MODELS OF NEUTRINO MASSES AND MIXINGS

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ABSTRACT

We briefly review models of neutrino masses and mixings. In view of the existing experimental ambiguities many possibilities are still open. After an overview of the main alternative options we focus on the most constrained class of models based on three widely split light neutrinos within SUSY Grand Unification.

1. Introduction

At present there are many alternative models of neutrino masses. This variety is in part due to the considerable existing experimental ambiguities. The most crucial questions to be clarified by experiment are whether the LSND signal will be confirmed or will be excluded and which solar neutrino solution will eventually be established. If LSND is right we need four light neutrinos, if not we can do with only the three known ones. Other differences are due to less direct physical questions like the possible cosmological relevance of neutrinos as hot dark matter. If neutrinos are an important fraction of the cosmological density, say $\Omega_\nu \sim 0.1$, then the average neutrino mass must be considerably heavier than the splittings that are indicated by the observed atmospheric and solar oscillation frequencies. For example, for three light neutrinos, only models with almost degenerate neutrinos, with common mass $|m_\nu| \approx 1\ eV$, are compatible with a large hot dark matter component. On the contrary hierarchical three neutrino models have the largest neutrino mass fixed by $m \approx \sqrt{\Delta m^2_{atm}} \approx 0.05\ eV$. In most models the smallness of neutrino masses is related to the fact that $\nu's$ are completely neutral (i.e. they carry no charge which is exactly conserved), they are Majorana particles and their masses are inversely proportional to the large scale where the lepton number L conservation is violated. Majorana masses can arise from the see-saw mechanism, in which case there is some relation with the Dirac masses, or from higher dimension non renormalisable operators which come from a different sector of the lagrangian density than other fermion mass terms.

In my lecture first I will briefly summarise the main categories of neutrino mass models and give my personal views on them. Then, I will argue in favour of the most constrained set of models, where there are only three widely split neutrinos, with masses dominated by the see-saw mechanism and inversely proportional to a large mass close to the Grand Unification scale M_{GUT}. In this framework neutrino masses are a probe into the physics of GUT's and one can aim at a comprehensive discussion of all fermion masses. This is for example possible in models based on

$SU(5) \otimes U(1)_{flavour}$ or on $SO(10)$ (we always consider SUSY GUT's). This will also lead us to consider the status of GUT models in view of the experimental bounds on p decay, which are now very severe also for SUSY models, and of well known naturality problems, like the doublet-triplet splitting problem. So we will discuss "realistic" as opposed to minimal models, including a description of the pattern of all fermion masses. We will also mention some recent ideas on a radically different concept of SUSY $SU(5)$ where the symmetry is valid in 5 dimensions but is broken by compactification and not by some Higgs system in the 24 or larger representation. In this version of $SU(5)$ the doublet-triplet splitting problem is solved elegantly and p decay can naturally be suppressed or even forbidden by the compactification mechanism.

This review is in part based on work that I have done over the recent months with Ferruccio Feruglio and Isabella Masina [1], [2], [3], [4], [5], [6], [7].

2. Neutrino Masses and Lepton Number Violation

Neutrino oscillations imply neutrino masses which in turn demand either the existence of right-handed neutrinos (Dirac masses) or lepton number L violation (Majorana masses) or both. Given that neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the conclusion that L must be violated. In fact, it is only in terms of lepton number violation that the smallness of neutrino masses can be explained as inversely proportional to the very large scale where L is violated, of order M_{GUT} or even M_{Planck}.

Once we accept L violation we gain an elegant explanation for the smallness of neutrino masses which turn out to be inversely proportional to the large scale where lepton number is violated. If L is not conserved, even in the absence of ν_R, Majorana masses can be generated for neutrinos by dimension five operators of the form

$$O_5 = \frac{L_i^T \lambda_{ij} L_j H H}{M} \tag{1}$$

with H being the ordinary Higgs doublet, λ a matrix in flavour space and M a large scale of mass, of order M_{GUT} or M_{Planck}. Neutrino masses generated by O_5 are of the order $m_\nu \approx v^2/M$ for $\lambda_{ij} \approx$ O(1), where $v \sim$ O(100 GeV) is the vacuum expectation value of the ordinary Higgs.

We consider that the existence of ν_R is quite plausible because all GUT groups larger than SU(5) require them. In particular the fact that ν_R completes the representation 16 of SO(10): $16 = \bar{5}+10+1$, so that all fermions of each family are contained in a single representation of the unifying group, is too impressive not to be significant. At least as a classification group SO(10) must be of some relevance. Thus in the following we assume that there are both ν_R and lepton number violation. With these assumptions the see-saw mechanism [8] is possible which leads to:

$$m_\nu = m_D^T M^{-1} m_D \tag{2}$$

That is, the light neutrino masses are quadratic in the Dirac masses and inversely proportional to the large Majorana mass. Note that for $m_\nu \approx \sqrt{\Delta m^2_{atm}} \approx 0.05\ eV$ and $m_\nu \approx m_D^2/M$ with $m_D \approx v \approx 200\ GeV$ we find $M \approx 10^{15}\ GeV$ which indeed is an impressive indication for M_{GUT}.

If additional non renormalisable terms from O_5 are comparatively non negligible, they should simply be added. After elimination of the heavy right-handed fields, at the level of the effective low energy theory, the two types of terms are equivalent. In particular they have identical transformation properties under a chiral change of basis in flavour space. The difference is, however, that in the see-saw mechanism, the Dirac matrix m_D is presumably related to ordinary fermion masses because they are both generated by the Higgs mechanism and both must obey GUT-induced constraints. Thus if we assume the see-saw mechanism more constraints are implied. In particular we are led to the natural hypothesis that m_D has a largely dominant third family eigenvalue in analogy to m_t, m_b and m_τ which are by far the largest masses among u quarks, d quarks and charged leptons. Once we accept that m_D is hierarchical it is very difficult to imagine that the effective light neutrino matrix, generated by the see-saw mechanism, could have eigenvalues very close in absolute value.

3. Four Neutrino Models

The LSND signal [9] has not been confirmed by KARMEN [10]. It will be soon double-checked by MiniBoone [9]. Perhaps it will fade away. But if an oscillation with $\Delta m^2 \approx 1\ eV^2$ is confirmed then, in presence of three distinct frequencies for LSND, atmospheric [11], [12] and solar [13], [14] neutrino oscillations, at least four light neutrinos are needed. Since LEP has limited to three the number of "active" neutrinos (that is with weak interactions, or equivalently with non vanishing weak isospin, the only possible gauge charge of neutrinos) the additional light neutrino(s) must be "sterile", i.e. with vanishing weak isospin. Note that ν_R that appears in the see-saw mechanism, if it exists, is a sterile neutrino, but a heavy one.

A typical pattern of masses that works for 4-ν models consists of two pairs of neutrinos [15],[16] the separation between the two pairs, of order 1 eV, corresponding to the LSND frequency. The upper doublet would be almost degerate at $|m|$ of order 1 eV being only split by (the mass difference corresponding to) the atmospheric ν frequency, while the lower doublet is split by the solar ν frequency. This mass configuration can be compatible with an important fraction of hot dark matter in the universe. A complication is that the data appear to be incompatible with pure 2-ν oscillations for $\nu_e - \nu_s$ oscillations for solar neutrinos and for $\nu_\mu - \nu_s$ oscillations for atmospheric neutrinos (with ν_s being a sterile neutrino). There are however viable alternatives. One possibility is obtained by using the large freedom allowed by the presence of 6 mixing angles in the most general 4-ν mixing matrix. If 4 angles are significantly different from zero, one can go beyond pure 2-ν oscillations and, for example, for

solar neutrino oscillations ν_e can transform into a mixture of $\nu_a + \nu_s$, where ν_a is an active neutrino, itself a superposition of ν_μ and ν_τ [15]. A different alternative is to have many interfering sterile neutrinos: this is the case in the interesting class of models with extra dimensions, where a whole tower of Kaluza-Klein neutrinos is introduced. This picture of sterile neutrinos from extra dimensions is exciting and we now discuss it in some detail [17], [18].

The context is theories with large extra dimensions. Gravity propagates in all dimensions (bulk), while SM particles live on a 4-dim brane. As well known [19], this can make the fundamental scale of gravity m_s much smaller than the Planck mass M_P. In fact, for $d = n + 4$, if R is the compactification radius we have a geometrical volume factor that suppresses gravity so that: $(m_s R)^n = (M_P/m_s)^2$ and, as a result, m_s can be as small as $\sim 1\ TeV$. For neutrino phenomenology we need a really large extra dimension with $1/R \lesssim 0.01\ eV$ plus $n-1$ smaller ones with $1/\rho \gtrsim 1\ TeV$. Then we define m_5 by $m_5 R = (M_P/m_s)^2$, or $m_5 = m_s(m_s\rho)^{n-1}$. In string theories of gravity there are always scalar fields associated with gravity and their SUSY fermionic partners (dilatini, modulini). These are particles that propagate in the bulk, have no gauge interactions and can well play the role of sterile neutrinos. The models based on this framework [20] have some good features that make them very appealing at first sight. They provide a "physical" picture for ν_s. There is a KK tower of recurrences of ν_s:

$$\nu_s(x,y) = \frac{1}{\sqrt{R}} \sum_n \nu_s^{(n)}(x) \cos\frac{ny}{R} \tag{3}$$

with $m_{\nu_s} = n/R$. The tower mixes with the ordinary light active neutrinos in the lepton doublet L:

$$L_{mix} = h\frac{m_s}{M_P} L \nu_s^{(n)} H \tag{4}$$

where H is the Higgs doublet field. Note that the geometrical factor m_s/M_P, which automatically suppresses the Yukawa coupling h, arises naturally from the fact that the sterile neutrino tower lives in the bulk. Note in passing that ν_s mixings must be small due to existing limits from weak processes, supernovae and nucleosynthesis, so that the preferred solution for 4-ν models is MSW-(small angle). The interference among a few KK states makes the spectrum compatible with solar data [17], [18]:

$$P(\nu_e \to X) = \sum_n \frac{m_e^2}{M_e^2 + \frac{n^2}{R^2}} \tag{5}$$

provided that $1/R \sim 10^{-2} - 10^{-3}\ eV$ or $R \sim 10^{-3} - 10^{-2}\ cm$, that is a really large extra dimension barely compatible with existing limits [21].

In spite of its good properties there are problems with this picture, in my opinion. The first property that I do not like of models with large extra dimensions is that the connection with GUT's is lost. In particular the elegant explanation of the smallness of neutrino masses in terms of the large scale where the L conservation is violated

in general evaporates. Since $m_s \sim 1\ TeV$ is relatively small, what forbids on the brane an operator of the form $\frac{1}{m_s} L_i^T \lambda_{ij} L_j H H$ which would lead to by far too large ν masses? One must assume L conservation on the brane and that it is only broken by some Majorana masses of sterile ν's in the bulk, which I find somewhat ad hoc. Another problem is that we would expect gravity to know nothing about flavour, but here we would need right-handed partners for ν_e, ν_μ and ν_τ. Also a single large extra dimension has problems, because it implies [22] a linear evolution of the gauge couplings with energy from $0.01\ eV$ to $m_s \sim 1\ TeV$. But more large extra dimensions lead to

$$P(\nu_e \to X) \;=\; \sum_n \frac{m_e^2}{M_e^2 + \frac{n^2}{R^2}} \;=\; \int dn n^{d-1} \frac{m_e^2}{M_e^2 + \frac{n^2}{R^2}} \tag{6}$$

For $d > 2$ the KK recurrences do not decouple fast enough (the divergence of the integral is only cut off at m_s) and the mixing becomes very large. Perhaps a compromise at $d = 2$ is possible.

In conclusion the models with large extra dimension are interesting because they are speculative and fascinating but the more conventional framework still appears more plausible at closer inspection.

4. Three Neutrino Models

We now assume that the LSND signal will not be confirmed, that there are only two distinct neutrino oscillation frequencies, the atmospheric and the solar frequencies, which can be reproduced with the known three light neutrino species (for reviews of three neutrino models see [4], [23] where a rather complete set of references can be found). The two frequencies, are parametrised in terms of the ν mass eigenvalues by

$$\Delta_{sun} \propto m_2^2 - m_1^2, \qquad \Delta_{atm} \propto m_3^2 - m_{1,2}^2 \tag{7}$$

The numbering 1,2,3 corresponds to our definition of the frequencies and in principle may not coincide with the family index although this will be the case in the models that we favour. Given the observed frequencies and our notation in eq. (7), there are three possible patterns of mass eigenvalues:

$$\begin{aligned} \text{Degenerate} &: |m_1| \sim |m_2| \sim |m_3| \\ \text{Inverted hierarchy} &: |m_1| \sim |m_2| >> |m_3| \\ \text{Hierarchical} &: |m_3| >> |m_{2,1}| \end{aligned} \tag{8}$$

We now discuss pro's and con's of the different cases and argue in favour of the hierarchical option.

4.1. Degenerate Neutrinos

At first sight the degenerate case is the most appealing: the observation of nearly maximal atmospheric neutrino mixing and the possibility that also the solar mixing

is large (at present the MSW-(large angle) solution of the solar neutrino oscillations appears favoured by the data) suggests that all ν masses are nearly degenerate. Moreover, the common value of $|m_\nu|$ could be compatible with a large fraction of hot dark matter in the universe for $|m_\nu| \sim 1 - 2\ eV$. In this case, however, the existing limits on the absence of neutrino-less double beta decay ($0\nu\beta\beta$) imply [24] double maximal mixing (bimixing) for solar and atmospheric neutrinos. In fact the quantity which is bound by experiments is the 11 entry of the ν mass matrix, which is given by [4]:

$$m_{ee} \ = \ m_1 cos^2\theta_{12} \ + \ m_2 sin^2\theta_{12} \ \lesssim \ 0.3 - 0.5\ eV \tag{9}$$

To satisfy this constraint one needs $m_1 = -m_2$ (recall that the sign of fermion masses can be changed by a phase redefinition) and $cos^2\theta_{12} \sim sin^2\theta_{12}$ to a good accuracy (in fact we need $sin^2 2\theta_{12} > 0.96$ in order that $|cos 2\theta_{12}| \ = \ |cos^2\theta_{12} - sin^2\theta_{12}| < 0.2$). Of course this strong constraint can be relaxed if the common mass is below the hot dark matter maximum. It is true in any case that a signal of $0\nu\beta\beta$ near the present limit (like a large relic density of hot dark matter) would be an indication for nearly degenerate ν's.

In general, for naturalness reasons, the splittings cannot be too small with respect to the common mass, unless there is a protective symmetry [25]. This is because the wide mass differences of fermion masses, in particular charged lepton masses, would tend to create neutrino mass splittings via renormalization group running effects even starting from degenerate masses at a large scale. For example, the vacuum oscillation solution for solar neutrino oscillations would imply $\Delta m/m \sim 10^{-9} - 10^{-11}$ which is difficult to obtain. In this respect the MSW-(large angle) solution would be favoured, but, if we insist that $|m_\nu| \sim 1 - 2\ eV$, it is not clear that the mixing angle is sufficiently maximal.

It is clear that in the degenerate case the most likely origin of ν masses is from dim-5 operators $O_5 = L_i^T \lambda_{ij} L_j HH/M$ and not from the see-saw mechanism $m_\nu = m_D^T M^{-1} m_D$. In fact we expect the ν-Dirac mass m_D to be hierarchical like for all other fermions and a conspiracy to reinstaure a nearly perfect degeneracy between m_D and M, which arise from completely different physics, looks very unplausible. Thus in degenerate models, in general, there is no direct relation with Dirac masses of quarks and leptons and the possibility of a simultaneous description of all fermion masses within a grand unified theory is more remote [26].

4.2. Inverted Hierarchy

The inverted hierarchy configuration $|m_1| \sim |m_2| >> |m_3|$ consists of two levels m_1 and m_2 with small splitting $\Delta m_{12}^2 \ = \ \Delta m_{sun}^2$ and a common mass given by $m_{1,2}^2 \sim \Delta m_{atm}^2 \sim 2.5 \cdot 10^{-3}\ eV^2$ (no large hot dark matter component in this case). One particularly interesting example of this sort [27], which leads to double maximal

mixing, is obtained with the phase choice $m_1 = -m_2$ so that, approximately:

$$m_{diag} \;=\; M[1,-1,0] \tag{10}$$

The effective light neutrino mass matrix

$$m_\nu \;=\; U m_{diag} U^T \tag{11}$$

which corresponds to the mixing matrix of double maximal mixing $c = s = 1/\sqrt{2}$:

$$U_{fi} = \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & +1/\sqrt{2} \end{bmatrix} \quad . \tag{12}$$

is given by:

$$m_\nu \;=\; \frac{M}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad . \tag{13}$$

The structure of m_ν can be reproduced by imposing a flavour symmetry $L_e - L_\mu - L_\tau$ starting from $O_5 = L_i^T \lambda_{ij} L_j H H / M$. The $1-2$ degeneracy remains stable under radiative corrections. The preferred solar solutions are vacuum oscillations or the LOW solution. The MSW-(large angle) could be also compatible if the mixing angle is large enough. The required dominance of O_5 leads to the same comments as the degenerate models of the previous section.

4.3. Hierarchical

We now discuss the class of models which we consider of particular interest because this is the most constrained framework which allows a comprehensive combined study of all fermion masses in GUT's. We assume three widely split ν's and the existence of a right-handed neutrino for each generation, as required to complete a 16-dim representation of $SO(10)$ for each generation. We then assume dominance of the see-saw mechanism $m_\nu = m_D^T M^{-1} m_D$. We know that the third-generation eigenvalue of the Dirac mass matrices of up and down quarks and of charged leptons is systematically the largest one. It is natural to imagine that this property will also be true for the Dirac mass of ν's: $diag[m_D] \sim [0, 0, m_{D3}]$. After see-saw we expect m_ν to be even more hierarchical being quadratic in m_D (barring fine-tuned compensations between m_D and M). The amount of hierarchy, $m_3^2/m_2^2 = \Delta m^2_{atm}/\Delta m^2_{sun}$, depends on which solar neutrino solution is adopted: the hierarchy is maximal for vacuum oscillations and LOW solutions, is moderate for MSW in general and could become quite mild for the upper Δm^2 domain of the MSW-(large angle) solution. A possible difficulty is that one is used to expect that large splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2 by 2 matrix context

the requirement of large splitting and large mixings leads to a condition of vanishing determinant. For example the matrix

$$m \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \quad . \tag{14}$$

has eigenvalues 0 and $1 + x^2$ and for x of 0(1) the mixing is large. Thus in the limit of neglecting small mass terms of order $m_{1,2}$ the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the 2 by 2 subdeterminant 23 of the 3 by 3 mixing matrix approximately vanishes. The problem is to show that this vanishing can be arranged in a natural way without fine tuning. Once near maximal atmospheric neutrino mixing is reproduced the solar neutrino mixing can be arranged to be either small of large without difficulty by implementing suitable relations among the small mass terms.

It is not difficult to imagine mechanisms that naturally lead to the approximate vanishing of the 23 sub-determinant. For example [27], [28], assume that one ν_R is particularly light and coupled to μ and τ. In a 2 by 2 simplified context if we have

$$M \propto \begin{bmatrix} \epsilon & 0 \\ 0 & 1 \end{bmatrix}; \qquad M^{-1} \approx \begin{bmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{bmatrix} \tag{15}$$

then for a generic m_D we find

$$m_\nu \;=\; m_D^T M^{-1} M_D \sim \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \;=\; \frac{1}{\epsilon} \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix} \tag{16}$$

A different possibility that we find attractive is that, in the limit of neglecting terms of order $m_{1,2}$ and, in the basis where charged leptons are diagonal, the Dirac matrix m_D, defined by $\bar{R} m_D L$, takes the approximate form:

$$m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{bmatrix} \quad . \tag{17}$$

This matrix has the property that for a generic Majorana matrix M one finds:

$$m_\nu = m_D^T M^{-1} m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{bmatrix} \quad . \tag{18}$$

The only condition on M^{-1} is that the 33 entry is non zero. But when the approximately vanishing matrix elements are replaced by small terms, one must also assume that no new $o(1)$ terms are generated in m_ν by a compensation between small terms in m_D and large terms in M. It is important for the following discussion to observe that m_D given by eq. (17) under a change of basis transforms as $m_D' -> V^\dagger m_D U$ where V and U rotate the right and left fields respectively. It is easy to check that in

order to make m_D diagonal we need large left mixings (i.e. large off diagonal terms in the matrix that rotates left-handed fields). Thus the question is how to reconcile large left-handed mixings in the leptonic sector with the observed near diagonal form of V_{CKM}, the quark mixing matrix. Strictly speaking, since $V_{CKM} = U_u^\dagger U_d$, the individual matrices U_u and U_d need not be near diagonal, but V_{CKM} does, while the analogue for leptons apparently cannot be near diagonal. However nothing forbids for quarks that, in the basis where m_u is diagonal, the d quark matrix has large non diagonal terms that can be rotated away by a pure right-handed rotation. We suggest that this is so and that in some way right-handed mixings for quarks correspond to left-handed mixings for leptons.

In the context of (Susy) SU(5) there is a very attractive hint of how the present mechanism can be realized. In the $\bar{5}$ of SU(5) the d^c singlet appears together with the lepton doublet (ν, e). The (u, d) doublet and e^c belong to the 10 and ν^c to the 1 and similarly for the other families. As a consequence, in the simplest model with mass terms arising from only Higgs pentaplets, the Dirac matrix of down quarks is the transpose of the charged lepton matrix: $m_D^d = (m_D^l)^T$. Thus, indeed, a large mixing for right-handed down quarks corresponds to a large left-handed mixing for charged leptons. At leading order we may have:

$$m_d = (m_l)^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & 0 & 1 \end{bmatrix} v_d \tag{19}$$

In the same simplest approximation with 5 or $\bar{5}$ Higgs, the up quark mass matrix is symmetric, so that left and right mixing matrices are equal in this case. Then small mixings for up quarks and small left-handed mixings for down quarks are sufficient to guarantee small V_{CKM} mixing angles even for large d quark right-handed mixings. If these small mixings are neglected, we expect:

$$m_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} v_u \tag{20}$$

When the charged lepton matrix is diagonalized the large left-handed mixing of the charged leptons is transferred to the neutrinos. Note that in SU(5) we can diagonalize the u mass matrix by a rotation of the fields in the 10, the Majorana matrix M by a rotation of the 1 and the effective light neutrino matrix m_ν by a rotation of the $\bar{5}$. In this basis the d quark mass matrix fixes V_{CKM} and the charged lepton mass matrix fixes neutrino mixings. It is well known that a model where the down and the charged lepton matrices are exactly the transpose of one another cannot be exactly true because of the e/d and μ/s mass ratios. It is also known that one remedy to this problem is to add some Higgs component in the 45 representation of SU(5) [29]. A different kind of solution [30] will be described later. But the symmetry under

transposition can still be a good guideline if we are only interested in the order of magnitude of the matrix entries and not in their exact values. Similarly, the Dirac neutrino mass matrix m_D is the same as the up quark mass matrix in the very crude model where the Higgs pentaplets come from a pure 10 representation of SO(10): $m_D = m_u$. For m_D the dominance of the third family eigenvalue as well as a near diagonal form could be an order of magnitude remnant of this broken symmetry. Thus, neglecting small terms, the neutrino Dirac matrix in the basis where charged leptons are diagonal could be directly obtained in the form of eq. (17).

5. Simple Examples with Horizontal Abelian Charges

We discuss here some explicit examples of the mechanism under discussion in the framework of a unified Susy $SU(5)$ theory with an additional $U(1)_F$ flavour symmetry [31]. If, for a given interaction vertex, the $U(1)_F$ charges do not add to zero, the vertex is forbidden in the symmetry limit. But the symmetry is spontaneously broken by the vev v_f of a number of "flavon" fields with non vanishing charge. Then a forbidden coupling is rescued but is suppressed by powers of the small parameters v_f/M with the exponent larger for larger charge mismatch. We expect $v_f \gtrsim M_{GUT}$ and $M \lesssim M_P$. Here we discuss some aspects of the description of fermion masses in these models. In the following sections we will consider how to imbed these concepts within more complete and realistic $SU(5)$ models. We will also discuss the need and the options to go beyond minimal models.

In these models the known generations of quarks and leptons are contained in triplets Ψ^a_{10} and $\Psi^a_{\bar{5}}$, $(a = 1, 2, 3)$ transforming as 10 and $\bar{5}$ of $SU(5)$, respectively. Three more $SU(5)$ singlets Ψ^a_1 describe the right-handed neutrinos. In SUSY models we have two Higgs multiplets, which transform as 5 and $\bar{5}$ in the minimal model. We first assume that they have the same charge. The simplest models are obtained by allowing all the third generation masses already in the symmetric limit. This is realised by taking vanishing charges for the Higgses and for the third generation components Ψ^3_{10}, $\Psi^3_{\bar{5}}$ and Ψ^3_1. We can arrange the unit of charge in such a way that the Cabibbo angle, which we consider as the typical hierachy parameter of fermion masses and mixings, is obtained when the suppression exponent is unity. Remember that the Cabibbo angle is not too small, $\lambda \sim 0.22$ and that in $U(1)_F$ models all mass matrix elements are of the form of a power of a suppression factor times a number of order unity, so that only their order of suppression is defined. As a consequence, in practice, we can limit ourselves to integral charges in our units, for simplicity (for example, $\sqrt{\lambda} \sim 1/2$ is already almost unsuppressed).

After these preliminaries let's first try a simplest model with all charges being non negative and containing one single flavon of negative charge. For example, we could take [32] (see also [33])

$$\Psi_{10} \quad \sim \quad (4, 2, 0) \tag{21}$$

$$\Psi_{\bar{5}} \sim (2,0,0) \quad (22)$$

$$\Psi_1 \sim (4,2,0) \quad (23)$$

In this case a typical mass matrix has the form

$$m = \begin{bmatrix} y_{11}\lambda^{q_1+q_1'} & y_{12}\lambda^{q_1+q_2'} & y_{13}\lambda^{q_1+q_3'} \\ y_{21}\lambda^{q_2+q_1'} & y_{22}\lambda^{q_2+q_2'} & y_{23}\lambda^{q_2+q_3'} \\ y_{31}\lambda^{q_3+q_1'} & y_{32}\lambda^{q_3+q_2'} & y_{33}\lambda^{q_3+q_3'} \end{bmatrix} v \quad (24)$$

where all the yij are of order 1 and q_i and q_i' are the charges of 10,10 for m_u, of $\bar{5}$,10 for m_d or m_l^T, of 1,$\bar{5}$ for m_D (the Dirac ν mass), and of 1,1 for M, the RR Majorana ν mass. Note the two vanishing charges in $\Psi_{\bar{5}}$. They are essential for this mechanism: for example they imply that the 32, 33 matrix elements of m_D are of order 1. It is important to observe that m can be written as:

$$m = \lambda^q y \lambda^{q'} \quad (25)$$

where $\lambda_q = diag[\lambda_{q_1}, \lambda_{q_2}, \lambda_{q_3}]$ and y is the y_{ij} matrix. As a consequence when we start from the Dirac ν matrix: $m_D = \lambda^{q_1} y_D \lambda^{q_{\bar{5}}}$ and the RR Majorana matrix $M = \lambda^{q_1} y_M \lambda^{q_1}$ and write down the see-saw expression for $m_\nu = m_D^T M^{-1} m_D$, we find that the dependence on the q_1 charges drops out and only that from $q_{\bar{5}}$ remains. On the one hand this is good because it corresponds to the fact that the effective light neutrino Majorana mass matrix $m_\nu \sim L^T L$ can be written in terms of $q_{\bar{5}}$ only. In particular the 22,23,32,33 matrix elements of m_ν are of order 1, which implies large mixings in the 23 sector. On the other hand the sub determinant 23 is not suppressed in this case, so that the splitting between the 2 and 3 light neutrino masses is in general small. In spite of the fact that m_D is, in first approximation, of the form in eq. (17) the strong correlations between m_D and M implied by the simple charge structure of the model destroy the vanishing of the 23 sub determinant that would be guaranteed for generic M. Models of this sort have been proposed in the literature [32], [33]. The hierarchy between m_2 and m_3 is considered accidental and better be moderate. The preferred solar solution in this case is MSW-(small angle) because if m_1 is suppressed the solar mixing angle is typically small.

Models with natural large 23 splittings are obtained if we allow negative charges and, at the same time, either introduce flavons of opposite charges or stipulate that matrix elements with overall negative charge are put to zero. We now discuss a model of this sort [3]. We assign to the fermion fields the set of F-charges given by:

$$\Psi_{10} \sim (3,2,0) \quad (26)$$

$$\Psi_{\bar{5}} \sim (3,0,0) \quad (27)$$

$$\Psi_1 \sim (1,-1,0) \quad (28)$$

We consider the Yukawa coupling allowed by $U(1)_F$-neutral Higgs multiplets φ_5 and $\varphi_{\bar{5}}$ in the 5 and $\bar{5}$ $SU(5)$ representations and by a pair θ and $\bar{\theta}$ of $SU(5)$ singlets with $F = 1$ and $F = -1$, respectively.

In the quark sector we obtain :

$$m_u = (m_u)^T = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u \ , \qquad m_d = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_d \ , \tag{29}$$

from which we get for the eigenvalues the order-of-magnitude relations:

$$\begin{aligned} m_u : m_c : m_t &= \lambda^6 : \lambda^4 : 1 \\ m_d : m_s : m_b &= \lambda^6 : \lambda^2 : 1 \end{aligned} \tag{30}$$

and

$$V_{us} \sim \lambda \ , \qquad V_{ub} \sim \lambda^3 \ , \qquad V_{cb} \sim \lambda^2 \ . \tag{31}$$

Here $v_u \equiv \langle \varphi_5 \rangle$, $v_d \equiv \langle \varphi_{\bar{5}} \rangle$ and λ, arising from the $\bar{\theta}$ vev, is, as above, of the order of the Cabibbo angle. For non-negative F-charges, the elements of the quark mixing matrix V_{CKM} depend only on the charge differences of the left-handed quark doublet [31]. Up to a constant shift, this defines the choice in eq. (26). Equal F-charges for $\Psi_{\bar{5}}^{2,3}$ (see eq. (27)) are then required to fit m_b and m_s. We will comment on the lightest quark masses later on.

At this level, the mass matrix for the charged leptons is the transpose of m_d:

$$m_l = (m_d)^T \tag{32}$$

and we find:

$$m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1 \tag{33}$$

The O(1) off-diagonal entry of m_l gives rise to a large left-handed mixing in the 23 block which corresponds to a large right-handed mixing in the d mass matrix. In the neutrino sector, the Dirac and Majorana mass matrices are given by:

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & \lambda' & \lambda' \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \ , \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix} \bar{M} \ , \tag{34}$$

where λ' is related to θ and $\bar{M}$ denotes the large mass scale associated to the right-handed neutrinos: $\bar{M} \gg v_{u,d}$.

After diagonalization of the charged lepton sector and after integrating out the heavy right-handed neutrinos we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\bar{M}} \tag{35}$$

where we have taken $\lambda \sim \lambda'$. The O(1) elements in the 23 block are produced by combining the large left-handed mixing induced by the charged lepton sector and the

large left-handed mixing in m_D. A crucial property of m_ν is that, as a result of the sea-saw mechanism and of the specific $U(1)_F$ charge assignment, the determinant of the 23 block is *automatically* of $O(\lambda^2)$ (for this the presence of negative charge values, leading to the presence of both λ and λ' is essential [2]).

It is easy to verify that the eigenvalues of m_ν satisfy the relations:

$$m_1 : m_2 : m_3 = \lambda^4 : \lambda^2 : 1 \quad . \tag{36}$$

The atmospheric neutrino oscillations require $m_3^2 \sim 10^{-3}$ eV^2. From eq. (35), taking $v_u \sim 250$ GeV, the mass scale $\bar{M}$ of the heavy Majorana neutrinos turns out to be close to the unification scale, $\bar{M} \sim 10^{15}$ GeV. The squared mass difference between the lightest states is of $O(\lambda^4)$ m_3^2, appropriate to the MSW solution to the solar neutrino problem. Finally, beyond the large mixing in the 23 sector, m_ν provides a mixing angle $s \sim (\lambda/2)$ in the 12 sector, close to the range preferred by the small angle MSW solution. In general U_{e3} is non-vanishing, of $O(\lambda^3)$.

In general, the charge assignment under $U(1)_F$ allows for non-canonical kinetic terms that represent an additional source of mixing. Such terms are allowed by the underlying flavour symmetry and it would be unnatural to tune them to the canonical form. The results quoted up to now remain unchanged after including the effects related to the most general kinetic terms, via appropriate rotations and rescaling in the flavour space.

Obviously, the order of magnitude description offered by this model is not intended to account for all the details of fermion masses. Even neglecting the parameters associated with the CP violating observables, some of the relevant observables are somewhat marginally reproduced. For instance we obtain $m_u/m_t \sim \lambda^6$ which is perhaps too large. However we find it remarkable that in such a simple scheme most of the 12 independent fermion masses and the 6 mixing angles turn out to have the correct order of magnitude. Notice also that this model prefers large values of $\tan\beta \equiv v_u/v_d$. This is a consequence of the equality $F(\Psi_{10}^3) = F(\Psi_5^3)$ (see eqs. (26) and (27)). In this case the Yukawa couplings of top and bottom quarks are expected to be of the same order of magnitude, while the large m_t/m_b ratio is attributed to $v_u \gg v_d$ (there may be factors O(1) modifying these considerations, of course). Alternatively, to keep $\tan\beta$ small, one could suppress m_b/m_t by adopting different F-charges for the $\Psi_{\bar{5}}^3$ and Ψ_{10}^3 or for the 5 and $\bar{5}$ Higgs, as we will see in the next section.

A common problem of all $SU(5)$ unified theories based on a minimal higgs structure is represented by the relation $m_l = (m_d)^T$ that, while leading to the successful $m_b = m_\tau$ boundary condition at the GUT scale, provides the wrong prediction $m_d/m_s = m_e/m_\mu$ (which, however, is an acceptable order of magnitude equality). We can easily overcome this problem and improve the picture [30] by introducing an additional supermultiplet $\bar{\theta}_{24}$ transforming in the adjoint representation of $SU(5)$ and possessing a negative $U(1)_F$ charge, $-n$ $(n > 0)$. Under these conditions, a posi-

tive F-charge f carried by the matrix elements $\Psi^a_{10}\Psi^b_{\bar{5}}$ can be compensated in several different ways by monomials of the kind $(\bar{\theta})^p(\bar{\theta}_{24})^q$, with $p+nq=f$. Each of these possibilities represents an independent contribution to the down quark and charged lepton mass matrices, occurring with an unknown coefficient of O(1). Moreover the product $(\bar{\theta}_{24})^q\varphi_{\bar{5}}$ contains both the $\bar{5}$ and the $\overline{45}$ $SU(5)$ representations, allowing for a differentiation between the down quarks and the charged leptons. The only, welcome, exceptions are given by the O(1) entries that do not require any compensation and, at the leading order, remain the same for charged leptons and down quarks. This preserves the good $m_b = m_\tau$ prediction. Since a perturbation of O(1) in the subleading matrix elements is sufficient to cure the bad $m_d/m_s = m_e/m_\mu$ relation, we can safely assume that $\langle\bar{\theta}_{24}\rangle/M_P \sim \lambda^n$, to preserve the correct order-of-magnitude predictions in the remaining sectors.

A general problem common to all models dealing with flavour is that of recovering the correct vacuum structure by minimizing the effective potential of the theory. It may be noticed that the presence of two multiplets θ and $\bar{\theta}$ with opposite F charges could hardly be reconciled, without adding extra structure to the model, with a large common VEV for these fields, due to possible analytic terms of the kind $(\theta\bar{\theta})^n$ in the superpotential. We find therefore instructive to explore the consequences of allowing only the negatively charged $\bar{\theta}$ field in the theory.

It can be immediately recognized that, while the quark mass matrices of eqs. (29) are unchanged, in the neutrino sector the Dirac and Majorana matrices get modified into:

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & 0 & 0 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \quad , \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} \bar{M} \quad . \tag{37}$$

The zeros are due to the analytic property of the superpotential that makes impossible to form the corresponding F invariant by using $\bar{\theta}$ alone. These zeros should not be taken literally, as they will be eventually filled by small terms coming, for instance, from the diagonalization of the charged lepton mass matrix and from the transformation that put the kinetic terms into canonical form. It is however interesting to work out, in first approximation, the case of exactly zero entries in m_D and M, when forbidden by F.

The neutrino mass matrix obtained via see-saw from m_D and M has the same pattern as the one displayed in eq. (35). A closer inspection reveals that the determinant of the 23 block is identically zero, independently from λ. This leads to the following pattern of masses:

$$m_1 : m_2 : m_3 = \lambda^3 : \lambda^3 : 1 \quad , \qquad m_1^2 - m_2^2 = \mathrm{O}(\lambda^9) \quad . \tag{38}$$

Moreover the mixing in the 12 sector is almost maximal:

$$\frac{s}{c} = \frac{\pi}{4} + \mathrm{O}(\lambda^3) \quad . \tag{39}$$

For $\lambda \sim 0.2$, both the squared mass difference $(m_1^2 - m_2^2)/m_3^2$ and $\sin^2 2\theta_{sun}$ are remarkably close to the values required by the vacuum oscillation solution to the solar neutrino problem. This property remains reasonably stable against the perturbations induced by small terms (of order λ^5) replacing the zeros, coming from the diagonalization of the charged lepton sector and by the transformations that render the kinetic terms canonical. We find quite interesting that also the just-so solution, requiring an intriguingly small mass difference and a bimaximal mixing, can be reproduced, at least at the level of order of magnitudes, in the context of a "minimal" model of flavour compatible with supersymmetric SU(5). In this case the role played by supersymmetry is essential, a non-supersymmetric model with $\bar{\theta}$ alone not being distinguishable from the version with both θ and $\bar{\theta}$, as far as low-energy flavour properties are concerned.

6. From Minimal to Realistic SUSY SU(5)

In this section, following the lines of a recent study [6], we address the question whether the smallest SUSY SU(5) symmetry group can still be considered as a basis for a realistic GUT model. The minimal model has large fine tuning problems (e.g. the doublet-triplet splitting problem) and phenomenological problems from the new improved limits on proton decay [34]. Also, analyses of particular aspects of GUT's often leave aside the problem of embedding the sector under discussion into a consistent whole. So the problem arises of going beyond minimal toy models by formulating sufficiently realistic, not unnecessarily complicated, relatively complete models that can serve as benchmarks to be compared with experiment. More appropriately, instead of "realistic" we should say "not grossly unrealistic" because it is clear that many important details cannot be sufficiently controlled and assumptions must be made. The model we aim at should not rely on large fine tunings and must lead to an acceptable phenomenology. This includes coupling unification with an acceptable value of $\alpha_s(m_Z)$, given α and $sin^2\theta_W$ at m_Z, compatibility with the bounds on proton decay, agreement with the observed fermion mass spectrum, also considering neutrino masses and mixings and so on. The success or failure of the programme of constructing realistic models can decide whether or not a stage of gauge unification is a likely possibility.

We indeed have presented in ref.[6] an explicit example of a "realistic" SU(5) model, which uses a $U(1)_F$ symmetry as a crucial ingredient. In this model the doublet-triplet splitting problem is solved by the missing partner mechanism [35] stabilised by the flavour symmetry against the occurrence of doublet mass lifting due to non renormalisable operators. Relatively large representations (50, $\bar{50}$, 75) have to be introduced for this purpose. A good effect of this proliferation of states is that the value of $\alpha_s(m_Z)$ obtained from coupling unification in the next to the leading order perturbative approximation receives important negative corrections from threshold

effects near the GUT scale arising from mass splittings inside the 75. As a result, the central value changes from $\alpha_s(m_Z) \approx 0.129$ in minimal SUSY SU(5) down to $\alpha_s(m_Z) \approx 0.116$, in better agreement with observation. At the same time, an increase of the effective mass that mediates proton decay by a factor of typically 20-30 is obtained to optimize the value of $\alpha_s(m_Z)$. So finally the value of the strong coupling is in better agreement with the experimental value and the proton decay rate is smaller by a factor 400-1000 than in the minimal model (in addition the rigid relation of the minimal model between mass terms and proton decay amplitudes is released, so that the rate can further be reduced) . The presence of these large representations also has the consequence that the asymptotic freedom of SU(5) is spoiled and the associated gauge coupling becomes non perturbative below M_P. We argue that this property far from being unacceptable can actually be useful to obtain better results for fermion masses and proton decay. The same $U(1)_F$ flavour symmetry that stabilizes the missing partner mechanism explains the hierarchical structure of fermion masses. In the neutrino sector, mass matrices similar to those discussed in the previous section are obtained. In the present particular version maximal mixing also for solar neutrinos is preferred.

While we refer to the original paper for a complete discussion, here we only summarise the fermion mass sector of the model, which is of relevance for neutrinos. At variance with the previous models we adopt in this case different $U(1)_F$ charges for the Higgs field $H \sim 5$ and $\bar{H} \sim \bar{5}$:

$$F(H) = -2 \quad \text{and} \quad \mathrm{F}(\bar{\mathrm{H}}) = 1, \tag{40}$$

For matter fields

$$\begin{aligned} F(\Psi_{10}) &= (4, 3, 1) \\ F(\Psi_{\bar{5}}) &= (5, 2, 2) \\ F(\Psi_1) &= (1, -1, 0) \end{aligned} \tag{41}$$

The Yukawa mass matrices are, in first approximation, of the form:

$$m_u = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u/\sqrt{2} \quad , \tag{42}$$

$$m_d = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_d \lambda^4/\sqrt{2} \quad = m_l^T, \tag{43}$$

$$m_\nu = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & 0 & 0 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u/\sqrt{2} \quad , \tag{44}$$

$$m_{maj} = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} M \quad , \tag{45}$$

For a correct first approximation of the observed spectrum we need $\lambda \approx \lambda_C \approx 0.22$, λ_C being the Cabibbo angle. These mass matrices closely match those of the previous section, with two important special features. First, we have here that $tan\beta = v_u/v_d \approx m_t/m_b\lambda^4$, which is small. The factor λ^4 is obtained as a consequence of the Higgs and matter fields charges F, while previously the H and $\bar{H}$ charges were taken as zero. We recall that a value of $\tan\beta$ near 1 is an advantage for suppressing proton decay. Of course the limits from LEP that indicate that $tan\beta \gtrsim 2-3$ must be and can be easily taken into account. Second, the zero entries in the mass matrices of the neutrino sector occur because the negatively F-charged flavon fields have no counterpart with positive F-charge in this model. Neglected small effects could partially fill up the zeroes. As already explained these zeroes lead to near maximal mixing also for solar neutrinos. A problematic aspect of this zeroth order approximation to the mass matrices is the relation $m_d = m_l^T$. The necessary corrective terms can arise from the neglected higher order terms from non renormalisable operators with the insertion of n factors of the 75, which break the transposition relation between m_d and m_l. With reasonable values of the coefficients of order 1 we obtain double nearly maximal mixing and $\theta_{13} \sim 0.05$. The preferred solar solutions are LOW or vacuum oscillations.

7. SU(5) Unification in Extra Dimensions

Recently it has been observed that the GUT gauge symmetry could be actually realized in 5 (or more) space-time dimensions and broken down to the the Standard Model (SM) by compactification [a]. In particular a model with N=2 Supersymmetry (SUSY) and gauge SU(5) in 5 dimensions has been proposed [37] where the GUT symmetry is broken by compactification on $S^1/(Z_2 \times Z_2')$ down to a N=1 SUSY-extended version of the SM on a 4-dimensional brane. In this model many good properties of GUT's, like coupling unification and charge quantization are maintained while some unsatisfactory properties of the conventional breaking mechanism, like doublet-triplet splitting, are avoided. In a recent paper of ours [7] we have elaborated further on this class of models. We differ from ref. [37] (and also from the later reference [38]) in the form of the interactions on the 4-dimensional brane. As a consequence we not only avoid the problem of the doublet-triplet splitting but also directly suppress or even forbid proton decay, since the conventional higgsino and gauge boson exchange amplitudes are absent, as a consequence of $Z_2 \times Z_2'$ parity assignments on matter fields on the brane. Most good predictions of SUSY SU(5) are thus maintained without unnatural fine tunings as needed in the minimal model. We find that the

[a] Grand unified supersymmetric models in six dimensions, with the grand unified scale related to the compactification scale were also proposed by Fayet [36].

relations among fermion masses implied by the minimal model, for example $m_b = m_\tau$ at M_{GUT} are preserved in our version of the model, although the Yukawa interactions are not fully SU(5) symmetric. The mechanism that forbids proton decay still allows Majorana mass terms for neutrinos so that the good potentiality of SU(5) for the description of neutrino masses and mixing is preserved. This class of models offers a new perspective on how the GUT symmetry and symmetry-breaking could be realized.

8. SO(10) Models

Models based on $SO(10)$ times a flavour symmetry are more difficult to construct because a whole generation is contained in the 16, so that, for example for $U(1)_F$, one would have the same value of the charge for all quarks and leptons of each generation, which is too rigid. But the mechanism discussed sofar, based on asymmetric mass matrices, can be embedded in an $SO(10)$ grand-unified theory in a rather economic way [39], [23]. The 33 entries of the fermion mass matrices can be obtained through the coupling $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H$ among the fermions in the third generation, $\mathbf{16}_3$, and a Higgs tenplet $\mathbf{10}_H$. The two independent VEVs of the tenplet v_u and v_d give mass, respectively, to t/ν_τ and b/τ. The keypoint to obtain an asymmetric texture is the introduction of an operator of the kind $\mathbf{16}_2\mathbf{16}_H\mathbf{16}_3\mathbf{16}'_H$. This operator is thought to arise by integrating out an heavy $\mathbf{10}$ that couples both to $\mathbf{16}_2\mathbf{16}_H$ and to $\mathbf{16}_3\mathbf{16}'_H$. If the $\mathbf{16}_H$ develops a VEV breaking $SO(10)$ down to $SU(5)$ at a large scale, then, in terms of $SU(5)$ representations, we get an effective coupling of the kind $\bar{\mathbf{5}}_2\mathbf{10}_3\bar{\mathbf{5}}_H$, with a coefficient that can be of order one. This coupling contributes to the 23 entry of the down quark mass matrix and to the 32 entry of the charged lepton mass matrix, realizing the desired asymmetry. To distinguish the lepton and quark sectors one can further introduce an operator of the form $\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H\mathbf{45}_H$, $(i,j=2,3)$, with the VEV of the $\mathbf{45}_H$ pointing in the $B-L$ direction. Additional operators, still of the type $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_H\mathbf{16}'_H$ can contribute to the matrix elements of the first generation. The mass matrices look like:

$$m_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} v_u \quad , \qquad m_d = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' & -\epsilon/3 & 1 \end{bmatrix} v_d \quad , \tag{46}$$

$$m_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} v_u \quad , \qquad m_e = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma+\epsilon & 1 \end{bmatrix} v_d \quad . \tag{47}$$

They provide a good fit of the available data in the quarks and the charged lepton sector in terms of 5 parameters (one of which is complex). In the neutrino sector one obtains a large θ_{23} mixing angle, $\sin^2 2\theta_{12} \sim 6.6 \cdot 10^{-3}$ eV^2 and θ_{13} of the same order of θ_{12}. Mass squared differences are sensitive to the details of the Majorana mass matrix.

Looking at models with three light neutrinos only, i.e. no sterile neutrinos, from a more general point of view, we stress that in the above models the atmospheric neutrino mixing is considered large, in the sense of being of order one in some zeroth order approximation. In other words it corresponds to off diagonal matrix elements of the same order of the diagonal ones, although the mixing is not exactly maximal. The idea that all fermion mixings are small and induced by the observed smallness of the non diagonal V_{CKM} matrix elements is then abandoned. An alternative is to argue that perhaps what appears to be large is not that large after all. The typical small parameter that appears in the mass matrices is $\lambda \sim \sqrt{m_d/m_s} \sim \sqrt{m_\mu/m_\tau} \sim 0.20 - 0.25$. This small parameter is not so small that it cannot become large due to some peculiar accidental enhancement: either a coefficient of order 3, or an exponent of the mass ratio which is less than 1/2 (due for example to a suitable charge assignment), or the addition in phase of an angle from the diagonalization of charged leptons and an angle from neutrino mixing. One may like this strategy of producing a large mixing by stretching small ones if, for example, he/she likes symmetric mass matrices, as from left-right symmetry at the GUT scale. In left-right symmetric models smallness of left mixings implies that also right-handed mixings are small, so that all mixings tend to be small. Clearly this set of models [40] tend to favour moderate hierarchies and a single maximal mixing, so that the SA-MSW solution of solar neutrinos is preferred.

9. Conclusion

By now there are rather convincing experimental indications for neutrino oscillations. If so, then neutrinos have non zero masses. As a consequence, the phenomenology of neutrino masses and mixings is brought to the forefront. This is a very interesting subject in many respects. It is a window on the physics of GUTs in that the extreme smallness of neutrino masses can only be explained in a natural way if lepton number is violated. Then neutrino masses are inversely proportional to the large scale where lepton number is violated. Also, the pattern of neutrino masses and mixings can provide new clues on the long standing problem of quark and lepton mass matrices. The actual value of neutrino masses is important for cosmology as neutrinos are candidates for hot dark matter: nearly degenerate neutrinos with a common mass around 1- 2 eV would significantly contribute to the matter density in the universe.

While the existence of oscillations appears to be on a solid ground, many important experimental ambiguities remain. For solar neutrinos it is not yet clear which of the solutions, MSW-SA, MSW-LA, LOW and VO, is true, and the possibility also remains of different solutions if not all of the experimental input is correct (for example, energy independent solutions are resurrected if the Homestake result is modified). Finally a confirmation of the LSND alleged signal is necessary, in order to know if 3

light neutrinos are sufficient or additional sterile neutrinos must be introduced. We argued in favour of models with 3 widely split neutrinos. Reconciling large splittings with large mixing(s) requires some natural mechanism to implement a vanishing determinant condition. This can be obtained in the see-saw mechanism if one light right-handed neutrino is dominant, or a suitable texture of the Dirac matrix is imposed by an underlying symmetry. In a GUT context, the existence of right-handed neutrinos indicates SO(10) at least as a classification group. The symmetry group at M_{GUT} could be either (Susy) SU(5) or SO(10) or a larger group. We have presented a class of natural models where large right-handed mixings for quarks are transformed into large left-handed mixings for leptons by the approximate transposition relation $m_d = m_e^T$ which is approximately realised in SU(5) models. We have shown that these models can be naturally implemented by simple assignments of $U(1)_F$ horizontal charges.

In conclusion the fact that some neutrino mixing angles are large, while surprising at the start, was eventually found to be well be compatible, without any major change, with our picture of quark and lepton masses within GUTs. In fact, it provides us with new important clues that can become sharper when the experimental picture will be further clarified.

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