NEUTRINO OSCILLATION PHYSICS AT A NEUTRINO FACTORY

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ABSTRACT

The potential of a neutrino factory at determining the neutrino mass matrix is reviewed.

1. Introduction

The atmospheric plus solar neutrino data can be easily accommodated in a three-family neutrino mixing scenario. For Dirac neutrinos, the corresponding matrix describing the neutrino mixing depends generically on four physical parameters: three angles (θ_{12}, θ_{13} and θ_{23}) and a CP-odd phase (δ). We use the standard parametrization of ref. [1]. Oscillation experiments are sensitive to two neutrino mass differences and the parameters of the CKM matrix. The challenge of a future ν-physics program is to determine all these quantities.

For convenience, we identify[a] the solar mass difference with Δm^2_{12} and the atmospheric one with Δm^2_{23}. The experimental results indicate that $|\Delta m^2_{23}| \gg |\Delta m^2_{12}|$ in most of the allowed parameter space [2]. This hierarchical pattern implies a high degree of decoupling between the solar and atmospheric oscillations: while the oscillation probabilities at atmospheric distances are controlled mainly by $\Delta m^2_{23}, \theta_{23}$ and θ_{13}, the oscillations at solar distances are controlled by $\Delta m^2_{12}, \theta_{12}$ and θ_{13}. Note that the angle θ_{13} is the connection between the two. If this angle is zero, the decoupling is complete and the solar and atmospheric anomalies are described by two independent two-by-two family mixing processes: $(\theta_{12}, \Delta m^2_{12})$ and $(\theta_{23}, \Delta m^2_{23})$ respectively. At present, the most valuable information on the angle θ_{13} comes from Chooz reactor experiment [3]. This experiment has set an stringent upper limit $\sin^2\theta_{13} < 0.05$, which implies that the above mentioned decoupling indeed occurs to a good accuracy.

In this situation, planned solar experiments (SNO, KamLAND and Borexino) will give us further information on $(\theta_{12}, \Delta m^2_{12})$, but remain insensitive to such a small θ_{13}. Hopefully, they will clarify the present situation by selecting the true solution among the three MSW regions presently allowed: LMA-MSW, SMA-MSW or LOW. Particularly important for the potential of the neutrino factory is the exploration of the LMA-MSW range. In this respect, the most relevant information will come from the long baseline reactor experiment KamLAND [4]. Recent analyses [5] have shown that this experiment can achieve sensitivities to Δm^2_{12} and $\sin^2 2\theta_{12}$ of a few per cent in three years of data taking.

[a] $\Delta m^2_{ij} \equiv m^2_j - m^2_i$ throughout the paper.

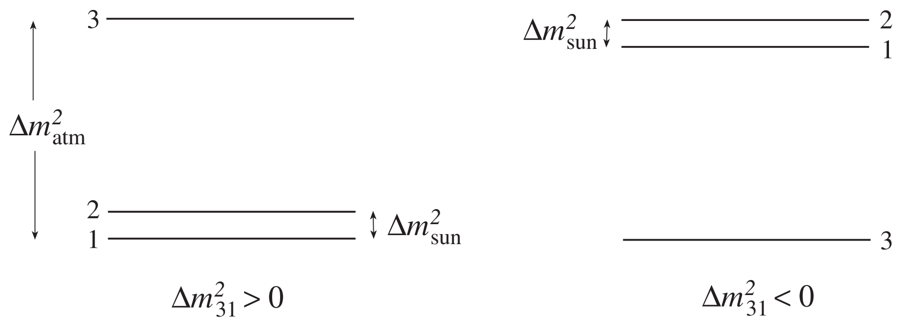

Figure 1: Patterns of neutrino masses

Similarly, the next generation of atmospheric experiments [6] will be mainly sensitive to $(\sin^2 2\theta_{23}, \Delta m^2_{23})$. It is expected that these quantities will be measured at Minos with a precision of $O(10\%)$ [7]. On the other hand, the sensitivity to the angle θ_{13} will not improve significantly the present upper bound imposed by Chooz.

Summarizing, because of the large degree of decoupling between atmospheric and solar oscillations, more precise neutrino oscillation experiments will be necessary to determine the remaining unknowns in the neutrino mass matrix that are in principle accesible to neutrino oscillations:

- The angle θ_{13} and the CP-odd phase δ
- The sign of Δm^2_{23}, which determines whether the three-family neutrino spectrum is of the "hierarchical" or "degenerate" type as depicted in Fig. 1.

The most sensitive observables are the transition probabilities involving $\nu_e(\bar{\nu}_e)$, in particular $\nu_e(\bar{\nu}_e) \to \nu_\mu(\bar{\nu}_\mu)$. This is precisely the golden measurement at the ν factory [9,10]. Such a facility is unique in providing high energy and intense $\nu_e(\bar{\nu}_e)$ beams coming from positive (negative) muons which decay in the straight sections of a muon storage ring [8]. Since these beams contain also $\bar{\nu}_\mu(\nu_\mu)$ (but no $\nu_\mu(\bar{\nu}_\mu)$!), the transitions of interest can be measured by searching for "wrong–sign" muons: negative (positive) muons appearing in a massive detector with good muon charge identification capabilities.

A lot of work has been devoted already to quantifying the reach of a neutrino factory at determining these unknowns [10]-[30]. I will describe here the results of [18,28], corresponding to a neutrino factory providing 10^{21} useful μ^+, and the same number of μ^-, decays. The muon energy is fixed to 50GeV. A large magnetized iron detector of 40KTon is considered [31], for which a detailed study of efficiencies and backgrounds to the wrong–sign muon search has been performed.

2. Measurement of θ_{13} and δ

The subleading transitions $\nu_e \to \nu_\mu$ are forbidden in the decoupling limit in which the effects of θ_{13} and Δm^2_{12} are neglected at terrestrial distances. As the precision in the measurement of this transition increases, the effects of the small angle θ_{13} or/and the solar splitting are revealed. Depending on whether each one or both of these parameters are non negligible, we get the following approximate formulae in vacuum for the subleading transition:

I. $\theta_{13} > 0, \quad \Delta_{12} = 0$

$$P_{\nu_e\nu_\mu} = s^2_{23}\ \sin^2 2\theta_{13}\ \sin^2\left(\frac{\Delta_{23}\,L}{2}\right) \tag{1}$$

II. $\theta_{13} = 0, \quad \Delta_{12} > 0$

$$P_{\nu_e\nu_\mu} = c^2_{23}\ \sin^2 2\theta_{12}\ \sin^2\left(\frac{\Delta_{12}\,L}{2}\right) \tag{2}$$

III. $\theta_{13} > 0, \quad \Delta_{12} > 0$

$$\begin{aligned} P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} &= s^2_{23}\ \sin^2 2\theta_{13}\ \sin^2\left(\frac{\Delta_{23}\,L}{2}\right) \\ &+ c^2_{23}\ \sin^2 2\theta_{12}\ \sin^2\left(\frac{\Delta_{12}\,L}{2}\right) \\ &+ \tilde J\ \cos\left(\pm\delta - \frac{\Delta_{23}\,L}{2}\right)\ \frac{\Delta_{12}\,L}{2}\ \sin\left(\frac{\Delta_{23}\,L}{2}\right) \end{aligned} \tag{3}$$

where $\tilde J \equiv \cos\theta_{13}\ \sin 2\theta_{13}\ \sin 2\theta_{23}\ \sin 2\theta_{12}$, $\Delta_{ij} \equiv \Delta m^2_{ij}/2E_\nu$.

At the neutrino factory, the baselines will be large enough so that Earth matter effects in the neutrino propagation are sizeable and have to be taken into account. Approximate expressions for the probabilities to second order in the small parameters θ_{13} and Δ_{12} have also been derived in [18], in the approximation of constant Earth matter density. The result is

I. $\theta_{13} = 0, \quad \Delta_{12} = 0$

$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} = s^2_{23}\ \sin^2 2\theta_{13}\ \left(\frac{\Delta_{23}}{B_\mp}\right)^2\ \sin^2\left(\frac{B_\mp\,L}{2}\right) \tag{4}$$

II. $\theta_{13} = 0, \quad \Delta_{12} > 0$

$$P_{\nu_e\nu_\mu} = c^2_{23}\ \sin^2 2\theta_{12}\ \left(\frac{\Delta_{12}}{A}\right)^2\ \sin^2\left(\frac{A\,L}{2}\right) \tag{5}$$

III. $\theta_{13} > 0, \quad \Delta_{12} > 0$

$$P_{\nu_e\nu_\mu(\bar\nu_e\bar\nu_\mu)} = s_{23}^2 \, \sin^2 2\theta_{13} \, \left(\frac{\Delta_{23}}{B_\mp}\right)^2 \, \sin^2\left(\frac{B_\mp\, L}{2}\right) + c_{23}^2 \, \sin^2 2\theta_{12} \, \left(\frac{\Delta_{12}}{A}\right)^2 \, \sin^2\left(\frac{A\, L}{2}\right) + \tilde{J} \, \left(\frac{\Delta_{12}}{A}\right) \left(\frac{\Delta_{23}}{B_\mp}\right) \cos\left(\pm\delta - \frac{\Delta_{23}\, L}{2}\right) \, \sin\left(\frac{A\, L}{2}\right) \sin\left(\frac{B_\mp\, L}{2}\right), \quad (6)$$

where $B_\mp \equiv |A \mp \Delta_{13}|$ ($\mp$ for neutrinos and antineutrinos respectively) and the matter parameter, A, is given in terms of the average electron number density, $n_e(L)$, as $A \equiv \sqrt{2} G_F n_e(L)$.

It is clear that the different approximations of eqs.(4-6) are relevant for the different solutions to the solar neutrino problem:

- SMA-MSW, LOW, VO solutions $\rightarrow$ case I
- LMA-MSW $\rightarrow$ cases II or III

Sensitivity to leptonic CP-violation can only be achieved in case III, where the effects of θ_{13} and Δ_{12} are both non negligible. Thus only if the solution to the solar anomaly lies in the LMA-MSW range there is hope to measure the CP-odd phase δ. In all cases however a precise measurement of the subleading transitions will give a measurement or a strong constraint on the angle θ_{13}.

In Fig. 9 we show the expected sensitivity to this angle at a neutrino factory in the SMA-MSW, LOW or VO scenarios. The improvement with respect to present bounds can be of 3-4 orders of magnitude.

If the LMA-MSW is the true solution the main challenge is the simultaneous measurement of θ_{13} and δ. It has been shown [18,28] that, at fixed E_ν and L, the measurement of the probabilities $P_{\nu_e\nu_\mu}$ and $P_{\bar\nu_e\bar\nu_\mu}$ is insufficient to fix both parameters: there are two equally probable solutions both in vaccuum and in matter [28]. Given the true set of parameters $(\bar\theta_{13}, \bar\delta)$, the second set in vacuum can be derived from eqs. (1-3):

$$\delta = 180^\circ - \bar\delta \qquad \theta_{13} = \bar\theta_{13} + \frac{Y}{X} \cos\bar\delta \cos\left(\frac{\Delta_{13} L}{2}\right), \quad (7)$$

where $Y \equiv \sin 2\theta_{12} \, \sin 2\theta_{23} \, \frac{\Delta_{12}\, L}{2} \, \sin\left(\frac{\Delta_{23}\, L}{2}\right)$ and $X \equiv s_{23}^2 \, \sin^2\left(\frac{\Delta_{23}\, L}{2}\right)$. Only for the particular values $\bar\delta = \pm 90^\circ$, the two solutions degenerate into one. Note that either both of the solutions break or both preserve CP. In matter this is no longer true. In Fig. 3 we show the value of δ for the fake solution as a function of $\bar\delta$ using eqs. (4-6), for three reference baselines together with the vacuum result. In matter the fake

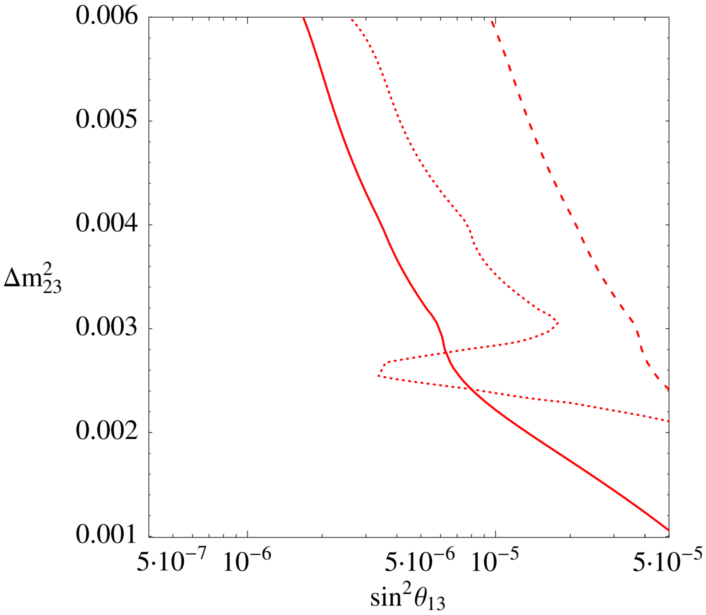

Figure 2: *Asymptotic sensitivity to* $\sin^2\theta_{13}$ *at* 90% *CL for L=732 km (dashed lines), 3500 km (solid lines) and 7332 km (dotted lines). Realistic backgrounds and efficiencies are included.*

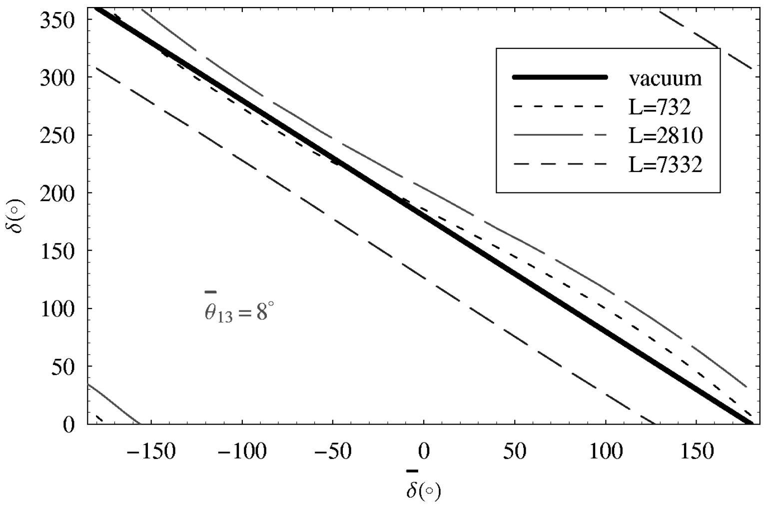

Figure 3: *Degenerate value of* δ *as a function of true value* $\bar{\delta}$, *for* $\bar{\theta}_{13} = 8^\circ$ *and three different baselines; the vacuum result* $\delta = \pi - \bar{\delta}$ *is also shown.*

solution might break CP when the true one does not or viceversa. As in vacuum, $|\delta - \bar{\delta}|$ becomes maximal near $\bar{\delta} = 0^\circ, 180^\circ$, and minimal close to $\bar{\delta} = \pm 90^\circ$.

At short baselines such that $\Delta_{23} L \ll 1$, disentangling both parameters is much more difficult. The reason is that in this case the probabilities $P_{\nu_e\nu_\mu}$ and $P_{\bar{\nu}_e\bar{\nu}_\mu}$ are approximately the same, since both the intrinsic CP-odd terms and matter effects are subleading in L. Indeed, in an expansion in L eqs. (3) and (6) reduce to

$$P_{\nu_e\nu_\mu} \simeq P_{\bar{\nu}_e\bar{\nu}_\mu} \simeq s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{23} L}{2}\right)^2 + c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12} L}{2}\right)^2 + \tilde{J} \cos\delta \frac{\Delta_{12} L}{2}\frac{\Delta_{23} L}{2} + O(L)^3. \quad (8)$$

As a result, instead of two degenerate solutions, there is a continuous curve of solutions of equal probability of the form:

$$\theta_{13} \simeq \bar{\theta}_{13} - \frac{Y}{2X}\left(\cos\delta - \cos\bar{\delta}\right). \quad (9)$$

Figs. 4 and show the result of a simultaneous χ^2 fit of θ_{13} and δ. For details we refer the reader to [18,28]. The results for three reference baselines of $L = 732, 2810, 7332$ km and various central values of $\bar{\theta}_{13}$ and $\bar{\delta}$ are compared. At the shorter baseline, $L = 732$km, the correlation described by eq. (9) is clearly seen. In the intermediate baseline, $L = 2810$km the degenerate solutions appear in some cases as isolated contours when $|\delta - \bar{\delta}|$ is large (for $\bar{\delta} = 0°, 180°$) or they merge into enlongated contours when it is smaller (for $\bar{\delta} = -90°, 90°$). The result is a very non uniform error in δ and θ_{13} depending on the value of $\bar{\delta}$. Finally, at the longer baseline, $L = 7332$km, the sensitivity to δ is almost lost with this setup.

The optimal baseline choice to measure CP violation is clearly the intermediate one [18]. However, the energy dependence of the signal is not significant enough, with the setup considered, to fully resolve the degeneracies and this results in unsatisfactorily large errors for δ and θ_{13}. It is important to stress that this problem stems to a large extent from the fact that efficiencies are very low for neutrino energies below 10GeV (because of the large experimental backgrounds) while the energy dependence of the wrong–sign muon signals is most significant at lower energies. Indeed it is easy to show that for large enough neutrino energies, the energy dependence is trivial and does not help to resolve the correlations [27].

The combination of two baselines however can help considerably in disentangling the right solution. In Fig. 6, we show the three possible combinations of two of the reference baselines. The two-fold degeneracy does not disappear completely in the combination of the two shorter baselines, while it does in the remaining two combinations. It is interesting that, while the shortest and longest baselines by themselves are not appropriate for CP studies, their combination is quite promising, due to the very different pattern shown by the correlation between θ_{13} and δ in each of them.

In this analysis, we have assumed that the remaining oscillation parameters and the average Earth matter density are exactly known. Clearly this will not be the case. Given the difficulty of extracting CP violating effects, it is an important question to

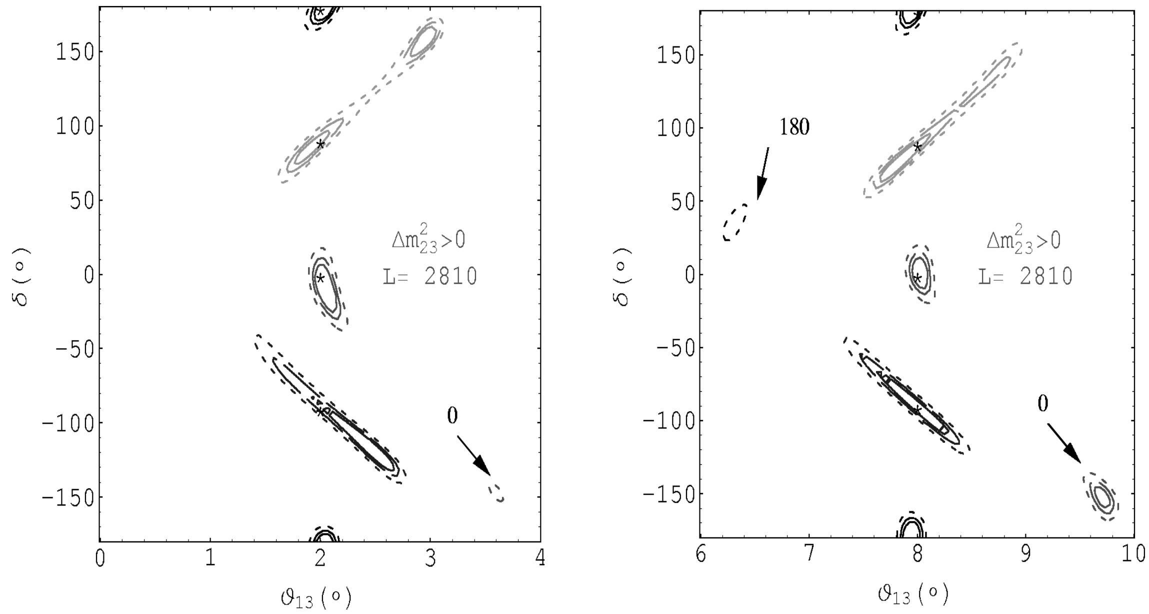

Figure 4: *Simultaneous fits of δ and θ_{13} at L = 2810 km for different central values (indicated by the stars) of $\bar{\delta} = -90°, 0°, 90°, 180°$ and $\bar{\theta}_{13} = 2°$ (left), $8°$ (right). The value of $\bar{\delta}$ for the degenerate solutions is also indicated. The contours correspond to 1σ, 90% and 99% confidence level.*

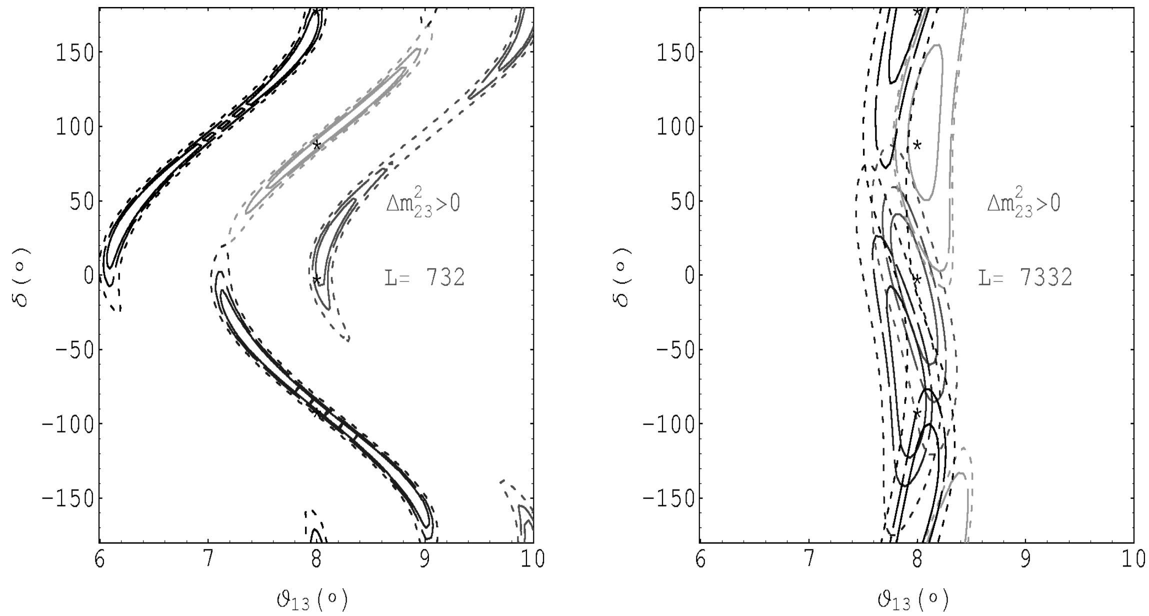

Figure 5: *Simultaneous fits of δ and θ_{13} at $L = 732$ km (left) and $L = 7332$ km (right) for different central values of $\bar{\delta} = -90°, 0°, 90°, 180°$ and $\bar{\theta}_{13} = 8°$.*

understand what will be the effect of the expected uncertainties in these parameters in the measurement of θ_{13} and δ. Recently this problem has been addressed by several groups from different points of view [26,28,29,30)].

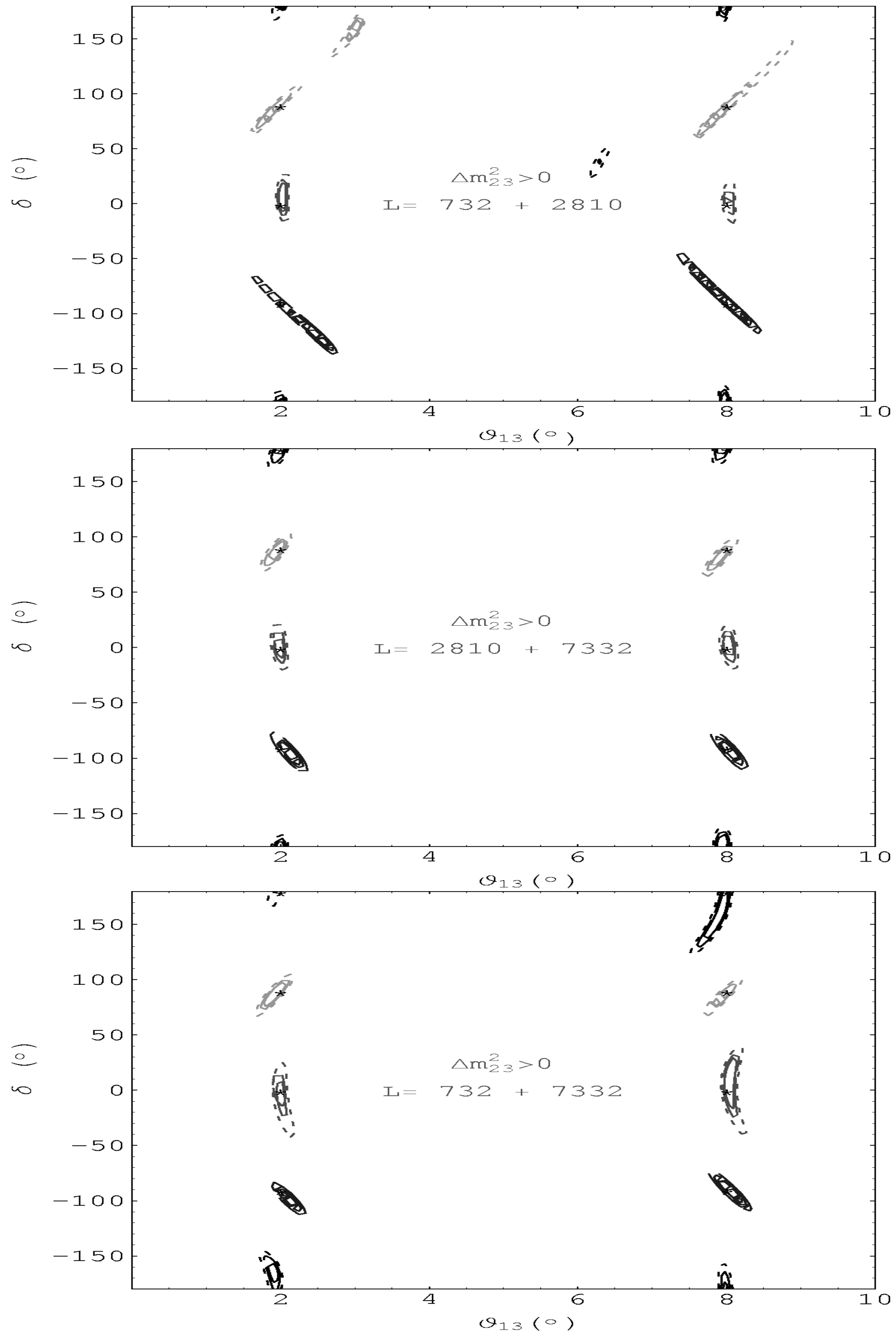

Figure 6: Fits of δ and θ_{13} combining any two baselines.

Concerning the error on the atmospheric parameters, the muon disappearance measurements at the neutrino factory itself can achieve a precision of the $O(1\%)$ in $(\sin^2 2\theta_{23}, \Delta m^2_{23})$ [15,32]. If the LMA-MSW is the right solution of the solar neutrino deficit, the corresponding solar parameters $(\sin^2 2\theta_{12}, \Delta m^2_{12})$, will be determined at

KamLAND at the level of a few per cent longer before the neutrino factory. The effect of these small uncertainties on the fits described above will not be important. The most sizeable effect is that of the uncertainty in θ_{23}, which will mainly affect the error on θ_{13}.

Less clear is the situation with the average Earth matter density along the neutrino path. The results of [28,30] show that if this quantity can be determined to the level of a few per cent, also in this case the effect on the measurement of θ_{13} and δ is not important. However if the uncertainty is of $O(10\%)$ or larger, it affects considerably the extraction of δ. There seems to be no consesus yet as how well this quantity can be determined from geological measurements. This is clearly a very important issue to clarify.

As an illustration in Fig. 7 we compare the fits for the optimal combination of baselines with and without these uncertainties, with an error on the matter density of 1%. For details of the analysis we refer to ref. [28]. Note that the effect of the theoretical errors is larger at larger θ_{13}.

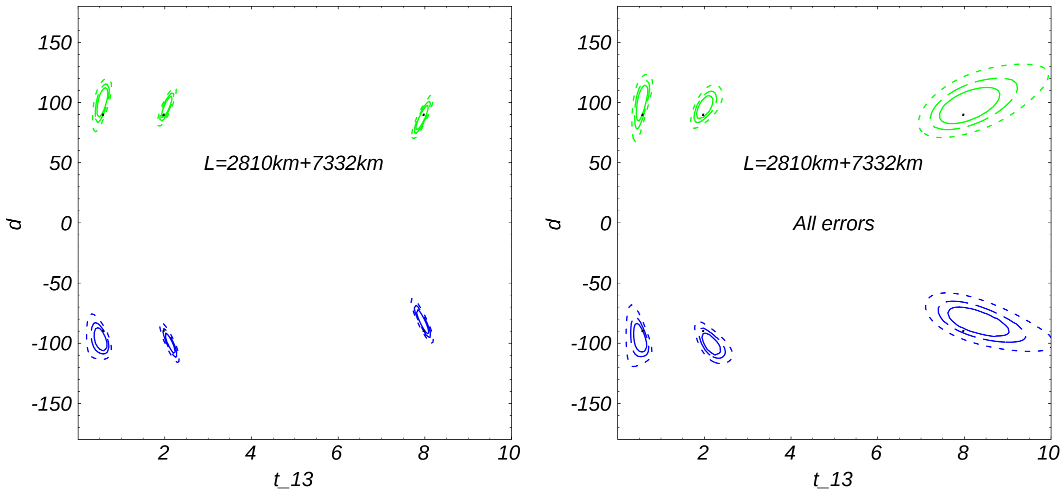

Figure 7: Fits of δ and θ_{13} combining two baselines assuming $\theta_{23}, \Delta m^2_{23}, \theta_{12}, \Delta m^2_{12}, A$ known (left) and including their expected uncertainties (right).

3. Measurement of the sign(Δm^2_{23})

The measurement of the sign(Δm^2_{23}) in vaccuum is not easy. The reason is that, as is clear from eqs. (1-3), this sign only enters in case III, which is only relevant if the LMA-MSW is the right solution of the solar anomaly. Furthermore, it is easy to check that a change in the sign of Δm^2_{23} in eq. (3) can be compensated by a change in the phase $\delta \to \pi - \delta$. This implies that from the measurement of the subleading transitions for neutrinos and antineutrinos in vacuum it is impossible to distinguish $(\mathrm{sign}(\Delta m^2_{23}) = +, \delta)$ from $(\mathrm{sign}(\Delta m^2_{23}) = -, \pi - \delta)$.

Fortunately, the baselines and neutrino energies at the neutrino factory will be large enough so that Earth matter effects in the neutrino propagation are sizeable. The sign of Δm^2_{23} is in principle measurable even when the effects of the solar mass difference are negligible, ie. for all solutions to the solar anomaly, provided θ_{13} is large enough. Note that in case I, a change in the sign changes the probability for neutrinos to that of antineutrinos, so sensitivity to the sign is equivalent to the sensitivity to the fake CP odd asymmetry induced by matter effects. In Fig. 8 we plot the significance of this asymmetry as a function of the baseline. Although the best sensitivity occurs at $O(7000)$ km, the asymmetry is measurable already at much shorter distances $\geq O(1000)$km.

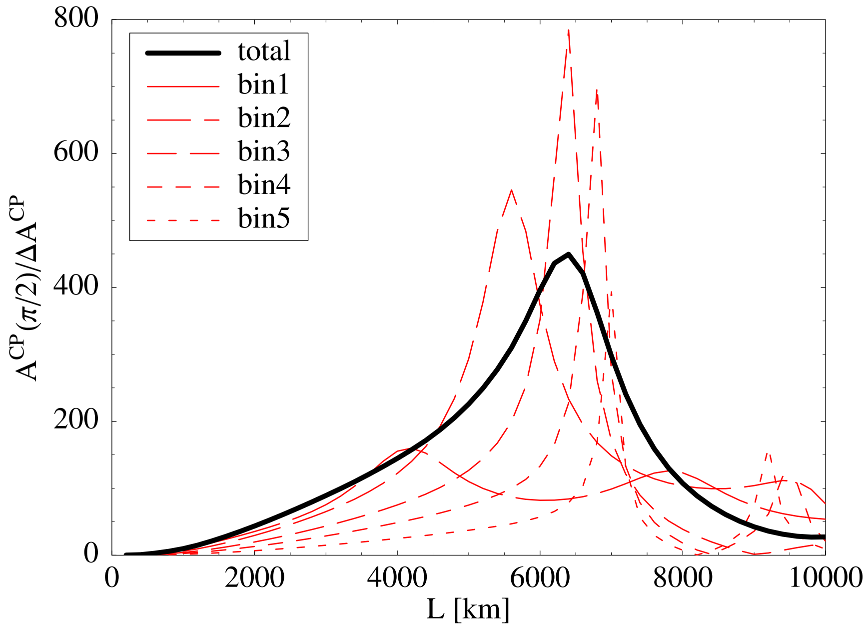

Figure 8: *Significance of the fake CP asymmetry as a function of the baseline in the SMA-MSW scenario. The curves with different dashing correspond to different energy bins, while the thick curve is the average.*

4. Conclusions

We have discussed the physics potential of the neutrino factory in exploring the lepton flavour sector of the Standard Model in a three family mixing scenario that accomodates the solar and atmospheric neutrino deficits. Such a facility can improve by several orders of magnitude the sensitivity to the angle θ_{13}, which links the atmospheric and solar anomalies, and may discover leptonic CP violation at a baseline of

$O(3000)$km, if the solution to the solar neutrino problem is confirmed to lie in the LMA-MSW regime, and the angle θ_{13} is larger than a few tenths of a degree. Within this range, the sensitivity to CP-violation is lost only for the smaller values of the solar mass difference allowed by the LMA-MSW scenario, as shown by the rough exclusion plot of Fig. 9. Finally, the discrimination between the two possible patterns of neutrino masses of Fig. 1 can be achieved thanks to the relevance of Earth matter effects at the neutrino energies of $O(10GeV)$ and baselines $\geq$ 1000km.

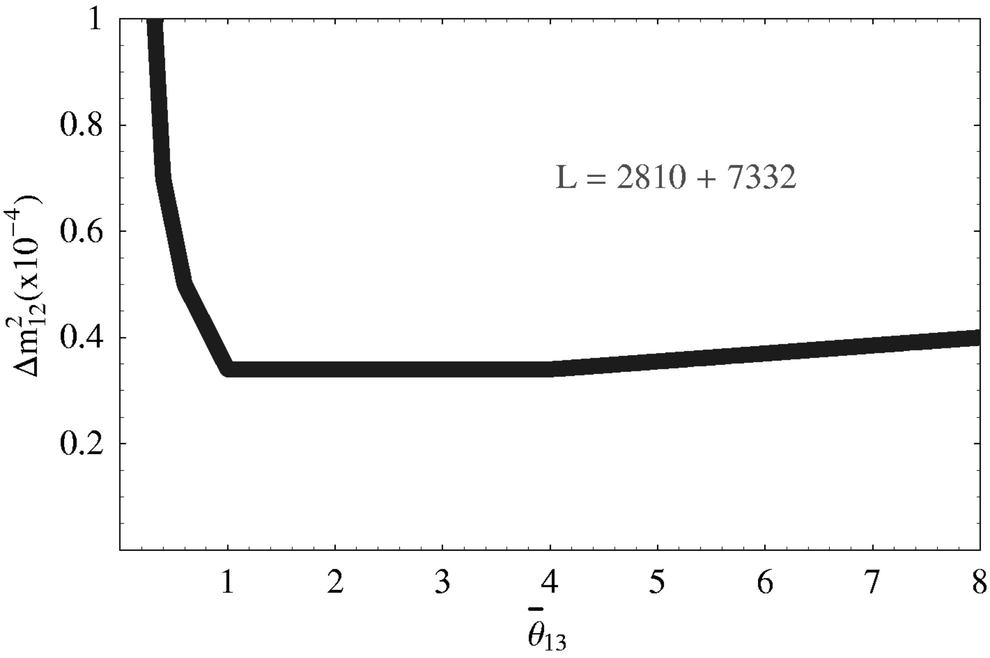

Figure 9: *Sensitivity reach for CP violation (limit up to which it is possible to distinguish $\bar{\delta} = 90°$ from $\bar{\delta} = 180°$ or $0°$ at 99%CL) on the plane $(\Delta m^2_{12}, \bar{\theta}_{13})$ for the combination of baselines $L = 2810$ and 7332 km. All errors are included.*

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