A: Neutrino Induced Charged Lepton Oscillations

1. KARMEN ANOMALY
2. (g−2) of the Muon

B: Neutrino Sources From The Core of the Earth

by

- Elisabetta Sassaroli (MIT)
- Allan Widom (Northeastern)
- Yogendra Srivastava (Perugia)

+(S. Palit on Karmen)

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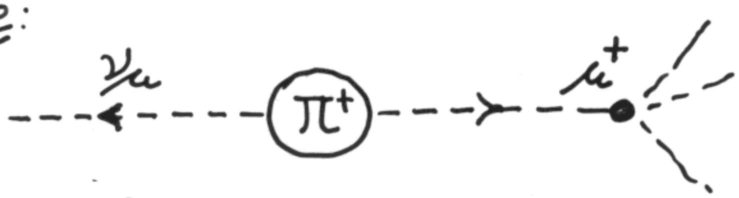
References (1÷5) showed the phenomenon of Induced Oscillations

example:



ν_μ : $\left.\begin{matrix}\nu_1\\ \nu_2\\ \nu_3\end{matrix}\right\}$ superposition ⟶ throws μ's into superposition by 4 mom conservation

$$P^\lambda(\text{pion}) = p_1^\lambda(\text{muon}) + k_1^\lambda(\nu \text{ of } m_1)$$
$$= p_2^\lambda(\;''\;) + k_2^\lambda(\;''\; m_2)$$
$$= p_3^2(\;''\;) + k_3^\lambda(\;''\; m_3)$$

$k_1^2 = m_1^2$; $k_2^2 = m_2^2$; $k_3^2 = m_3^2$ (diff ν masses)

$p_1^2 = p_2^2 = p_3^2 = M_\mu^2$ (same μ mass)

But ! since $P^2 = M_\pi^2$

$$\therefore \quad p_1^\lambda \neq p_2^\lambda \neq p_3^\lambda$$

Muon is a superposition of diff mom states (same mass M_μ)

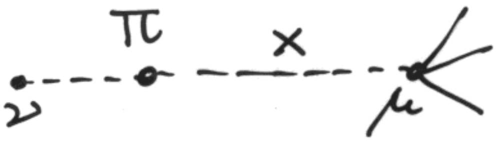
→ Interference pattern in the decay distribution of the muon.

Muon Momentum Distribution: $\pi \to \mu \nu_\mu$

Let $\quad |\nu_f\rangle = \sum_a R_{fa} |a\rangle$

flavour index f : $e \quad \mu \quad \tau$

mass index a : $1, 2, 3$



Muon survival probability (at the secondary space time vertex x)

$$P_{muon}(x) \approx e^{-\Gamma\sqrt{x^2}} \sum_{\substack{j=1\\ l=1}}^{3} |R_{\mu j}|^2 \cdot |R_{\mu l}|^2 \cdot \cos[(p_j - p_l)\cdot x]$$

It would show an interference pattern if neutrino's have mass and if they mix.

KARMEN ANOMALY

- KARMEN group measured ν_e produced from μ^+ decaying at rest

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

- They found an oscillatory counting rate.

- For simplicity, consider 2 flavours only

$$\langle \bar{\nu}_\mu | = \cos\phi \langle \nu_1 | + \sin\phi \langle \nu_2 |$$

kets for mass eigenstates m_1 & m_2

- View μ^+ decay as a 2 step process

1. $\mu^+ \rightarrow W^+_{eff} + \bar{\nu}_\mu$
2. $W^+_{eff} \rightarrow e^+ + \nu_e$

- The virtual effective W^+_{eff} (far off the W mass shell) wave function propagates as

$$W^+_{eff,\alpha}(x) = \int \underbrace{\mathcal{D}_{\alpha\beta}(M_W; x-y)}_{\text{W-propagator}} \underbrace{J^{+\beta}(y)}_{\text{charged current}} (d^4y)$$

$$\simeq g_W \cdot \left(\frac{1}{M_W^2}\right) \bar{\Psi}(x) \gamma_\alpha (1-\gamma_5) N_\mu(x).$$

- Muon wave function $\overline{\Psi}$: 1 particle to vacuum matrix element of the muon field operator $\overline{\psi}$:

$$\overline{\Psi}(x) = \langle 0 | \overline{\psi}(x) | \mu^+ \rangle$$

- The muon neutrino wave function N_μ is the vacuum to 1 particle matrix element of the muon neutrino field operator ν_μ:

$$N_\mu(x) = \langle \overline{\nu}_\mu | \nu_\mu(x) | 0 \rangle$$

where the muon neutrino field operator ν_μ is given by

$$\nu_\mu(x) = \cos\phi \; \nu_1(x) + \sin\phi \; \nu_2(x)$$

$$\therefore \quad \boxed{N_\mu(x) = \cos^2\phi \; N_1(x) + \sin^2\phi \; N_2(x)}$$

↗ muon neutrino wave function has an oscillatory form.

- This through 4-momentum conservation throws "<u>the other particle</u>" (here W^+_{eff}) into an oscillatory superposition of amplitudes

$$W^+_{eff}(x) = \cos^2\phi \; W^+_1(x) + \sin^2\phi \; W^+_2(x)$$

with $\quad W^+_{i\alpha}(x) \sim \overline{\Psi}(x) \gamma_\alpha (1-\gamma_5) N_j \quad (j=1,2)$

- Let p^λ be the initial 4 mom of the muon and neutrino 4-mom $k_j^2 = m_j^2$ $(j=1,2)$

↳ 2 different 4-mom of the W^+

- $P^\lambda_{eff,j} = (p - k_j)^\lambda$ $\quad j=1,2$

effective mass $M_{eff,W}$: $M^2_{eff,W} = P^2_{eff,W_1} = P^2_{eff,W_2}$

- $2(k_2 - k_1)\cdot p = (m_1^2 - m_2^2)$

- In the muon rest frame, the two possible energies ϵ_1 & ϵ_2 have a Bohr transition frequency $\omega = |\epsilon_2 - \epsilon_1| = \frac{(m_2^2 - m_1^2)}{2M_\mu}$

- Consider the virtual W^+_{eff} wave function which is a product of a neutrino wavefn. $N_\mu(x)$ and a muon wave fn. $\Psi(x)$

$N_\mu(x)$ ⇓ the usual neutrino oscillation probab

$$= |\cos^2\phi\, e^{-i\epsilon_1 t} + \sin^2\phi\, e^{-i\epsilon_2 t}|^2$$

$$= 1 - \sin^2 2\phi \sin^2(\omega t/2)$$

$\Psi(x)$ → the muon decay probability $e^{-\Gamma t}$

∴ W^+ survival amplitude has both

- W^+ survival probability satisfies the differential equation

$$-\left(\frac{dP}{dt}\right) = Z\, e^{-\Gamma t}\left[1 - \sin^2 2\phi\, \sin^2(\omega t/2)\right]$$

- The constant Z is found by normalization

$$-\int_0^\infty \left(\frac{dP}{dt}\right) dt = P(0) - P(\infty) = P(0) = 1$$

- **<u>Likelihood Analysis of K Data</u>:**

  - ν_e were detected by KARMEN in time bins $\Delta t_i = 0.5\,\mu s$
  - We expect a mean # of detected ν_e from μ decay (in each bin) to be given by

$$\mu_i(\omega, \phi) = -\mathcal{N}\left[\frac{dP(t_i\,;\omega,\phi)}{dt}\right](\Delta t_i)$$

where $\mathcal{N} = \sum_i \mu_i \longrightarrow$ total # of obs. ν_e

  - If events in each bin obey Poisson statistics then the probability distribution for a counting sequence of n_i events in the ith bin is given by

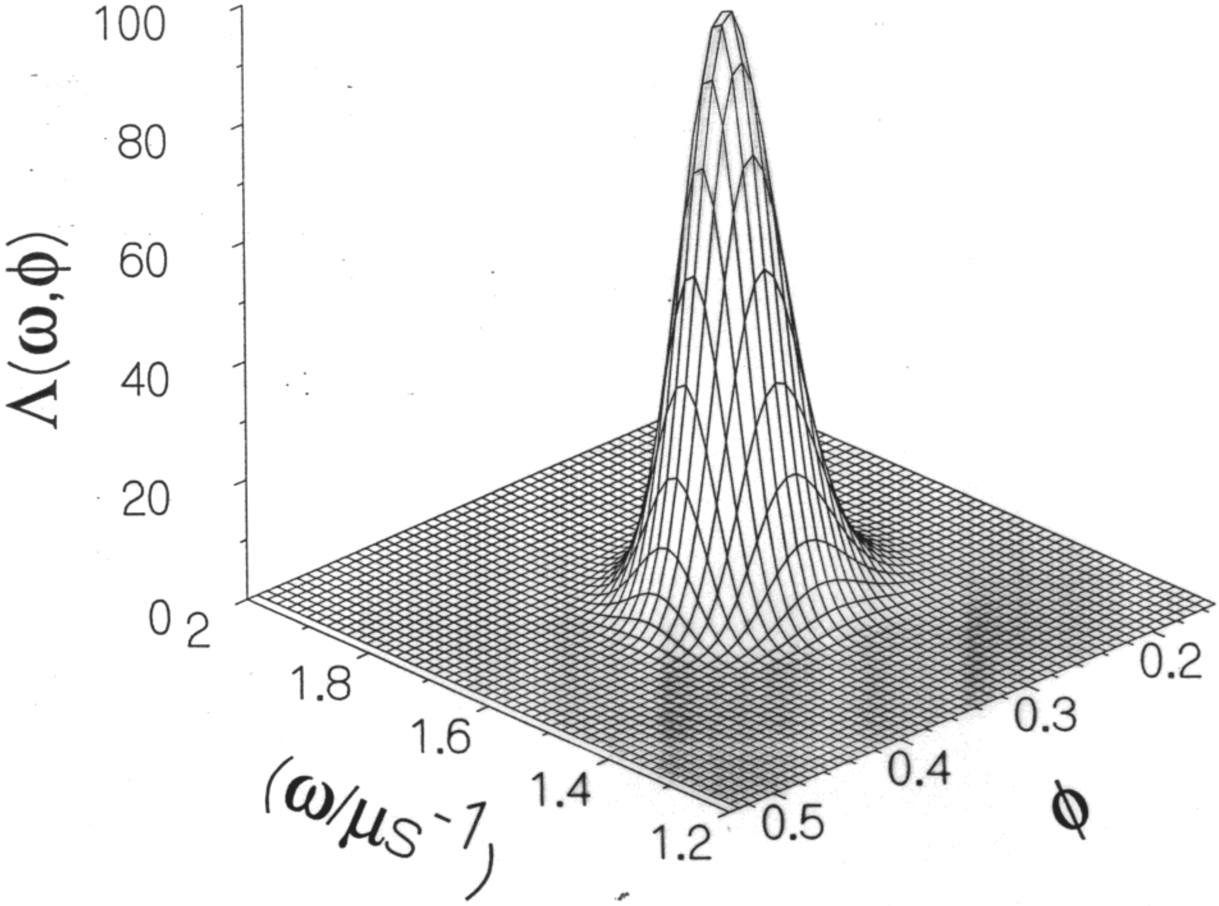
$$\mathcal{P}[n\,;\omega,\phi] = \prod_i \left(\frac{e^{-\mu_i(\omega,\phi)}\,\mu_i(\omega,\phi)^{n}}{n_i!}\right)$$

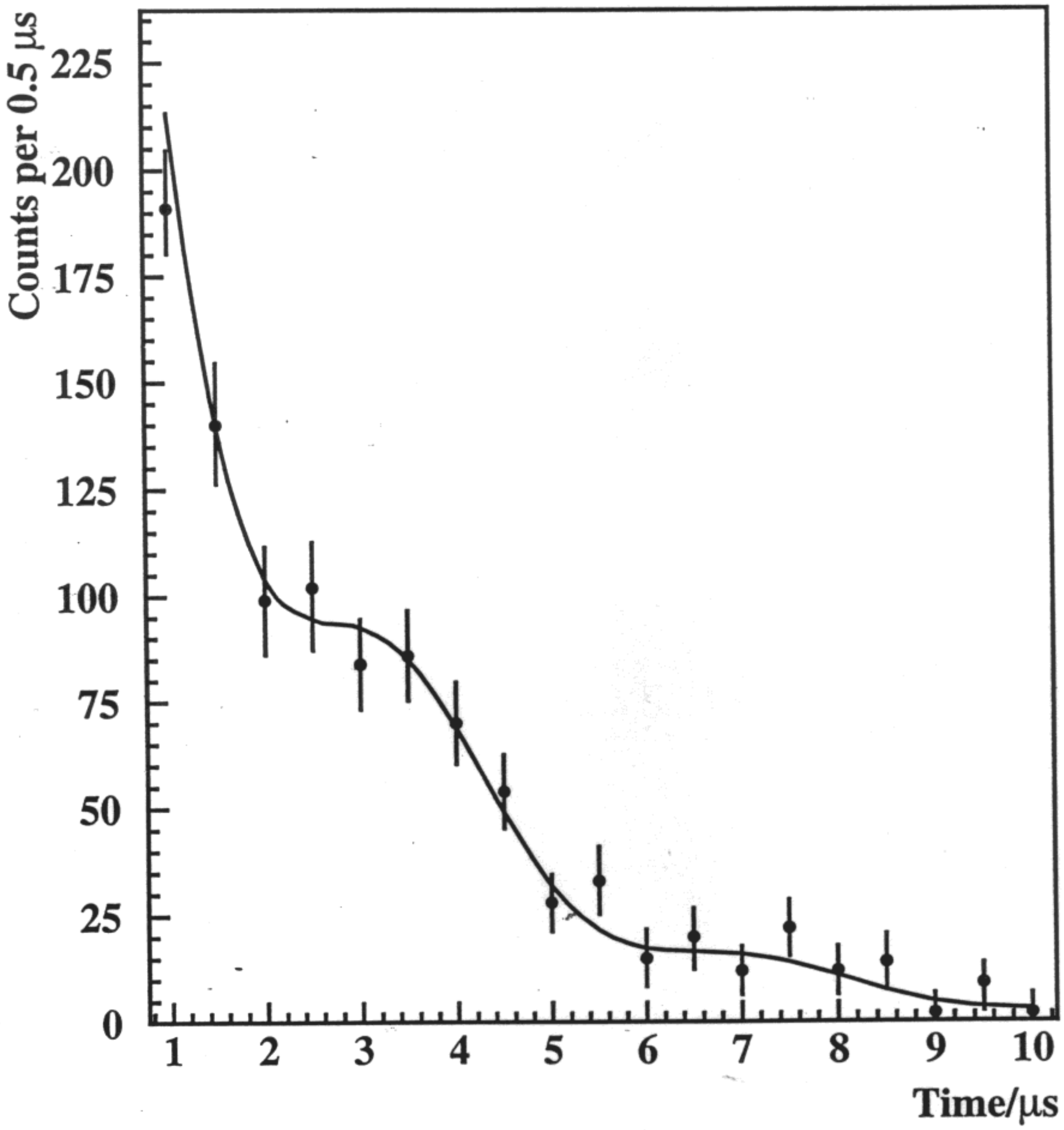
  - Likelihood fn. for the data

$$\Lambda(\omega,\phi) = \underset{\text{const.}}{K}\;\mathcal{P}[n_{data}\,;\omega,\phi]$$

Under the best circumstances, Λ should exhibit a single maximum in the theoretical $(\omega, \phi$

LIKELIHOOD FUNCTION

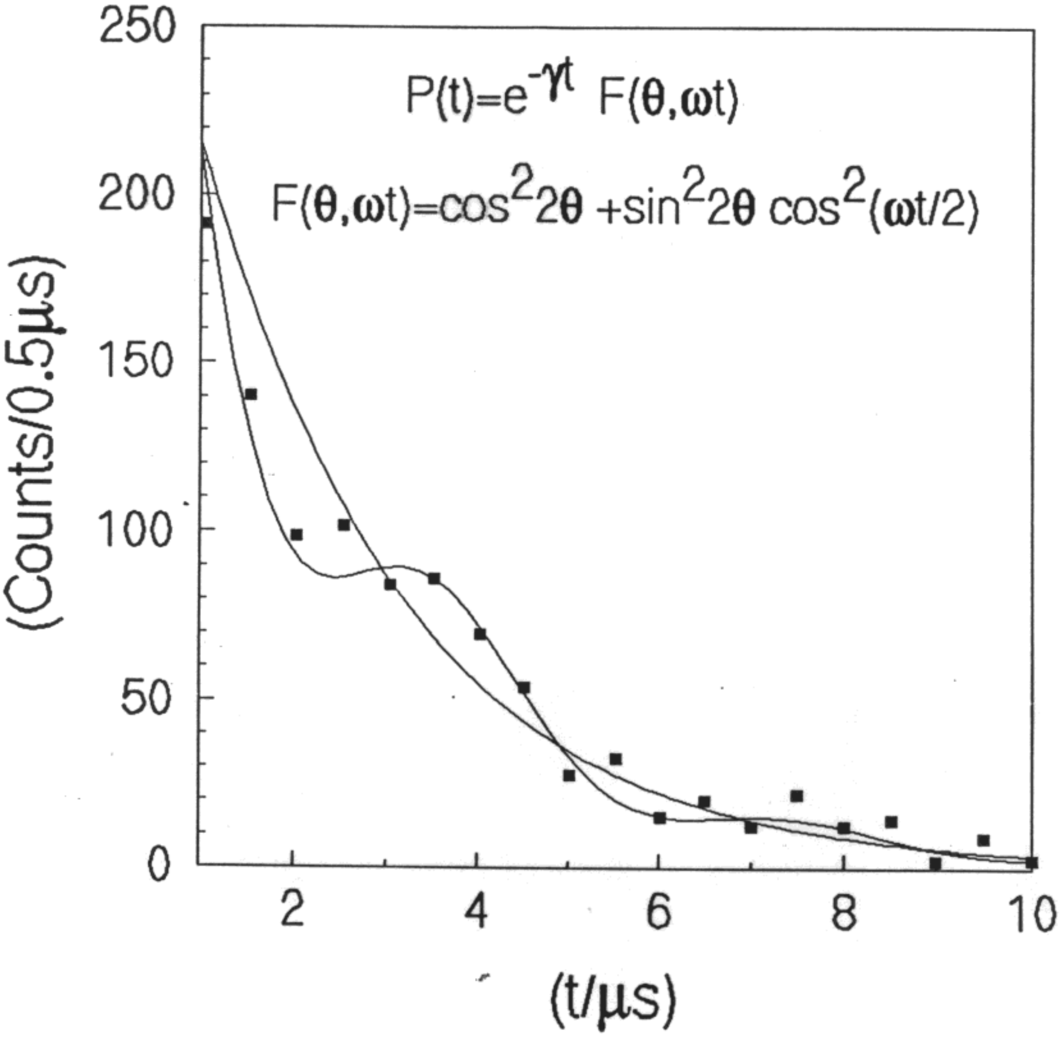




$$\Delta m^2 \simeq (0.22 \pm .02)\, eV^2$$

$$\sin^2 2\phi \simeq 0.34 \pm 0.10$$

Karmen Anomaly



$Z = 2(\cos\theta\sin\theta)^2/(\cos^4\theta+\sin^4\theta)$

$\omega = (1.6/\mu s),\ \gamma^{-1} = 2.2\,\mu s,\ Z = 0.32$

A2 (g-2) of the Muon:

- (i) Schwinger (in 1948) showed that the mass of the muon is split

$$\mathcal{M} = M - \kappa\left(\frac{e}{2M}\right)\left(\frac{1}{2} F_{\lambda\rho}\sigma^{\lambda\rho}\right)$$

- In an external B-field

$$\mathcal{M}_{\pm} = M \mp \Delta M$$

$$\boxed{\Delta M = \kappa\left(\frac{e}{2M}\right) B}$$

$$\kappa = (g-2)/2 \simeq \frac{\alpha}{2\pi} + \cdots\cdots$$

- The survival probability of the muon & ∴ the # of positrons producd oscillates in real time as

$$P_{muon}(t) = e^{-(\Gamma t M/E)}\left[\frac{1 + A\cos(\Omega t)}{1 + A}\right]$$

$(\Omega = \Delta M)$

- (ii) Now the μ's are derived from π decay ∴ there is a ν-mixing induced.

Superposition of 2 different energies

$$E_{\pm} = \bar{E} \pm \left(\frac{\hbar\Omega'}{2}\right)$$

due to 2 different ν masses.

- The $K_{(i)}$ and $\nu_{(ii)}$ induced oscillations get combined as

$$\frac{dW_{TOT}(E)}{dE} = \int (d\epsilon) \left[\frac{dW_1(E+\epsilon)}{dE}\right]\left[\frac{dW_2(\epsilon)}{d\epsilon}\right]$$

∴ total muon survival amplitude is a product of respective survival amplitudes

$$S_{TOT}(t) = S_1(t) \cdot S_2(t)$$

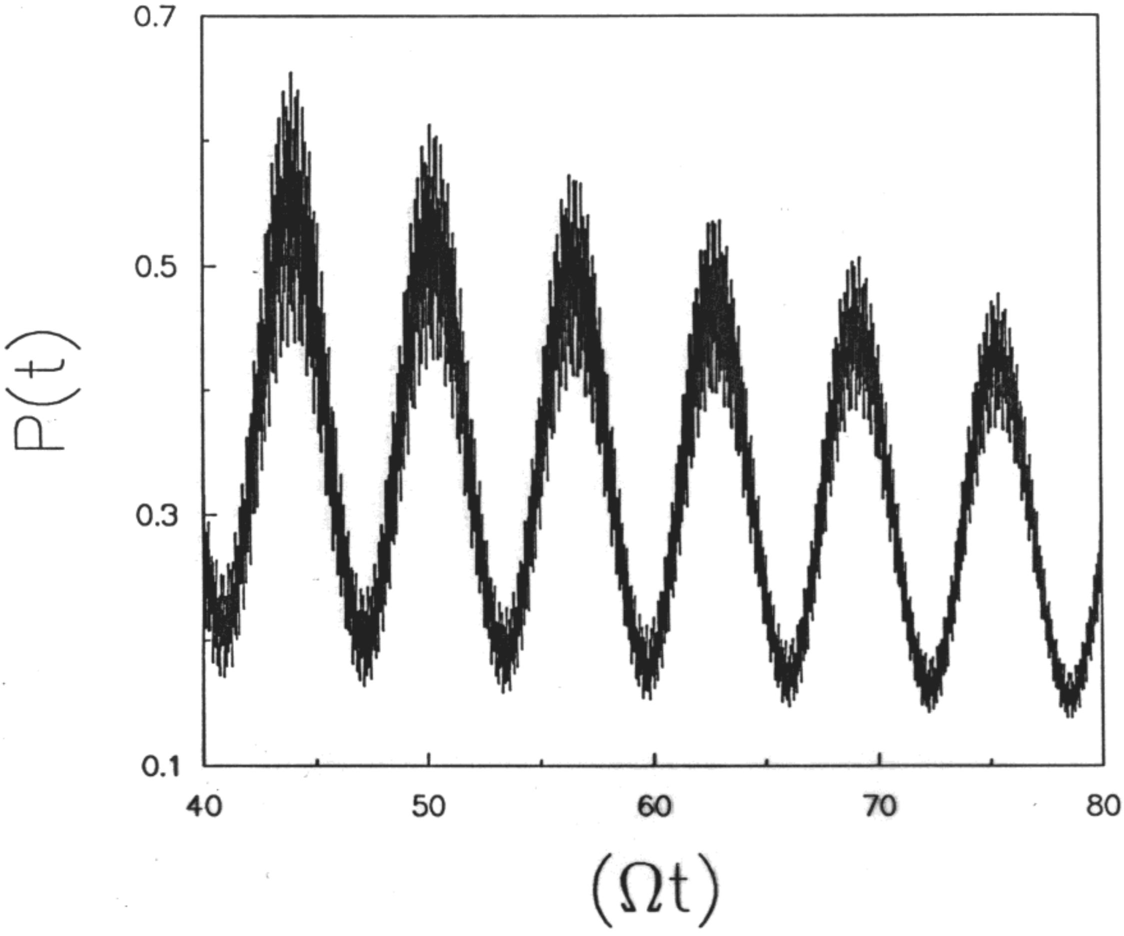
- Muon total survival probability

$$P_{TOT}(muon; t) = \left[1 - \sin^2 2\phi \, \sin^2(\Omega' t)\right] P(muon; t)$$

$$\Omega' \simeq E_\pi \left(\frac{\Delta m}{M_\pi}\right)^2$$

- During the first few life times it gives a considerable high frequency oscillation.
- This effect may already have been seen in the 70's CERN muon (g-2) experiments by Picasso et al under the name of detector paralysis: the first lifetime data were simply thrown away.

Double Muon Oscilllations



B: Neutrino's From The Core Of The Earth

- Shell structure of the Earth & radioactive processes within the Earth are poorly understood. Early discussions were by Darwin & Rutherford who corrected Kelvin.

- In his classic treatise Jeffreys states:

  "---- it would be interesting to consider what would happen if the present radioactive elements with their quantities adapted to 4×10^9 years ago, were uniformly distributed ----- it is quite possible that radioactive heating could produce fusion in a fraction of the age of the Earth."

Electron Neutrino Sources from the Core of the Earth

A. Widom and E. Sassaroli
Physics Department, Northeastern University, Boston, Massacusetts, U.S.A

Y.N. Srivastava
Physics Department & INFN, University of Perugia, Perugia, Italy

The physical interpretation of extensive measurements of electron neutrinos (in laboratories located on or somewhat below the Earth's surface) often require geophysical notions concerning the possible neutrino sources. Here, we discuss the notion that the Earth's core is a substantial source of low energy electron neutrinos.

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Our knowledge about the internal geophysical structure of the Earth has been summarized in the classic work of Jeffreys [1]. In his work can be found the basic physics of how the structured spherical shells of our planet have been deduced. One employs the measured sound wave propagation due to the seismic crunching and crackling of Earth quakes. The standard geophysical model [2,3] of the Earth is pictured (approximately to scale) below in Fig.1.

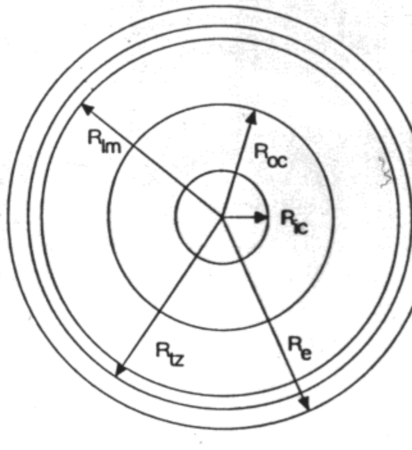


FIG. 1. The shell structure of the Earth includes an inner core with radius R_{ic}, an outer core with radius R_{oc}, a lower mantle with radius R_{lm}, a transition zone with radius R_{tz} and a thin crust to the surface at radius R_e.

Less well developed are our notions of radioactive processes within the core and shell structure of the Earth. Early discussions on the nature of nuclear physics within the Earth are due to Darwin [4] and Rutherford [5]. Later Jeffreys states [6] "... it would be interesting to consider what would happen if the present radioactive elements, with their quantities adapted to 4×10^9 years ago, were uniformly distributed ..., it is quite possible that radioactive heating could produce fusion in a fraction of the age of the Earth."

Just as the solar neutrino flux [7] constitutes *direct evidence* of nuclear reactions within the Sun, an observed geophysical neutrino flux would constitute *direct evidence* of nuclear reactions within the Earth. Experimental geophysical data will exist in laboratories located on or somewhat below the Earth's surface with large fiducial volume neutrino detectors. The idea is to measure the differential neutrino flux $d^2\Phi$ (per unit time per unit area) within a neutrino energy interval dE and incident from a solid angle direction within $d\Omega$. If θ is the angle between a line drawn from the laboratory to the center of the earth and the direction of the solid angle $d\Omega$, and if there exist spherically symmetric sources, then the differential flux has the functional form

$$\left(\frac{d^2\Phi}{dEd\Omega}\right) = \mathcal{F}(E, \cos\theta). \tag{1}$$

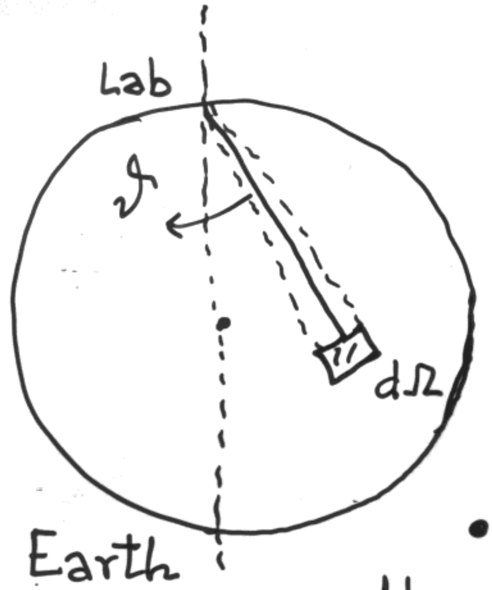
If $\eta_e(r, E)d^3\mathbf{r}$ denotes the number of neutrinos produced within an earth volume $d^3\mathbf{r}$ per unit time with an energy less than E, then the flux of geophysical neutrinos seen in a laboratory located on the Earth's surface is described (in an angular range $0 < \theta < \pi/2$) by

$$\mathcal{F}_e(E, \cos\theta) =$$

$$R_e \int_0^{2\cos\theta} \left(\frac{d\eta_e(r = R_e\sqrt{1 + x^2 - 2x\cos\theta}, E)}{dE}\right) dx. \tag{2}$$

Under the assumption that $\eta_e(r, E)$ is proportional to the mass density $\rho_e(r)$ in the Earth, it is possible to plot numerically the angular distribution of geophysical neutrinos $(d\Phi_e/d\Omega)$. We have carried out such a calculation as shown in Fig.2 below. The mass density in the numerical integrals are taken from tables provided by Birch [8]. For the purpose of conversion into a neutrino flux, we have assumed one part per million of the nuclei β-decay with a mean life-time of 4×10^9 years. These nuclear physics numbers are in reasonable agreement with the estimates of Jeffreys.

We find (using the above estimates) that in an Earth bound laboratory the total geophysical electron neutrino flux is at least comparable to the total solar electron neutrino flux. The theoretical angular distribution of the geophysical neutrino flux is (of course) broader. Nevertheless, the magnitudes are sufficient to imply some periodic modulations in what has previously been regarded



- Let $\eta_e(r, E)(d\underset{\sim}{r}^3)$

  = # of ν produced within an Earth volume $(d\underset{\sim}{r}^3)$ per unit time with energy $< E$

- Then,

  the flux of geophysical neutrinos seen in a laboratory on the Earth's surface is given by

$$\mathcal{F}_e(E, \cos\vartheta) = R_e \int_0^{2\cos\vartheta} (dx) \; \frac{d\eta_e\left(r = \sqrt{1 + x^2 - 2\cos\vartheta}\, R_e; E\right)}{dE}$$

- Assume that $\eta_e(r, E) \propto \rho_e(r)$ mass density of the Earth $\rightarrow$ plot numerically the angular distribution of geophysical neutrinos $\left(\frac{d\Phi_e}{d\Omega}\right)$
- Mass density are taken from Tables by Birch
- For conversion, we have assumed 1 part per million of the nuclei to β-decay with a mean life time of 4×10^9 yrs. in agreement with Jeffreys.
- Total geophysical neutrino flux is comparable to the total electron neutrino flux from the Sun.

as a purely solar neutrino flux. Modulations will occur whenever the neutrino beam from the Sun is parallel (or anti parallel) to the neutrino beam from the Earth's core.

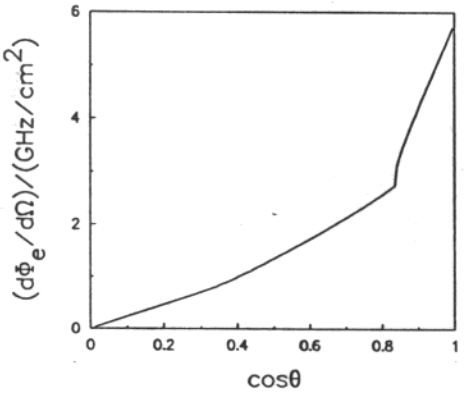


FIG. 2. The angular distribution of geophysical neutrinos in a model in which the source intensity of neutrino production is proportional to the mass density. The geophysical mass density used is due to Birch[8].

The peak in $(d\Phi_e/d\Omega)$ as $\cos\theta \to 1$ is due to the fact that the core of the Earth contains roughly half of the Earth's mass and thereby half of the Earth's neutrino sources in the model under consideration. The discontinuity of the mass density at the core radius R_{oc} is clearly visible from the structure of the peak.

While the nuclear physics estimates within the core of the Earth are not as accurate as the nuclear physics estimates within the core of the Sun, the above considerations may be valid at least in order of magnitude, and are surely worthy of further study. Recent geophysical investigations [9] indicate a substantial concentration of Uranium (and other radio active nuclei) in the Earth's core, since Uranium is totally miscible at the elevated temperatures of the Earth's core material. This recent geophysical work supersedes an earlier miscibility gap argument for the bulk of nuclear reactions to occur in the mantle and/or above the mantle. Further discussions may be found in the literature [10]. Given that nuclear reactions take place in both the core and mantle, as well as on the surface of the Earth, it should be of no surprise that *nature* and *not nuclear physicists* built the first nuclear reactors. These naturally occurring nuclear reactors have been relatively recently discovered [11].

Atmospheric neutrinos also require geophysical reasoning to trace their source. It has been assumed that cosmic ray protons first produce pions which then decay via

$$\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu, \tag{3}$$

or

$$\pi^- \to \mu^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu. \tag{4}$$

From Eqs.(2) and (3), and in the absence of neutrino oscillations, one expects [12] from atmospheric neutrinos a ratio of

$$\left(\frac{\Phi_a(\nu_\mu + \bar{\nu}_\mu)}{\Phi_a(\nu_e + \bar{\nu}_e)}\right) \approx 2. \tag{5}$$

Initial atmospheric neutrino experiments [13,14] were aimed at deducing neutrino oscillation magnitudes via deviations from Eq.(5).

The notion of geophysical neutrino sources within the Earth's core was ignored in these experiments. However, an "excess" (above and beyond the factor of two) of low energy electron neutrinos heading upward from the earth has been observed. In the most recent Super Kamiokande [15] experiments, the electron neutrino excess was quite pronounced. Note, in this regard, that our θ is related to θ_{SK} used in the Super Kamiokande experiment by $\theta = \pi - \theta_{SK}$; i.e.

$$\cos\theta = -\cos\theta_{SK}. \tag{6}$$

If we consider neutrinos with energy $E < 0.4GeV$, then we estimate (from the excess electron neutrino data at angles $0.6 < \cos\theta = (-\cos\theta_{SK}) < 1.0$) the ratio

$$\left(\frac{\Phi_e(\nu_e, \cos\theta > 0.6\ , E < 0.4GeV)}{\Phi_a(\nu_e, \cos\theta > 0.6\ , E < 0.4GeV)}\right) \approx 0.3, \tag{7}$$

wherein the excess electron neutrinos are here presumed to be of geophysical origin from within the Earth's core.

We hope that the hypotheses (and the experimental data) discussed here concerning geophysical electron neutrino sources within the Earth's core, will inspire future investigations. It appears within current neutrino detection technology to map, via Eqs.(1) and (2), geophysical nuclear reaction sources inside the Earth's core. Such a technique appears presently as a unique probe of such distributions.

[1] H. Jeffreys, *The Earth*, 6th edition, Cambridge University Press, Cambridge (1976).

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[4] G. H. Darwin, *Scientific Papers* Vol. 4, 547, Cambridge University Press, Cambridge (1905).

[5] Lord Rutherford and B. B. Boltwood, Am. J. Sci 22, 1, (1906).

[6] H. Jeffreys, op. cit. page 428.

- Geophysical ν_e flux is much broader
- Nevertheless, magnitudes are sufficient to imply some periodic modulations in what has previously been regarded as purely solar ν flux.
- Modulations will occur whenever the ν beam from the Sun is ↑↑ or ↑↓ to the ν beam from the Earth's core.
- The peak in $\left(\frac{d\Phi_e}{d\Omega}\right)$ at $\cos\vartheta \to 1$ is due to Earth's core containing rougly half the Earth's mass and ∴ half of the Earth's neutrino ~~beams~~ sources in the model under consideration
- <u>Nota Bene</u>:

Recent results ('84 vintage) indicate the possibility of a substantial concentration of Uranium (and other radio active nuclei) in the Earth's core since U is totally <u>misible</u> at the elevated temperatures of the Earth's core material.

- This recent result supercedes earlier miscibility argument for the bulk of nuclear reactions to occur in the mantle and/or above the mantle.
- Given that nuclear reactions take place both in the core & mantle, as well as on the surface of the Earth, it should be no surprise that Nature and not Nuclear physicists built the first nuclear reactors on Earth. Such naturally occurring nuclear reactors were discovered in the 70's
- It appears within the current neutrino detection technology to map geophysical nuclear reaction sources within the Earth's core.

Such a technique appears presently as a Unique probe (jus as the Solar ν's do for direct probe of nuclear reactions for the Sun)

of who or what lives

in the

Earth's Core.