Implication of Neutrino oscillations for Gauge unification

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1) Present status of neutrino oscillations

2) The SO(10) unified theory with intermediate symmetry $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$

3) See-saw mechanism with a "natural" M^R

4) Conclusions

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NEUTRINO OSCILLATIONS, INVENTED ALMOST HALF CENTURY AGO BY BRUNO PONTECORVO, ARE ADVOCATED TO EXPLAIN ATMOSPHERIC AND SOLAR NEUTRINO PHENOMENA WITH

$$\Delta m_a^2 = 2.5 \cdot 10^{-3} (eV)^2$$

AND MAXIMAL $\nu_\mu - \nu_\tau$ MIXING AND UN UPPER LIMIT FOR ν_e CONTENT BY CHOOZ

$|U_{e3}|^2 < .05$ AND

$$\Delta m_s^2 \simeq 2.7 \cdot 10^{-5} (eV)^2$$

AND LARGE MIXING ANGLE FOR THE ACTUALLY FAVOURED LMA MSW SOLUTION

IN CONCLUSION A MIXING ③

MATRIX NOT FAR FROM THE

BIMAXIMAL FORM

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & +\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & +\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

PROPOSED BY GEORGI AND GLASHOW

THE KNOWLEDGE OF 3 MIXING

ANGLES (FOR SIMPLICITY WE ASSUME CP)

AND OF 2 Δm^2 IS NOT

SUFFICIENT TO DETERMINE THE

EFFECTIVE MAJORANA MASS MATRIX

OF ν_L's, WHICH NEEDS THE

KNOWLEDGE OF $|m_1|$ AND OF THE

RELATIVE SIGNS OF THE m_i's

THE QUANTUM NUMBERS WITH RESPECT TO

$$G = SU(3) \times SU(2) \times U(1)$$

OF A FAMILY OF FUNDAMENTAL FERMIONS ARE A STRONG HINT FOR GAUGE UNIFICATION

	$\mathrm{Tr}\, Y$	$\mathrm{Tr}\, Y^2$	$\mathrm{Tr}\, T_3^2$	$\mathrm{Tr}\, \mathcal{T}_3^2$
$(3, 2, +\frac{1}{6})$	$+1$	$+\frac{1}{6}$	$+\frac{3}{2}$	1
$(\bar{3}, 1, -\frac{2}{3})$	-2	$+\frac{4}{3}$	0	$\frac{1}{2}$
$(1, 1, +1)$	$+1$	$+1$	0	0
	$=0$	$=\frac{5}{2}$	$=\frac{3}{2}$	$=\frac{3}{2}$
$(1, 2, -\frac{1}{2})$	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$(\bar{3}, 1, +\frac{1}{3})$	$+1$	$+\frac{1}{3}$	0	$\frac{1}{2}$
	$=0$	$=\frac{5}{6}$	$=\frac{1}{2}$	$=\frac{1}{2}$

THE STATES ARE GROUPED IN A $10 + \bar{5}$ REPRESENTATION OF $SU(5)$

THE EXTENSION TO $SO(10)$ REQUIRES THE EXISTENCE OF $\bar{\nu}^L$'s SINCE

$$16 \to 10 + \bar{5} + 1$$

A $SO(10)$ THEORY WITH THE SPONTANEOUS SYMMETRY BREAKING PATTERN

$$SO(10) \xrightarrow[M_X]{210} SU(4)_{PS} \times SU(2)_L \times SU(2)_R$$

$$\xrightarrow[M_R]{126} G \equiv SU(3) \times SU(2) \times U(1)$$

IMPLIES

$$M_X = 4.7 \cdot 10^{15}\ \text{GeV} > 1.6 \cdot 10^{15}\ \text{GeV}$$

$$M_R = 2.8 \cdot 10^{11}\ \text{GeV}$$

AND M_R, THE SCALE OF SPONTANEOUS SYMMETRY BREAKING OF $B-L$ IS RELATED TO THE VALUES OF THE MATRIX ELEMENTS OF THE MAJORANA MASS MATRIX OF ν_R's

THE SEE-SAW FORMULA

$$M^L = - M_D^T (M^R)^{-1} M_D$$

GIVES THE MASS MATRIX OF THE ν_L's IN TERMS OF THE DIRAC NEUTRINO MASS MATRIX AND THE MAJORANA MASS MATRIX OF THE ν_R's

IN SO(10) WITH ONLY ONE 10 FOR THE ELECTRO-WEAK HIGGS ONE SHOULD FIND AT M_X

$$m_t = m_e = m_\tau = m^D_{\nu\tau}$$

AND TRIVIAL CKM MATRIX AND M_D

WITH TWO OR MORE 10 ONE SHOULD FIND $m_e = m_\tau$
$m_t = m^D_{\nu\tau}$ AND M_D EQUAL TO THE CKM MATRIX

BY TAKING

$$m_D = \frac{m_\tau}{m_e}\,\mathrm{diag}\,(m_u, m_c, m_t)$$

AND A ZEE-LIKE MATRIX FOR

$$M_R \equiv \begin{pmatrix} 0 & \rho & \nu \\ \rho & 0 & \mu \\ \nu & \mu & 0 \end{pmatrix}$$

WE FIND ALMOST OPPOSITE VALUES FOR THE TWO SMALLEST m_i

$$m_1 + m_2 = -\frac{m_1 m_2}{m_3}$$

BY REQUIRING MAXIMAL MIXING FOR $\nu_\mu - \nu_\tau$ WE FIND ALMOST THE BIMAXIMAL MIXING MATRIX PREVIOUSLY MENTIONED

THE HIGHEST MATRIX ELEMENT OF M^R IS $(M^R)_{23} = \left(\frac{m_\tau}{m_e}\right)^2 \frac{m_c m_t}{\sqrt{4 m_i^2}} \simeq 3 \cdot 10^{14}\,\mathrm{GeV}$

IN GOOD AGREEMENT WITH THE SCALE OF B-L SPONTANEOUS BREAKING OF THE SO(10) THEORY PREVIOUSLY DISCUSSED

SIMILAR PROPERTIES, ALMOST BIMAXIMAL MIXING, QUASI OPPOSITE VALUES OF THE LIGHTEST NEUTRINO MASSES AND MODERATE VALUES FOR THE HIGHEST MATRIX ELEMENT OF M^R ARE FOUND IN OTHER SO(10) SEE-SAW MODELS (JEZABECK AND SUNINO, STECH)

THE TWO LAST PROPERTIES ARE RELATED

$$|\mathrm{Det}(M_R)| = \left(\frac{m_\tau}{m_b}\right)^6 \frac{m_u^2 m_c^2 m_t^2}{|m_1 m_2 m_3|}$$

$$\simeq \frac{\Delta m_3^2}{|m_1 m_2|} \, 4 \cdot 10^{30} (\mathrm{GeV})^3$$

TO UNDERSTAND THE SOURCE OF THESE PROPERTIES WE HAVE CONSIDERED THE INVERSE SEE-SAW FORMULA

$$M^R = - M^0 \left(M^L\right)^{-1} M^0_T$$

WITH A DIAGONAL M^0

$$M^0 = \frac{m_\tau}{m_e} \,\mathrm{diag}\left(m_u, m_c, m_t\right)$$

AND BIMAXIMAL MIXING FOR M^L

WE GET

$$\left(M^R\right)_{11} = -\frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left(\frac{m_\tau m_u}{m_e}\right)^2$$

$$\left(M^R\right)_{12} = \frac{1}{2\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)\left(\frac{m_\tau}{m_e}\right)^2 m_u m_c$$

$$\left(M^R\right)_{13} = -\frac{1}{2\sqrt{2}}\left(\frac{1}{m_1} - \frac{1}{m_2}\right)\left(\frac{m_\tau m_c}{m_e}\right)^2 m_u m_t$$

$$\left(M^R\right)_{22} = -\frac{1}{4}\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{2}{m_3}\right)\left(\frac{m_\tau m_c}{m_e}\right)^2$$

$$\left(M^R\right)_{23} = -\frac{1}{4}\left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{2}{m_3}\right)\left(\frac{m_\tau}{m_e}\right)^2 m_c m_t$$

$$\left(M^R\right)_{33} = -\frac{1}{4}\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{2}{m_3}\right)\left(\frac{m_\tau m_t}{m_e}\right)^2$$

TO AVOID

$$\frac{(M^R)_{11}}{(M^R)_{33}} \simeq \frac{2\, m_\mu^2}{m_t^2} \simeq 1.3 \cdot 10^{-9}$$

WE TAKE

$$\frac{1}{m_1} + \frac{1}{m_2} + \frac{2}{m_3} = 0$$

AND GET FOR THE LARGEST MATRIX ELEMENT OF M^R

$$\left|(M^R)_{23}\right| = \left(\frac{m_\tau}{m_e}\right)^2 \frac{m_c\, m_t}{m_3} \simeq 9 \cdot 10^{11}\,\mathrm{GeV}$$

CONCLUSIONS

1) THE FREEDHOM LEFT BY THE STUDY OF NEUTRINO OSCILLATIONS IN THE DETERMINATION OF M^L CAN BE USED TO REQUIRE A "NATURAL" M^R

2) WITH A DIAGONAL m_ν^D AND A BIMAXIMAL MIXING MATRIX FOR ν^L's THIS PROPERTY IS OBTAINED WITH $M^R_{33} = 0$ AND WITH THE LARGEST MATRIX ELEMENT OF M^R, $(M^R)_{23} \simeq 9 \cdot 10^{11}$ GeV IN GOOD AGREEMENT WITH THE SCALE OF B-L SYMMETRY BREAKING FOUND IN THE SO(10) THEORY WITH PATI-SALAM INTERMEDIATE SYMMETRY