THE INFLATIONARY UNIVERSE

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ABSTRACT

We feel lucky that at the time of this Conference there is more and more evidence that there was an inflationary epoch very early in the history of the universe. In this talk we will describe how we might be able to get some understanding of the paradigm of inflation.

1. Introduction

It is commonly believed that the Universe underwent an early era of cosmological inflation. The flatness and the horizon problems of the standard big bang cosmology are elegantly solved if, during the evolution of the early Universe, the energy density is dominated by some vacuum energy and comoving scales grow quasi-exponentially. The prediction of the simplest models of inflation is a flat Universe, *i.e.* $\Omega_{tot} = 1$ with great precision.

Inflation [1)] has also become the dominant paradigm for understanding the initial conditions for structure formation and for Cosmic Microwave Background (CMB) anisotropy. In the inflationary picture, primordial density and gravity-wave fluctuations are created from quantum fluctuations "redshifted" out of the horizon during an early period of superluminal expansion of the universe, where they are "frozen" as perturbations in the background metric. Metric perturbations at the surface of last scattering are observable as temperature anisotropy in the CMB.

The first and most impressive confirmation of the inflationary paradigm came when the CMB anisotropies were firmly detected by the COBE satellite in 1992. Subsequently, it became clear that the measurements of the spectrum of the CMB anisotropy can provide very detailed information about the fundamental cosmological parameters and other crucial parameters for particle physics. At the time of this Conference, this era of precision CMB anisotropy measurements has just begun!. This is thanks to the the results of two balloon-borne experiments, Boomerang [2,3)] – which was flown over Antarctica in 1999 – and MAXIMA-1 [4,5)]. The observed first acoustic peak at $\ell \sim 200$ (already detected by previous experiments, see e.g. [6,7)]) indicates that the curvature of the Universe is consistent with flatness, thus providing another convincing confirmation of the fact that the early Universe experienced a period of superluminal acceleration.

2. Perturbations Produced During Inflation

One of the striking features of the cosmic background radiation (CBR) temperature fluctuations is the growing evidence that the fluctuations are acausal. The CBR fluctuations were largely imprinted at the time of last-scattering, about 300,000 years after the bang. However, there seems to be fluctuations on length scales much larger than 300,000 light years! How could a causal process imprint correlations on scales larger than the light-travel distance since the time of the bang? The answer is inflation.

In order to see how inflation solves this problem, first consider the evolution of the Hubble radius with the scale factor $a(t)$ (here and throughout the paper "RD" is short for radiation dominated, and "MD" implies matter dominated)

$$R_H \equiv H^{-1} = \left(\frac{\dot{a}}{a}\right)^{-1} \propto \rho^{-1/2} \propto \begin{cases} a^2 & \text{(RD)} \\ a^{3/2} & \text{(MD)}. \end{cases} \tag{1}$$

In a $k=0$ matter-dominated universe the age is related to H by $t=(2/3)H^{-1}$, so $R_H=(3/2)t$. In the early radiation-dominated universe $t=(1/2)H^{-1}$, so $R_H=2t$. On length scales smaller than R_H it is possible to move material around and make an imprint upon the universe. Scales larger than R_H are "beyond the Hubble radius," and the expansion of the universe prevents the establishment of any perturbation on scales larger than R_H.

Next consider the evolution of some physical length scale λ. Clearly, any physical length scale changes in expansion in proportion to $a(t)$.

Correlations on physical length scales λ larger than $R_H(t)$ are often called *acausal.*

Now let us form the dimensionless ratio $L \equiv \lambda/R_H$. If L is smaller than unity, the length scale is smaller than the Hubble radius and it is possible to imagine some microphysical process establishing perturbations on that scale, while if L is larger than unity, no microphysical process can account for perturbations on that scale. Since $R_H = a/\dot{a}$, and $\lambda \propto a$, the ratio L is proportional to $\dot{a}$, and $\dot{L}$ scales as $\ddot{a}$, which in turn is proportional to $-(\rho+3p)$. There are two possible scenarios for $\dot{L}$ depending upon the sign of $\rho+3p$:

$$\dot{L} \begin{cases} <0 \rightarrow & R_H \text{ grows faster than } \lambda, & \text{happens for } \rho+3p>0 \\ >0 \rightarrow & R_H \text{ grows more slowly than } \lambda, & \text{happens for } \rho+3p<0. \end{cases} \tag{2}$$

In the standard MD or RD universe, $\rho+3p>0$, and R_H grows faster than λ. Perturbations that appear to be noncausal at last scattering can be produced if sometime during the early evolution of the universe the expansion was such that $\ddot{a}>0$. If $\ddot{a}>0$, then the Hubble radius will increase more slowly than any physical scale increases during inflation. Physical length scales larger than the Hubble radius at the time of last scattering would have been smaller than the Hubble radius during the accelerated era. Therefore it is possible to imprint correlations during inflation as a scale passes out of the horizon and have it appear as an acausal perturbation at last scattering.

From Einstein's equations, the "acceleration" is related to the energy density and the pressure as $\ddot{a} \propto -(\rho + 3p)$. Therefore, an acceleration (positive $\ddot{a}$) requires an unusual equation of state with $\rho + 3p < 0$. This is the condition for "accelerated expansion" or "inflation."

2.1. Simple Dynamics of Inflation: The Inflaton

In building inflation models [8] it is necessary to find a mechanism by which a universe dominated by vacuum energy can make a transition from the inflationary universe to a matter-dominated or radiation-dominated universe. There is some unknown dynamics causing the expansion rate to change with time. There may be several degrees of freedom involved in determining the expansion rate during inflation, but the simplest assumption is that there is only one dynamical degree of freedom responsible for the evolution of the expansion rate.

If there is a single degree of freedom at work during inflation, then the evolution from the inflationary phase may be modeled by the action of a scalar field ϕ evolving under the influence of a potential $V(\phi)$. Let's imagine the scalar field is displaced from the minimum of its potential. If the energy density of the universe is dominated by the potential energy of the scalar field ϕ, known as the *inflaton*, then $\rho + 3p$ will be negative. The vacuum energy disappears when the scalar field evolves to its minimum. The amount of time required for the scalar field to evolve to its minimum and inflation to end (or even more useful, the number of e-folds of growth of the scale factor) can be found by solving the classical field equation for the evolution of the inflaton field, which is simply given by $\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$.

2.2. Quantum fluctuations

In addition to the classical motion of the inflaton field, during inflation there are quantum fluctuations. Since the total energy density of the universe is dominated by the inflaton potential energy density, fluctuations in the inflaton field lead to fluctuations in the energy density. Because of the rapid expansion of the universe during inflation, these fluctuations in the energy density are frozen into super-Hubble-radius-size perturbations. Later, in the radiation or matter-dominated era they will come within the Hubble radius as if they were *noncausal* perturbations.

The spectrum and amplitude of perturbations depend upon the nature of the inflaton potential. One starts with the action for gravity (the Einstein–Hilbert action) plus a minimally-coupled scalar inflaton field ϕ:

$$S = -\int d^4x \sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right] . \tag{3}$$

Here R is the Ricci curvature scalar. Quantum fluctuations result in perturbations in the metric tensor and the inflaton field

$$g_{\mu\nu} \rightarrow g_{\mu\nu}^{FRW} + \delta g_{\mu\nu} \quad ; \qquad \phi \rightarrow \phi_0 + \delta\phi , \tag{4}$$

where $g_{\mu\nu}^{FRW}$ is the Friedmann–Robertson–Walker metric, and $\phi_0(t)$ is the classical solution for the homogeneous, isotropic evolution of the inflaton. The action describing the dynamics of the small perturbations can be written as

$$\delta_2 S = \frac{1}{2}\int d^4x \left[\partial_\mu u \partial^\mu u + z^{-1}\frac{d^2 z}{d\tau^2}\ u^2\right] \quad ; \quad z = a\dot{\phi}/H , \tag{5}$$

i.e., the action in conformal time τ ($d\tau^2 = a^2(t)dt^2$) for a scalar field in Minkowski space, with mass-squared $m_u^2 = -z^{-1}d^2z/d\tau^2$. Here, the scalar field u is a combination of metric fluctuations $\delta g_{\mu\nu}$ and scalar field fluctuations $\delta\phi$. This scalar field is related to the amplitude of the density perturbation.

The simple matter of calculating the perturbation spectrum for a noninteracting scalar field in Minkowski space will give the amplitude and spectrum of the density perturbations. The problem is that the solution to the field equations depends upon the background field evolution through the dependence of the mass of the field upon z. Different choices for the inflaton potential $V(\phi)$ results in different background field evolutions, and hence, different spectra and amplitudes for the density perturbations. Although the scalar and tensor spectra depend upon $V(\phi)$, for most potentials they can be characterized by Q_{RMS}^{PS} (the amplitude of the scalar and tensor spectra on large length scales added in quadrature), n (the scalar spectral index describing the best power-law fit of the primordial scalar spectrum), r (the ratio of the tensor-to-scalar contribution to C_2 in the angular power spectrum), and n_T (the tensor spectral index describing the best power-law fit of the primordial tensor spectrum). For single-field, slow-roll inflation models, there is a relationship between n_T and r, so in fact there are only three independent variables. Furthermore, the amplitude of the fluctuations often depends upon a free parameter in the potential, and the spectra are normalized by Q_{RMS}^{PS}. This leads to a characterization of a wide-range of inflaton potentials in terms of two numbers, n and r.

2.3. Models of inflation

Probably the first step in sorting out different models is a classification scheme. One proposed classification scheme has two main types. Type-I inflation models are models based on a single inflaton field, slowly rolling under the influence of an inflaton potential $V(\phi)$. Anything that is not Type I is denoted as a Type-II model.

There are subclasses within Type I. Type-Ia models are "large-field" models, where the inflaton field starts large and evolves toward its minimum. Examples of large-field models are chaotic inflation and power-law inflation. Type-Ib models are "small-field" models, where the inflaton field starts small and evolves to its minimum at larger values of the inflaton field. Examples of small-field models are new inflation and natural inflation. Finally, hybrid-inflation models are classified as Type-Ic models. In Type-Ia and Type-Ib models the vacuum energy is approximately zero at the end of inflation. Hybrid models have a significant vacuum energy at the end of inflation. Hybrid inflation is usually terminated by a first-order phase transition or by the action of a second scalar field. A more accurate description of large-field and small-field potential is the sign of the second derivative of the potential: large-field models have $V'' > 0$ while small-field models have $V'' < 0$.

Of course a classification scheme is only reasonable if there are some observable quantities that can differentiate between different schemes. It turns out that the different Type-I models fill in different regions of the n–r. For a given spectral index n, small-field models have a smaller value of r.

2.4. What we see on the sky is the dynamics of inflation

As we said, inflation produces both scalar and tensor perturbations, and each generate microwave anisotropies. If $V(\phi)$ is known, the perturbation spectra can be computed exactly in linear perturbation theory through integration of the relevant mode equations. Measuring the spectral index n and r will therefore give us a chance to be introduced to the inflaton dynamics!

Two crucial ingredients are the primordial scalar and tensor perturbation spectra $A_S(k)$ and $A_T(k)$. The scalar and tensor perturbations depend upon the behavior of the expansion rate during inflation, which in turn depends on the value of the scaler inflaton field ϕ. In order to track the change in the expansion rate, we define slow-roll parameters ϵ, η, and ξ as

$$\epsilon(\phi) \equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H'(\phi)}{H(\phi)}\right]^2 \qquad \eta(\phi) \equiv \frac{m_{\rm Pl}^2}{4\pi}\frac{H''(\phi)}{H(\phi)} \qquad \xi(\phi) \equiv \frac{m_{\rm Pl}^2}{4\pi}\left(\frac{H'(\phi)H'''(\phi)}{H^2(\phi)}\right)^{1/2} , \tag{6}$$

where here the prime superscript implies $d/d\phi$. If the slow-roll parameters are small, ϵ, η, and ξ can be expressed in terms of derivatives of the inflaton potential:

$$\epsilon(\phi) = \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \qquad \eta(\phi) = \frac{m_{\rm Pl}^2}{8\pi}\frac{V''}{V} \qquad \xi^2(\phi) = \left(\frac{m_{\rm Pl}^2}{8\pi}\right)^2\frac{V'V'''}{V^2} . \tag{7}$$

As long as the slow-roll parameters are small compared to unity, the scalar and tensor perturbation amplitudes $A_S(k)$ and $A_T(k)$ are given by

$$A_S^2(k) \simeq \frac{4}{25\pi}\left(\frac{H}{m_{\rm Pl}}\right)^2 \epsilon^{-1}\left[1-(2C+1)\epsilon+C\eta\right]^2 \quad (8)$$

$$A_T^2(k) \simeq \frac{4}{25\pi}\left(\frac{H}{m_{\rm Pl}}\right)^2 \left[1-(C+1)\epsilon\right]^2 , \quad (9)$$

where H, ϵ, and η are to be determined at the value of ϕ when $k = aH$ during inflation, and where $C = -2 + \ln 2 + \gamma \simeq -0.73$ is a numerical constant, γ being the Euler constant. These equations are the basis of the reconstruction process. From the expressions for A_S^2 and A_T^2 one can express the scalar and tensor spectral indices, defined as

$$n(k) - 1 \equiv \frac{d\ln A_S^2}{d\ln k} \qquad n_T(k) = \frac{d\ln A_T^2}{d\ln k} \quad (10)$$

in terms of slow-roll parameters. Inflation therefore predicts adiabatic, gaussian perturbations with a flat spectrum. Indeed, if slow-roll is valid, one expects the spectral index to be significantly close to unity.

We have recently investigated the observational constraints from the current CMB anisotropy measurements on the simplest inflation models, characterized by a single scalar field ϕ, in the parameter space consisting of scalar spectral index n and tensor/scalar ratio r [9]. If we include constraints on the baryon density from big bang nucleosynthesis (BBN), we have shown that the favored inflationary models have negligible tensor amplitude and a "red" tilt, with a best fit of $n \simeq 0.93$, which is consistent with the simplest "small-field" inflation models, but rules out large-field models at the 1σ level. Without including BBN constraints, a broader range of models are consistent with the data. The best fit (assuming negligible reionization) is a scale-invariant spectrum, $n \simeq 1$, which includes large-field and hybrid scenarios. Large-field models (such as chaotic and power-law inflation) with tilt $n < 0.9$ are strongly disfavored in all cases.

3. Conclusions and Perspectives

It was once said that the words "precision" and "cosmology" could not both appear in a sentence containing an even number of negatives. However that statement is now out of date, or at the very least, very soon will be out of date. A number of new instruments will come on line in the next few years and revolutionize cosmology.

There is now a world-wide campaign to pin down the microwave anisotropies. Long-duration balloon flights are already changing the situation. Finally, in the next decade two satellites, a NASA mission —the Microwave Anisotropy Probe (MAP)— and an ESA mission—PLANCK, will culminate in a determination of the spectrum with errors much smaller than the present errors. Of course we don't know what the shape of the spectrum will turn out to be, but we can anticipate errors as small as shown in the figure.

With errors of this magnitude, fitting the spectrum will allow determination of n and r, useful for inflation, as well as determination of the cosmological parameters (H_0, Ω_0, Ω_B, Λ, *etc.*) to a few percent. Future observations promise to significantly narrow the allowed regions of the parameter space, and will potentially make it possible to rule out entire classes of inflation models.

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5. References

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