A NEW FIT TO SOLAR NEUTRINOS USING EXTRA DIMENSIONS

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ABSTRACT

A neutrino mass-mixing scheme which explains qualitatively all present evidence for neutrino mass (the solar and atmospheric neutrino deficits, LSND, and hot dark matter), and also makes possible heavy-element nucleosynthesis by supernovae, requires at least one light sterile neutrino. String-inspired models with sub-millimeter extra dimensions provide naturally light sterile neutrinos, as is needed to explain the solar ν_e deficit. This bulk sterile neutrino provides a better fit to the solar data than conventional models by having vacuum oscillations of the ν_e to its zero mode and MSW oscillations to its first few Kaluza-Klein modes. While the prediction of the Super-Kamiokande energy spectrum gives a fit probability of 73%, the superior energy resolution of SNO's charged-current spectrum will determine whether this neutrino scheme is correct and can demonstrate that an extra dimension of $\sim 60\mu$m exists. Should this be the case, there are important implications for supernovae, ultra-high-energy cosmic rays, double beta decay, and dark matter.

1. Introduction

The four-neutrino scheme in which the solar ν_e deficit is explained by $\nu_e \to \nu_s$ (where ν_s is a sterile neutrino), the atmospheric ν_μ/ν_e anomaly is attributed to $\nu_\mu \to \nu_\tau$, and the heavier ν_μ and ν_τ share the role of hot dark matter was originally proposed[1] in order to explain those three phenomena. Later the LSND experiment,[2] which observed $\bar{\nu}_\mu \to \bar{\nu}_e$, provided a measure of the mass difference between the nearly degenerate $\nu_e - \nu_s$ and $\nu_\mu - \nu_\tau$ pairs and required the three mass differences that were already present in that neutrino scheme. Exactly this same pattern of neutrino masses and mixings appears necessary to allow production of heavy elements (A $\gtrsim$ 100) by type II supernovae.[3] The neutrino properties required to ensure heavy-element nucleosynthesis in the neutrino-heated ejecta of supernovae provide independent evidence for (1) at least one light sterile neutrino, (2) a near maximally-mixed ν_μ–ν_τ doublet split from the lower mass ν_e–ν_s doublet, (3) ν_μ–ν_e mixing $\gtrsim 10^{-4}$, and (4) a splitting between the doublets (measured by the ν_μ–ν_e mass difference) $\gtrsim 1$ eV^2, favoring the upper part of the LSND range.

While qualitatively this neutrino scheme seems to explain all existing neutrino phenomena, solar neutrino observations are now sufficiently constraining that even the most viable $\nu_e \to \nu_s$ explanation, a small-angle MSW transition, appears to be in difficulty.[4] Although providing better fits to the solar data, even active-active

transitions in a three-neutrino scheme do not give a quantitatively good explanation of those data. In this paper we point out that there is a way to achieve an excellent fit and rescue the apparently needed four-neutrino scheme if large extra dimensions exist. This is motivated by the latest developments in string theories, which have made plausible the interesting possibility[5] that there are such large extra space dimensions ($\lesssim$mm).

To strengthen the motivation for introducing such an exotic solution to the solar neutrino problem, the next section deals with the particle physics evidence for the four-neutrino scheme, with particular emphasis on the new and much more compelling analysis of the LSND experiment,[6] and the section after that treats the need for the same neutrino scheme to make possible the supernova r process. Then two extra-dimension models are briefly and qualitatively introduced, one having the usual low fundamental string scale of a few TeV, and the other having a much higher scale ($\sim 10^6$ TeV), which has desirable phenomenological, and probably theoretical, advantages. Next, the solar data are fit using a scheme which either model justifies. While active neutrinos are confined to a brane, the sterile neutrino occupies the bulk, the large size of which is responsible for the suppression of its mass, solving an inherent problem of sterile neutrinos. The small mass difference between the ν_e and the zero mode of the sterile neutrino causes vacuum oscillations, while low Kaluza-Klein excitations, which have a mass of the inverse extra dimension size, give small-angle MSW oscillations. The parameters of the model are determined by the average rates of the three types of solar experiments, and then the energy spectrum is predicted. This prediction agrees well (73% probability) with the Super-Kamiokande spectrum,[4] but the crucial test will come from SNO, which will have far better energy resolution. The model parameters, which give quite different mass-difference/mixing-angle regions from usual vacuum or MSW fits, provide important implications for gravity experiments, double beta decay, ultra-high-energy cosmic rays, supernovae, and dark matter, and these issues occupy the last section.

2. LSND's Added Support for the Four-Neutrino Scheme

Although not the original motivation for the four-neutrino scheme,[1] the three very different mass differences required by the solar and atmospheric anomalies and the LSND experiment[2] is usually cited as the reason to have, in addition to the three light active neutrinos allowed by the Z^0 width, a light neutrino (ν_s) not having the usual weak interaction. As evidence for neutrino oscillation explanations for the solar and atmospheric phenomena have become stronger, it is essential to evaluate the LSND result. A new analysis[6] of data from that experiment strengthens the conclusion that $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations were seen via events of the type $\bar{\nu}_e p \to e^+ n$, with the e^+ being identified by both scintillation and Cherenkov light from a 167-metric-ton tank of doped mineral oil, and the n detected by the 2.2 MeV γ from $np \to d\gamma$.

The 1996 publication[2] by LSND claiming a positive result was based on data collected in 1993–5 using a water target in which 798-MeV protons produced mainly π^+, of which 97% were brought to rest, providing a $\bar{\nu}_\mu$ beam from the subsequent decay at rest of the μ^+. During 1996–8, data were obtained at a lower rate in parasitic operation using a high-Z target. The latter data sets had larger cosmic-ray backgrounds, and conference reports using them had some disturbing distributions, probably indicating a problem of accidental electron-γ coincidences. This was likely because, along with the higher background, the R distribution (a measure of the likelihood that the e–γ was real as opposed to accidental) was the one published distribution—not any event spatial distributions, as often supposed—which was statistically worrisome.

The new analysis deals with all the data, and the various distributions from the 1996–8 period all agree well with those from 1993–5, since now the R distribution gives a smooth fit to the data over the whole energy range from 60 even down to 20 MeV. From 20–36 MeV there is added background from accidentals from $\nu_e{}^{12}\mathrm{C} \rightarrow e^- X$, but this now gives no problem. The R value is obtained using laser events for accidentals and Monte Carlo data for reals, checked by cosmic-ray neutron events. Three ingredients go into R: the γ energy, the distance between the e and the γ, and the neutron capture time.

In the new LSND analysis[6] a simultaneous likelihood fit of an event is made to the position, energy, track direction, track length, and fraction of Cherenkov (vs. scintillation) light utilizing the light amplitudes and arrival times at each of the 1220 8" photomultipliers looking into the tank of mineral oil and scintillator. The light amplitude and times were formerly used separately in determining these output quantities. A systematic skewing of event positions and angles was eliminated by taking into account an exponential tail on the Gaussian charge distribution from the photomultipliers. The improvement in position resolution in the new analysis reduced the most likely e–γ distance from 74 to 55 cm, and the accidental γ rate is proportional to the cube of this distance. The 1996 publication[2] used a cut at $R > 30$, whereas the new analysis[6] uses $R > 10$, where for a given analysis the larger the value of R the more likely the e and γ are correlated. For those two R cuts from different analyses the accidental rate has decreased from 0.6% to 0.3%, while the correlated efficiency has increased from 23% to 39%. Using the $R > 10$ cut there were 86 beam-on events, 36.9 ± 1.5 beam-off background (which has a small error, since data are collected during 94% of the time between pulses), 16.9 ± 2.3 expected ν background, giving an excess of $32.2 \pm 9.4 \pm 2.3$ events. The probability that $36.9 + 16.9 = 53.8$ background events fluctuate up to the observed 86 is $< 1 \times 10^{-4}$, taking into account all errors. A different way to state the result is to fit the R distribution, instead of using a cut. This gives $87.9 \pm 22.4 \pm 6.0$ events above expected backgrounds and corresponds to an oscillation probability of $(0.264 \pm 0.067 \pm 0.045)\%$, which is consistent with, but smaller than, the 1996 result[2] of $(0.31 \pm 0.12 \pm 0.05)\%$.

The results on $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ are quite compelling, and confidence in them is increased by

the excellent agreement with results on conventional processes, such as ν_eC and ν_μC to ground and excited states. Unfortunately it was decided to make the analysis "global" and use not only the 20–60 MeV data for decay-at-rest $\bar{\nu}_\mu \to \bar{\nu}_e$, but also the 60–200 MeV data for π^+ decay-in-flight $\nu_\mu \to \nu_e$. The latter process has only one signal, since it is detected by $\nu_e{}^{12}\mathrm{C} \to e^- X$, and hence suffers from much higher backgrounds. Furthermore, unlike the $\bar{\nu}_\mu \to \bar{\nu}_e$ case for which large numbers of electrons from cosmic μ decays provide calibrations and optimization of the analysis, the higher energy $\nu_\mu \to \nu_e$ has no reference process and must depend upon uncertain extrapolations from lower energy. Thus the published[7] LSND analysis of $\nu_\mu \to \nu_e$ was extremely complex, and while yielding confirmatory evidence for oscillations $(18.1 \pm 6.6 \pm 4.0)$ events above background, or an oscillation probability of $(0.26 \pm 0.10 \pm 0.05)\%$, the results could never be used on their own to claim observation of oscillations. As might be expected, the "global" analysis—which is optimized for $\bar{\nu}_\mu \to \bar{\nu}_e$—sees no significant evidence for $\nu_\mu \to \nu_e$, yielding $(8.1 \pm 12.2 \pm 1.7)$ events above background, or an oscillation probability of $(0.10 \pm 0.16 \pm 0.04)\%$.

The lack of sufficient $\nu_\mu \to \nu_e$ events skews the results when expressed in a mass-squared-difference (Δm^2), mixing-angle ($\sin^2 2\theta$) plot, favoring lower Δm^2 values. The often neglected island around 5–7 eV^2 is diminished in probability, whereas in the published[7] $\nu_\mu \to \nu_e$ case it was actually the most favored region. Probably a better plot to use, even if out of date, is that of Eitel,[8] which analyzes together LSND and KARMEN[9] results. This incredibly computer-intensive work has been not entirely correctly utilized in the new LSND plot. Eitel's work still gives the most representative picture if shifted to slightly smaller mixing angles, corresponding to the LSND oscillation probability shift of 0.31 to 0.26%, since KARMEN now has more events said to be consistent with background. Except for some gaps, such as around 3–5 eV^2, all Δm^2 values from 0.2 to 10 eV^2 are possible.

3. Supernova Evidence for the Four-Neutrino Scheme

There was an apparent conflict between the production of the heaviest elements in the neutrino-heated material ejected relatively long ($\sim$ 10s) after the explosion of a Type II or Type I b/c supernova and at least the larger Δm^2 values from LSND. Limits were placed by this r process of rapid neutron capture on ν_μ–ν_e mixing because energetic ν_μ ($\langle E \rangle \approx 25$ MeV) coming from deep in the supernova core could convert via an MSW transition to ν_e inside the region of the r-process, producing ν_e of much higher energy than the thermal ν_e ($\langle E \rangle \approx 11$ MeV). The latter, because of their charged-current interactions, emerge from farther out in the supernova where it is cooler. Since the cross section for $\nu_e n \to e^- p$ rises as the square of the energy, these converted energetic ν_e would deplete neutrons, stopping the r-process. Calculations[10] of this effect limit $\sin^2 2\theta$ for $\nu_\mu \to \nu_e$ to $\lesssim 10^{-4}$ for $\Delta m^2_{e\mu} \gtrsim 2$ eV^2, in conflict with at least the higher mass region of the LSND results.

More recently, serious problems have been found with the r process itself. First, simulations[11] have revealed the r-process region to be insufficiently neutron-rich, since about 10^2 neutrons are required for each seed nucleus, such as iron. This was bad enough, but the recent realization of the full effect of α-particle formation has created a disaster for the r process.[12] At a radial region inside where the r process should occur, all available protons swallow up neutrons to form the very stable α particles, following which $\nu_e n \to e^- p$ reactions reduce the neutrons further and create more protons which make more α particles, and so on. The depletion of neutrons rapidly shuts off the r process, and essentially no nuclei above $A = 95$ are produced.

To solve this problem the ν_e flux has to be removed before the r process site, while leaving a very large ν_e flux at a smaller radius for material heating and ejection. The apparent miracle of having a huge ν_e flux disappear before it reaches the radius of the supernova where α particles form can be accomplished[3] if there is (1) a sterile neutrino, (2) approximately maximal $\nu_\mu \to \nu_\tau$ mixing, (3) $\nu_\mu \to \nu_e$ mixing $\gtrsim 10^{-4}$, and (4) an appreciable ($\gtrsim 1$ eV^2) mass-squared difference between ν_s and the ν_μ–ν_τ. This is precisely the neutrino mass pattern required to explain the solar and atmospheric anomalies and the LSND result, plus providing some hot dark matter!

Such a mass-mixing pattern creates two level crossings. The inner one, which is outside the neutrinosphere (beyond which neutrinos can readily escape) is near where the $\nu_{\mu,\tau}$ potential $\propto (n_{\nu_e} - n_n/2)$ goes to zero. Here n_{ν_e} and n_n are the numbers of ν_e and neutrons, respectively. The $\nu_{\mu,\tau} \to \nu_s$ transition which occurs depletes the dangerous high-energy $\nu_{\mu,\tau}$ population. Outside of this level crossing, another occurs where the density is appropriate for a matter-enhanced MSW transition corresponding to whatever $\Delta m^2_{e\mu}$ LSND is observing. Because of the $\nu_{\mu,\tau}$ reduction at the first level crossing, the dominant process in the MSW region reverses from the deleterious $\nu_{\mu,\tau} \to \nu_e$, becoming $\nu_e \to \nu_{\mu,\tau}$ and dropping the ν_e flux. For an appropriate value of $\Delta m^2_{e\mu}$, the two level crossings are separate but sufficiently close so that the transitions are coherent. Then with adiabatic transitions (as calculations show) and maximal ν_μ–ν_τ mixing, the neutrino flux emerging from the second level crossing is 1/4 ν_μ, 1/4 ν_τ, and 1/2 ν_s, and no ν_e.

Note that the $\bar{\nu}_e$ flux is unaffected at the level crossings, so $\bar{\nu}_e p \to e^+ n$ enhances the neutron number in the r process region, since the protons have not been depleted by α particle formation. It should be emphasized that this mechanism is quite robust, not depending on details of the supernova dynamics, especially as it occurs quite late in the explosive expansion.

It is essential that the two level crossings be in the correct order, and this provides a requirement on $\Delta m^2_{e\mu}$, since the MSW transition depends on density and hence on radial distance from the protoneutron star. Detailed calculations have been made for $\Delta m^2_{e\mu} \sim 6$ eV^2, which works very well. Possibly $\Delta m^2_{e\mu}$ as low as 2 eV^2 or maybe even 1 eV^2 would work, but that is speculative. At any rate, the mass difference needed in this scheme, which is the only one surely consistent with all manifestations of neutrino

mass and which rescues the r process,[13] implies appreciable hot dark matter.

4. Invoking Extra Large Dimensions

An idea which has caused a lot of interest lately is that one or more of the extra dimensions required by string theories may be of a size which is observable and which would have a lot of consequences for our present universe. The possible observations, however, seem to require higher accelerator energies than presently available or difficult gravity experiments at senstitivities not yet reached. Instead, the case is made here that existing data, or that available soon, can show evidence for extra dimensions in an unexpected sector, the observation of neutrinos from the sun.

While it is said qualitatively that $\nu_e \to \nu_s$ can explain the solar ν_e deficit by either small-angle MSW or vacuum oscillations, quantitatively the solar experiments are so numerous and precise that the fits to either solution are rather poor. Even three-neutrino schemes, which use active-active transitions (e.g., $\nu_e \to \nu_\mu$), do only marginally better. One can be easily misled by these fit results because a very bad fit to one type of data (e.g., the rates of the three types of experiments) can get ignored when its few degrees of freedom are insignificant when included in the fit are the many degrees of freedom from an energy spectrum, zenith-angle data, etc.

If there are large extra dimensions, then ν_s becomes a particle which can exist in the extra dimension(s), or in other words it inhabits the bulk, while active neutrinos are confined to the brane. There could be more than one extra dimension and more than one brane, but for simplicity the discussion here is limited to one of each. A characteristic of a bulk particle (such as the graviton) is that it is really a series of states; this Kaluza-Klein tower has mass values $m_n \approx n/R$, where R is the size of the extra dimension.

While many papers have been written about bulk sterile neutrinos, since this provides a means of getting sterile neutrinos of small mass which have some mixing with active neutrinos, generally these theories do not produce three Δm^2 or attempt to explain all evidence for neutrino mass. The two models described here briefly and just qualitatively were developed by R.N. Mohapatra, whose contribution to these Proceedings goes into more detail. These models provide the desired phenomenology to have both vacuum and MSW oscillations for solar $\nu_e \to \nu_s$, while also giving suitable masses and mixings for the active neutrinos compatible with the four-neutrino scheme motivated above. One model has the usual string scale of a few TeV,[14] necessitating at least two extra large dimensions (although only one enters the considerations here), while the other has a very high string scale, $\sim 10^6$ TeV, and hence only one extra dimension need be large.

The low-scale model was developed first, and the fit to the solar data[15] is described in this basis, so somewhat more information on this is provided here. It is based on a mechanism[16] in which one or more gauge singlet neutrinos in the bulk couple to

lepton doublets in the brane, and after electroweak symmetry breaking this coupling leads to Dirac neutrino masses which are suppressed by the ratio $M_*/M_{P\ell}$, where $M_{P\ell}$ is the Planck mass and M_* is the string scale. This is sufficient to explain small neutrino masses and owes its origin to the large bulk volume that suppresses the effective Yukawa couplings of the Kaluza-Klein modes of the bulk neutrino to the brane fields. In this class of models, naturalness of small neutrino mass requires that one must assume the existence of a global B-L symmetry in the theory, since that will exclude the undesirable higher dimensional operators from the theory. In particular this leads to a neutrino mass $m_\nu = h\frac{M^* v}{M_{P\ell}} \sim 10^{-5}$ eV, where ν is the scale of $SU(2)_L$ breaking, and h is the Yukawa coupling. The ν_e and ν_s (zero mode, or ground state, hence really $\nu_{s,o}$) are two, two-component spinors that form the Dirac fermion with mass m_ν.

In order to fit neutrino data, one needs to include new physics in the brane that will generate a Majorana mass matrix for the three standard model neutrinos of the form δ_{ab} (where $a, b = e, \mu, \tau$). We assume that $\delta_{\mu\tau}$ is much bigger than the other elements. As a result the $\nu_{\mu,\tau}$ in effect decouple from the $\nu_{e,s}$ and do not affect the mixing between the bulk neutrino modes and the ν_e. Further, it leads to maximal mixing in the $\mu - \tau$ sector as is needed to understand the atmospheric neutrino data. If we choose $\delta_{\mu\tau} \sim$eV, then this provides an explanation of the LSND observations. In the rest of this discussion we focus only on the $\nu_e - \nu_s$ sector and how we fit the solar neutrino data.

The new physics chosen to accomplish this is to introduce two singlets to the Higgs sector on the brane to produce Majorana neutrino masses by radiative effects at the two-loop level. The radiative effects also split the ν_e–$\nu_{s,o}$ Dirac neutrino into two Majorana fermions, introducing a very small mass difference; the two mass eigenstates are maximally mixed and have the values $\nu_{1,2} \approx (\nu_e \pm \nu_{s,o})/\sqrt{2}$. Thus as the ν_e produced in a weak interaction process evolves, it oscillates to the $\nu_{s,o}$ state with an oscillation length of the order of the Sun-Earth distance, giving vacuum oscillations (VO). Since the ν_e mixes also with the Kaluza-Klein (KK) modes of the bulk neutrinos with a $\Delta m^2 \sim 10^{-5}$ eV^2, this brings in the MSW resonance transition of ν_e to $\nu_{s,n}$ modes at higher energies.

In this model the mixing angle is given by $\cos\theta = 1/N$, where

$$N^2 = 1 + (\pi m_o R)^2 + (m_n/m_o)^2$$

for the nth KK state. Since $m_o R \ll 1$, the second term can be neglected. For the zero mode, $N^2 \approx 1 + (m_o/m_o)^2 = 2$, or $\theta \approx \pi/4$, which is the maximal mixing required by experiment. For $n \geq 1$, $m_n \approx n/R$, so $N^2 \approx 1 + (n/m_o R)^2 \approx (n/m_o R)^2$, which gives small mixing angles.

A second way to achieve the same phenomenology is possible using a much higher string scale. In this class of models,[17] one postulates that the theory in the brane is left-right symmetric so that it contains B-L as a local symmetry, which is more

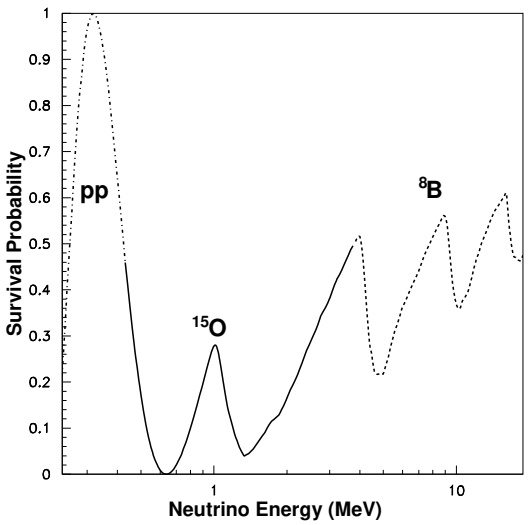

Figure 1: Energy dependence of the ν_e survival probability when $R = 58\mu$m, $mR = 0.0094$, $\delta_{ee} = 0.84 \times 10^{-7}$ eV. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the pp reaction, the solid part assumes ^{15}O radial dependence, and the dashed part assumes ^{8}B radial dependence.

desirable than having it as a global symmetry. The gauge group of the model is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ with quark and lepton doublets assigned as usual in a left-right symmetric manner and Higgs fields belonging to a didoublet field $\phi(2,2,0)$ and B-L doublets $\chi_{L,R}$. The use of this model to give suitable active neutrino masses (now through a type of see-saw mechanism) and mixings and to produce the same phenomenology as in the low-scale model cannot be described in a few lines, but it is spelled out in Ref. 18. The point does need to be emphasized here, however, that if there is only one large extra dimension, R, and all small extra dimensions are $\sim M_*^{-1}$, then $M_{Pl}^2 \approx M_*^3 R$ leads to $M_* \sim 10^6$ TeV if $R \sim$ mm.[17)]

5. Fitting the Solar Data*

The models provide a naturally small mass difference between the mass eigenstates formed from the ν_e and the $\nu_{s,0}$, giving vacuum oscillations. The first node of the survival probability function due to VO can be used to suppress ^{7}Be neutrinos. Going up in energy toward ^{8}B neutrinos, the survival probability, which in the VO case would have risen to very near one, is suppressed by the small-angle MSW transitions to the different KK excitations of the bulk sterile neutrinos, as is clear from Fig. 1. This is a new way to fit the solar neutrino data in models with large extra dimensions and is the main observation of this report.

*Converting the theory to a fit was done by S.J. Yellin.

For comparison with experimental results, tables of detector sensitivity for the Chlorine and Gallium experiments were taken from Bahcall's web site.[19] The Super-Kamiokande detector sensitivity[20] was used with appropriate smearing of the neutrino-electron elastic scattering cross section. Calculations of electron neutrino survival probability, averaged over the response of detectors, were compared with measurements. While theoretical uncertainties in the solar model and detector response were included in the computation[21] of χ^2, the measurement results given here include only experimental statistical and systematic errors added in quadrature. The Chlorine survival probability, from Homestake,[22] is 0.332 ± 0.030. Gallium results[23] for SAGE, GALLEX and GNO were combined to give a survival probability of 0.579 ± 0.039. The 5.0–20 MeV, 1258-day Super-K experimental survival probability[4] is 0.451 ± 0.016. The best fits were with $R \approx 58\mu m$, mR around 0.0094, and a mass splitting term, $\delta_{ee} \sim 0.84 \times 10^{-7}$ eV, corresponding to $\delta m^2 \sim 0.53 \times 10^{-11}$ eV^2 for VO. These parameters give average survival probabilities for Chlorine, Gallium, and water of 0.383, 0.533, and 0.450, respectively, and the energy dependence shown in Fig. 1. Here the coupling between ν_e and the first KK excitation replaces $\sin^2 2\theta$ by $4m^2R^2 = 0.00035$.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of 0.63 MeV$/(2n+1)$ for $n = 0, 1, 2, \ldots$ Thus Fig. 1 shows nearly zero ν_e survival near 0.63 MeV, partly eliminating the ^{7}Be contribution at 0.862 MeV, and giving a dip at the lowest neutrino energy. Note that the pattern of two eigenstates very close in mass persists for the Kaluza-Klein excitations as well. These MSW resonances start causing the 3rd and 4th eigenstates to be significantly occupied above $\sim$ 0.8 MeV, the 5th and 6th eigenstates above $\sim$ 3.7 MeV, the 7th and 8th above $\sim$ 8.6 MeV, and the 9th and 10th above $\sim$ 15.2 MeV. Fig. 1 shows dips in survival probability just above these energy thresholds.

The expected energy dependence of the ν_e survival probability is compared with Super-K data[4] in Fig. 2. The uncertainties are statistical only. The parameters used in making Fig. 2 were chosen to provide a good fit ($\chi^2 = 3.4$) to only the total rates; they were not adjusted to fit this spectrum. Combining spectrum data with rates[24] gives $\chi^2 = 14.0$ for the spectrum predicted from the fit to total rates. With 18 degrees of freedom, the probability of $\chi^2 > 14.0$ is 73%. If instead the fit were to an undistorted energy spectrum the χ^2 would be 19.0. If VO were eliminated, the best fit to the rates gives $\chi^2 = 4.4$, whereas the same parameters applied to the spectrum yield $\chi^2 = 18.7$, corresponding to a probability of 41%.

Despite the contribution of VO, the seasonal effects are very small and will be hard to observe, as shown in Table 1. On the other hand, the SNO experiment is about to release its energy spectrum obtained from charged-current interactions, which give far better energy resolution than the neutrino-electron scattering observed by Super-Kamiokande, and the characteristic shape of the spectrum above 7 MeV should be seen if this idea is correct.

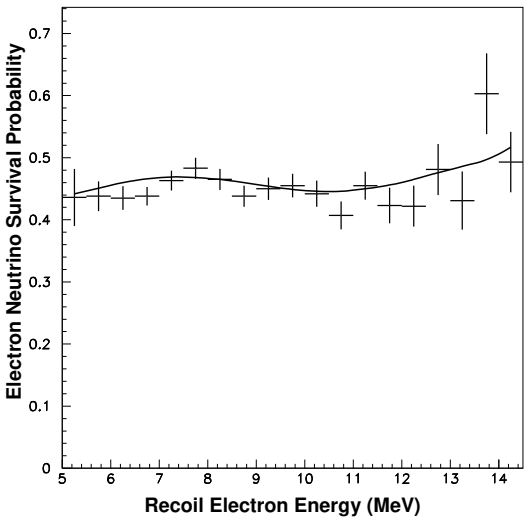

Figure 2: Super-Kamiokande energy spectrum: measured[4] results based on 1285 days (error bars) and predicted (curve) for the same parameters as in Fig. 1. The curve is not a fit to these data.

Table 1: Predicted seasonal variations in ν_e fluxes, excluding the $1/r^2$ variation. The model assumed the same parameters as were used for Fig. 1.

$\theta - \theta_0$	Chlorine	Gallium	Water
0 (January 2)	0.3885	0.5362	0.4602
$\pm\pi/2$	0.3861	0.5328	0.4600
π (July 4)	0.3838	0.5278	0.4598

6. Consequences of this Fit to the Solar Data

The parameters required to fit the average rates of the three types of solar neutrino experiments, if confirmed by the SNO energy spectrum, would have some obvious consequences other than demonstrating that the four-neutrino scheme is correct and that at least one large extra dimension exists. For instance, the mass eigenstate which is mainly electron neutrino is 3×10^{-5} eV, which is undetectable directly or by neutrinoless double beta decay. The latter process measures an effective neutrino mass, but even the contributions to that from the ν_μ and ν_τ must be sufficiently small as to make that very unlikely to be observed, although some other conjectured processes not involving neutrino mass could cause neutrinoless double beta decay.

The effect of the 0.06 mm extra dimension size should be detectable by gravity experiments in the not too distant future, since the present best limit[25] on such

effects is less than a factor of four from that value. This would give experimenters a definite goal for which to design.

Such a relatively large extra dimension size raises issues about cosmological and supernova limits from the effects of high Kaluza-Klein states of both the sterile neutrinos[26] and gravitons.[27] While these constraints are necessarily somewhat suspect because the two regimes, the hot early universe and the supernova core, are very complex and are not yet fully understood, nevertheless if taken seriously, especially the graviton limits may pose a problem for the low-scale model, although there are extenuating circumstances. Usually these constraining arguments assume a model of n dimensions, each of size R, which is not true of either of our models. For sterile neutrino limits, the phenomenology presented here is aided because there is a single Kaluza-Klein tower based on a very small mass, the VO Δm^2 is an order of magnitude smaller than usual, and for MSW the equivalent $\sin^2 2\theta$ value is more than an order of magnitude smaller than for standard fits. Furthermore, for both sterile neutrinos and gravitons the universe reheat temperature could be very low, since anything above 0.7 MeV has cosmological validity, reducing production of high KK states. These are very complicated issues and under much discussion, but should it turn out that the low-scale model does not seem to satisfy constraints, the same phenomenology is obtained by the theoretically more desirable high-scale model, and that appears to avoid all of these limits, as it certainly also does if the graviton-ν_s interactions in the bulk[28] are a problem.

The huge density of KK states which can be produced if enough energy is available provides an explanation of ultra high energy cosmic rays beyond the GZK cut-off.[29] Neutrinos have long been suggested[30] as the source of these air showers, but providing a sufficiently large interaction cross section has been the problem. Achieving this without some observable low-energy effect has been the difficulty, but these narrowly ($\sim 10^{-3}$ eV) spaced KK states provide such a high density at $> 10^{19}$ eV that hadronic-type cross sections can be obtained.

The means of rescuing the r process described in Section 3 still works for the bulk ν_s, actually assuring the adiabaticity of the $\nu_{\mu,\tau} \to \nu_s$ level crossing. In contrast to the usual concern that the ν_s would provide too much supernova energy loss, it may actually aid the blow-up of the supernova, since at early times there is a region behind the stalled shock where the interaction potential goes to zero, and the many KK ν_s states can reconvert to active electron neutrinos, depositing energy just where and when it is needed. The details of this process are being worked out with George Fuller and his students.

Finally there is the intriguing possibility that the KK states of the sterile neutrino may provide the main component of the dark matter. This is also being worked on with George Fuller and his students and is bound up with the question of the reheat temperature; can it be high enough to produce sufficient neutrino states without overproducing gravitons? This appears to be true for the high-scale model. Some

preliminary calculations have given an interesting mix: very little hot dark matter, and about half warm and half cold dark matter. That combination should produce good agreement with structure measurements over a considerable range of scale.

7. Conclusions

The recent reanalysis of the LSND experiment greatly strengthens the case for three different neutrino mass differences, forcing the need for a sterile neutrino. It is quite remarkable that the profound problems of producing the heaviest elements by supernovae can be solved in a manner which requires no adjustment of parameters if the arrangement of masses and mixings of neutrinos is exactly that required to explain the solar ν_e deficit, the atmospheric neutrino anomaly, and the observations of the LSND experiment (or alternatively the need for hot dark matter). On the other hand, this apparently successful four-neutrino scheme fails quantitatively (as do other models) to explain all the solar ν_e data, unless the essential sterile neutrino is a bulk neutrino of extra large dimensions. The resulting Kaluza-Klein tower of states provides both MSW and vacuum oscillations, explaining the otherwise confusing solar data. This excellent fit to the data may be providing the first experimental evidence for large extra dimensions, but the SNO energy spectrum should settle this issue soon. A positive result will have wide-ranging consequences for gravity experiments, double beta decay, ultra-high-energy cosmic rays, supernovae, and dark matter.

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