STERILE NEUTRINOS, MIRROR UNIVERSE AND LARGE EXTRA DIMENSIONS

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ABSTRACT

There are several reasons to suspect that there may be one or more ultra light sterile neutrinos in nature, in addition to the familiar three active neutrinos ν_e, ν_μ, ν_τ. Two classes of models that provide natural ways to understand an ultralight sterile neutrino are: (i) the mirror universe scenario where all the particles and forces of nature are duplicated and (ii) models which contain large hidden dimensions with size of the order of a millimeter. Both models are inspired by string theories and provide new ways to understand the solar neutrino observations simultaneously with the atmospheric and the LSND data. We argue that in theories with large extra dimensions, the Kaluza-Klein (KK) states add new oscillation channels for the electron neutrino emitted from the solar core and provide a new fit to the solar neutrino data, for both the rates and the energy distributions. The model predicts characteristic dips in the neutrino energy distribution that can be tested by the SNO solar neutrino experiment.

1. Introduction

At present there appear to be three classes of experiments that provide evidence for neutrino oscillations: solar neutrino searches[1)], atmospheric neutrino data[2)] and an accelerator search for oscillations by the LSND experiment[3)]. A simultaneous understanding of all these data seems to require the existence of an ultralight neutrino (beyond the three known ones: ν_e, ν_μ, ν_τ), which must be sterile with respect to weak interactions. Within this four-neutrino scheme[4)], the solar ν_e deficit is explained by $\nu_e \to \nu_s$ (where ν_s is a sterile neutrino), the atmospheric ν_μ/ν_e anomaly is attributed to $\nu_\mu \to \nu_\tau$, and the LSND[3)] results are explained by the $\nu_e - \nu_\mu$ oscillation predicted in the model. The heavier near-degenerate ν_μ and ν_τ are required by the LSND results to be in the eV range and can therefore share the role of hot dark matter. Exactly this same pattern of neutrino masses and mixings appears necessary to allow production of heavy elements (A $\gtrsim$ 100) by type II supernovae [5)]. While qualitatively this neutrino scheme[6)] seems to explain all existing neutrino phenomena, solar neutrino observations are now sufficiently constraining that the small-angle MSW $\nu_e \to \nu_s$ explanation appears to be in some difficulty[7)], and seemingly one must go to some length [8)] in order to try to rescue this scheme. Although providing better fits to the solar data, even active-active transitions in a three-neutrino scheme do not give a quantitatively good explanation of all those data. The theoretical and phenomenological challenge then is to find a scheme which has an ultralight sterile neutrino and at the same time provides a fit to the solar neutrino data.

There are two classes of models which provide natural frameworks for understanding the ultralight sterile neutrinos. The first, the mirror universe scheme is discussed in section II and the second based theories with large extra dimensions is discussed in section III, IV and V. In section VI, I discuss the recent work with D. Caldwell and S. Yellin[9] on an attempt to fit the solar neutrino data in the second class of models; in section VII, I present some concluding remarks.

2. Mirror universe model

Let us start with a brief overview of the mirror matter models [10,11]. The basic idea of the model is: *simply duplicate the standard model or any extension of it in the gauge symmetric Lagrangian and allow for the possibility that symmetry breaking may be different in the two sectors.* (See table below) We assume that there is an exact mirror symmetry connecting the Lagrangians (prior to symmetry breaking) describing physics in each sector. Clearly the $W's, \gamma's$ etc in each sector differ from those in the other as do the quarks and leptons. When the symmetry breaking scale is different in the two sectors, we will call this the asymmetric mirror model[10]. The QCD scale being an independent scale in the theory could be arbitrary. We will allow both the weak scale and the QCD scale of the mirror sector to differ from those of the familiar sector[12] and assume the same common ratio ζ for both scales i.e. $< H' > / < H >= \Lambda'/\Lambda \equiv \zeta$.

$$
\begin{array}{ccc}
u,\ d,\ e,\ \nu_e & \leftrightarrow & u',\ d',\ e',\ \nu'_e \\
W,\ Z,\ \gamma,\ G & \leftrightarrow & W',\ Z',\ \gamma',\ G' \\
\phi, \nu_R, ... & \leftrightarrow & \phi',\ \nu'_R, ...
\end{array}
$$

$\leftarrow$Gravity$\rightarrow$

It is assumed that the two sectors in the universe are connected by only gravitational interactions. It was shown in [10,11] that gravity induces nonrenormalizable operators that generate mixings between the familiar and the mirror neutrinos. This is one of the ingredients in the resolution of neutrino puzzles. To get an idea of how this works, note that the lepton operators induced due to nonperturbative gravitational effects have the form $\frac{LHLH}{M_{Pl}}$, $\frac{LHL'H'}{M_{Pl}}$ and $\frac{L'H'L'H'}{M_{Pl}}$. After spontaneous breakdown (i.e. $< H >= v$ and $< H' >= v'$, we get for the mass matrix mixing the first generation neutrinos from each sector to have the form (in the basis ν_e, ν'_e):

$$M = \frac{v^2}{M_{Pl}} \begin{pmatrix} 1 & \zeta \\ \zeta & \lambda\zeta^2 \end{pmatrix} \tag{1}$$

where $\zeta \equiv v'/v$ defined above. To solve the solar neutrino puzzle via small angle MSW solution, we will choose $\lambda \sim 1$ and $\zeta \approx 10 - 30$. This gives the sterile neutrino mass

of order 10^{-3} eV or so, choosing $M_{Pl} \simeq 10^{18}$ GeV, which is also in the right range to explain the solar neutrino puzzle. Note that if the small angle solution is disfavored from the neutrino energy distribution as is suggested by the Super-Kamiokande data, then one may try new fits with the help of the extra two sterile neutrino states, which in principle can also be very light. We do not pursue this any further. It is however worth commenting that this model has a number of very interesting implications for the dark matter content of the universe. For instance, one can show that in the simple asymmetric inflation picture, the mirror proton has all th right properties to be the dark matter of the universe[13] and may even resolve some possible puzzles of dark matter field such as the explanation of the microlensing anomaly, cusp problem of collisionless dark matter etc.[12].

These models arise naturally in string theories with 3branes embedded in the higher dimensions[14], if two standard models are in the two branes. Gravity propagates in the full higher dimensional space time and therefore couples the two sectors.

3. Large extra dimensions and ultra light sterile neutrinos

One of the important predictions of string theories is the existence of more than three space dimensions. Recent progress in the understanding of the nonperturbative aspects of string theories have opened up the possibility that some of the hidden extra dimensions could be large[14,15] without contradicting observations. In particular, models where some of the extra dimensions have sizes as large as a millimeter and where the string scale is in the few TeV range have attracted a great deal of phenomenological attention in the past two years[15]. The basic assumption of these models, inspired by the D-branes in string theories, is that the space-time has a brane-bulk structure, where the brane is the familiar (3+1) dimensional space-time, with the standard model particles and forces residing in it, and the bulk consists of all space dimensions where gravity and other possible gauge singlet particles live. One could of course envision (3+d+1) dimensional D-branes where d-space dimensions have miniscule ($\leq TeV^{-1}$) size. The main interest in these models has been due to the fact that the low string scale provides an opportunity to test them using existing collider facilities.

A major challenge to these theories comes from the neutrino sector, the first problem being how one understands the small neutrino masses in a natural manner. The conventional seesaw[16] explanation which is believed to provide the most satisfactory way to understand this, requires that the new physics scale (or the scale of $SU(2)_R \times U(1)_{B-L}$ symmetry) be around 10^9 to 10^{12} GeV or higher, depending on the Dirac masses of the neutrinos whose magnitudes are not known. If the highest scale of the theory is a TeV, clearly the seesaw mechanism does not work, so one must look for alternatives. The second problem is that if one considers only the standard model group in the brane, operators such as $LHLH/M_*$ could be induced by string theory

in the low energy effective Lagrangian. For TeV scale strings this would obviously lead to unacceptable neutrino masses.

One mechanism suggested in Ref.[17] is to postulate the existence of one or more gauge singlet neutrinos, ν_B, in the bulk which couple to the lepton doublets in the brane. After electroweak symmetry breaking, this coupling can lead to neutrino Dirac masses, which are suppressed by the ratio $M_*/M_{P\ell}$, where $M_{P\ell}$ is the Planck mass and M_* is the string scale. This is sufficient to explain small neutrino masses and owes its origin to the large bulk volume that suppresses the effective Yukawa couplings of the Kaluza-Klein (KK) modes of the bulk neutrino to the brane fields. In this class of models, naturalness of small neutrino mass requires that one must assume the existence of a global B-L symmetry in the theory, since that will exclude the undesirable higher dimensional operators from the theory.

An alternative possibility[18] is to consider the brane theory to have an extended gauge symmetry which contains B-L symmetry as a subgroup. Phenomenological considerations, however, require that the local $B-L$ scale and hence the string scale be of order of 10^9 GeV or so. The extra dimensions in these models could also be large. Indeed, it is interesting that if there were only one large extra dimension, R (and all small extra dimensions are $\sim M_*^{-1}$), the formula

$$M_{P\ell}^2 \simeq M_*^3 R \tag{2}$$

leads to $M_* \simeq 10^9$ GeV if R$\sim$ mm[18]. While in these models, there is no strict need to introduce the bulk neutrinos to understand the small masses of known neutrinos, if we wanted to include the sterile neutrinos, one must add the ν_B. The high scale models may also have certain other advantages which we will see as we proceed.

Regardless of which path one chooses for understanding small neutrino masses, a very desirable feature of these models is that if the size of extra dimensions is of order a millimeter, the KK excitations of the bulk neutrino have masses of order 10^{-3} eV, which is in the range needed for a unified understanding of oscillation data[4], as already noted.

4. TeV scale models

To discuss the mechanisms in a concrete setting, let us first focus on TeV scale models. Here, one postulates a bulk neutrino, which is a singlet under the electroweak gauge group. Let us denote the bulk neutrino by $\nu_B(x^\mu, y)$. The bulk neutrino is represented by a four-component spinor and can be split into two chiral Weyl 2-component spinors as $\nu_B^T = (\chi^T, -i\phi^\dagger\sigma_2)$. The 2-component spinors χ and ϕ can be decomposed in terms of 4-dimensional Fourier components as follows:

$$\chi(x,y) = \frac{1}{\sqrt{2R}}\chi_{+,0} + \frac{1}{\sqrt{R}}\sum_{n=1}^{\infty}\left(\chi_{+,n} cos\frac{n\pi y}{R} + i\chi_{-,n} sin\frac{n\pi y}{R}\right). \tag{3}$$

There is a similar expression for ϕ. It has a five dimensional kinetic energy term and a coupling to the brane field $L(x^\mu)$. The full Lagrangian involving the ν_B is

$$\mathcal{L} = i\bar{\nu}_B\gamma_\mu\partial^\mu\nu_B + \kappa\bar{L}H\nu_{BR}(x, y=0) + i\int dy\ \bar{\nu}_{BL}(x,y)\partial_5\nu_{BR}(x,y) + h.c., \qquad (4)$$

where H denotes the Higgs doublet, and $\kappa = h\frac{M_*}{M_{P\ell}}$ is the suppressed Yukawa coupling. This leads to a Dirac mass for the neutrino[17] given by:

$$m = \frac{hv_{wk}M_*}{M_{P\ell}}, \qquad (5)$$

where v_{wk} is the scale of $SU(2)_L$ breaking. In terms of the 2-component fields, the mass term coming from the fifth component of the kinetic energy connects the fields χ_+ with ϕ_- and χ_- with ϕ_+, whereas it is only the ϕ_+ (or $\nu_{B,R,+}$) which couples to the brane neutrino $\nu_{e,L}$. Thus as far as the standard model particles and forces go, the fields ϕ_- and χ_+ are totally decoupled, and we will not consider them here. The mass matrix that we will write below therefore connects only ν_{eL}, $\phi_{+,n}$ and $\chi_{-,n}$.

From Eq. 5, we conclude that for $M_* \sim 10$ TeV, this leads to $m \simeq 10^{-4}h$ eV. It is encouraging that this number is in the right range to be of interest in the discussion of solar neutrino oscillation if the Yukawa coupling h is appropriately chosen. Furthermore, this neutrino is mixed with all the KK modes of the bulk neutrino, with a mixing mass $\sim \sqrt{2}m$; since the nth KK mode has a mass $nR^{-1} \equiv n\mu$, the mixing angle is given by $\sqrt{2}mR/n$. Note that for $R \sim 0.1mm$, this mixing angle is of the right order to be important in MSW transitions of solar neutrinos.

It is worth pointing out that this suppression of m is independent of the number and radius hierarchy of the extra dimensions, provided that our bulk neutrino propagates in the whole bulk. For simplicity, we will assume that there is only one extra dimension with radius of compactification as large as a millimeter, and the rest with much smaller compactification radii. The smaller dimensions will only contribute to the relationship between the Planck and the string scale, but their KK excitations will be very heavy and decouple from the neutrino spectrum. Thus, all the analysis can be done as in five dimensions.

In order to make this model applicable to resolving the observed oscillation phenomena, we have to extend the model as has been noted in [19]. Even if one wanted to understand the solar neutrino oscillation using this picture, one would have difficulty fitting all the rates in Gallium, Chlorine and the water Cherenkov data while simultaneously explaining the recoil energy distribution in the Super-Kamiokande data.

One way to do this would be to include new physics in the brane. We parameterize this in terms of an effective Majorana neutrino mass matrix in the brane:

$$\mathcal{M} = \begin{pmatrix} \delta_{ee} & \delta_{e\mu} & \delta_{e\tau} \\ \delta_{e\mu} & \delta_{\mu\mu} & m_0 \\ \delta_{e\tau} & m_0 & \delta_{\tau\tau} \end{pmatrix}. \qquad (6)$$

The origin of this pattern of brane neutrino masses have been discussed in [20]. If we assume that $m_0 \gg \delta_{ij}$; as a result, the $\nu_{\mu,\tau}$ decouple and do not affect the mixing between the bulk neutrino modes and the ν_e, and in the subsequent analysis we consider the remaining modes. Their mass matrix in the basis $(\nu_e, \nu^{(0)}_{BR,+}, \nu^{(1)}_{BL,-}, \nu^{(1)}_{BR,+}, \nu^{(2)}_{BL,-}, \nu^{(2)}_{BR,+}, ..)$ is given by:

$$\mathcal{M} \;=\; \mathcal{M}_{TeV} \;\equiv\; \begin{pmatrix} \delta_{ee} & m & 0 & \sqrt{2}m & 0 & \sqrt{2}m & \cdot & \cdot \\ m & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & \mu & 0 & 0 & \cdot & \cdot \\ \sqrt{2}m & 0 & \mu & 0 & 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 2\mu & \cdot & \cdot \\ \sqrt{2}m & 0 & 0 & 0 & 2\mu & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}. \tag{7}$$

One can evaluate the eigenvalues and the eigenstates of this matrix. The former are the solutions of the transcendental equation:

$$m_n \;=\; \delta_{ee} + \frac{\pi m^2}{\mu} cot\left(\frac{\pi m_n}{\mu}\right). \tag{8}$$

The equation for eigenstates is

$$\tilde{\nu}_n = \frac{1}{N_n}\left[\nu_e + \frac{m}{m_n}\nu^{(0)}_{B,+} + \sum_k \sqrt{2}m\left(\frac{m_n}{m_n^2 - k^2\mu^2}\nu^{(k)}_{B,-} + \frac{k\mu}{m_n^2 - k^2\mu^2}\nu^{(k)}_{B,+}\right)\right], \tag{9}$$

where we have used the notation $\pm$ for the left- and right-handed parts of the KK modes of the bulk neutrino in the two-component notation and dropped the L, R subscripts, the sum over k runs through the KK modes, and N_n is the normalization factor given by

$$N_n^2 = 1 + m^2\pi^2R^2 + \frac{(m_n - \delta_{ee})^2}{m^2}. \tag{10}$$

From Eq. 8 we see that there are two eigenvalues $m_\pm$ near m when $\delta_{ee} \ll m \ll \mu$, and these are given by $m_\pm \simeq \frac{\delta_{ee}}{2} \pm m$. These are the lowest two levels, and their mass difference square is given by $\sim 2m\delta_{ee}$ eV^2. From Eq. 9, we see that they are maximally mixed. Therefore, if $\delta_{ee} \sim 10^{-7}$ eV, then the transition between these levels can lead to vacuum oscillation ("VO") of the solar neutrinos. This will be one of the ingredients of our new solution, as we see below.

5. Local $B - L$ symmetry models:

A second way to achieve the same phenomenology is possible using a much higher string scale. In this class of models [18], one postulates that the theory in the brane is left-right symmetric so that it contains the B-L as a local symmetry. The gauge

group of the model is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with field content for leptons given by left and right doublets $\psi_{L,R} \equiv (\nu, \ell)_{L,R}$ and in the Higgs sector bidoublet $\phi = (2, 2, 0)$, doublets $\chi_{L,R}$. As in the previous case we choose a single bulk neutrino. We then impose the following Z_2 symmetry on the Lagrangian under which $\chi_L \to -\chi_L$, $\nu_{B,R} \to -\nu_{B,R}$ and all other fields are invariant. Note that under this symmetry, the 5th coordinate $y \to -y$. The invariant Lagrangian is

$$\mathcal{L} = h_\ell \frac{(\psi_R \chi_R)^2 + (\psi_L \chi_L)^2}{M_*} + \bar{\psi}_L \phi \psi_R + \frac{f}{M_*^{1/2}} [\bar{\psi}_{eR} \tilde{\chi}_R + \bar{\psi}_{eL} \tilde{\chi}_L] \nu_B(x, y = 0) \quad (11)$$

$$+ i \int dy \; \bar{\nu}_{BL}(x, y) \partial_5 \nu_{BR}(x, y) + h.c., \quad (12)$$

where f and h_ℓ are Yukawa couplings and $\tilde{\chi}_{L,R} = i\tau_2 \chi^*_{L,R}$. We then break the right-handed symmetry with $\langle \chi^0_R \rangle = v_R$, while at the same time keeping $\langle \chi_L \rangle = 0$. We expect v_R to be of order of the string scale, M_*. The Lagrangian involving the electron neutrino and the bulk neutrinos then becomes

$$\mathcal{L} = f \frac{v_R^2}{M_*} \nu_R \nu_R + m \bar{\nu}_{eL} \nu_{eR} + \alpha \bar{\nu}_{eR} \nu_{BL}(x, y = 0) + i \int dy \; \bar{\nu}_{BL}(x, y) \partial_5 \nu_{BR}(x, y) + h.c., \quad (13)$$

where $\alpha \simeq \frac{h_\ell M_* v_R}{M_{P\ell}}$ and $m = h_\ell v_{wk}$. In this section we discuss only the $\nu_e - \nu_B$ sector and address the full three generation mixing in Appendix B. In what follows, the Majorana mass of the ν_R is denoted by M, i.e., $M = f \frac{v_R^2}{M_*}$. This leads to the mass matrix of Eq. 14 which mixes the brane neutrinos with the KK modes of ν_B. For $\langle \chi^0_R \rangle = v_R \gg \langle \phi \rangle, \mu$, which we assume, ν_{eR} has a Majorana mass, M, which is much bigger than any other masses in the theory (ignoring real superheavy KK modes), and ν_{eR} decouples. One can then write an effective Lagrangian at low energies (i.e., $E \ll v_R$) using the seesaw mechanism. The effective mass matrix for the light modes can be written down using the same notation for the KK modes of ν_B as in the previous section. In the basis $(\nu_{eL}, \nu^{(0)}_{BL,+}, \nu^{(1)}_{BL,+}, \nu^{(1)}_{BR,-}, \nu^{(2)}_{BL,+}, \nu^{(2)}_{BR,-}, \cdot, \cdot)$ it is given by:

$$\mathcal{M} = \mathcal{M}_{B-L} \equiv \frac{1}{M} \begin{pmatrix} m^2 & \alpha m & \sqrt{2} m\alpha & 0 & \sqrt{2} m\alpha & 0 & \cdot & \cdot \\ \alpha m & \alpha^2 & \sqrt{2}\alpha^2 & 0 & \sqrt{2}\alpha^2 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha m & \sqrt{2}\alpha^2 & 2\alpha^2 & M\mu & 2\alpha^2 & 0 & \cdot & \cdot \\ 0 & 0 & M\mu & 0 & 0 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha m & \sqrt{2}\alpha^2 & 2\alpha^2 & 0 & 2\alpha^2 & 2M\mu & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 2M\mu & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}. \quad (14)$$

It is easy to see that the lowest eigenvalue of this matrix is zero. The transcendental equation describing the rest of the eigenvalues is

$$\frac{m^2}{M} + \frac{\alpha^2}{M} \frac{\pi m_n}{\mu} cot \frac{\pi m_n}{\mu} = m_n. \quad (15)$$

The next lowest eigenvalue solution of this equation is $m_1 \simeq \frac{m^2+\alpha^2}{M}$. For $m, \alpha \sim 1-5$ MeV (similar to the first generation fermion mass) and $M \simeq 10^9$ GeV, we get this eigenvalue to be of order $10^{-6} - 2.5 \times 10^{-5}$ eV. Its square is therefore in the range where the VO solution to the solar neutrino puzzle can be applied. Also for $m \simeq \alpha$, the mixing angle between the zero eigenvalue mode and this mode is maximal.

In the diagonalization, we have ignored the radiative corrections that allow us to extrapolate to the weak scale the above mass matrix which is valid at the string scale. Going to the weak scale, M_Z = mass of the Z^0 meson, there are two kinds of contributions that dominate the radiative corrections: one arising from the top quark coupling to the standard model Higgs doublet in the effective $LHLH$ operator induced by the seesaw mechanism[21], and a second one which can arise from self couplings of the Higgs fields (for the nonsupersymmetric version of the model). The top quark contribution replaces the parameter m in the matrix in Eq. 14 by $m\left(1 + \frac{6h_t^2}{16\pi^2} ln(M_*/M_Z)\right)$ in the off-diagonal terms, whereas the self-scalar coupling contributes only to the m^2 term. Thus the m^2 term in Eq. 14 is replaced by $m^2\left(1 + \frac{6h_t^2}{16\pi^2} ln(M_*/M_Z) + \frac{2\lambda}{16\pi^2} ln(M_*/M_Z)\right)^2$. (This expression will have more terms involving extra scalar self couplings if there is more than one Higgs doublet in the low energy theory). Let us denote the rest of the radiative corrections by ϵ in the m^2 term and ϵ' in the off-diagonal ones. Such radiative corrections could come from one loop contributions at the scale M itself. The magnitude of the radiative corrections also could increase if the low energy theory below M_* has more than one Higgs doublet, as noted. It is therefore not implausible to assume that the radiative corrections are significant. With redefinition of the off-diagonal m terms, $\mathcal{M}$ takes the same form as in Eq. 14, except the m^2 term is replaced with $m^2 + \Delta$, with $\Delta = (m^2/16\pi^2)(24h_t^2\lambda + 4\lambda^2 + \epsilon - \epsilon')ln(M_*/M_Z)$. The characteristic equation for the m_n becomes

$$\frac{m^2+\Delta}{M} + \frac{\alpha^2\pi(m_n - \Delta/M)}{M\mu} cot\frac{\pi m_n}{\mu} = m_n. \tag{16}$$

It is now easy to see that if $\Delta << (\alpha^2 + m^2)$, the zero eigenvalue is replaced by $\sim \frac{\alpha^2\Delta}{M(\alpha^2+m^2)}$. Clearly, we want Δ closer to m^2, α^2 to obtain our desired parameters. For this purpose, we choose parameters λ and the other radiative corrections (i.e., ϵ, ϵ') appropriately, so that the two lowest eigenvalues are of almost equal magnitude, and the parameters of the next section can be reproduced. This situation can be realized more easily if the theory is nonsupersymmetric all the way to the string scale. If there is supersymmetry all the way down to the TeV scale, then the scalar self coupling contributions "kick" in only below M_{SUSY} and one has to stretch parameters and perhaps require a two-Higgs doublet model below the SUSY scale to realize the parameters used below.

The detailed diagonalization of this matrix has been carried out in [20]. The mixing of the ν_e with the n-th eigen mode (with $m_n \sim n\mu$) is $\approx \frac{m\alpha}{Mn\mu}$. For $m\alpha/M \sim 10^{-5}$

eV and $\mu \sim 10^{-3}$ eV, this mixing is of order of a percent and is therefore in the interesting range for a possible MSW transition of the solar neutrinos.

6. Solar neutrino data fit by a combination of vacuum and MSW oscillations

In this section, we discuss the question of how to understand the solar neutrino data in these models, while at the same time explaining the atmospheric as well as the LSND data. There have been several recent papers that have addressed the issue of explaining observed oscillation data in models with large extra dimensions [9,22,23,19,24,25,26,27,28]. In particular, in Ref.[19] it has been shown that while the overall features of the solar and atmospheric data can be accommodated in scenarios with standard model in the brane and three bulk neutrinos, it is not possible to explain simultaneously the LSND observation for the $\nu_\mu - \nu_e$ oscillation probability, and one must incorporate new physics in the brane. In the models presented in [9,20], new physics is assumed in the brane such that the ν_μ and ν_τ are heavier due to its effect and it is the ν_e that is most sensitive to the presence of the bulk neutrino. We consider only one bulk neutrino as already mentioned. The $\nu_\mu - \nu_\tau$ mass difference is generated via the radiative corrections in the TeV scale models, and the seesaw mechanism in the local B-L models. Since we arrange the models so that the mass of the $\nu_{\mu,\tau}$ pair is in the eV range, this provides a way to accomodate the LSND results. Let us therefore focus on the solar data.

A first glance at the values of the parameters of the model such as m in Eq.(3) and $R^{-1} \sim 10^{-3}$ eV suggests that perhaps one should seek a solution of the solar neutrino data in these models using the small-angle MSW mechanism[22]. However, the present Super-Kamiokande recoil energy distribution seems to disfavor such an interpretation, although any definitive conclusion should perhaps wait till more data accumulates. In any case if the present trend of the data near the higher energy region of the solar neutrino spectrum from Super-Kamiokande persists, it is likely to disfavor the small-angle MSW solution and tend to favor a vacuum oscillation. However, the latter does not give correct rates for the three types of solar neutrino experiments. Note that for sterile neutrinos the large-mixing-angle MSW solution does not work.

As discussed in [9], our solution to the solar neutrino data consists of two components: one involving the vacuum oscillation of ν_e to ν_B^0 and the second one involving the MSW transition of the higher energy ν_e's to higher KK modes of the ν_B. The vacuum oscillation part is straightforward, and in order to get a better fit we have to adjust the $\Delta m^2_{\nu_e - \nu_B^0}$. On the other hand, to discuss the MSW effect for the case of bulk neutrinos, we need to include the effect of solar matter on the infinite dimensional neutrino mass matrix.

To have a physical understanding of our strategy, note that the simplest way to reconcile the rates for the three classes of solar experiments is to "kill" the ^{7}Be

neutrinos, reduce the ^{8}B neutrinos by half and leave the pp neutrinos alone. To achieve this using pure vacuum oscillation, one may put a node of the survival probability function P_{ee} around 0.86 MeV. However, for an arbitrary node number, the oscillatory behaviour of P_{ee} before and after 0.86 MeV cannot in general satisfy the other two requirements mentioned above; specifically, if there are more nodes prior to 0.8 MeV, the Gallium pp neutrinos get suppressed. If one uses the first node to "kill ^{7}Be", then for ^{8}B neutrino energies the P_{ee} is close to one and not half as would be desirable. The strategy generally employed is not to have a node at the precise ^7Be energy but rather somewhat away so that it reduces ^{7}Be to a value above zero. This requires that one must reduce the ^{8}B neutrinos by much more than 50%, so one can fit Chlorine data. The water data then requires an additional contribution, which, in the case of active vacuum oscillation (VO), is provided by the neutral current cross section, amounting to about 16% of the charged current one. Thus in a pure two-neutrino oscillation picture, VO comes close to working for oscillation to active neutrinos but certainly does not work for active to sterile oscillation. It is here that the large extra dimensions come to the rescue.

In our model, both vacuum oscillations and MSW oscillations are important. This is because the lowest mass pair of neutrinos is split by a very small mass difference, whereas the KK states have to be separated by $\sim 10^{-3}$ eV We can then use the first node of P_{ee} to suppress the ^{7}Be. Going up in energy toward ^{8}B neutrinos, the survival probability, which in the VO case would have risen to very near one, is now suppressed by the small-angle MSW transitions to the different KK excitations of the bulk neutrino. Furthermore, the presence of multiple dips arising from sequential MSW transitions helps to flatten the spectrum in a desirable manner. This is the essence of our new way to fit the solar neutrino observations[9]. Clearly, it is important to include the matter effect in the diagonalization of the neutrino mass matrix as has been discussed in [20]. We do not go into the details; instead, we briefly discuss qualitatively the origin of the dips in the survival probability of the solar neutrino, since it plays a key role in flattening the spectrum. The pattern of dips in our model is shown in Fig.1.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of 0.63 MeV/(2n+1) for n = 0, 1, 2, ... Thus Fig. 1 shows nearly zero ν_e survival near 0.63 MeV, partly eliminating the ^7Be contribution at 0.862 MeV, and giving a dip at the lowest neutrino energy. Increasing δ_{ee} moves the low energy dip to the right into Gallium's most sensitive pp energy range, making the fit worse. Decreasing δ_{ee} increases Gallium, but hurts the Chlorine fit by moving the higher energy vacuum oscillation dip further to the left of the ^7Be peak. Note that the pattern of two eigenstates very close in mass persists for the Kaluza-Klein excitations as well. These MSW resonances start causing the third and fourth eigenstates to be significantly occupied above ~ 0.8 MeV, the fifth and sixth eigenstates above ~ 3.7 MeV, the 7'th and 8'th above ~ 8.6 MeV, and the 9'th and 10'th above ~ 15.2 MeV.

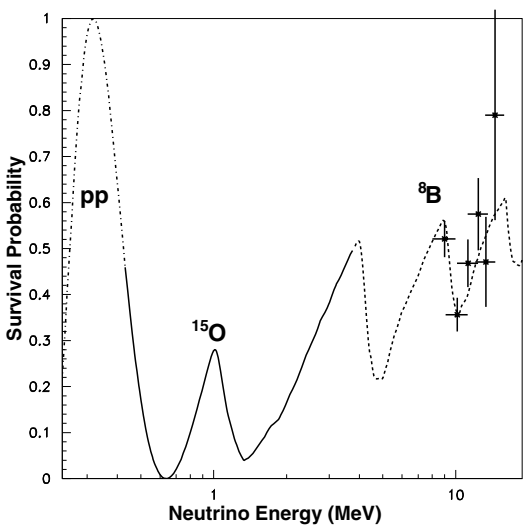

Figure 1: Energy dependence of the ν_e survival probability when $R = 58\mu$m, $mR = 0.0094$, $\delta_{ee} = 0.84 \times 10^{-7}$ eV. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the pp reaction, the solid part assumes ^{15}O radial dependence, and the dashed part assumes ^{8}B radial dependence.

Fig. 1 shows dips in survival probability just above these energy thresholds.

To understand the origin of the dips, note that the MSW resonance condition is given by

$$m_{res}^2 \simeq \frac{4G_F\rho}{\sqrt{2}m_p}E_{res} \tag{17}$$

As a result, as the neutrino energy E increases, the resonance condition is satisfied for higher and higher KK modes of the bulk neutrino. The survival probability at and after the resonance is given by

$$P_{ee} \simeq e^{-\frac{\pi m_n^2 sin^2 2\theta}{2E}R_{eff}} \tag{18}$$

for $E \geq E_{res}$ and $P_{ee} \sim 1$ for $E < E_{res}$. The cumulative survival probability is given by $P = P^{(1)}P^{(2)} \cdots P^{(n)}$. Note that due the exponential being larger than one at the resonance (which can be chaecked by putting numbers), as soon as a new KK level is crossed, the P_{ee} dips to a value much less than one and then starts to rise as E increases giving rise to the dip structure. The net of effect of the combined dip structure is to flatten the spectrum and bringing it into cordance with data.

The typical values of the survival probability within the ^{8}B region ($\sim$ 6 to $\sim$ 14 MeV) are quite sensitive to the value of mR. As can be seen from Eq. 10, higher mR increases $1/N_n \approx m/m_n \approx mR/n$ for various n, and thereby increases

ν_e coupling to higher mass eigenstates, strengthens MSW resonances, and lowers ν_e survival probability.

One may also seek fits with δ_{ee} constrained to be very small, thereby eliminating vacuum oscillations. We found $\chi^2 = 4.4$ for the best such fit to the total rates only. The same parameters then used with the Super-K spectrum gave $\chi^2 = 18.7$, corresponding to a probability of 41%.

Finally, it is also worth noting that if no such dip structure is observed in the SNO data, this would constrain models with large extra dimensions. In particular this would imply that either the mixing with KK modes is very small i.e. m is small or the radius of the extra dimension is small. In particular, we find

$$R^{-2} \geq \frac{2\sqrt{2}G_F\rho}{m_p} 15MeV \simeq 10^{-4} \;\; eV^2 \tag{19}$$

This upper limit on the size of the extra dimension is more stringent than the current limits from gravity experiments, which give $R^{-2} \geq 10^{-6}$ eV^2.

7. Consequences and comments

In conclusion, the presence of large extra dimensions with neutrinos in them would provide an interesting reolution of the neutrino oscillation puzzles. In particular, it predicts a characteristic energy spectrum of Fig. 1 which should be observable by SNO.

If the relatively large dimension size is confirmed by SNO, this will raise issues about cosmological and supernova limits from the effects of high Kaluza-Klein states of both sterile neutrinos[23,24,29] and gravitons[30]. These constaints are much stronger than those from new contributions to low energy weak processes[31], but are somewhat suspect because the two regimes, the early universe and the supernova core, are very complex and not yet fully understood. Also, for sterile neutrino limits, the phenomenology presented here is aided by there being a single Kaluza-Klein tower based on an exceedingly small mass, the VO Δm^2 is an order of magnitude smaller than usual, and for MSW the equivalent $\sin^2 2\theta$ is more than an order of magnitude smaller than for standard fits.

On theoretical side, a legitimate question is whether these considerations can provide any clue to the magnitude of the string scale M_*. There appears to be several problems with theories with $M_* \sim 1 - 10$ TeV. For instance for these models, the universe re-heat temperature could be very low[33] e.g. as low as an MeV for $M_* \sim 10$ TeV, in order to prevent the over-closure of the universe due to production of heavier KK states. This would clearly put our successful understanding of nucleosynthesis in jeopardy, let alone questions such as the origin of matter etc. In high string scale, theories, on the other hand, the reheat temperature can be as large as 10-100 GeV depending on the value of M_* and the number of large extra dimensions (the less the

better). For the neutrino scenarios, these arguments would seem to prefer the local B-L models over the global B-L case. There may also be other reasons to prefer high scale (or local B-L) theories e.g.[32].

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8. References

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32) M. Fabrichese, M. Piai, G. Tasinato, hep-ph/0012227 argue that due to the gravitational potential of the brane fields in the 5th direction, there is a new contribution to the bulk neutrinos in the neutrino mass matrix. This can affect the mixing angle of the brane to bulk neutrinos. If we take the estimate given in reference [32], we find that this effect for the zero mode of the bulk neutrino dominates the usual matter effect in the solar core by a factor $10^5 \left(\frac{TeV}{M_*}\right)^3$.

Clearly for $M_* \leq 50$ TeV, it dominates. It is then clear that the effect is completely negligible in the local B-L model.

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