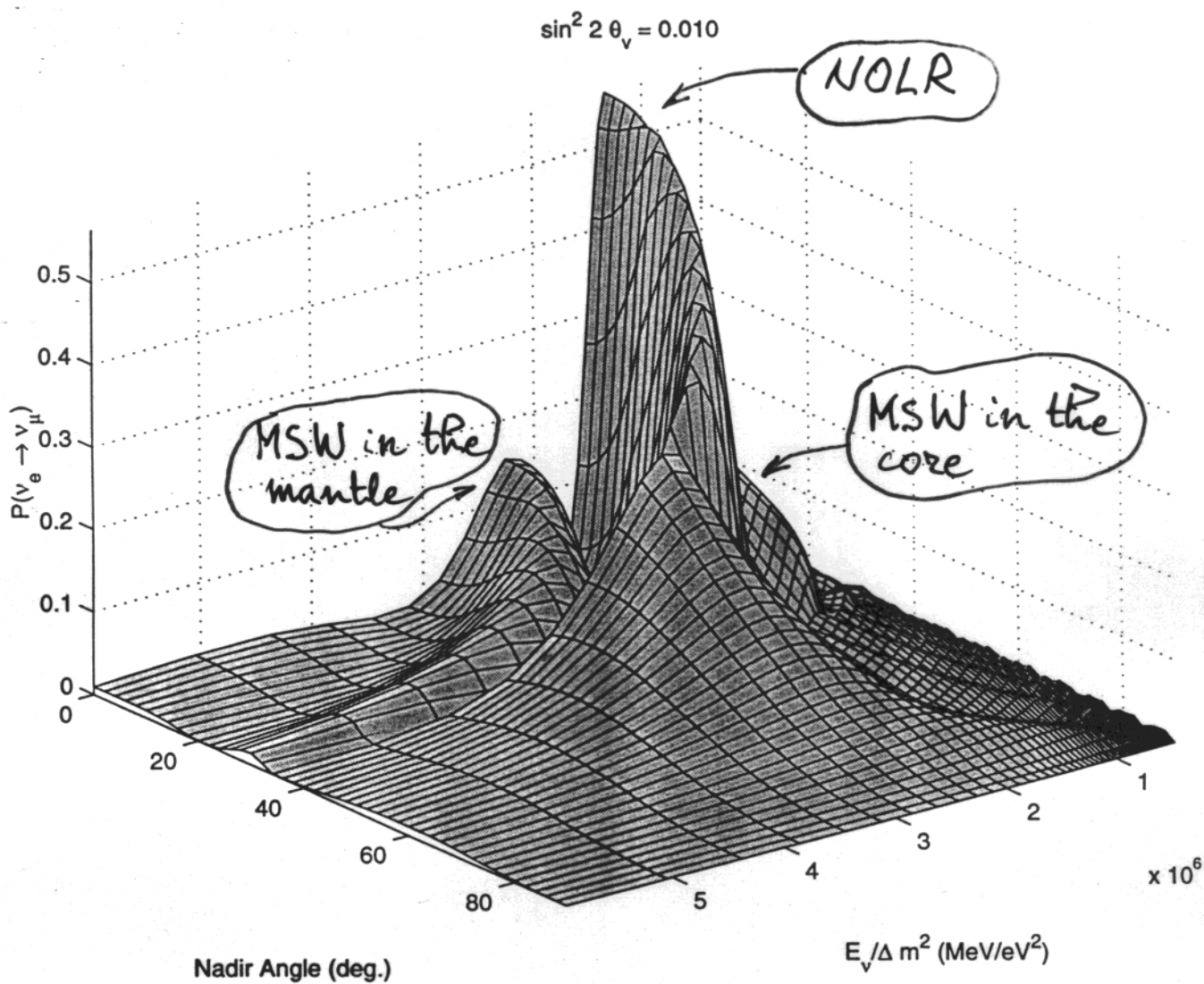


$$\vec{B}_0 : B_{0y} = 0, \text{ const.};$$

$$\vec{B}_1 : B_{1x} = B_{1y} = 0, \text{ assumes } \textcircled{2} \text{ values; changes step-wise}$$

$$B_z = \begin{cases} B_0 \cos 2\theta - B_1 > 0, & t < t_1 \\ \text{const.} & t_2 \leq t \leq t_3 \\ B_0 \cos 2\theta - B_1 < 0, & t_4 \leq t < t_5 \end{cases}$$

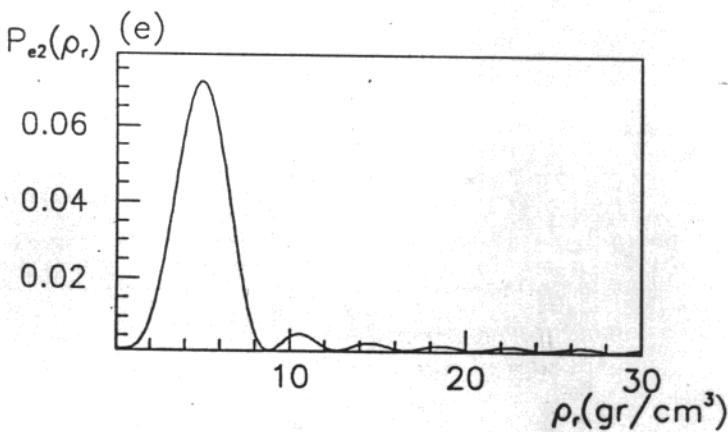
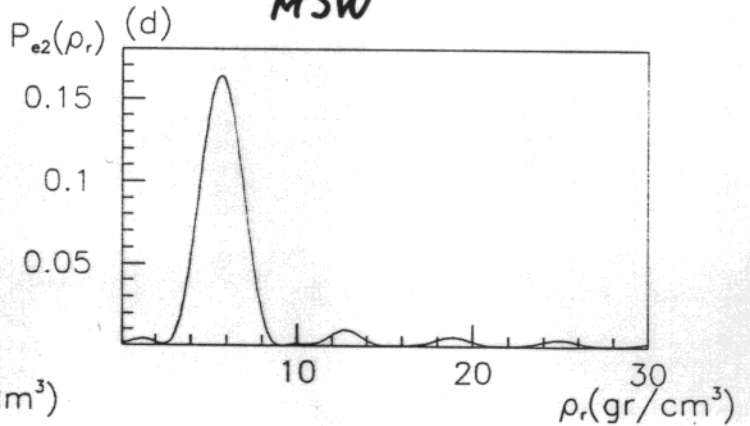
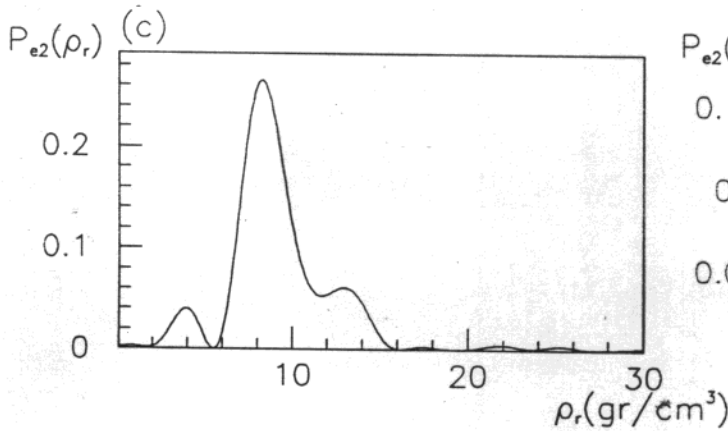
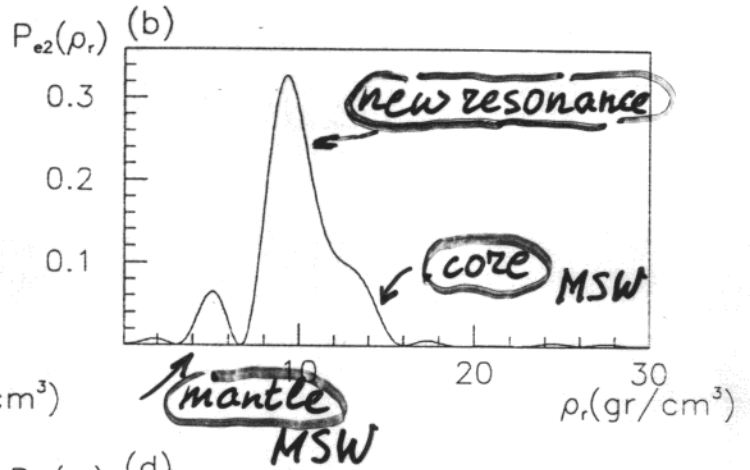
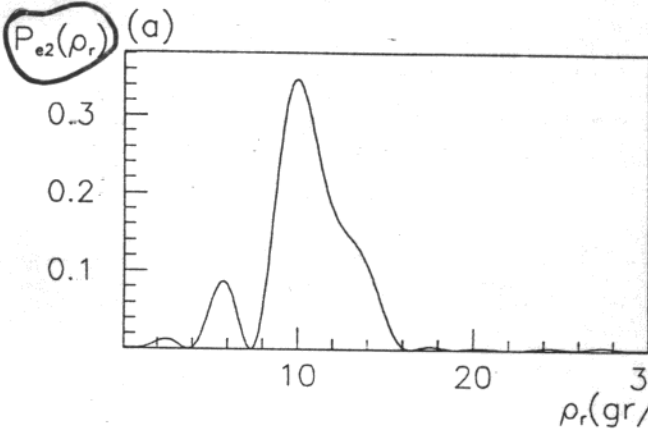
$\nu_\mu \rightarrow \nu_e, \nu_e \rightarrow \nu_\mu(z)$



$\nu_2 \rightarrow \nu_e$

$$E/\Delta m^2 = \frac{6.56 \times 10^6}{0.5 \rho_2 [\text{g/cm}^3]} \cos 2\theta \text{ MeV/eV}^2$$

Active



$\sin^2(2\theta_v) = 0.0060$

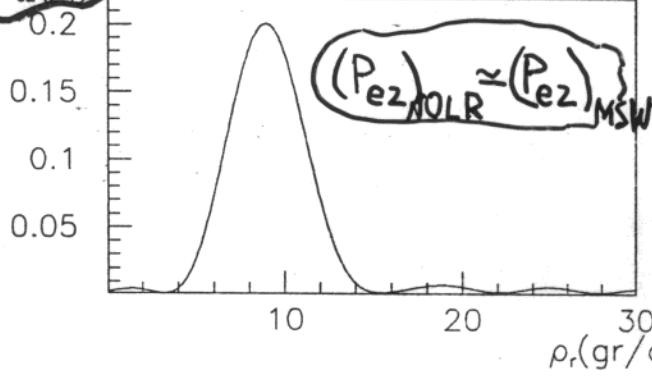
- (a) $h = 0^\circ$ Center Crossig
- (b) $h = 13^\circ$ SK Winter Solstice
- (c) $h = 23^\circ$ Half Core
- (d) $h = 33^\circ$ Core/Mantle Boundary
- (e) $h = 51^\circ$ Half Mantle

$\nu_2 \rightarrow \nu_e (\nu_e \rightarrow \nu_s)$ (M. MARIAS, S.V.P. '98)

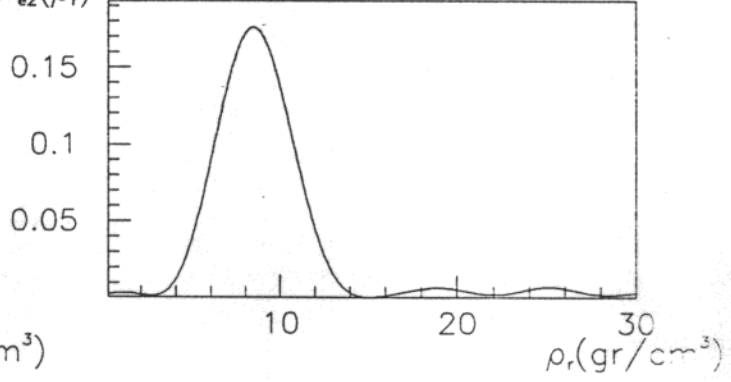
$$\boxed{E/\Delta m^2 = \frac{6.56 \times 10^6}{0.25 \rho_z [\text{g/cm}^3]} \text{ MeV/eV}^2}$$

Sterile

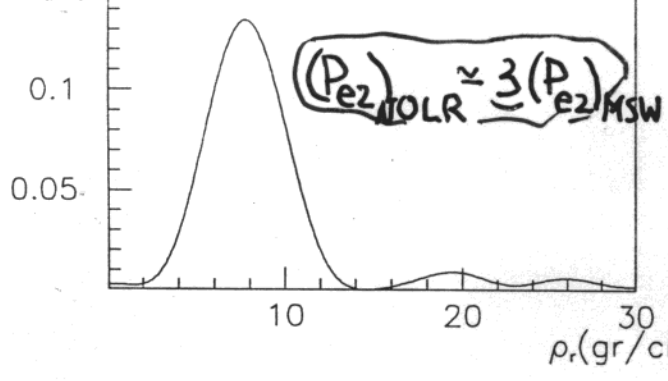
$P_{e2}(\rho_r)$ (a)



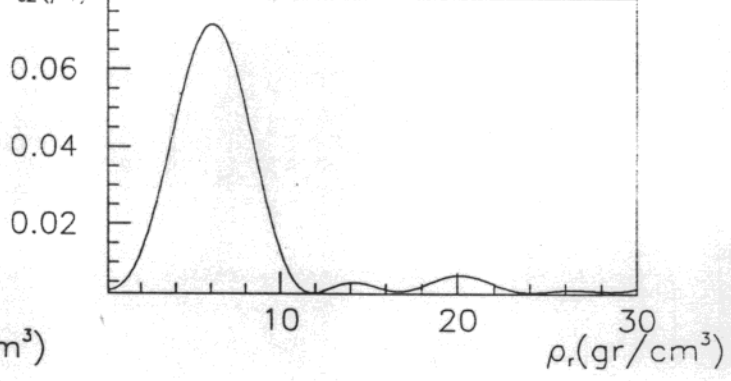
$P_{e2}(\rho_r)$ (b)



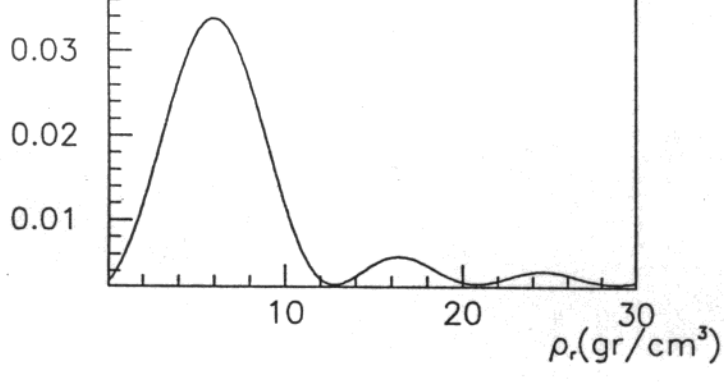
$P_{e2}(\rho_r)$ (c)



$P_{e2}(\rho_r)$ (d)



$P_{e2}(\rho_r)$ (e)



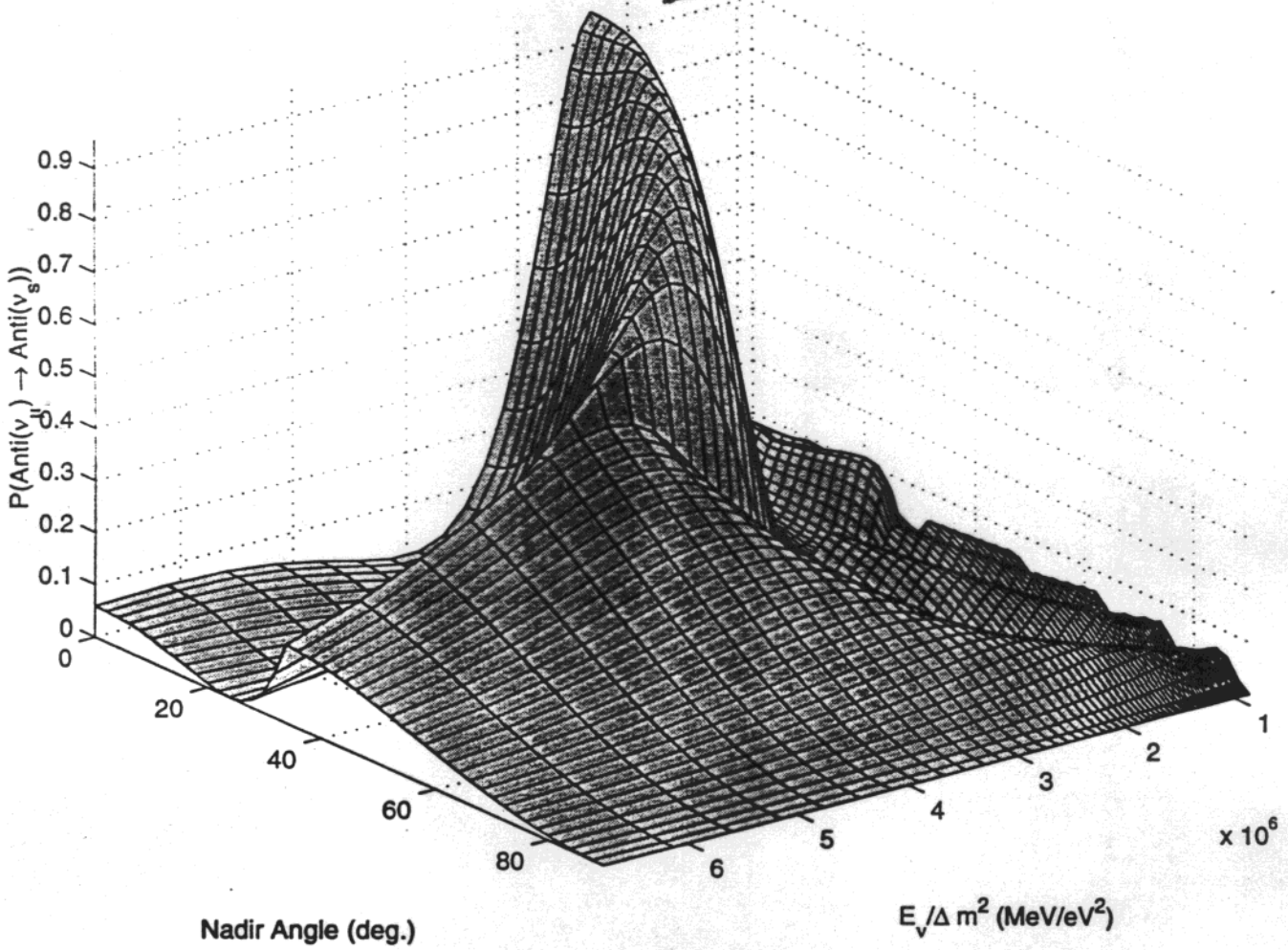
$\text{Sin}^2(2\theta_{\nu}) = 0.0100$

- (a) $h = 0^\circ$ Center Crossig
- (b) $h = 13^\circ$ SK Winter Solstice
- (c) $h = 23^\circ$ Half Core
- (d) $h = 33^\circ$ Core/Mantle Boundary
- (e) $h = 51^\circ$ Half Mantle

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_s$$

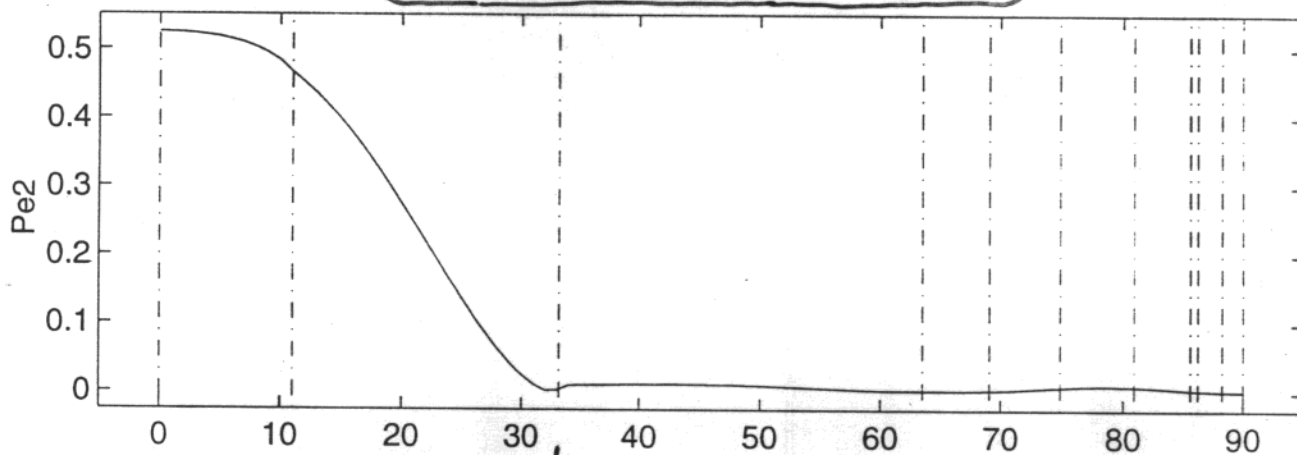
$$\sin^2 2\theta_\nu = 0.050$$

NOLR

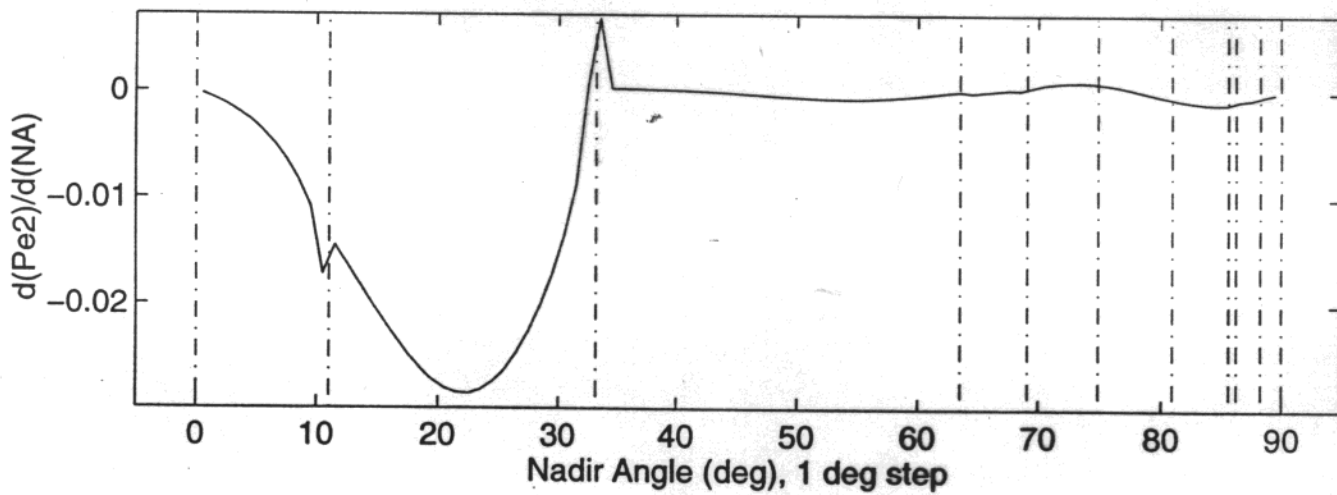


$$\gamma_e \rightarrow \gamma_{\mu}(z)$$

Active, SdTvS = 0.01, RhoR = 10 gr/cm³



core ←



The neutrino oscillation length resonance in $P_{e\bar{e}}$, $P(\nu_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu;\tau)})$

- exhibits strong dependence on E
- for Δm^2 from the SMA solution region takes place for $E \cong (5-12) \text{ MeV}$
- leads in the case of the SMA (MSW)

$\bar{\nu}_e \rightarrow \bar{\nu}_{\mu(\tau)}$ solution to a

~ 6 times bigger $D-N$ asymmetry in the "Core" sample of event in the SK detector than the asymmetry in the whole "Night" sample

- is sufficiently wide (it is wider than the MSW resonance)

$\Delta E/E_{\text{max}} \cong (0.3-0.4)$ and is $\sin^2 2\theta$ independent at $\sin^2 2\theta \leq 0.05$

The resonance takes place in the $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$, ($\bar{\nu}_e \rightarrow \bar{\nu}_{\mu}$) transitions of atmospheric neutrinos as well:

e.g., for $\left\{ \begin{array}{l} \Delta m^2 \sim 10^{-3} \text{ eV}^2, \\ \theta = 0^\circ \end{array} \right.$, $\sin^2 2\theta_{e\mu} \sim (0.01-0.10)$

max $P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)$ occurs at $E_{\nu} \cong 1.6 \text{ GeV}$

Table II. D - N Asymmetries for the Super - Kamiokande Detector.

N.	$\sin^2 2\theta_V$	Δm^2	f_B	$Y_e = 0.467$				$Y_e = 0.500$			
				$A_{D-N}^s \times 100$			$\frac{ A_{D-N}^C }{ A_{D-N}^N }$	$A_{D-N}^s \times 100$			$\frac{ A_{D-N}^C }{ A_{D-N}^N }$
				Night	Core	Mantle		Night	Core	Mantle	
1	0.0008	9.0e-6	0.4	-0.09	-0.54	-0.01	6.24	-0.12	-0.75	-0.01	6.25
2	0.0008	7.0e-6	0.4	-0.21	-1.26	-0.04	6.00	-0.22	-1.35	-0.04	6.14
3	0.0008	5.0e-6	0.4	-0.37	-1.29	-0.23	3.44	-0.35	-1.14	-0.23	3.26
4	0.0010	9.0e-5	0.4	3e-3	4e-3	3e-3	1.25	3e-3	4e-3	3e-3	1.33
5	0.0010	7.0e-6	0.4	-0.25	-1.50	-0.04	6.01	-0.26	-1.59	-0.04	6.12
6	0.0010	5.0e-6	0.4	-0.45	-1.52	-0.27	3.39	-0.43	-1.35	-0.27	3.14
7	0.0020	1.0e-5	0.5	-0.07	-0.41	-0.01	6.00	-0.10	-0.62	-0.01	6.20
8	0.0020	7.0e-6	0.5	-0.35	-2.10	-0.07	5.98	-0.36	-2.18	-0.07	6.06
9	0.0020	5.0e-6	0.5	-0.71	-2.30	-0.45	3.23	-0.67	-2.02	-0.45	3.01
10	0.0040	1.0e-5	1.0	0.15	0.79	0.04	5.42	0.22	1.28	0.04	5.82
11	0.0040	7.0e-6	0.7	-0.04	-0.12	-0.02	3.48	0.01	0.21	-0.02	21.00
12	0.0040	5.0e-6	0.7	-0.59	-1.36	-0.46	2.31	-0.56	-1.12	-0.47	2.00
13	0.0060	1.0e-5	1.5	0.72	3.98	0.17	5.56	1.05	6.20	0.17	5.90
14	0.0060	7.0e-6	1.0	1.06	6.46	0.17	6.13	1.26	7.60	0.17	6.03
15	0.0060	5.0e-6	0.7	0.47	2.93	0.06	6.19	0.46	2.82	0.06	6.13
16	0.0080	1.0e-5	1.5	1.63	8.93	0.37	5.47	2.36	13.59	0.37	5.76
17	0.0080	7.0e-6	1.5	3.04	17.00	0.53	5.59	3.39	19.10	0.53	5.63
18	0.0080	5.0e-6	1.0	2.55	10.26	1.22	4.02	2.44	9.54	1.22	3.91
19	0.0100	7.0e-6	1.5	5.72	29.85	1.05	5.22	6.28	32.85	1.05	5.23
20	0.0100	5.0e-6	1.0	5.60	19.84	3.03	3.54	5.36	18.38	3.03	3.43
21	0.0130	5.0e-6	1.5	11.69	36.33	6.89	3.11	11.21	33.72	6.89	3.01
22	0.3000	1.5e-5	2.0	10.73	13.25	10.31	1.24	11.03	15.29	10.30	1.39
23	0.3000	2.0e-5	2.0	7.64	9.60	7.31	1.26	7.79	10.66	7.31	1.37
24	0.3000	3.0e-5	2.0	4.74	5.54	4.61	1.17	4.78	5.85	4.61	1.22
25	0.3000	4.0e-5	2.0	3.29	3.93	3.18	1.20	3.31	4.07	3.19	1.23
26	0.4800	3.0e-5	1.5	5.95	6.81	5.81	1.15	5.99	7.08	5.81	1.18
27	0.4800	5.0e-5	1.5	2.98	3.50	2.89	1.17	2.99	3.57	2.89	1.19
28	0.5000	2.0e-5	1.5	9.48	10.97	9.24	1.16	9.60	11.79	9.23	1.23
29	0.5600	1.0e-5	1.5	23.65	34.9	21.64	1.48	27.50	40.77	25.11	1.48
30	0.6000	8.0e-5	1.0	1.42	1.64	1.38	1.15	1.42	1.65	1.38	1.16
31	0.7000	3.0e-5	1.0	6.54	7.26	6.41	1.11	6.56	7.43	6.41	1.13
32	0.7000	5.0e-5	1.0	3.37	3.90	3.29	1.16	3.38	3.95	3.29	1.17
33	0.7700	2.0e-5	1.0	9.89	10.27	9.83	1.04	9.94	10.59	9.83	1.07
34	0.8000	1.3e-4	0.7	0.57	0.69	0.55	1.21	0.57	0.69	0.55	1.21
35	0.9000	4.0e-5	0.7	4.53	5.12	4.42	1.13	4.53	5.17	4.42	1.14
36	0.9000	1.0e-4	0.7	1.15	1.33	1.13	1.15	1.15	1.33	1.13	1.16

FIGURES

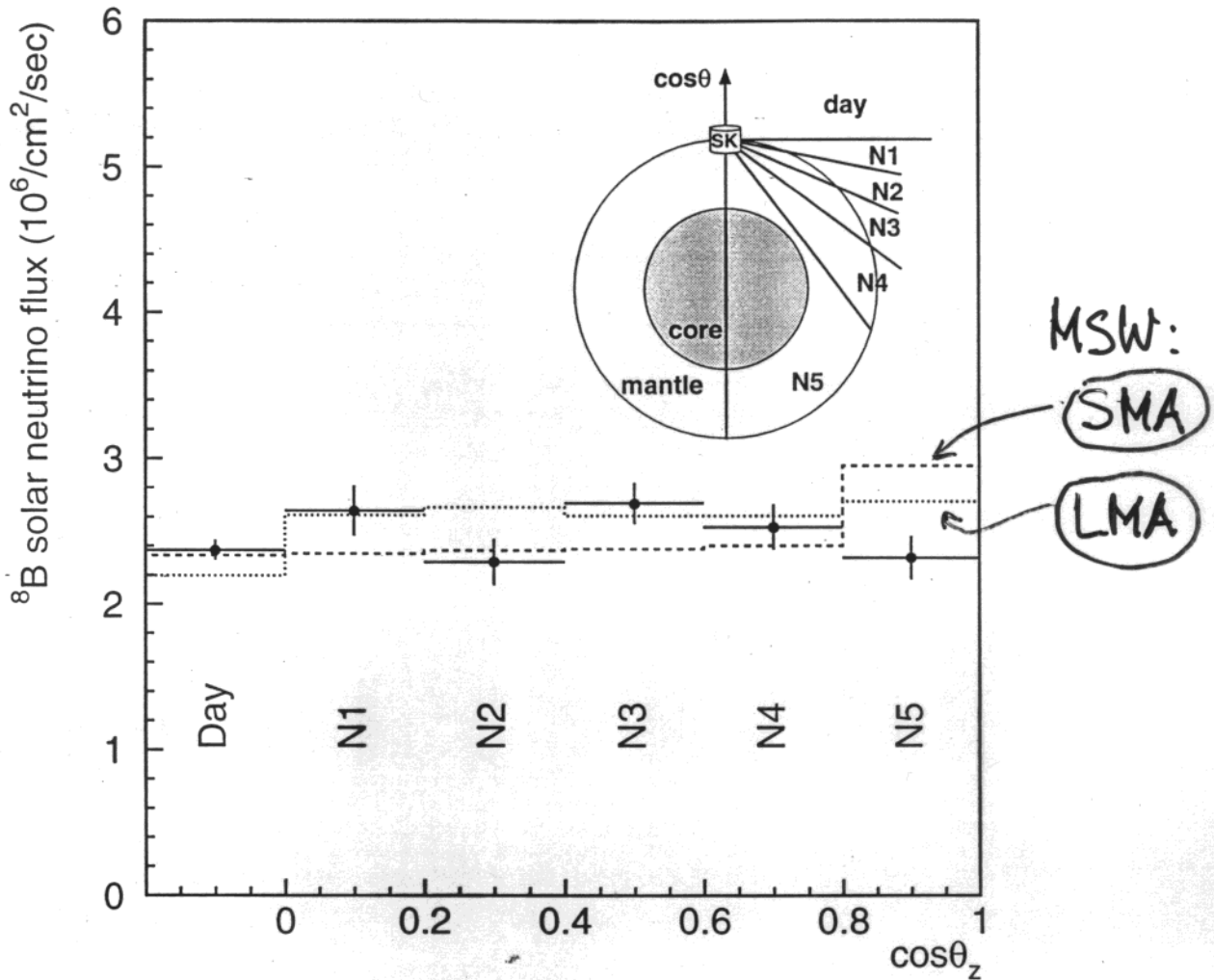


FIG. 1. Measured day/night solar neutrino fluxes as a function of the nadir of the Sun. Error bars represent statistical errors only. Night data is divided into 5 bins. Dotted histogram is the expected variation of a typical large angle solution and dashed histogram is that of a typical small angle solution.

MSW $\nu_e \rightarrow \nu_{\mu(\tau)}$

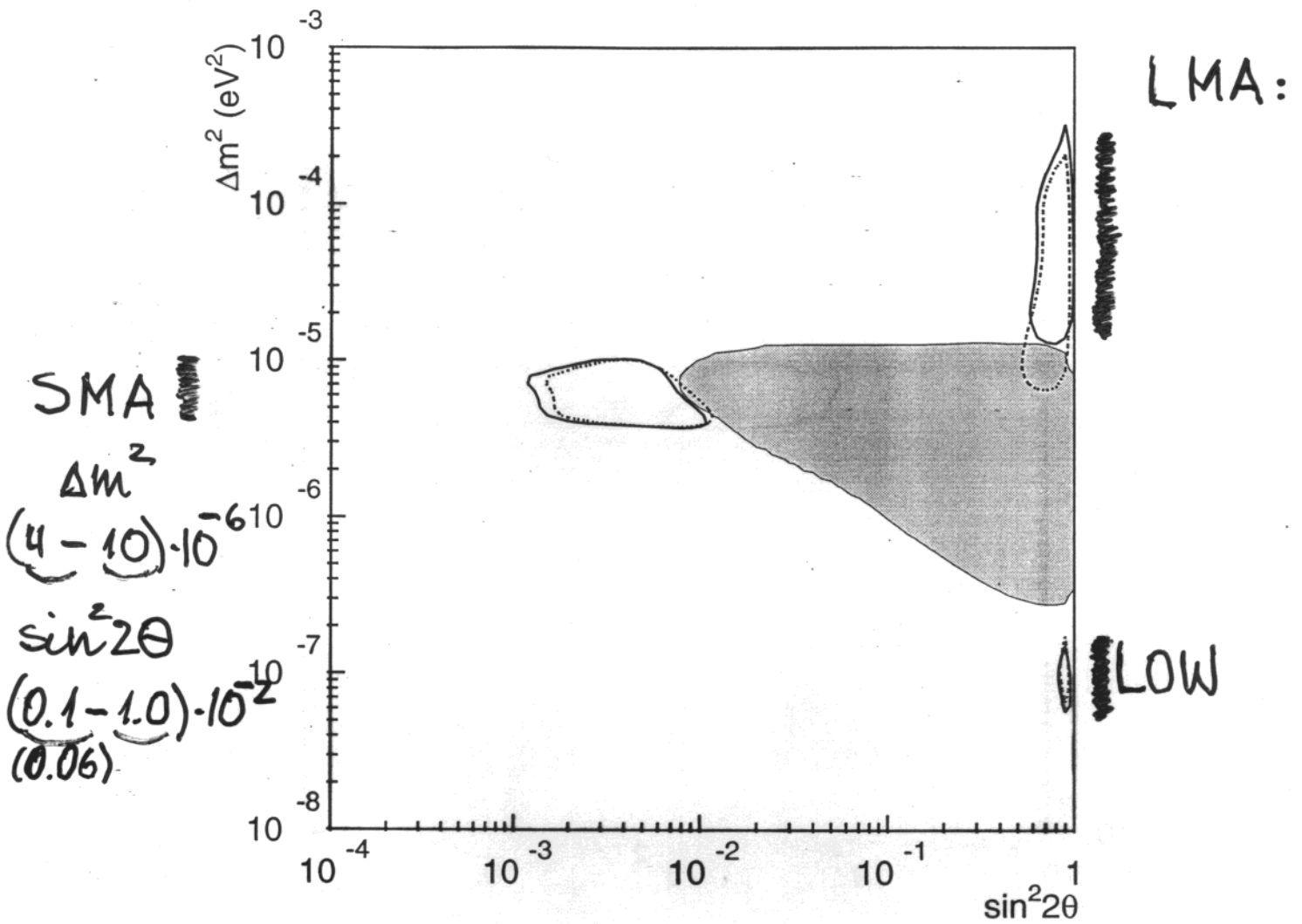


FIG. 2. Flux independent exclusion region by SK day/night variation for $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations. Exclusion probabilities larger than 99% are shown in the shaded area. Regions inside of the dotted lines are allowed at the 99% C.L. from the combined rate analysis of Homestake, SAGE, Gallex and SK-flux in comparison with the BP98 SSM [6]. Regions inside of the thick solid lines are allowed at the 99% C.L. from the combined rate analysis of the rates and the SK D/N variation.

LMA : $\Delta m^2 \cong (0.2 - 30) \cdot 10^{-5} \text{ eV}^2$ (99% C.L.)
 $\sin^2 2\theta \cong (0.6 - 0.96)$

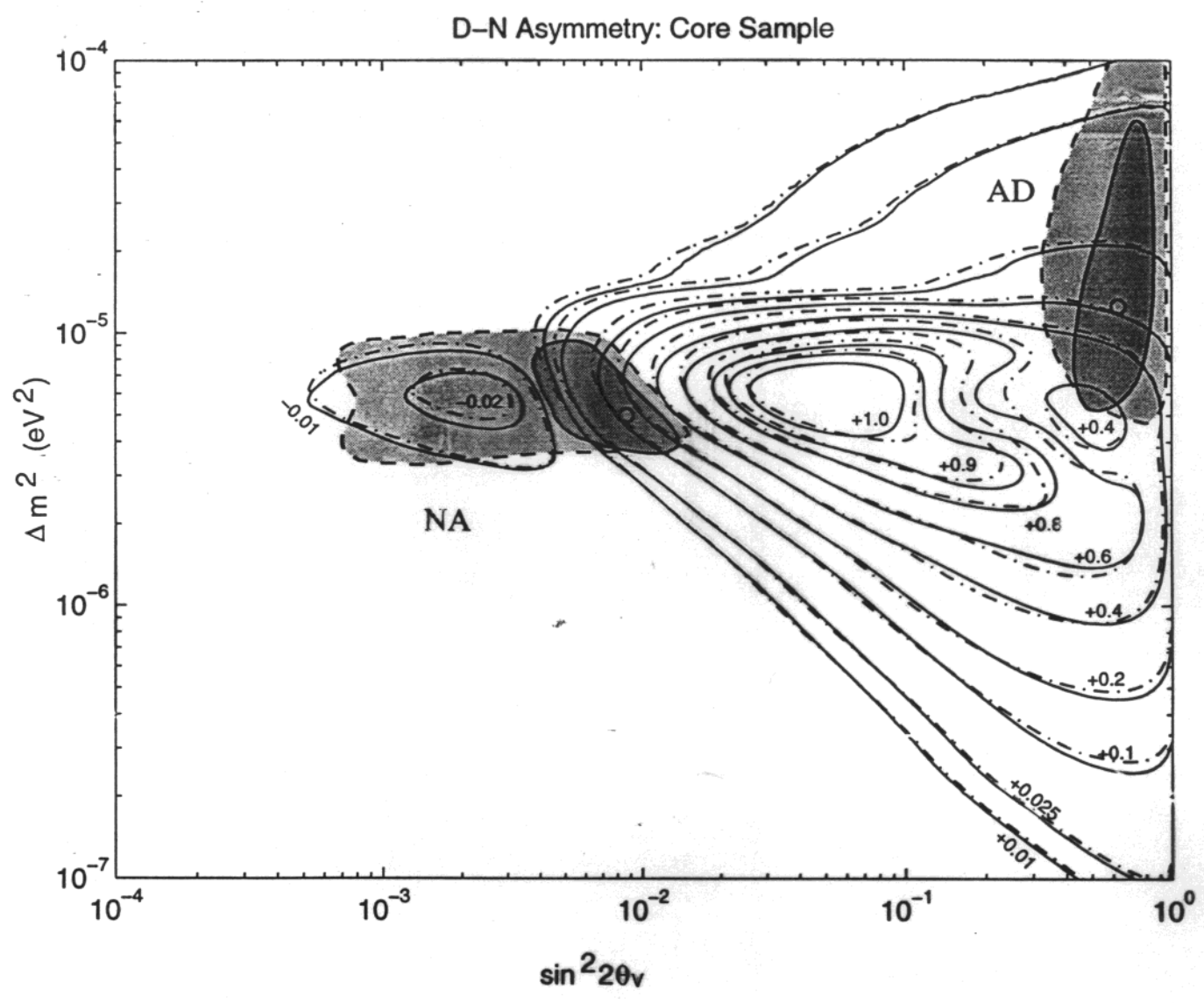


Figure 3c

The same resonance is present
in the

$$\nu_{\mu} \rightarrow \nu_e, \nu_e \rightarrow \nu_{\mu}$$

transitions of atmospheric neutrinos
crossing the Earth's core.

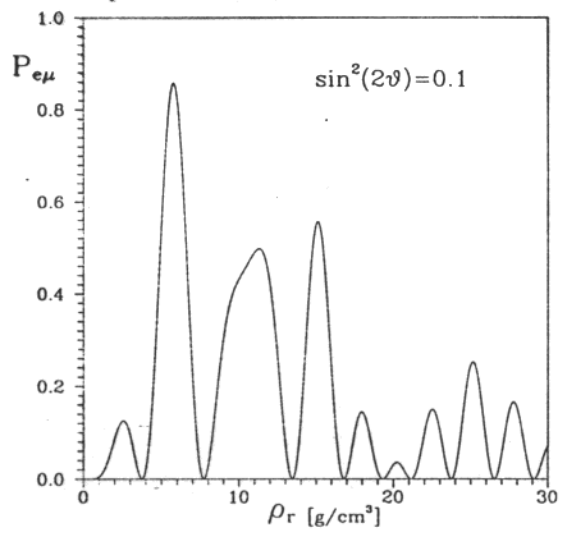
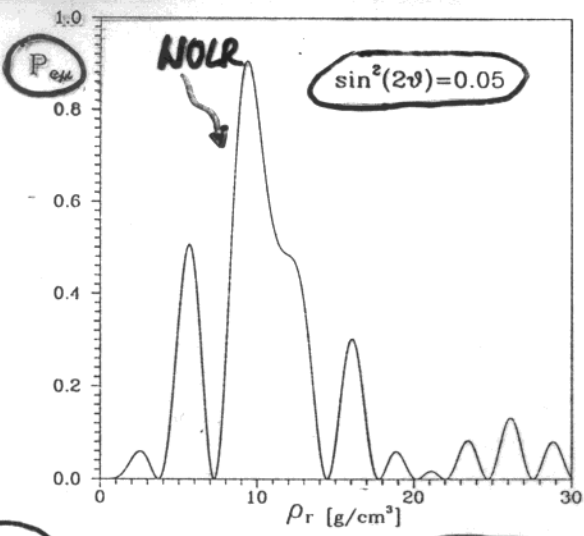
S.T.P., REPORT SISSA 31/98/EP

(Rep-ph/9805262)

M. MARIS, S.T.P., M. TCHIZOV.

Report SISSA 53/98/EP

(to be released)



$\Delta m_{atm}^2 \approx 10^{-3} eV^2$ } $E_{max} = 1.46 GeV$
 $\approx 5 \times 10^{-3} eV^2$ } $E_{SR} \approx 1.05 GeV$
 $\approx 5 \times 10^{-3} eV^2$ } $E_{max} \approx 7.3 GeV$

sub-GeV

sample of SK event

multi-GeV

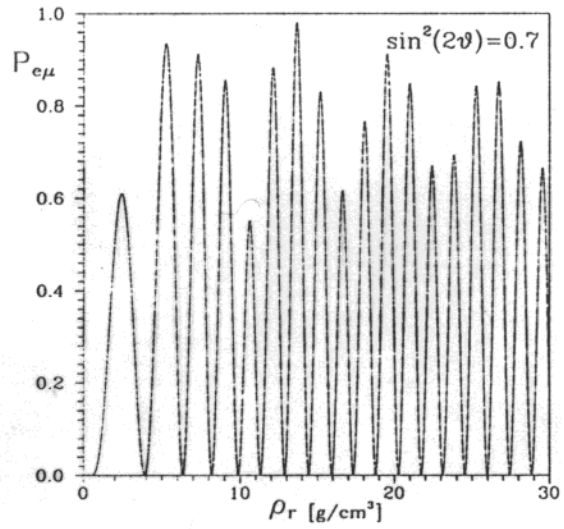
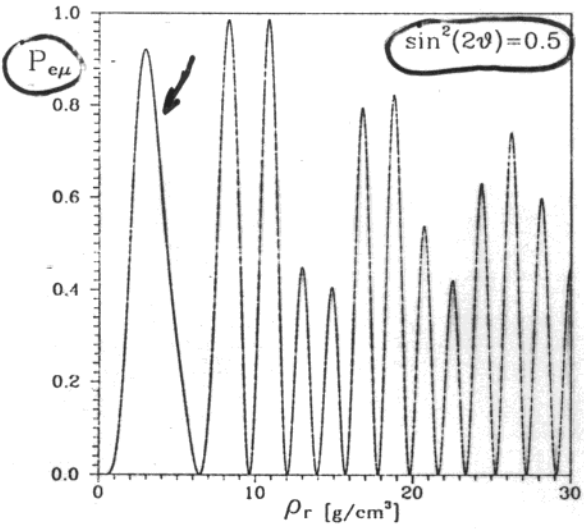


Figure 5a: $h=0$

ν_{ATM} : KAMIOKANDE, IMB, SOUDAN, SUPER-K

VERY STRONG EVIDENCES FOR $\nu_{\mu} \leftrightarrow \nu_{\tau}$ OSCILLATIONS FROM THE SK DATA ON

- Up - Down ASYMMETRY IN THE SUB-GeV AND MULTI-GeV SAMPLES OF μ -like EVENTS.

- ZENITH ANGLE DEPENDENCE OF THE RATES OF THE SUB-GeV AND MULTI-GeV μ -like EVENTS

NO SIMILAR EFFECTS WERE OBSERVED IN THE e -like SAMPLES OF EVENTS:

$$\nu_{\mu} \leftrightarrow \nu_{\tau} \quad \text{or}$$

$$\nu_{\mu} \leftrightarrow \nu_{\tau} \quad \text{DOMINATE!}$$

$$\Delta m^2 [\text{eV}^2] \quad 10^{-3} - 8 \cdot 10^{-3}$$

$$2 \cdot 10^{-3} - 7 \cdot 10^{-3}$$

$$\sin^2 2\theta \quad 0.86 - 1.0$$

$$0.86 - 1.0$$

90% C.L.

SK 736 days

L/E, PATH (i.e. ZENITH ANGLE) INDEPENDENT SUPPRESSION OF THE ATM. ν_{μ} -FLUX IS NOT COMPATIBLE WITH THE DATA.

Lipari, Lusignoli: '99

FCNC, ν -DECAY, GRAV. INDUCED OSCILLATIONS: DISFAVOR

Let us assume 3- ν mixing:

$$|\nu_{eL}\rangle = \sum_{k=1}^3 U_{ek}^* |\nu_{kL}\rangle, \quad \nu_k - \text{Dirac or Majorana.}$$

$l=e, \mu, \tau$ $\beta, (m_k)$

Two independent Δm^2 : say, $\Delta m_{21}^2, \Delta m_{31}^2$

Assume further that:

(A) one Δm^2 - in the range of the MSW or VO solution of the ν_0 -problem

$$\Delta m^2 \lesssim 10^{-4} \text{ eV}^2$$

(B) the second $\Delta m^2 \gtrsim 10^{-3} \text{ eV}^2$

relevant to the ν_{ATM} -anomaly; (another option: LSND effect) and Δm^2 can be in the range of reactor/accelerator ν -oscillation experiments...

(?) (C) ν_k masses provide the "hot" DM component ($\sim 30\%$ of DM)

$$\sum_{k=1}^3 m_k \sim (4.5 - 6.0) \text{ eV}$$

Assume, for example, that

ν_{ATM} - ANOMALY DUE TO $\nu_{\mu} \leftrightarrow \nu_{\tau}$ $\Delta m_{31}^2 \sim 10^3 \text{ eV}^2$

ν_{\odot} - PROBLEM DUE TO $\nu_e \rightarrow \nu_{\mu, \tau}$ $\Delta m_{21}^2 \lesssim 10^4 \text{ eV}^2$

$m_1 \ll m_2 \ll m_3$; $m_1 \approx m_2 \ll m_3$; $m_1 < m_2 < m_3$,
 $(m_1 \ll m_2 \approx m_3)$ $m_1 \approx m_2 \approx m_3$

- the same ν -oscillation phenomenology

DeRujula et al., '86
 Barger et al., '88
 Bilenky et al., '89

$$P_{ATM}(\nu_{\mu} \rightarrow \nu_{\tau}) \approx 2 |U_{\mu 3}|^2 |U_{\tau 3}|^2 \left[1 - \cos \frac{\Delta m_{31}^2 R}{2E} \right]$$

$$P_{ATM}^{VAC}(\nu_{\mu} \rightarrow \nu_e) \approx 2 |U_{\mu 3}|^2 |U_{e 3}|^2 \left[1 - \cos \frac{\Delta m_{31}^2 R}{2E} \right]$$

$$P_{\odot}(\nu_e \rightarrow \nu_e) = |U_{e 3}|^4 + (1 - |U_{e 3}|^2)^2 P_{\odot}^{(2\nu)}(\nu_e \rightarrow \nu_e)$$

G.S. Lim '87
 S.T.P. '88,
 Smirnov, '91

MSW or VO solution of the ν_{\odot} -problem:

$$|U_{e 3}|^2 \lesssim \begin{matrix} 0.1 \\ 0.05 \end{matrix}$$

compatible with the CHOOZ result.

$$P_{\odot}^{(2\nu)}(\nu_e \rightarrow \nu_e) : \Delta m_{21}^2 \cdot \sin^2 2\theta_{12} \approx 4 \frac{|U_{e1}|^2 |U_{e2}|^2}{(|U_{e1}|^2 + |U_{e2}|^2)^2}$$

$$\cos 2\theta_{12} \approx \frac{|U_{e1}|^2 - |U_{e2}|^2}{|U_{e1}|^2 + |U_{e2}|^2}$$

More specifically, for $\Delta m_{21}^2 \ll \Delta m_{31}^2$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

ν_{ATM}

"weak" connection via U_{e3} .

CHOOZ : $|U_{e3}|^2 \leq 0.05$, $|U_{e3}| \leq 0.22$
 + ν_0
 ($\Delta m_{31}^2 \geq 2 \cdot 10^{-3} \text{ eV}^2$)

Perhaps, $|U_{e3}| \ll 1$:

"very small"

Fogli, Lisi et al. '98
 Yasuda '98
 Barzger et al. '98
 Bilenky, Giunti,
 Grimus '98

LB Line: ICARUS, MINOS, K2K

$\nu_\mu \rightarrow \nu_e$ - high sensitivity to $|U_{e3}|$ (limit: $\sim 5 \cdot 10^{-2}$)

$$P_{\text{LBL}}(\nu_\mu \rightarrow \nu_e) \approx \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} P_{2\nu}^m(\Delta m_{31}^2, \theta_{13})$$

$$\sin^2 2\theta_{13} \approx 4|U_{e3}|^2(1 - |U_{e3}|^2)$$

$$P_{2\nu}^m(\Delta m_{31}^2, \theta_{13}) = \frac{1}{2} \sin^2 2\theta_{13}^m [1 - \cos \Delta E]$$

$$\Delta E = \frac{\Delta m_{31}^2}{2E} \sqrt{\left(1 - \frac{N_e}{N_e^{\text{res}}}\right)^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13}}$$

Interesting physics related to $|U_{e3}|$:

$|U_{e3}|$ drives the sub-dominant

$\nu_\mu \rightleftharpoons \nu_e$, $\nu_e \rightleftharpoons \nu_\tau$, $\nu_e \rightleftharpoons \nu_\mu$
oscillations

of the atmospheric ν_μ, ν_e

which can be strongly enhanced

by a new type of resonance (\neq MSW) in the Earth

$$P_{\text{ATM}}^{\text{VAC}}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \cong P_{\text{ATM}}^{\text{VAC}}(\bar{\nu}_e \rightarrow \bar{\nu}_{\mu(\tau)}) \sim \frac{|U_{e3}|^2}{\text{suppressed!}}$$

Consider $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu(\tau)}$
in the Earth: assume $\Delta m_{21}^2 \ll \Delta m_{31}^2$

Then

$$P_E^{3\nu}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \cong P_E^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \cong \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} P_E^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau)$$

S.T.P. '88

$$P_E^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) \cong \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2} P_E^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau)$$

← enhanced by NOLR

$$\sin^2 2\theta = 4|U_{e3}|^2(1 - |U_{e3}|^2)$$

$$\Delta m^2 = \Delta m_{31}^2$$

Consider transitions of $\bar{\nu}$'s
crossing the Earth core:

$$-1 \leq \cos \theta_2 \leq -0.8$$

S.T.P. '98;
CHIZHOV, MARIS, STP

E_ν - fixed:

$$\phi(\bar{\nu}_e) = \phi^0(\bar{\nu}_e) \left[1 + \left(\tau \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} - 1 \right) P_E^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) \right]$$

$$\tau \equiv \frac{\phi^0(\bar{\nu}_\mu)}{\phi^0(\bar{\nu}_e)} = \begin{cases} 2.0 - 2.5, & \text{sub-GeV} \\ 3.5 - 4.0, & \text{multi-GeV} \\ 4.5 \end{cases}$$

$$\Phi(\nu_e) \cong \Phi^0(\nu_e) \left[1 + \begin{matrix} \text{sub-GeV} \\ \left\{ \begin{matrix} 0.0 - 0.25 \\ 0.75 - 1.0 \end{matrix} \right\} P_E^{\nu_e \rightarrow \nu_\tau} \\ \text{multi-GeV} \end{matrix} \right]$$

If, for example

$$|U_{e3}|^2 \cong \textcircled{0.013}, \quad \sin^2 2\theta \cong \textcircled{0.05}$$

$$\max P_E^{\nu_e \rightarrow \nu_\tau} \cong \textcircled{0.94}$$

$$\text{at } \frac{1}{2}\text{-width} \cong \textcircled{0.45}$$

e-like

sub-GeV : $\cong 10\%$ excess - small

multi-GeV : $< \textcircled{(30-40)\%}$ excess - can be relatively large!

If the predictions for $\Phi^0(\nu_e)$ and $\Phi^0(\nu_\mu)$ are correct, the NOL resonance can produce sizeable excess of e-like events in the multi-GeV sample of SK data, provided $\Delta m_{31}^2 \gtrsim (2-3) \times 10^{-3} \text{ eV}^2$.

If an excess due to NOL resonance is observed in the sub-GeV sample of e-like events, then $\tau \gtrsim 3$, $\Delta m_{31}^2 \lesssim 2 \cdot 10^{-3} \text{ eV}^2$.

Similar analysis for $\Phi(\nu_\mu)$:

$$\Phi(\nu_\mu) \approx \Phi^0(\nu_\mu) \left[1 + S_{23}^4 \left((S_{23}^2 \tau)^{-1} - 1 \right) P_E^{2\nu} - 2 C_{23}^2 S_{23}^2 (1 - \text{Re}(e^{-i\alpha} A^{2\nu})) \right]$$

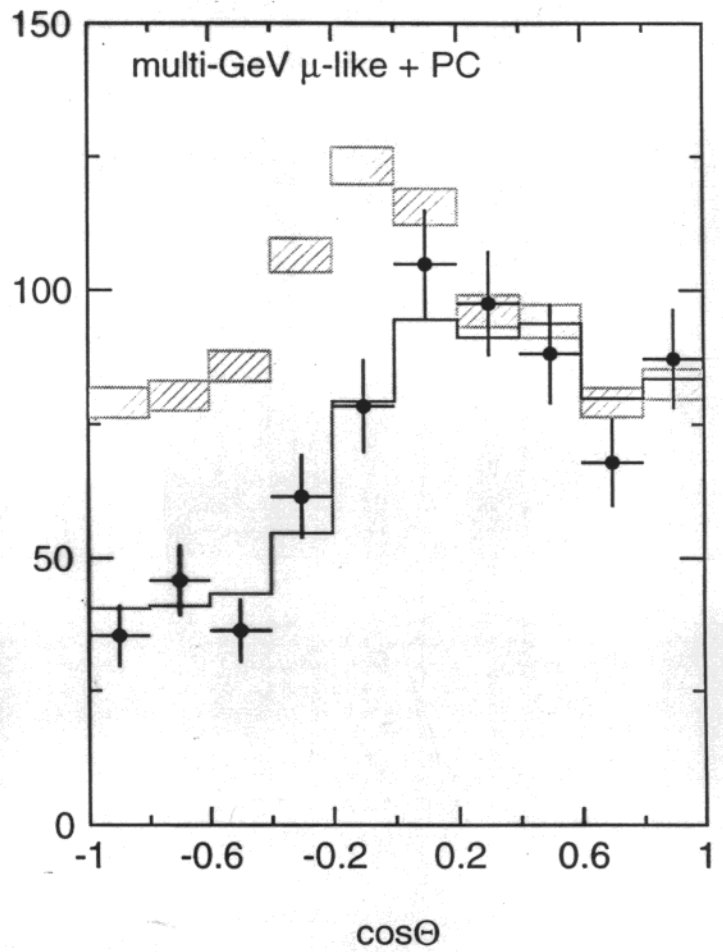
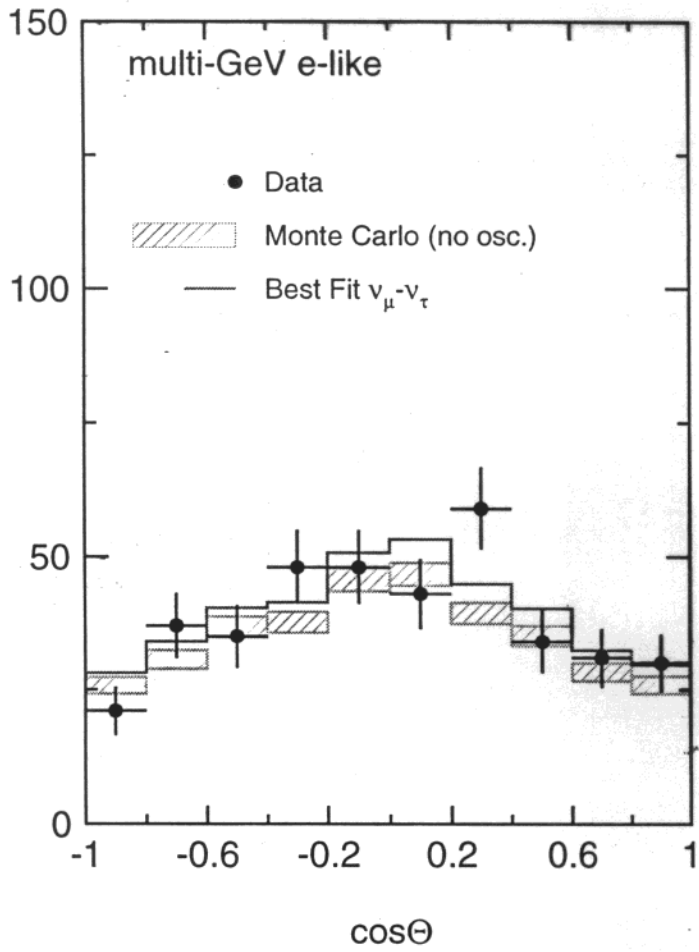
$$S_{23}^2 \equiv |U_{\mu 3}|^2 / (1 - |U_{e 3}|^2), \quad C_{23}^2 = 1 - S_{23}^2$$

$\alpha, A^{2\nu}$ - known

$$0.25 \leq S_{23}^4 \leq 0.5$$

$$S_{23}^4 \left[\frac{1}{S_{23}^2 \tau} - 1 \right] = \begin{cases} 0.0 \text{ to } (-0.22), \text{ SUB-GeV} \\ S_{23}^2 = 0.5; 0.7 \\ (-0.1) \text{ to } (-0.39), \text{ multi-GeV} \end{cases}$$

Thus, if the predictions for $\Phi^0(\nu_\mu; E, \theta_z)$ and $\Phi^0(\nu_e; E, \theta_z)$ are correct, the NOLR can contribute to the θ_z -dependence of the μ -like multi-GeV signal for $S_{23}^2 \approx 0.6-0.7$, and $\Delta m_{31}^2 \approx (2-3) \cdot 10^3 \text{ eV}^2$.



THE NATURE OF THE NOLR :

$$\begin{aligned}
 A_E(\nu_\mu \rightarrow \nu_e) &= A^{\text{man}}(\nu_\mu \rightarrow \nu_e) A^{\text{core}}(\nu_e \rightarrow \nu_e) A^{\text{man}}(\nu_e \rightarrow \nu_e) \\
 &+ A^{\text{man}}(\nu_\mu \rightarrow \nu_\mu) A^{\text{core}}(\nu_\mu \rightarrow \nu_\mu) A^{\text{man}}(\nu_\mu \rightarrow \nu_e) \\
 &+ A^{\text{man}}(\nu_\mu \rightarrow \nu_\mu) A^{\text{core}}(\nu_\mu \rightarrow \nu_e) A^{\text{man}}(\nu_e \rightarrow \nu_e) \\
 &+ A^{\text{man}}(\nu_\mu \rightarrow \nu_e) A^{\text{core}}(\nu_e \rightarrow \nu_\mu) A^{\text{man}}(\nu_\mu \rightarrow \nu_e)
 \end{aligned}$$

CHIZHOV, S.T.P.'92

MSW : terms in $P(\nu_\mu \rightarrow \nu_e)$

$$\sim |A^{\text{man}}(\nu_\mu \rightarrow \nu_e)|^2, \quad \bar{g}^{\text{res}} \sim \bar{g}^{\text{man}}$$

or

$$\sim |A^{\text{core}}(\nu_\mu \rightarrow \nu_e)|^2, \quad \bar{g}^{\text{res}} \sim \bar{g}^{\text{core}}$$

dominate!

$$\text{NOLR : } \sim 2 \text{Re} [A^{\text{man}}(\nu_\mu \rightarrow \nu_e) (A^{\text{core}}(\nu_\mu \rightarrow \nu_e))^* \dots]$$

dominates!

NOLR : constructive interference effect
 $\bar{g}^{\text{man}} < \bar{g}^{\text{res}} < \bar{g}^{\text{core}}$

S.T.P.,
 PLB434
 CHIZHOV,
 S.T.P.'98

$$(A) + (B) \equiv \cos(2\theta_m'' - 4\theta_m' + \theta) < 0$$

The "maximum" (supplementary) conditions are satisfied for

$$\sin^2 2\theta \lesssim 0.05$$

$$\bar{g}_m < g^{res} < \bar{g}_c$$

$$g^{res} < \bar{g}_m$$

SMA

LMA

MSW solutions

$$\Delta E^{(m)} = \frac{\Delta m^2}{2E} \left[\left(1 - \frac{\bar{g}_m(c)}{g^{res}} \right)^2 \cos^2 2\theta + \sin^2 2\theta \right]^{1/2}$$

$$\Delta E^{X'} = \pi(2k+1), \quad \Delta E^{X''} = \pi(2k'+1), \quad \Delta E^{(m)} = \frac{2\pi}{L_{osc}^{max(c)}}$$

$$k = k' = 0$$

$$\sin^2 2\theta \lesssim 0.02$$

$$\pi \left\{ \frac{1}{X'} + \frac{1}{X''} \right\} \approx \sqrt{2} G_F (\bar{g}_c Y_e^c - \bar{g}_m Y_e^m) \frac{\bar{N}_e^c - \bar{N}_e^m}{\bar{N}_e^c - \bar{N}_e^m}$$

For a given trajectory X' and X'' are fixed: the resonance conditions are constraints on $(L_{osc}^{max}, L_{osc}^c)$

The NOLR can be observed in the present and/or future ν_0 and/or ν_{atm} experiments. Detectors located at lower geographical latitudes are better suited for that.

A NEW ATMOSPHERIC NEUTRINO EXPERIMENT WITH BETTER E - AND θ_z -RESOLUTION IS HIGHLY DESIRABLE.

Ermilova et al., 1986 :

$$S(\tau) = \overset{\text{const.}}{\bar{S}} + \overset{\text{const.}}{S_1} \cos \omega \tau, \quad \tau = \pi x / \omega v$$

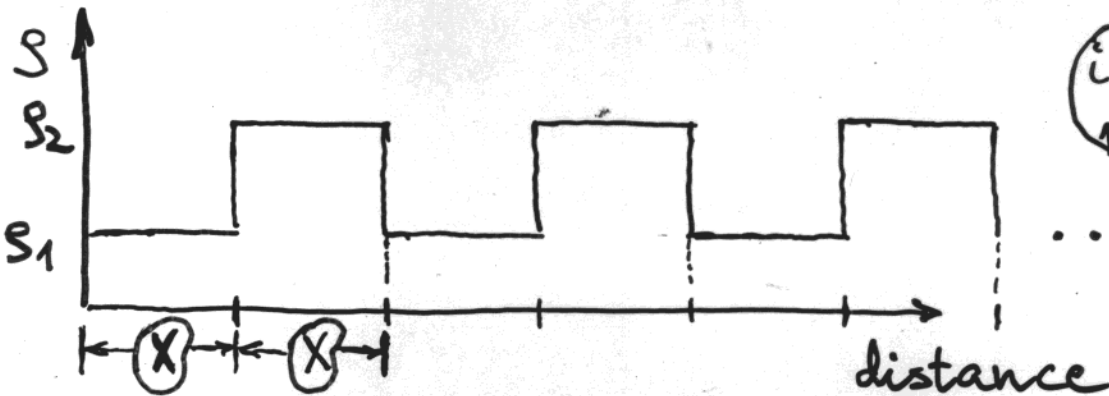
"... periodic dependence of S on the distance along the \mathcal{D} path."

True parametric resonance:

$$\bar{\omega} = \frac{n}{2} \omega, \quad \bar{\omega} = \cos 2\theta - \frac{lv}{l_0} = \cos 2\theta \left(1 - \frac{N_e}{N_e^{\text{res}}}\right)$$

Akhmedov, '88 : the case considered by Ermilova et al. +

"periodic step function" (1 page)



Looked for parametric resonance:
specific case

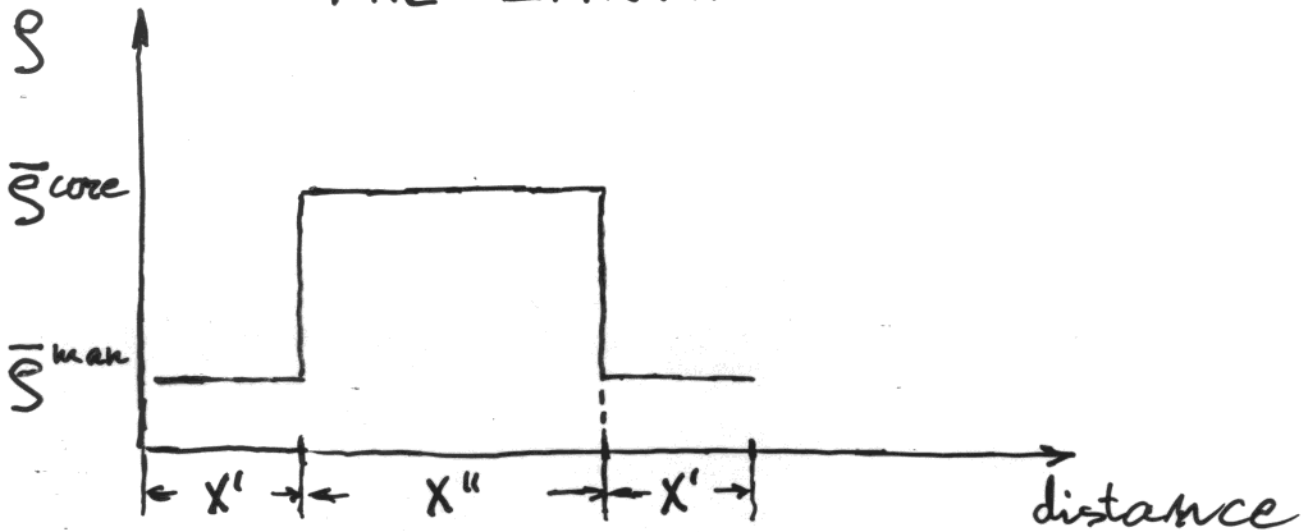
$$\sin^2 2\theta \ll 1$$

$$S_{1,2} \ll S_{\text{res}}$$

no NOL resonance!

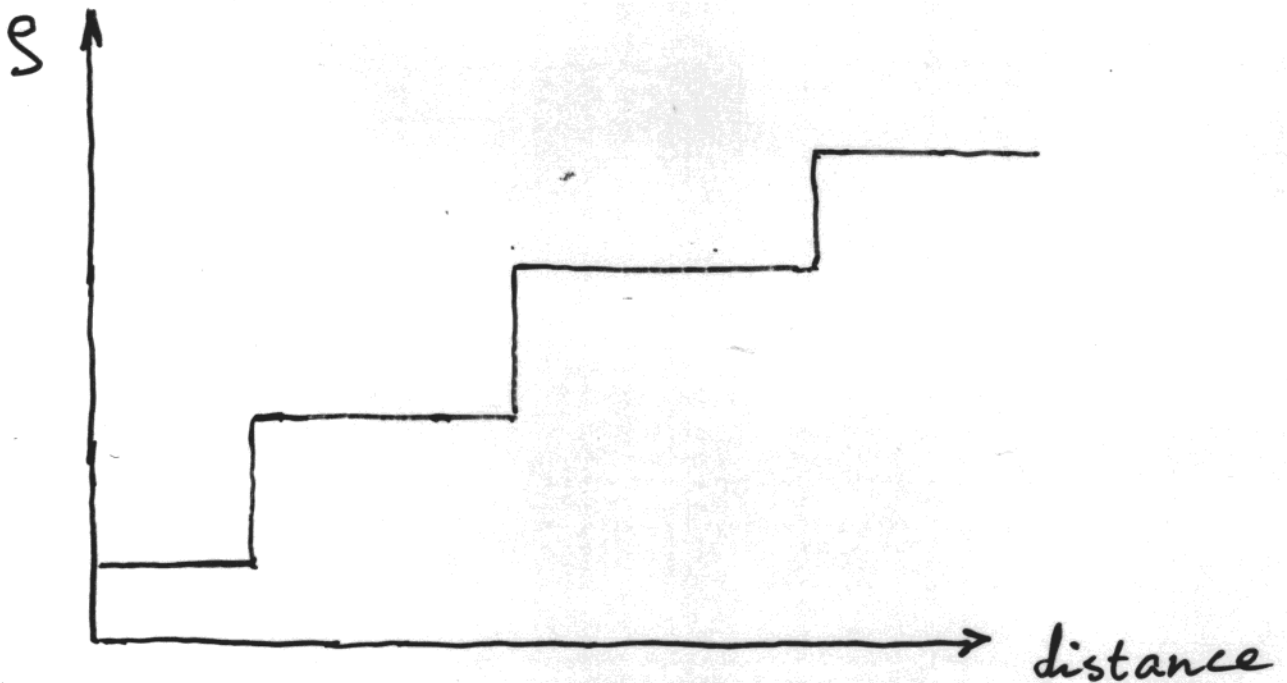
The Earth does not "look" like that!

THE EARTH



Not even $1 + \frac{1}{2}$ periods.

THE NOLR possible even if, e.g.,



Liu + Smirnov '98

(Liu + Mikheyev + Smirnov '98) :

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_s$$

$$\sin^2 2\theta \cong 1.0$$

$$\Delta m^2 / 2E \ll V_{\bar{\mu}s} = \sqrt{2} G_F \frac{1}{2} \bar{N}_n$$

$$R \cong 28^\circ$$

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_s) \cong 1$$

$$X' V_{\bar{\mu}s}^{\text{man}} \cong \sqrt{I}$$

$$X'' V_{\bar{\mu}s}^{\text{core}} \cong \sqrt{I}$$

The authors claimed:

- No similar effect in $\bar{\nu}_\mu(e) \rightarrow \bar{\nu}_e(\mu)$