Venice, 7 March 01

MODELS OF ν MASSES AND MIXINGS

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CERN

Our work on the subject:

G.A., F. Feruglio

- Phys. Letters B439(1998)112
- JHEP 11(1998)021
- Phys. Letters B451(1999)388
- Phys. Rep. 320(1999)295<- A REVIEW
- hep-ph/0102301<- NEW

G.A., F. Feruglio, I. Masina

- Phys. Letters B472(2000)382
- JHEP 11(2000)040

THERE ARE MANY ALTERNATIVE MODELS

- $\boxed{\geq 4\nu\text{'s}}$ (LSND)

  $\nu_{STERILE}$??

  atm — $m^2 \sim eV^2$

  LSND

  $\odot$

  $\boxed{3\nu\text{'s}}$ (NO LSND)

  HOT DARK MATTER

- DEGENERATE — $m^2 \sim eV^2$

  $0\nu\beta\beta$ CLOSE TO LIMIT

- INVERSE HIERARCHY

  $\odot$ — $\frac{1}{2}$ $m^2 \sim 10^{-3} eV^2$

  atm

  3

- NORMAL HIERARCHY

  3 $m^2 \sim 10^{-3} eV^2$

  atm

  2

  $\odot$ 1

  I WILL ARGUE FOR THIS CASE

  $$m_\nu \sim m^T_{Dirac} M^{-1} m_{Dirac}$$

  DOMINANCE OF SEE-SAW + GUT's

  CONNECTION TO q, ℓ MASSES VIA GUT'S

  MODELS AND IDEAS

- $SU(5) \otimes U(1)_{HORIZONTAL}$ MODELS
- FROM MINIMAL TO "REALISTIC" SU(5)
- SU(5) FROM EXTRA-DIMENSIONS

Neutrino masses are very small!

- Direct limits
- Cosmological limits (hot dark matter)
- ν oscillation data

----> $m_{\nu i} \leq 1\text{-}2$ eV

or: $m_\nu/m_e \leq 10^{-5}$, $m_\nu/m_t \leq 10^{-11}$

Most appealing explanation:

$$m_\nu \sim m^2/M$$

M: scale of L non conserv. $\sim M_{GUT}\text{-}M_{Pl}$
$m \leq m_t \sim v \sim 200$ GeV

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV
$m \sim v \sim 200$ GeV

-----> $M \sim 10^{15}$ GeV

Neutrino masses are a probe of GUT physics!

ν OSCILLATIONS MEASURE Δm^2

$$\Delta m^2_{atm} \simeq 3 \cdot 10^{-3} \, eV^2$$

$$\Delta m^2_{\odot} \lesssim 10^{-5} \, eV^2$$

- DIRECT LIMITS :

$$\begin{cases} m_{\text{"}\nu_e\text{"}} \lesssim \sim 5 \, eV \\ m_{\text{"}\nu_\mu\text{"}} \lesssim 170 \, KeV \\ m_{\text{"}\nu_\tau\text{"}} \lesssim 18 \, MeV \end{cases}$$

- COSMOLOGY

$$\sum_i m_{\nu_i} \lesssim 6 \, eV \quad [\Omega_\nu \lesssim 0.2]$$

↳ ALL $\nu_i \lesssim 2 eV$

WHY ν'S SO MUCH LIGHTER THAN q AND ℓ^- ?

$$\mathcal{L}_\nu = L\, h\, \nu_R\, H + h.c. + \xrightarrow{} m_D = h v \quad \text{(DIRAC)}$$

$$+ \nu_R^T M_R \nu_R +$$

$$+ \nu_L^T \frac{\lambda}{M_L} \nu_L H H \rightarrow \text{(MAJORANA)}$$

$$\hookrightarrow m = \frac{\lambda v^2}{M_L}$$

SEE-SAW MECHANISM:

$$m = \begin{array}{c c} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{pmatrix} \frac{\lambda v^2}{M_L} & m_D \\ m_D & M_R \end{pmatrix} \end{array}$$

$$|m_{light}| \simeq \frac{m_D^2}{M_R} \div \frac{\lambda v^2}{M_L}$$

$$m_{heavy} \simeq M_R$$

$$m^{eff} = \nu_L^T m_{light} \nu_L$$

IN GENERAL BOTH $O_5 \sim \nu_L^T \frac{\lambda^2}{M} \nu_L H H$

AND THE SEE-SAW MECHANISM

ARE OPERATIVE :

$$\nu_L^T m_\nu \nu_L = \nu_L^T m_D^T M^{-1} m_D \nu_L + \nu_L^T \frac{\lambda^2 v^2}{M} \nu_L$$

THE 2 TERMS HAVE THE SAME FORM, THE SAME TRANSF. PROPERTIES UNDER $\nu_L' = U \nu_L$, BUT DIFFERENT ORIGINS

[e.g. in GUT's m_D RELATED TO q & ℓ DIRAC MASSES]

THEY CAN BE OF COMPARABLE OR OF VERY DIFFENT SIZE

[e.g. $1/M_{GUT}$ vs $1/M_{P\ell}$]

ν_R is a heavy "sterile" neutrino:

sterile: no gauge int's

ν_R has: colour = t_3^W = Q = 0

ν_L is a light "active" neutrino:

LEP: $N_{\nu L} = 3$

Are there light sterile neutrinos?

----> Is LSND signal true?

LSND + Solar + Atm. Oscill's

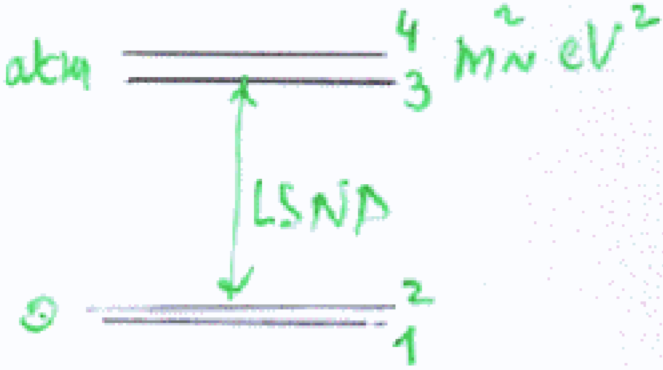
----> at least 4 light ν's

LSND not double checked (MiniBoone)
KARMEN did not confirm

Perhaps will fade away. But if right
LNSD + Solar + Atm -----> ≥ 4 ν's
or ≥ 1 light sterile ν's

4-ν Models

Typical configuration (Bilenky et al; Barger et al; Gonzales-Garcia et al)



•Can be compatible with hot dark matter, $m_{4,3}$~2 eV

• Pure two-neutrino ν_e<-->ν_s oscill's disfavoured for solar

Viable alternatives:

• 6->4 mixing angles: ν_e <--> $\nu_s + \nu_a$ (a: active=μ,τ) for solar Gonzales-Garcia et al; Fogli, Lisi

• A K-K tower of sterile ν_s (extra-dim models)

Since ν_s mixings better be small (limits from weak processes, supernovae, nucleosynthesis) the preferred solar neutrino solution is MSW-SA

Sterile ν's from extra dimensions

Context:

Large extra dimensions

Gravity propagates in all dim (bulk)

SM particles on a 4-dim brane

$$d = n + 4$$
$$(m_s R)^n = (M_P/m_s)^2$$
$$m_s \sim \mathrm{TeV}$$

Assume 1 very large dim: $1/R \leq 0.01$ eV
+ n-1 smaller ($1/\rho \geq$ TeV)

$$(m_s R)\,(m_s \rho)^{n-1} = (M_P/m_s)^2$$

or

$$(m_5 R) = (M_P/m_s)^2$$

ν_s: SUSY partners of gravitational moduli (string th.)

Also propagate in the bulk

(Arani-Hamed et al; Benakli and Smirnov, Dvali and Smirnov, Faraggi and Pospelov, Mohapatra et al, Ioannisian and Pilaftis, Ioannisian and Valle, Barbieri et al, Lukas et al; Dienes and Sarcevic, Caldwell et al;

Good Features

Caldwell et al. Mohapatra et al. Lukas et al

- A "physical" picture for ν_s.
- ν_s has KK recurrences:

$$\nu_s(x,y) = 1/\sqrt{R}\sum_n \nu_s^{(n)}(x)\cos(ny/R)$$

with: $m_{\nu s} = n/R$

and mixes with L:

$$h(m_s/M_P)\, L\nu_s^{(n)}\, H$$

[the suppression factor (m_s/M_P) is automatic from the bulk volume!]

- Interference among a few KK states make spectrum compatible with solar data

$$P(\nu_e\text{->}X) = \sum_n m^2_e/(M^2_e + n^2/R^2)$$

$1/R \sim 10^{-2} - 10^{-3}$ eV

$R \sim 10^{-3} - 10^{-2}$ cm: very close to limits!!

Problems

- GUT's? Connection with GUT's?
- What forbids (on the brane)

$1/m_s \, L^T \lambda LHH$??

Recall that m_s is small ~ TeV

- ν_e, ν_μ, ν_τ ??
- Only 1 large extra dim has problems

(linear evolution of couplings from 0.01 eV to TeV) Antoniadis, Bachas; Arkani Hamed et al

But more large extra dim

$$P(\nu_e\text{->}X) = \sum_n m^2_e/(M^2_e + n^2/R^2)$$
$$= \int m^2_e \, n^{d-1} \, dn \, /(M^2_e + n^2/R^2)$$

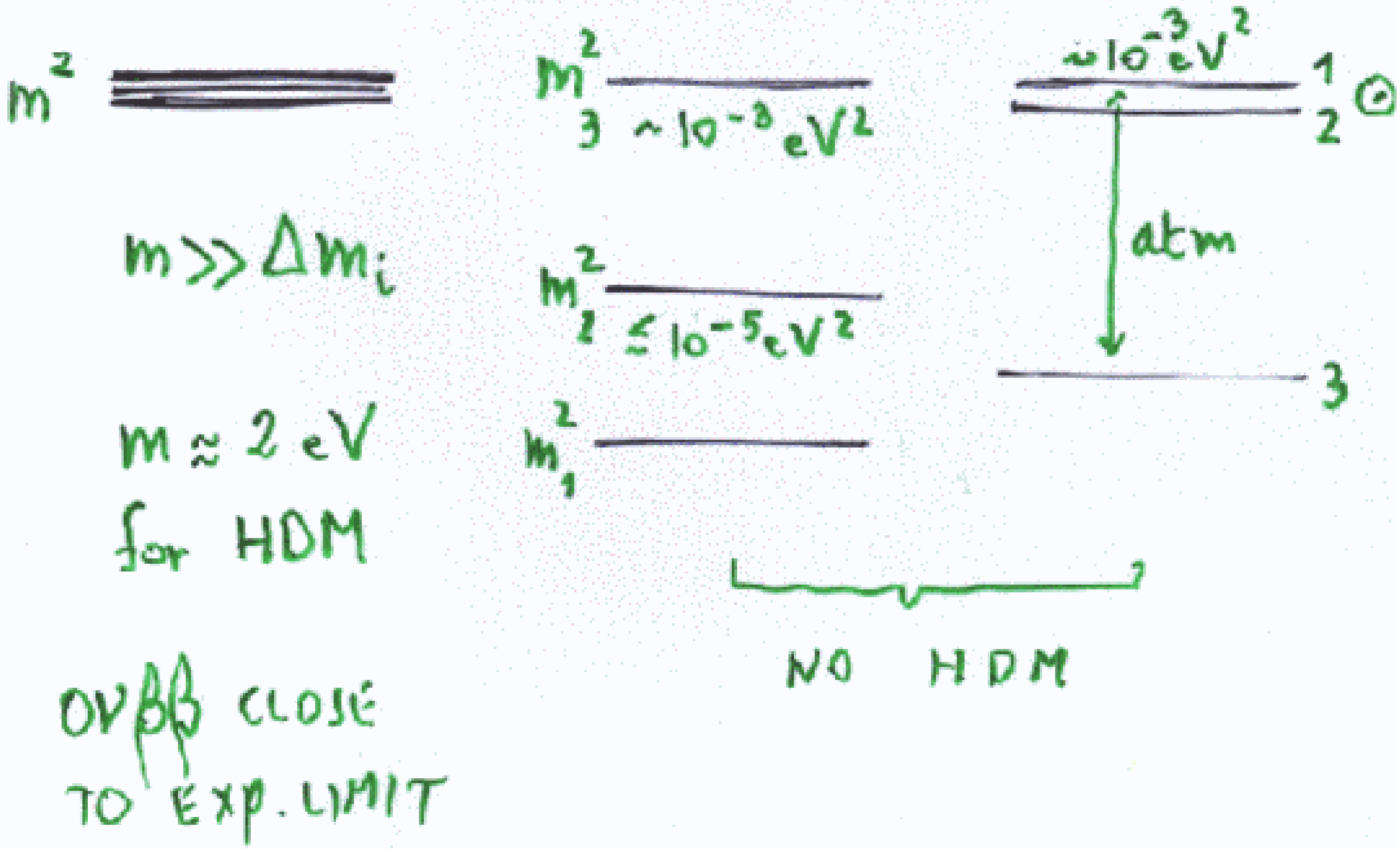
High KK states do not decouple fast enough, mixing large.

Compromise d=2?

3-ν Models

Possible configurations

Degenerate | Hierarchical | Inverted



THE U MATRIX

- 3 FLAVOURS
- 2 FREQUENCIES
- NO $e \to$ ANY FOR ΔV_{atm} (CHOOZ)
- MAXIMAL MIXING FOR ΔV_{atm}

CAN BE RELAXED IN 2nd APPROX.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c & \overset{\sin\theta}{-s} & \overset{U_{e3}=0 \text{ CHOOZ}}{0} \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

MAXIMAL ΔV_{atm} MIXING

NOTE: • ~~CP~~ NEGLECTED (U REAL)

• SOME SIGNS ARE CONVENTIONAL

$$\begin{cases} P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = \frac{1}{2}\sin^2 2\theta \sin^2 \Delta_{sun} \\ P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - \frac{1}{4}\sin^2 2\theta \sin^2 \Delta_{sun} \end{cases}$$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \qquad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

U IS ANALOGOUS TO V_{CKM}

$\Rightarrow$ SAME GENERAL FORM

e.g. MAIANI

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta} \\ \cdots & \cdots & s_{23}c_{23} \\ \cdots & \cdots & c_{23}c_{13} \end{bmatrix}$$

FOR: $s_{13} \sim 0$, $s_{23} = s_\gamma \sim \frac{1}{\sqrt{2}}$, $s_{12} = s$, $\delta \sim 0$

$c_{23} = c_\gamma \sim \frac{1}{\sqrt{2}}$, $c_{12} = c$

$$U = \begin{bmatrix} c & -s & 0 \\ sc_\gamma & cc_\gamma & -s_\gamma \\ ss_\gamma & cs_\gamma & c_\gamma \end{bmatrix}$$

FOR $s_\gamma \sim c_\gamma = \frac{1}{\sqrt{2}}$ SAME AS ABOVE APART FROM SIGNS

CONVENTION

MOST GENERAL m_ν

$$m_\nu \sim m_D^T M^{-1} m_D \sim U \begin{bmatrix} e^{i\varphi_1} m_1 & & \\ & e^{i\varphi_2} m_2 & \\ & & m_3 \end{bmatrix} U^T$$

$L^T m_\nu L$ $\qquad$ $\bar{R} m_D L$

9 NEW PARAM'S ADDED TO SM: 3 MASSES
3 MIXINGS
3 PHASES

GIVEN $|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$

AND $m_\nu^{diag} = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$

THEN $m_\nu = U\, m_\nu^{diag}\, U^T$

$$m_\nu = \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2)\frac{cs}{\sqrt{2}} & (m_1 - m_2)\frac{cs}{\sqrt{2}} \\ \cdots & \frac{m_3}{2} + \frac{m_1 s^2}{2} + \frac{m_2 c^2}{2} & \cdots \\ \cdots & -\frac{m_3}{2} + \frac{m_1 s^2}{2} + \frac{m_2 c^2}{2} & \frac{m_3}{2} + \frac{m_1 s^2}{2} + \frac{m_2 c^2}{2} \end{bmatrix}$$

THIS IS IN BASIS WHERE m_ℓ DIAGONAL

CHARGED LEPTONS

NOTE : m_ν IS SYMMETRIC

m_ν IS THE EFFECTIVE LIGHT ν MATRIX

MAJORANA

$$L^T m_\nu L \xrightarrow{\text{SEE-SAW}} m_\nu = m_D M^{-1} m_D^T$$

$\bar{L}R$
(DIRAC)

FOR EXAMPLE : ASSUME $|m_3| \gg |m_{1,2}|$

BY NEGLECTING SMALL TERMS OF ORDER $m_{1,2}/m_3$:

$$m_\nu^{diag} \simeq \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_3 \end{pmatrix}$$

$$m_\nu = U\, m_\nu^{diag}\, U^T = \quad \text{IN BASIS WHERE } m_\ell^D \text{ DIAGONAL}$$

$$= \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

$\hookrightarrow \det[23] = 0$

<u>NOTE</u> : THIS IS INDEPENDENT OF s

(BIMIXING : $s \cong \frac{1}{\sqrt{2}}$, MSW : s SMALL)

SIGN CONVENTIONAL : $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ ALSO OK!

ZEROTH ORDER APPROXIMATION

$$U = \begin{bmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

BASIS OF m_e^D DIAGONAL

$m_\nu = U \, \mathrm{Diag} \, U^T$

		m_{diag}	double maximal mixing $s = \frac{1}{\sqrt{2}}$	single maximal mixing $s = 0$
$\lvert m_3\rvert \gg \lvert m_{1,2}\rvert$	A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
$\lvert m_1\rvert \sim \lvert m_2\rvert \gg \lvert m_3\rvert$	B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
	B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
$\lvert m_1\rvert \approx \lvert m_2\rvert \approx \lvert m_3\rvert$	C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
	C3	Diag[1,1,-1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

G.A., F. FERUGLIO II

Table I : Zeroth order form of the neutrino mass matrix for double and single max mixing, according to the different possible hierarchies given in eq. (6).

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II /9809596 JHEP 11(1998)21
III /9812475

Degenerate ν 's

- Compatible with hot dark matter
  (m ~ 2 eV)

- Limits on m_{ee} from $0\nu\beta\beta$ imply double maximal mixing (bimixing) for atmospheric and solar oscill's:

  Vissan; Georgi,Glashow.

  $$m_{ee} = m_1\cos^2\theta_{12} + m_2\sin^2\theta_{12}$$

  $m_{ee} \leq 0.3\text{-}0.5$ eV (exp) needs

  $$m_1 = -\, m_2$$

  and

  $$\cos^2\theta_{12} \sim \sin^2\theta_{12}$$

  $$\boxed{\begin{array}{l} \sin^2 2\theta_{12} > 0.99 \\ \cos^2\theta_{12} - \sin^2\theta_{12} < 0.1 \end{array}}$$

- For naturalness $\Delta m/m$ cannot be too small
  (e.g. vacuum sol. $\Delta m/m \sim 10^{-11}$)
  MSW-LA would be preferred in this respect. But is θ_{12} sufficiently maximal?

FOR $|m_1| \sim |m_2| \sim |m_3| \sim 1$ eV

ONLY ONE POSSIBILITY

$\boxed{\text{BIMIXING!}}$ (C1 EQUIVALENT C2)

	m_{diag}	double maximal mixing	single maximal mixing
A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\cancel{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$
C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}}$
C3	Diag[1,1,-1]	$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$	$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$

Table I : Zeroth order form of the neutrino mass matrix for double and single max
ixing, according to the different possible hierarchies given in eq. (6).

•For degenerate ν's see-saw dominance is unlikely:

See-saw: $m_\nu = m^T{}_D M^{-1} m_D$

We expect m_D to be hierarchical as for q & l

(conspiracy between m_D and M unplausible)

More likely:

Degenerate ν's from dim-5 operators

$$1/M \;\; L^T \lambda L \; H \; H$$

unrelated to m_D and q & l.

•For m ~ 2 eV, v ~ 200 GeV, λ ~ 1:

M ~ 10^{13} GeV Somewhat low?

A MODEL WHICH IS SIMPLE TO STATE BUT DIFFICULT TO REALIZE

Fritzsch, Xhing

ASSUME THAN IN 1st APPROX.

$$m_{q,\ell^-} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[\text{DIAG.}]{U} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 3 \end{pmatrix}$$

LR DIRAC

"DEMOCRATIC" $S_L \times S_R$ SYMMETRY

IN SAME BASIS FOR ν's

PHASES NEEDED FOR $0\nu\beta\beta$

$$|m_\nu| = a\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + b\cancel{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}$$

BOTH ALLOWED BY S_L

LTL MAJORANA

ASSUME NEGLIGIBLE

THEN IN BASIS WHERE ℓ DIAGONAL [BY IMPOSING CHOOZ]

$$m'_\nu = U\, m_\nu\, U^T$$

APPROX. BIMIXING

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

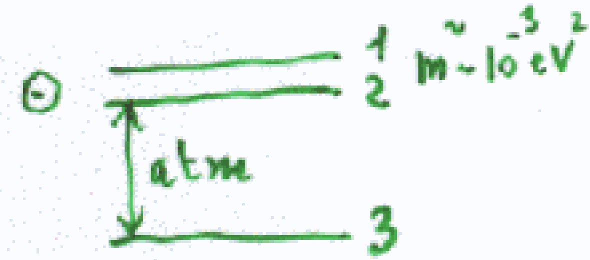
$$\sin^2 2\vartheta_{atm} = 4s^2c^2 = 4\,\frac{4}{6}\,\frac{1}{3} = 8/9$$

Inverted hierachy

Joshipura et al; Mohapatra et al; Jarlskog et al; Frampton and Glashow; Barbieri et al.........; Zee

Provides interesting models for bimixing.

$$U = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{bmatrix}$$



$$m_{diag} = M \begin{bmatrix} 1 & & \\ & -1 & \\ & & 0 \end{bmatrix}$$

$$U\, m_{diag}\, U^{+} = 1/\sqrt{2} \begin{bmatrix} 0 & M & M \\ M & 0 & 0 \\ M & 0 & 0 \end{bmatrix} \text{(in flavour basis)}$$

- From dim-5 $L^T LHH$
- Approximate L_e-L_μ-L_τ symmetry.
- 1-2 degeneracy stable under rad. correct's.
- Prefers VO for solar, but could be comp. with MSW-LA (is mixing large enough?)

OR LOW

Hierarchical neutrinos

•Assume 3 widely split light neutrinos

•SO(10)--->ν_R + assume see-saw dominant:

$$m_\nu \sim m_D{}^T M^{-1} m_D$$

Maximally constraining: Gut's relate q, l, ν masses!!

•For u, d, l Dirac mass matrices:
the 3rd generation eigenv. dominant.

•It is natural to assume this is also true for $m_D{}^\nu$: diag $m_D{}^\nu \sim (0,0,m_3)$

•After see-saw, $m_\nu \sim m_D{}^T M^{-1} m_D$, in general will be even more hierarchical:
fine tuned compensation between m_D and M unlikely.

•A possible problem:

We need both large m_3-m_2 splitting and large mixing in 2-3 sector.

$$m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}\ \mathrm{eV}$$

$$m_2 \sim (\Delta m^2_{sol})^{1/2} \sim \underset{\text{(MSW)}}{2\ 10^{-3}\ \mathrm{eV}} - \underset{\text{(VO)}}{10^{-5}\ \mathrm{eV}}$$

•The "theorem" that large Δm_{32} implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$) is not true in general:
All we need is (sub)det[23]~0

NOTE: for MSW-LA the splitting could be by a factor of ~10 only: a factor of 3 in m_D easily becomes a factor of 10 in

$$m_\nu \sim m_D^T M^{-1} m_D$$

Example

$$m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

$\mathrm{Det}[m_{23}] \sim 0$

Eigenvalues: $0,\ 1+x^2$

Mixing: $\sin^2(2\theta) = 4x^2/(1+x^2)^2$

For $x \sim 1$ large splitting and large mixing!

So we need mechanisms for $\mathrm{Det}[m_{23}] \sim 0$ automatic

MECHANISMS FOR $\mathrm{Det}[23] \sim 0$

$$\boxed{m_\nu = m^T M^{-1} m}$$

(1) A ν_R IS LIGHTEST AND COUPLED TO μ & τ : King, Allanach, Barbieri...

$$M \sim \begin{pmatrix} \epsilon & \\ & 1 \end{pmatrix} \to M^{-1} \sim \begin{pmatrix} 1/\epsilon & \\ & 1 \end{pmatrix} \approx \begin{pmatrix} 1/\epsilon & \\ & 0 \end{pmatrix}$$

$$m_\nu \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1/\epsilon & \\ & 0 \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix} \approx \frac{1}{\epsilon} \begin{pmatrix} a^2 & ac \\ ac & c^2 \end{pmatrix}$$

(2) M GENERIC, BUT $m \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{pmatrix}$

$$M^{-1} \sim \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$m_\nu \approx f \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix}$$

G.A., F. Feruglio

• Hierarchical neutrinos and see-saw dominance

$$m_\nu \sim m_D^T M^{-1} m_D$$

allow to relate q, l, ν masses and mixings-->
--> **GUT's models**

•The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

$$SU(5) \times U(1)_{horizontal}$$

•SO(10) models are often more predictive, but are based on specific textures from a set of special operators

ALBRIGHT, BARR
BUCCELLA et al
BABU et al

IMPORTANT HINT FROM SU(5)

LEFT-HANDED QUARKS : SMALL MIXINGS

$$V_{CKM} = U_u^\dagger U_d \simeq \begin{bmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

$\lambda \simeq \sin\vartheta_c \simeq 0.22$

$A \simeq 0.8$, $\sqrt{\rho^2+\eta^2} \sim 0.4$

BUT RIGHT-HANDED QUARKS CAN HAVE LARGE MIXINGS (UNKNOWN)

IN SU(5) LH FOR d QUARKS

$\Longleftrightarrow$ RH FOR ℓ^- LEPTONS

$$\bar{5} : (\underbrace{\bar{d}\ \bar{d}\ \bar{d}}_{R}\ \underbrace{\nu\ e^-}_{L})$$

$$m_D^d \sim \underset{\bar{5}}{\bar{d}_R}\ \overset{10}{d_L}$$

$$m_D^\ell \sim \underset{10}{\bar{\ell}_R}\ \overset{5}{\ell_L} \rightsquigarrow \boxed{m_D^\ell = \left(m_D^d\right)^T}\ !!$$

CANNOT BE EXACT

$$\boxed{m_d = m_e^T}$$

FOR EIGENVALUES IMPLIES:

$$\boxed{\frac{m_d}{m_e} = \frac{m_s}{m_\mu} = \frac{m_b}{m_\tau} = 1}$$ AT GUT SCALE

$m_b = m_\tau$ AT M_{GUT} IS OK!

RUNNING BY RENORM. GROUP

$$\frac{m_b}{m_\tau}(M_{GUT}) = 1 \longrightarrow \frac{m_b}{m_\tau}(\text{few GeV}) \simeq 3$$

BUT $\frac{m_d}{m_e} = \frac{m_s}{m_\mu}$ AT M_{GUT} IS BAD

CAN BE CORRECTED BY ADDING H_{45}

$$m_e^T = m_{H_5} - 3\, m_{H_{45}}$$ Georgi, Jarlskog

$$m_d = m_{H_5} + m_{H_{45}}$$

PROBLEM: AVOID SPOILING $m_b = m_\tau$

OR BY ADDING SMALL NON REN. TERMS

GENERATION OF TEXTURES: → HORIZONTAL U(1) CHARGES

MANY PAPERS Froggatt, Nielsen '79

A GENERIC MASS TERM:

$\bar{R}_1 m_{12} L_2 H$

q_1 U(1) CHARGE OF $\bar{R}_1$
q_2 " " " L_2
q_H " " " H

FORBIDDEN IF $q_1+q_2+q_H \neq 0$

BUT

$\bar{R}_1 m_{12} L_2 H \left(\frac{\theta}{M}\right)^{q_1+q_2+q_H}$ ALLOWED IF $q_\theta = -1$

vev $\theta = w$, $\frac{w}{M} = \lambda$ SMALL

$m_{12} \to M_{12}\, \lambda^{q_1+q_2+q_H}$

SO MORE U(1) CHARGE MISMATCH → → MORE SUPPRESSION

THERE CAN BE SEVERAL θ's

e.g. DIFFERENT λ, λ' FOR

$q_1+q_2 > 0$ (λ) OR < 0 (λ')

EXAMPLE : SU(5) MULTIPLETS WITH U(1) CHARGES

SUSY SU(5)

G.A., F Feruglio PLB451(1999)388

U(1) CHARGES:

$$\begin{cases} 10_i: & [3, 2, 0] \\ \bar{5}_i: & [3, 0, 0] \\ 1_i: & [1, -1, 0] \end{cases}$$

Higgs: $H(5), H(\bar{5})$

WITH U(1) CH. = 0

+ θ, $\bar{\theta}$ SU(5) SINGLETS OF CH. −1 AND +1

$\frac{\langle\theta\rangle}{M_{Pl}} \sim \lambda$

$$m_u = m_u^T \approx \underset{10_i\, 10_j}{\begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}} v_u \; ; \quad m_d = \overset{\bar{5}_i\, 10_j}{\begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}} v_d$$

(EACH ENTRY MULTIPLIED BY O(1) COEFF.)

$$m_d = m_e^T \; ; \quad m_{D\nu} \approx \underset{1_i\, \bar{5}_j}{\begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & \lambda' & \lambda' \\ \lambda^3 & 1 & 1 \end{bmatrix}} v_u$$

$$\lambda' = \frac{\langle\bar{\theta}\rangle}{M_{Pl}}$$

$$M = \underset{1_i\, 1_j}{\begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}} \bar{M}$$

$$V_{CKM}^{ij} \sim \lambda^{q_{10_i} - q_{10_j}}$$

$m_u : m_c : m_t \sim \lambda^6 : \lambda^4 : 1$ $\qquad V_{us} \sim \lambda$

$m_d : m_s : m_b \sim \lambda^6 : \lambda^2 : 1$ $\qquad V_{ub} \sim \lambda^3$

$m_e : m_\mu : m_\tau \sim \lambda^6 : \lambda^2 : 1$ $\qquad V_{cb} \sim \lambda^2$

$\tan\beta$ LARGE $\qquad \lambda = O(\sin\theta_c)$

AFTER SEE-SAW AND DIAG'N OF CHARGED LEPTONS (FOR $\lambda \sim \lambda'$)

$$m_\nu \sim \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\bar{M}}$$

- THIS IS CONSISTENT WITH THE $\bar{5}_i$ CHARGES $[3, 0, 0]$ $(m_\nu \sim \bar{5}_i \bar{5}_j)$
- Det $[2\,3] \sim o(\lambda^2)$ AUTOMATICALLY!
- EIGENVALUES : $m_1 : m_2 : m_3 = \lambda^4 : \lambda^2 : 1$

$$\hookrightarrow \lambda^4 \simeq \frac{m_2^2}{m_3^2} \sim \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}} \sim 3 \cdot 10^{-3} \sim (0.05)^2$$

($\Delta m^2_{sun} \sim$ MSW)

INDEED $\lambda \sim \sin\theta_c$ FOR MSW-SA

ALTERNATIVE $\lambda' = 0$

$$m_1 : m_2 : m_3 = \lambda^3 : \lambda^3 : 1$$

$$m_1^2 - m_2^2 \sim o(\lambda^9)$$

$$\frac{s_{12}}{c_{12}} \sim \frac{\pi}{4} + o(\lambda^3)$$

$\lambda \sim \sin\theta_c$ FOR LOW, VO

A SUSY GUT BENCHMARK MODEL

SHOULD POSSESS THE PROPERTIES

- COUPLING UNIFICATION
  - NO EXTRA LIGHT HIGGS DOUBLETS
  - M_{GUT} THRESHOLDS UNDER CONTROL (NOT TOO LARGE REPRES'NS)

- DOUBLET-TRIPLET SPLITTING NATURAL
  - $SU(5)$: $5, \bar{5}, (24), 50, \overline{50}, 75$
  - $SO(10)$: (DW): $10, 10', 16, \overline{16}', 45$

  } HIGGS

- CORRECT MASSES AND MIXINGS FOR q, l AND ν's

  IN PARTICULAR: $m_b = m_\tau$ at M_{GUT}

  BUT $m_s \neq m_\mu$, $m_d \neq m_e$

- COMPATIBLE WITH p DECAY

  (NEEDS ~ 0.1 FINE TUNING IN COUPLINGS)

Babu, Pati, Wilczek
Albright, Barr
G.A., Feruglio, Masina

A "REALISTIC" $SU(5) \otimes U(1)_Q$ MODEL

G.A., Feruglio, Masina JHEP 11(2000)040

D-T SPLITTING : MISSING PARTNER MECHANISM STABILIZED BY $U(1)_Q$

	Y	H	$\bar{H}$	H_{50}	$H_{\overline{50}}$	X
$SU(5)$	75	5	$\bar{5}$	50	$\overline{50}$	1
Q	0	-2	1	2	-1	-1

Masiero, Tamvakis ; Nanopoulos, Tavagida ; Berezhiani, Tavartkiladze ; ....

X : $SU(5)$ 1, $Q=-1$ — THE ONLY Q-CHARGED FIELD WITH $\langle X \rangle$ LARGE

$$SU(5) \xrightarrow[75]{} SU(3) \otimes SU(2) \otimes U(1)$$

$75 \leftarrow \langle Y \rangle \sim M_{GUT}$

EXACT SUSY :
- DOUBLETS MASSLESS
- $m_{TRIPLET} \sim \dfrac{\langle Y \rangle^2}{\langle X \rangle}$
- $H\bar{H} X^m Y^n$ $(m, n \geq 0)$ FORBIDDEN BY $U(1)_Q$

$\cancel{SUSY}$: $\langle X \rangle \sim \Lambda \leftarrow$ CUT OFF

COUPLING UNIFICATION

MINIMAL SUSY-SU(5)

$$\alpha_s(m_Z) \simeq 0.13 \pm 0.01$$

SOMEWHAT LARGE

LARGE THRESHOLD CORR'S FROM THE SPLITTED 75 MULTIPLET INDUCE LARGE DECREASE OF $\alpha_s(m_Z)$

(50, $\overline{50}$ NOT SPLIT)

MINIMAL : m_T LOW TO DECREASE α_s

REALISTIC : m_T HIGH TO INCREASE α_s

$$m_T\Big|_{\text{REALISTIC}} \approx 20\text{-}30\ m_T\Big|_{\text{MINIMAL}}$$

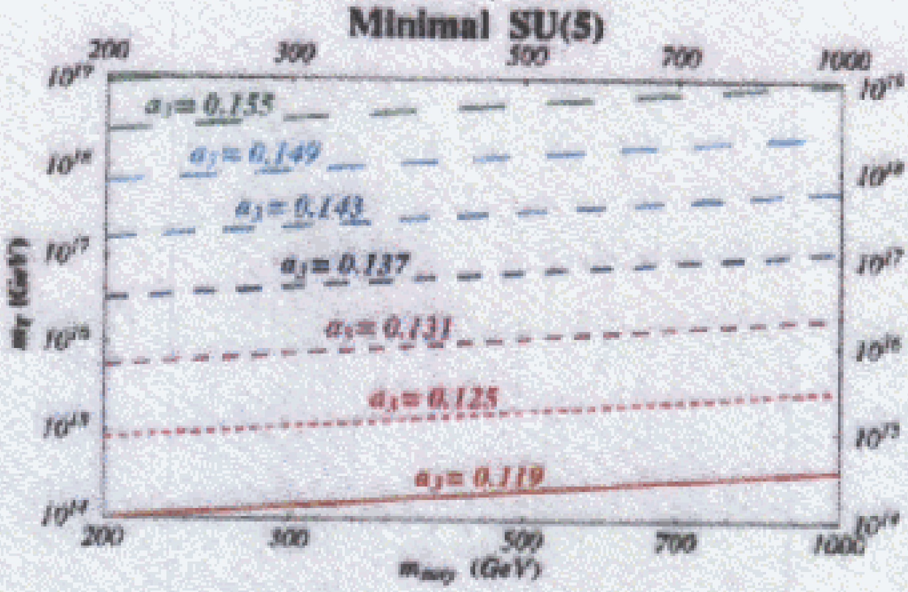
GOOD FOR p DECAY :

SUPPRESSION OF RATE BY 400-900!!

DUE TO 50, $\overline{50}$, 75 SU(5) NO MORE ASYMPT. FREE UP TO M_{Pl}

α_s BLOWS UP $\Lambda \sim$ 20-30 M_{GUT}

NOT NECESSARILY A BAD FEATURE.



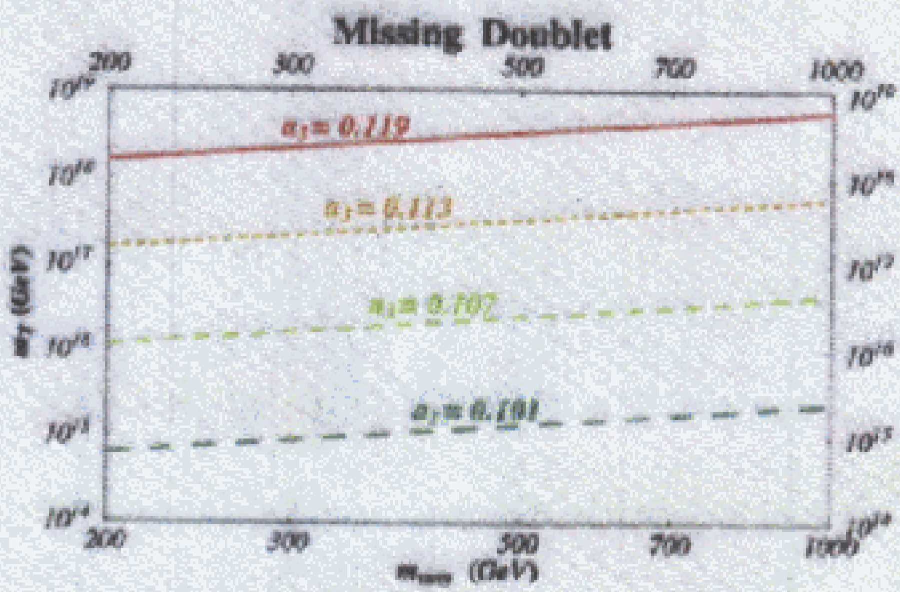


Figure 1: Contours of $\alpha_3 \equiv \alpha_s(m_Z)$ in the plane (m_{SUSY}, m_T), for the minimal $SU(5)$ and the missing doublet model. The SUSY spectrum is parametrized as in Table 1.

DETERMINED BY $U(1)_Q$ CHARGES

$Q(10) = (4, 3, 1)$ $\qquad Q(H) = -2$

$Q(\bar{5}) = (4, 2, 2)$ $\qquad Q(\bar{H}) = 1$

$Q(1) = (1, -1, 0)$ $\qquad Q(X) = -1$

$\hookrightarrow \lambda = \langle X \rangle / \Lambda$
$\approx \lambda_c \approx 0.22$

FIRST APPROX. (NO Y INSERTIONS)

$$m_U = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u \; ; \quad m_d = m_e^T = \begin{bmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{bmatrix} v_d \lambda^4$$

$$m_D^\nu = \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & 0 & 1 \\ \lambda & 0 & 1 \end{bmatrix} v_u \; ; \quad M_{RR} = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} M$$

QUARKS: $m_u, m_d, V_{CKM} \simeq$ OK

$tg\beta \approx O(1)$ NICE FOR p DECAY

CHARGED LEPTONS: $m_d = m_e^T \longleftrightarrow 10\ \bar{5}\ \bar{H}$

BROKEN BY: $\frac{1}{\Lambda}\, 10\ \bar{5}\ \bar{H} Y$

PREDICTIONS FOR ν'S

$$\begin{cases} \theta_{23} \sim \dfrac{\pi}{4} & (\text{atm.}) \\ \theta_{12} \sim \dfrac{\pi}{4} & (\text{Ge}) \\ \theta_{13} \sim 0.05 & \end{cases}$$

PREFER LOW OR VO SOLUTIONS OF $\nu_{\odot}$

PROTON DECAY

HIGGS TRIPLET EXCHANGE

$$W(\Delta B=\pm 1)=\frac{1}{m_T}\left[Q\hat{A}Q\,Q\hat{C}L+U^c\hat{B}E^c u^c\hat{D}D^c\right]$$

WITH RESPECT TO MINIMAL $SU(5)$:

- LARGER m_T BY FACTOR 20-30
- EXTRA TERMS

FOR EXAMPLE:

CONSTRAINED BY MASSES

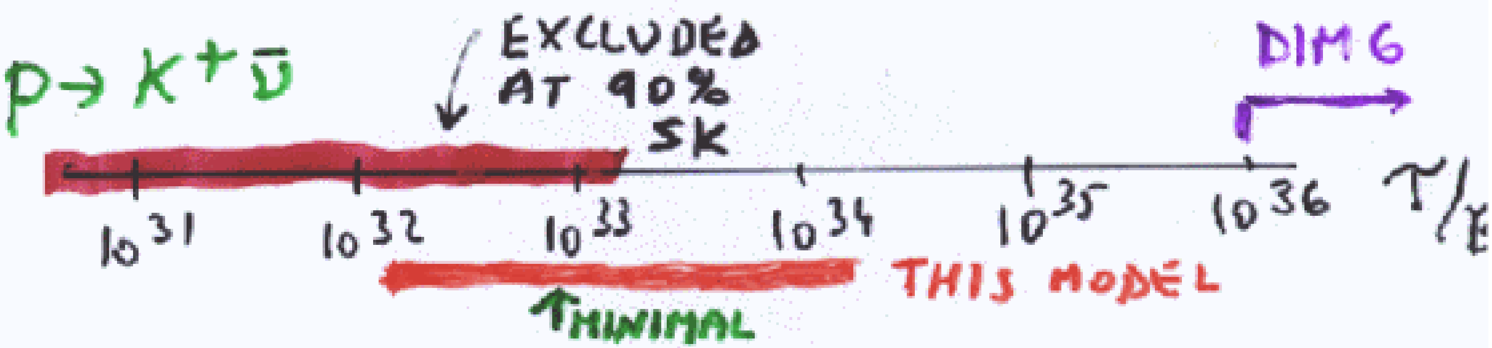
NOT ONLY $10\,G_u\,10\,5$ BUT ALSO

$10\,G_{\overline{50}}\,10\,\overline{50}$ IS ALLOWED

FREE FROM MASS CONSTRAINTS

$\langle\overline{50}\rangle=0$!

RESULTS



SIMILARLY FOR $p\to\pi^+\bar{\nu}$

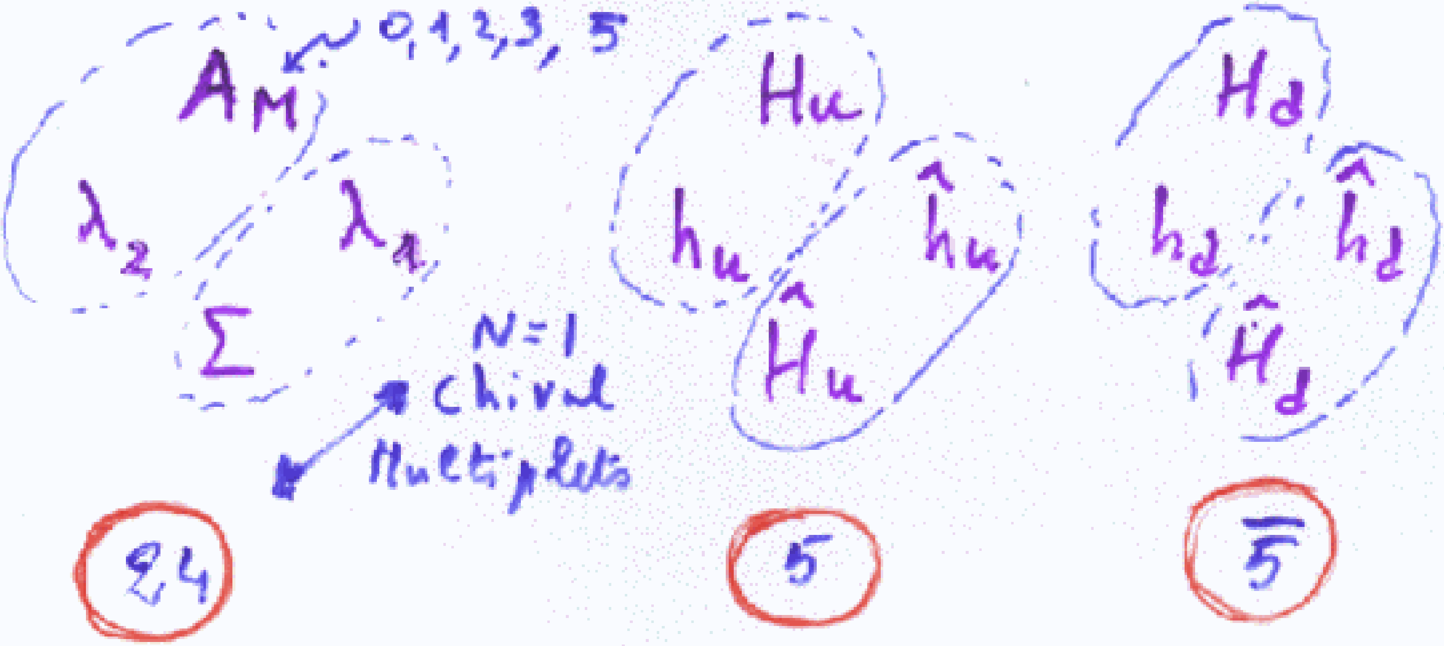
A DIFFERENT VIEW OF GUT'S

SU(5) FROM EXTRA DIMENSIONS

Fayet ; Kawamura ;
G.A., F. Feruglio, hep-ph/0102301

- IN 5 DIM : $N=2$ SUSY + SU(5)

  GAUGE 24 + 5 + $\bar{5}$ HIGGS $\boxed{\text{IN BULK}}$
  ($N=2$ HYPERMULTIPLETS)



- COMPACTIFICATION BY $S/(Z_2 \times Z_2')$

$$1/R \sim M_{GUT}$$

$N=2$ SUSY SU(5) $\Rightarrow$ $N=1$ SUSY $3 \otimes 2 \otimes 1$

- WE LIVE ON $x_5 \equiv y = 0$ BRANE

  $\Psi_{\bar{5}}, \Psi_{10}, \Psi_1$

Kawamura

$Z_2 \to P: \quad y \leftrightarrow -y$

$Z_2' \to P': \quad y' \leftrightarrow -y' \qquad y' = y + \frac{\pi R}{2}$

(or $y \to -y - \pi R$)

ϕ_{++} AND $\phi_{+-} \neq 0$ AT $y=0$

ONLY $\phi_{++}^{(0)}(x)$ MASSLESS

$$\begin{cases} \phi_{++}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_n \phi_{++}^{(2n)}(x) \cos\frac{2ny}{R} \\ \phi_{+-}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_n \phi_{+-}^{(2n+1)}(x) \cos\frac{2n+1}{R} y \\ \phi_{-+}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_n \phi_{-+}^{(2n+1)}(x) \sin\frac{2n+1}{R} y \\ \phi_{--}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_n \phi_{--}^{(2n+2)}(x) \sin\frac{2n+2}{R} y \end{cases}$$

BULK FIELDS:

$\bar{5}$- doublet

triplet

$P\ P'$	field	m
$+\ +$	$A_\mu^a, \lambda_2^a, H_u^D, H_d^D$	$\frac{2n}{R}$
$+\ -$	$A_\mu^{\hat{a}}, \lambda_2^{\hat{a}}, H_u^T, H_d^T$	$\frac{2n+1}{R}$
$-\ +$	$A_5^{\hat{a}}, \Sigma^{\hat{a}}, \lambda_1^{\hat{a}}, \hat{H}_u^T, \hat{H}_d^T$	$\frac{2n+1}{R}$
$-\ -$	$A_5^a, \Sigma^a, \lambda_1^a, \hat{H}_u^D, \hat{H}_d^D$ ($\mathbb{H} = H + h$)	$\frac{2n+2}{R}$

P: BREAKS $N=2 \to N=1$ SUSY

P': BREAKS $SU(5)$

$$\begin{cases} P' T^a P' = T^a \\ P' T^{\hat{a}} P' = -T^{\hat{a}} \end{cases}$$

NOTE $\partial_5 = (-,-)$ $\quad T^a$: generators of $3 \otimes 2 \otimes 1$

- $A_\mu^{a(0)}$, $\lambda_2^{a(0)}$ MASSLESS $N=1$ MULTIPLET

- $A_\mu^{a(2n)}$ EAT $\partial_5 A_5^{a(2n)}$ AND BECOME MASSIVE $(n>0)$ etc.

NO BAROQUE 24 HIGGS TO BREAK SU(5)

- DOUBLET-TRIPLET SPLITTING AUTOMATIC AND NATURAL

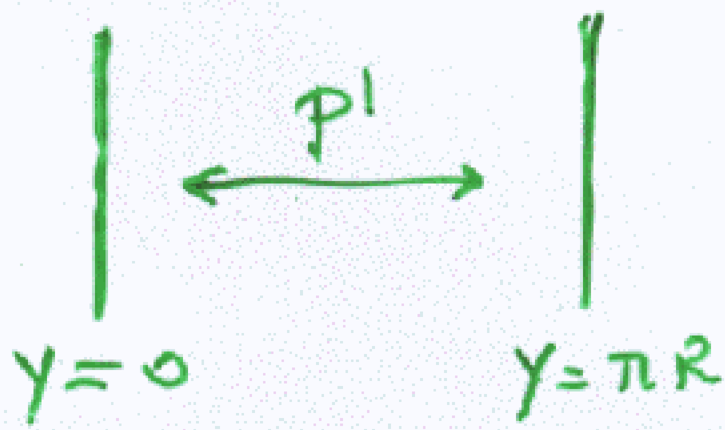
  $H_{u,d}^{D(0)}$ MASSLESS

  $H_{u,d}^{T(0)}$ MASS $1/R \sim M_{GUT}$

(NO HIGGS MASS TERM ALLOWED IN BULK BY $N=2$ SUSY.)

$\Psi_{10}, \Psi_{\bar{5}}, \Psi_1$ CHIRAL MULTIPLETS LIVE ON BRANE $y=0$ (OR $y=\pi R$)

(ALL COMPONENTS MASSLESS : NO KK RECURRENCES



P' BREAKS SU(5)

FIXED POINT OF P

G.A., FERUGLIO

P: BREAKS N=2 SUSY BUT PRESERVES SU(5)

N=1 SU(5) COUPLINGS AT $y=0$

EFFECTIVE 4 DIM ACTION

$$S = \int dy \left[\delta(y) + \delta(y-\pi R) \right] W(y)$$

ODD TERMS UNDER P' VANISH !!

$W(y) = y_u\, 10\, 10\, H_5 + y_d\, 10\, \bar{5}\, H_{\bar{5}} + y_R\, 10\, \bar{5}\, \bar{5} + \ldots$

$$W_{mass} = y_u\, Q U^c H_u^D + y_d\, Q D^c H_d + y_d\, L E^c H_d^D$$

MUST BE ALLOWED: Q, U^c, D^c / L, E^c — SAME P, P' PARITIES

p - DECAY $\quad H_U^T H_d^T$ $\quad$ ALLOWED

$Q Q H_U^T$, $U^c D^c H_d^T$ $\quad$ FORBIDDEN

$Q L H_d^T$, $U^c E^c H_u^T$

P DECAY IN GENERAL SUPPRESSED, BUT CAN BE FORBIDDEN BY:

$Q, U^c, D^c \quad + +$

$L, E^c\ (, \nu^c) \quad + -$

DIM 5 $\quad [Q Q Q L]_F$, $[U^c U^c D^c E^c]_F$

DIM 6 $\quad [Q Q U^{c\dagger} E^{c\dagger}]_D$, $[U^{c\dagger} D^{c\dagger} Q L]_D$

$Q D^c L$, $L E^c L$ $\quad$ ALL FORBIDDEN!

$$W^{(4)} = 2\int dy\, \delta(y) \Big[y_d \big(Q D^c H_d^D + L E^c H_d^D + Q L H_d^T \big) + y_u \big(Q U^c H_u^D + U^c E^c H_u^T \big) + y_R\, U^c D^c D^c \Big]$$

ν MASSES ALLOWED:

DIRAC $\quad L\,\nu^c H_u^D + D^c \nu^c H_u^T$

ALLOWED

MAJORANA $\quad M_R\,\nu^c \nu^c$

↳ SEE-SAW OK!

$$\frac{\lambda}{M_L}\, L^T L\, H_d^D H_d^D$$

ALLOWED

$$m_d = m_e^T$$

PRESERVED

GOOD QUALITATIVE POTENTIAL OF SU(5) GUT'S FOR ν'S MAINTAINED.

Conclusion

• Many crucial questions to be answered by experiments, e.g.

LNSD: true or false
Which solar ν solution?
How maximal is maximal mixing?.....

• Pending these questions many inventive models and elegant speculative solutions have been proposed.

•But the simplest and most constraining possibility is

3 hierarchical ν's
Dominance of see-saw

$$m_\nu \sim m_D{}^T M^{-1} m_D$$

(relation with q,l Dirac masses)
GUT models [SU(5), SO(10)...]

•Viable models in SU(5) using $m_d = m_l{}^T$