IMPLICATIONS OF NEUTRINO OSCILLATIONS FOR GAUGE UNIFICATION

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ABSTRACT

By assuming bimaximal mixing for neutrino mass matrix and $M^R_{33} = 0$ for the Majorana mass matrix of right-handed neutrinos, we get a value for its highest element consistent with the scale of the spontaneous symmetry breaking of B–L found in a SO(10) unified theory with Pati-Salam intermediate symmetry.

1. Present Status of Neutrino Oscillations

Neutrino oscillations, invented almost half century ago by Bruno Pontecorvo [1)], are advocated to explain atmospheric and solar neutrino phenomena with:

$$\Delta m^2_a = 2.5\,10^{-3}(eV)^2 \tag{1}$$

and maximal ν_μ-ν_τ mixing and an upper limit for ν_e content by Chooz:

$$|U_{e3}|^2 < .05 \tag{2}$$

The actually favoured solution to the "solar neutrino problem" is the large angle MSW solution with:

$$\Delta m^2_s \approx 2.7\,10^{-5}(eV)^2 \tag{3}$$

$$\sin^2 2\theta_s \approx .8 \tag{4}$$

The resulting mixing matrix is not far from the bimaximal form proposed by Vissani and Georgi and Glashow.
The knowledge of 3 mixing angles (for simplicity we assume CP) and of two Δm^2 is not sufficient to determine the effective Majorana mass matrix of ν_L, which needs the knowledge of $|m_1|$ and of the relative signs of the m_i.

The quantum numbers with respect to G = SU(3) x SU(2) x U(1) of a family of fundamental fermions are a strong hint for gauge unification.
In fact not only it is Tr Y = 0, which is a property of the generators of non-abelian Lie algebras, but this property holds also for two sets of multiplets, namely

$$(3, 2, +\frac{1}{6}) + (\bar{3}, 1, -\frac{2}{3}) + (1, 1, +1) \tag{5}$$

and

$$(\bar{3}, 1, +\frac{1}{3}) + (1, 2, -\frac{1}{2}) \tag{6}$$

for which the trace of the square of the generators of G have the same ratio, as it should be the case if G is imbedded in a simple group.
The two sets of states can be classified in the 10 + $\bar{5}$ of SU(5) [2], while the extension to SO(10) [3] requires the existence of $\bar{\nu}_L$, since its spinorial representation reduces under SU(5) according to

$$16 \rightarrow 10 + \bar{5} + 1. \tag{7}$$

A SO(10) theory with the spontaneous symmetry breaking pattern

SO(10) $\rightarrow$ SU(4) x SU(2) x SU(2) $\rightarrow$ G

with Higgs in the 210 + 126 representation reproduces the values of the gauge couplings if [4]:

$$M_X = 4.7\,10^{15} GeV \tag{8}$$

$$M_R = 2.8\,10^{11} GeV \tag{9}$$

where M_X and M_R are the scales of spontaneous symmetry breaking of SO(10) and of SU(4) x SU(2) x SU(2), respectively.
M_X is larger than the value $1.6\,10^{15}$, as required by the lower limit on proton decay and M_R is related to the values of the matrix elements of the Majorana mass matrix of the ν_R. The see-saw formula:

$$M^L = -M_D^T\,(M^R)^{-1}\,M_D \tag{10}$$

gives the mass matrix of the ν_L in terms of the Dirac neutrino mass matrix and of the Majorana mass matrix of the ν_R.
In SO(10) with only one 10 representation for the electro-weak Higgs, one should find at the scale M_X:

$$m_t = m_b = m_\tau = m_{D\nu_\tau} \tag{11}$$

and a trivial mixing matrix for quarks and Dirac leptons.
With two or more 10's, one should have the same mass matrix for the quarks with charge -1/3 and for the charged leptons and a Dirac mass matrix for neutrinos equal to the one for the quarks with charge 2/3. In particular for Dirac leptons one should get the mixing given by the CKM matrix.
By taking:

$$m_D = \frac{m_\tau}{m_b}\,diag(m_u, m_c, m_t) \tag{12}$$

and a Zee-like matrix [5] for M^R:

$$(M^R)_{11} = (M^R)_{22} = (M^R)_{33} = 0 \tag{13}$$

$$(M^R)_{12} = \rho \tag{14}$$

$$(M^R)_{13} = \nu \tag{15}$$

$$(M^R)_{23} = \mu \tag{16}$$

the assumption that $|m_3|$ is larger than the other two $|m_i|$ leads [6] to almost opposite values for the smallest eigenvalues:

$$m_1 + m_2 = -\frac{m_1 m_2}{m_3} \tag{17}$$

By requiring a maximal mixing for ν_μ-ν_τ, we find almost the bimaximal mixing matrix previously mentioned.
The highest matrix element of M^R is:

$$(M^R)_{23} = \frac{m_\tau^2 m_c m_t}{m_b^2 \sqrt{\Delta m_a^2}} \approx 9\,10^{11}\,GeV \tag{18}$$

in good agreement with the scale of B–L spontaneous symmetry breaking of the SO(10) theory with SU(4) x SU(2) x SU(2) intermediate summetry.
Similar properties, almost bimaximal mixing, almost opposite values for the lightest neutrino masses and moderate values for the highest matrix elements of M^R, are found in other SO(10) see-saw models [7].
The two last properties are related:

$$|Det(M^R)| = \frac{m_\tau^6 m_u^2 m_c^2 m_t^2}{m_b^6 |m_1 m_2 m_3|} = \frac{\Delta m_s^2}{|m_1 m_2|} 4\,10^{30}\,(GeV)^3 \tag{19}$$

The source of these properties may be found [8] by considering the inverse see-saw formula:

$$M^R = -M_D\,(M^L)^{-1}\,M_D^T \tag{20}$$

with the diagonal M_D of Eq.(13) and a bimaximal mixing for M^L.

$$(M^R)_{11} = -\frac{1}{2}(\frac{m_\tau m_u}{m_b})^2(\frac{1}{m_1} + \frac{1}{m_2}) \tag{21}$$

$$(M^R)_{12} = \frac{1}{2\sqrt{2}}\frac{m_\tau^2 m_u m_c}{m_b^2}(\frac{1}{m_1} - \frac{1}{m_2}) \tag{22}$$

$$(M^R)_{13} = -\frac{1}{2\sqrt{2}}\frac{m_\tau^2 m_u m_t}{m_b^2}(\frac{1}{m_1} - \frac{1}{m_2}) \tag{23}$$

$$(M^R)_{22} = -\frac{1}{4}(\frac{m_\tau m_c}{m_b})^2(\frac{1}{m_1} + \frac{1}{m_2} + \frac{2}{m_3}) \tag{24}$$

$$(M^R)_{23} = -\frac{1}{4}\frac{m_\tau^2 m_c m_t}{m_b^2}(\frac{1}{m_1} + \frac{1}{m_2} - \frac{2}{m_3}) \tag{25}$$

$$(M^R)_{33} = -\frac{1}{4}(\frac{m_\tau m_t}{m_b})^2(\frac{1}{m_1} + \frac{1}{m_2} + \frac{2}{m_3}) \tag{26}$$

To avoid:

$$\frac{M^R_{11}}{M^R_{33}} \approx \frac{2m_u^2}{m_t^2} \approx 1.3\,10^{-9} \tag{27}$$

we take:

$$\frac{1}{m_1} + \frac{1}{m_2} + \frac{2}{m_3} = 0 \tag{28}$$

and get for the largest matrix element of M^R:

$$|M^R_{23}| = \frac{m_\tau^2 m_c m_t}{m_b^2 m_3} \approx 9\,10^{11}\,GeV \tag{29}$$

2. Conclusion

The freedom left by the study of neutrino oscillations in the determination of M^L can be used to require a "natural" M^R.
With a diagonal $m_{D\nu}$ and a bimaximal mixing matrix for ν_L, this property is obtained with $M^R_{33} = 0$ and with the largest matrix element of M^R, $M^R_{23} \approx 9\,10^{11}\,GeV$ in good agreement with the scale of B -L symmetry breaking found in the SO(10) theory with Pati-Salam intermediate symmetry [9].

3. References

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