

# Global Analysis of Neutrino Oscillation Data

GLF

1. Introduction: neutrino oscillation evidence
2. The "standard"  $3\nu$  interpretation
3.  $3\nu$  oscillations: atmospheric neutrinos
4.  $3\nu$  oscillations: solar neutrinos
5. Conclusions

# 1. Introduction

## Neutrino oscillation evidence

Two kinds of observables:

- total neutrino event rates

⇒ give information on "averaged" neutrino oscillation probability  $\langle P_{\alpha\beta} \rangle$

- neutrino event spectra (as a function of E, L, L/E, or t)

⇒ give information on  $\frac{\partial P_{\alpha\beta}}{\partial x}$   $x = \begin{cases} E \\ L \\ L/E \\ t \\ \dots \end{cases}$

crucial to assess oscillations unambiguously !!!

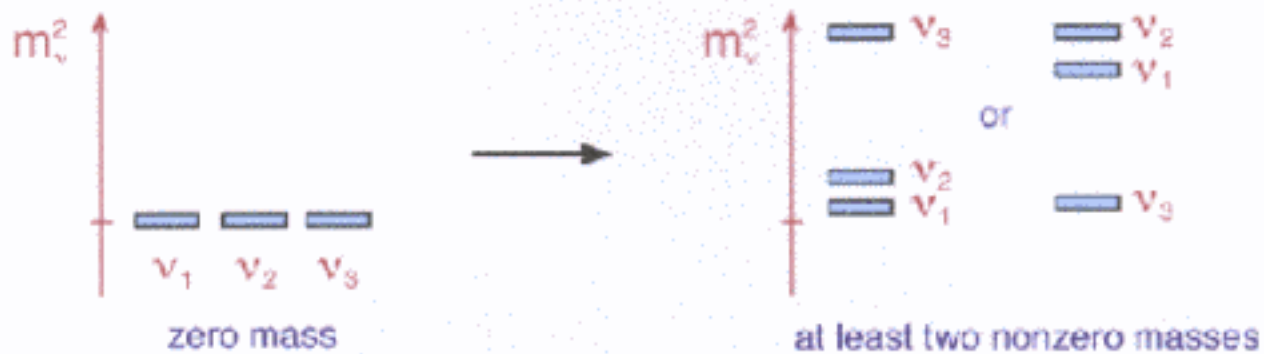
## Present evidence

	total rate	spectra
LSND	$P(\nu_\mu \rightarrow \nu_e) > 0$ (controversial)	no significant info
solar	$P(\nu_e \rightarrow \nu_e) < 1$ (robust)	no significant indication for $\frac{\partial P_{ee}}{\partial(t, E, L)} \neq 0$
atmospheric	$P(\nu_\mu \rightarrow \nu_\mu) < 1$ (robust)	$\frac{\partial P_{\mu\mu}}{\partial L} \neq 0$ (very robust) $\frac{\partial P_{\mu\mu}}{\partial E} \neq 0$ (robust)

Limiting our attention to the “robust” information, we can describe it within a “standard” 3ν interpretation

## 2. The "standard" 3ν interpretation

The two strongest sources of evidence for neutrino oscillations (**atmospheric** and **solar** ν anomalies) can be accommodated in a "standard" framework (3ν oscillations) that requires a minimal modification of the electroweak model:



$$U = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

no mixing

$$U = U(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$$

at least two nonzero  
mixing angles ( $\omega, \psi$ )

We assume then

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U(\theta_{ij}, \delta_{CP}) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

with in the following

$$\omega = \theta_{12}$$

$$\phi = \theta_{13}$$

$$\psi = \theta_{23}$$

status and prospects of such interpretative framework will be discussed in this talk

# Parameters probed by oscillations

- 1  $\delta m^2 \stackrel{\text{def}}{=} m_2^2 - m_1^2$        $\delta m^2 \ll m^2$  from phenomenology
- 2  $m^2 \stackrel{\text{def}}{=} |m_3^2 - m_{1,2}^2|$       with  $\pm m^2$  both scenarios allowed
- 3  $\omega = \theta_{12} \in [0, \pi/2]$
- 4  $\varphi = \theta_{13} \in [0, \pi/2]$
- 5  $\psi = \theta_{23} \in [0, \pi/2]$
- 6  $\delta = \delta_{\text{CP}} \in [0, \pi]$

## MINIREVIEW

PARAMETER	STATUS	PROSPECTS
$m^2$	$\sim 3 \times 10^{-3} \text{ eV}^2$ within a factor of $\sim 2$	Good. It will be better and better determined by atm. and LBL expts.
$\delta m^2$	$\neq 0$ but multiple ranges allowed (MSW, QV)	Selection of one of the different solutions will take years
$\psi$	$s_\psi^2 \sim 1/2$ within a factor of $\sim 2$	as for $m^2$
$\omega$	$\neq 0$ but multiple ranges allowed (MSW, QV)	as for $\delta m^2$
$\varphi$	$s_\varphi^2 \leq \text{few \%}$ (CHOOZ) but no reason for $s_\varphi^2 = 0!$	Its determination will be one of the major challenges for future reactor, atmospheric and LBL experiments.
$\delta$	unconstrained. Effects suppressed by $\delta m^2/m^2$	very bad before $\nu$ factories



# Graphical representation of parameter space

$\delta m^2 \ll m^2$  implies that:

- ①  $\delta_{CP} \sim$  **unobservable** effects doubly suppressed by  $\begin{cases} \delta m^2/m^2 \rightarrow 0 \\ \sin^2 \theta \rightarrow 0 \end{cases}$

we will assume  $U$  real. Difficult to prove  $\delta_{CP} \neq 0$  in future exps.

- ② **Solar  $\nu$**  (up to terms of the order  $\delta m^2/m^2$  the parameter space is spanned only by three variables:

$$(\delta m^2, \omega, \phi) \Leftrightarrow (\delta m^2, U_{e1}^2, U_{e2}^2, U_{e3}^2) \quad \text{with} \quad U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$$

or equivalently (unitarity)

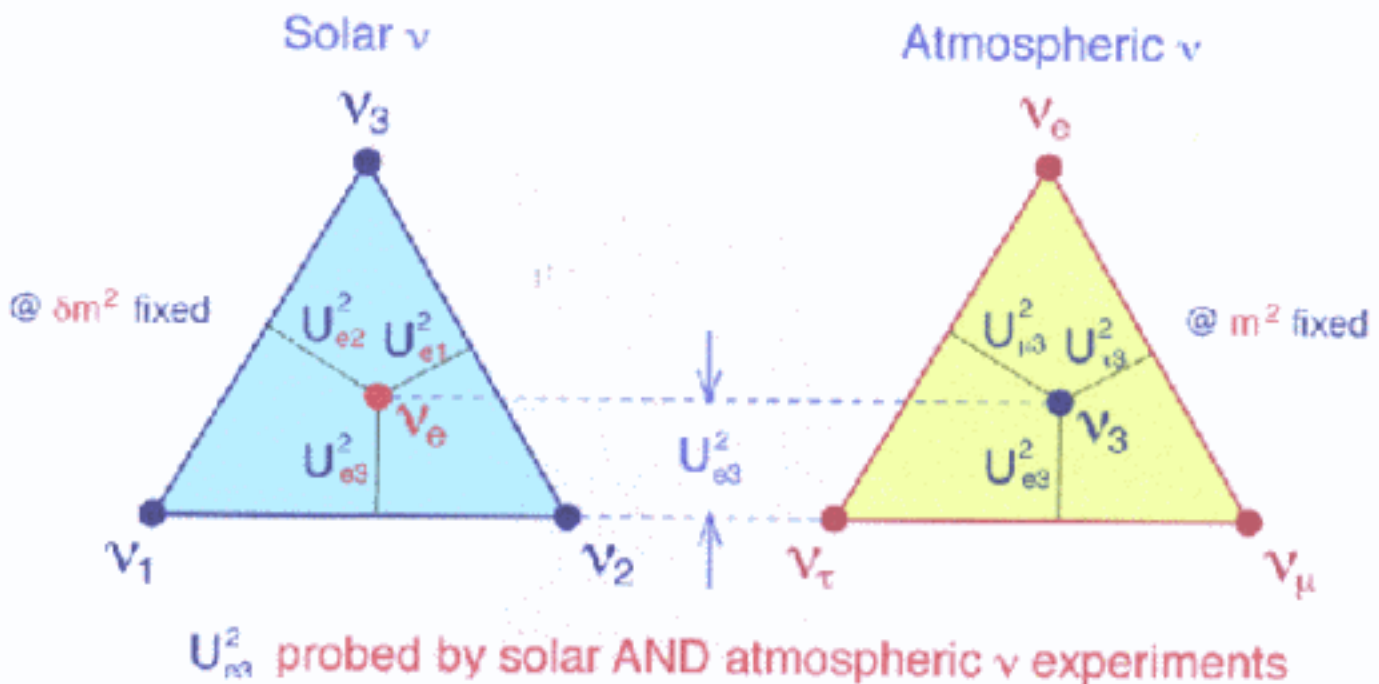
Relevant the **mass composition** of  $\nu_e$ :  $\nu_e = U_{e1} \nu_1 + U_{e2} \nu_2 + U_{e3} \nu_3$

- ③ **Atmospheric  $\nu$**  (or, in general, terrestrial  $\nu$ ): up to terms of the order  $\delta m^2/m^2$  the parameter space is spanned by only three variables:

$$(m^2, \psi, \phi) \Leftrightarrow (m^2, U_{\mu 3}^2, U_{\tau 3}^2, U_{e3}^2) \quad \text{with} \quad U_{\mu 3}^2 + U_{\tau 3}^2 + U_{e3}^2 = 1$$

or equivalently (unitarity)

Relevant the **flavour composition** of  $\nu_3$ :  $\nu_3 = U_{e3} \nu_e + U_{\mu 3} \nu_\mu + U_{\tau 3} \nu_\tau$



### 3. $3\nu$ oscillations: atmospheric neutrinos

$$\begin{aligned} \nu_3 &= U_{e3} \nu_e + U_{\mu 3} \nu_\mu + U_{\tau 3} \nu_\tau = \\ &= s_\psi \nu_e + c_\psi (s_\psi \nu_\mu + c_\psi \nu_\tau) \end{aligned}$$

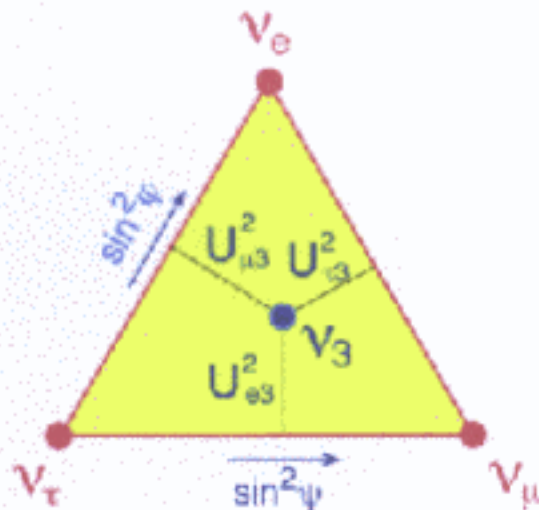
@  $m^2$  fixed

being

$$U_{e3}^2 = s_\psi^2$$

$$U_{\mu 3}^2 = c_\psi^2 s_\psi^2$$

$$U_{\tau 3}^2 = c_\psi^2 c_\psi^2$$



The analysis includes:

- The latest (December 2000: 79.5 kTy) **SK** data:

- 10 SubGeV **e**-like bins
- 10 SubGeV **μ**-like bins
- 10 MultiGeV **e**-like bins
- 10 MultiGeV **μ**-like bins
- 5 stopping upgoing **μ** bins
- 10 through-going **μ** bins

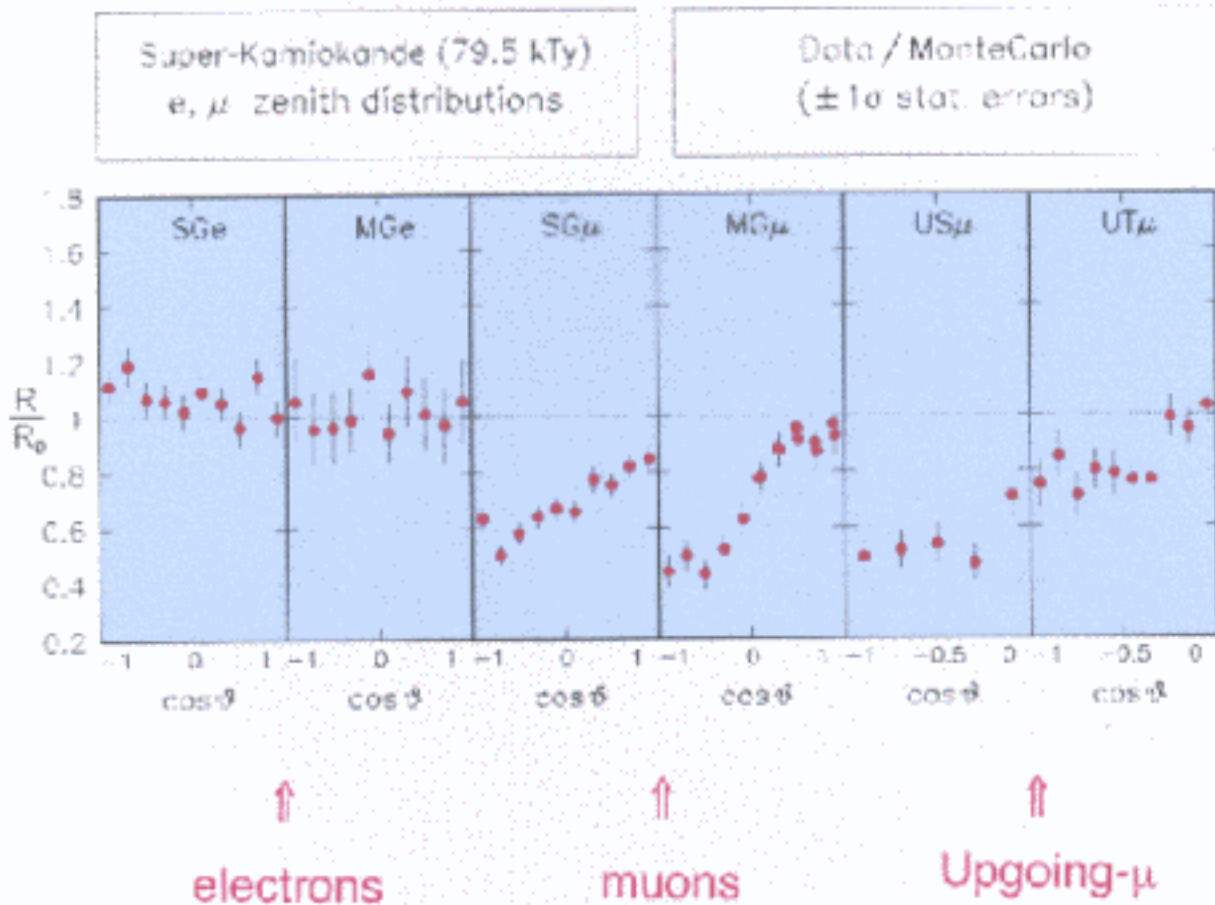
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- The latest (1999) **CHOOZ** total rate

1 data point

# SK zenith distributions (Dec. 2000 data: 79.5 kTy)

(normalized to **NO OSCILLATION** in each bin)



## Comments:

- **electrons:** no significant deviation from a flat shape (any excess disappeared)
- **up-muons:** UT $\mu$  possibly affected by fluctuations (shape not statistically stable yet)

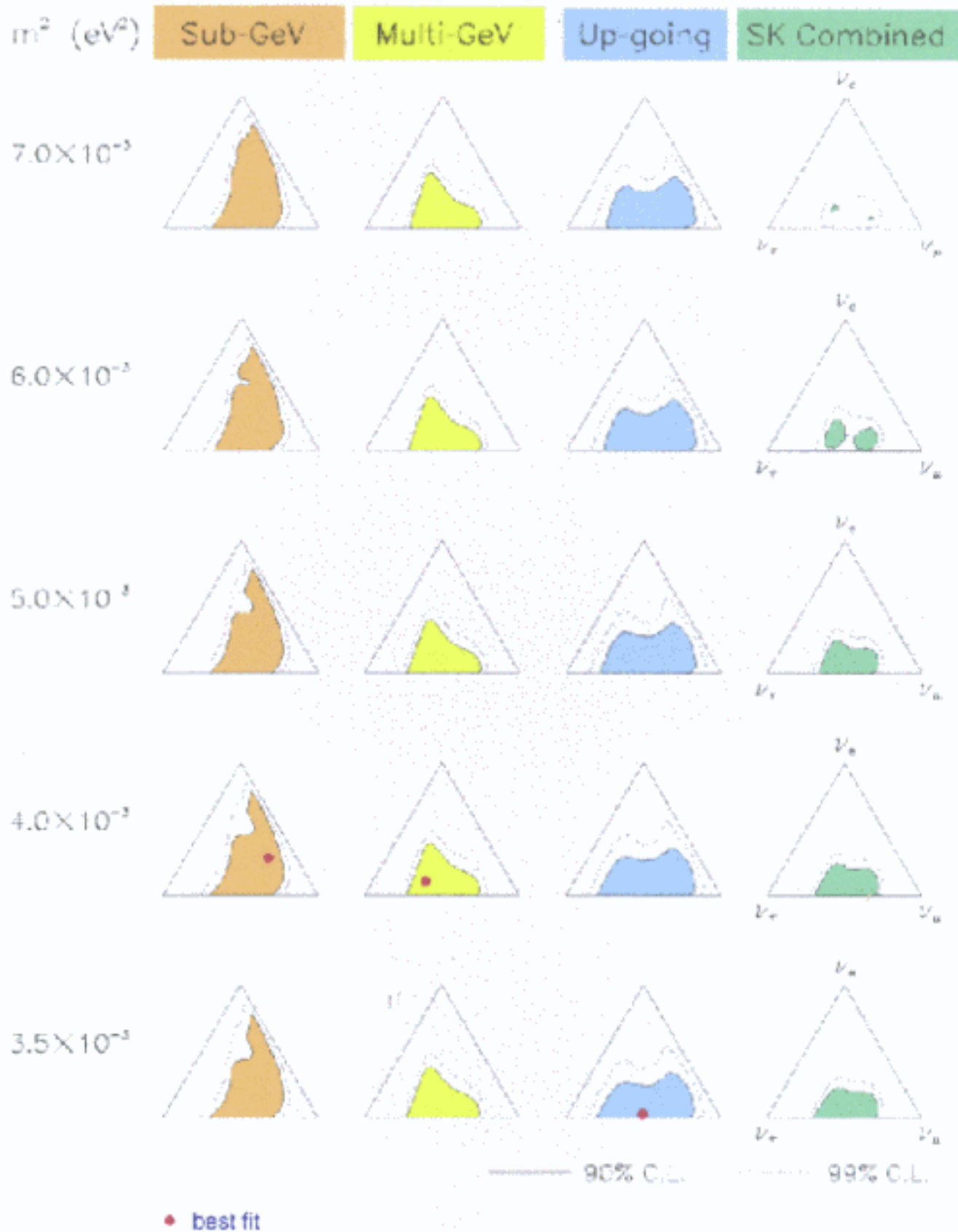
## CHOOZ

$$\frac{R_{\text{exp}}}{R_{\text{theo}}} = 1.01 \pm 2.8\% (\text{stat}) \pm 2.7\% (\text{syst})$$



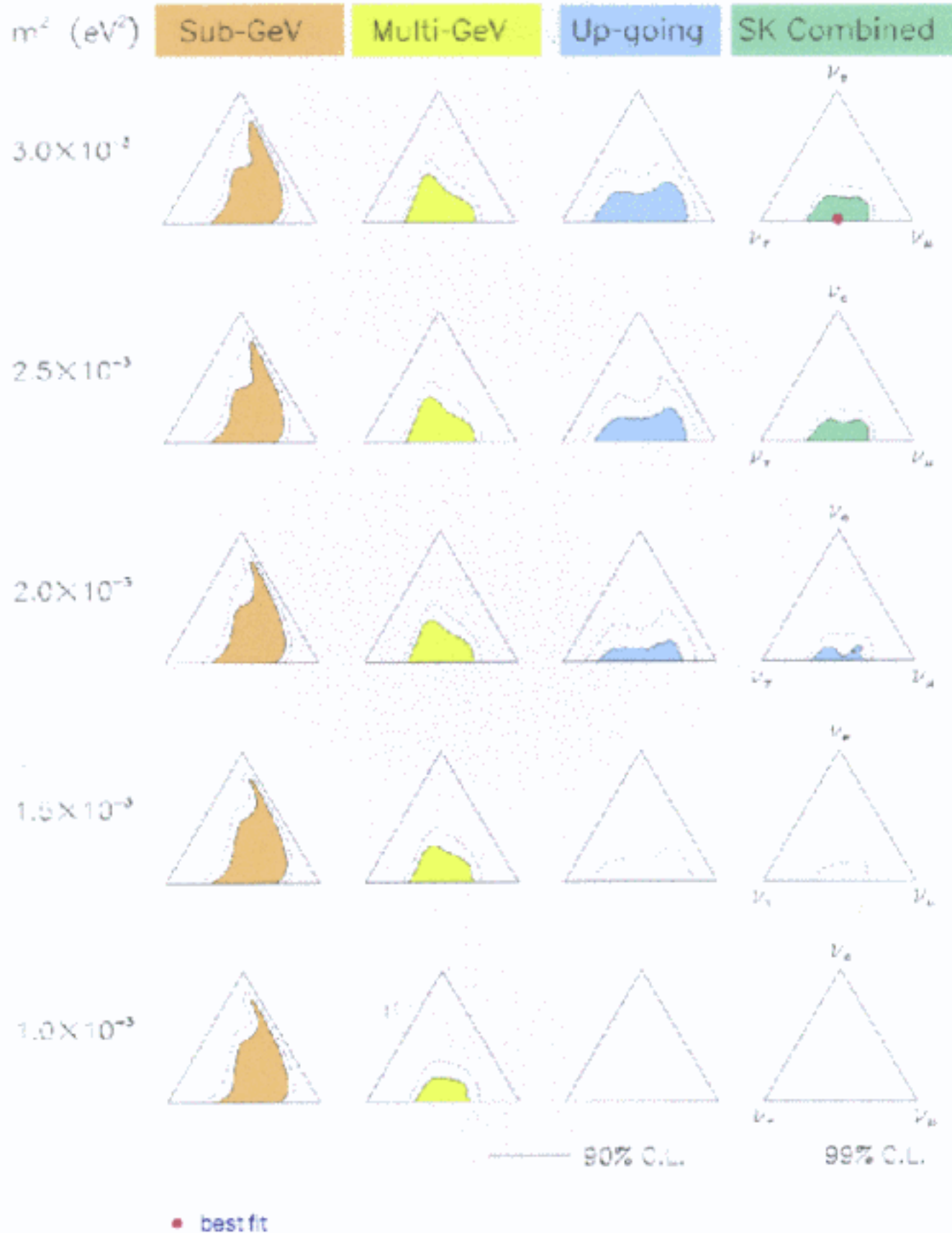
# Allowed regions in a three-flavour approach

(Dec. 2000 SK data: 79.5 kTy)



# Allowed regions in a three-flavour approach

(Dec. 2000 SK data: 79.5 kTy)



# Combining Superkamiokande and CHOOZ

(Dec. 2000 SK data: 79.5 kTY)

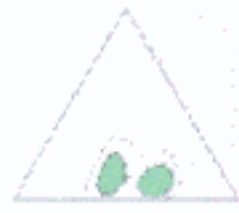
$m^2$  ( $eV^2$ )

SK

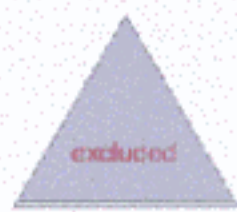
CHOOZ

SK+CHOOZ

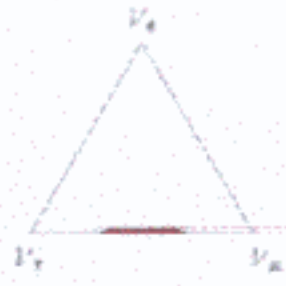
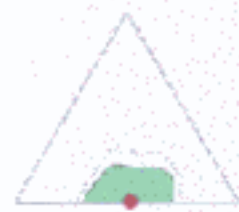
$6.0 \times 10^{-3}$



$4 \times 10^{-3}$

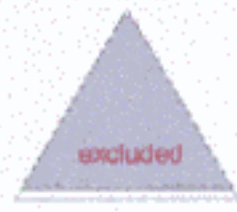


$3.0 \times 10^{-3}$

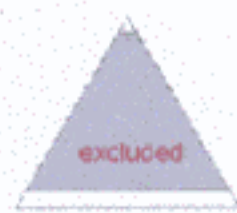


$U_{e3}^2$

$2.0 \times 10^{-3}$



$1.5 \times 10^{-3}$



— 90% C.L.

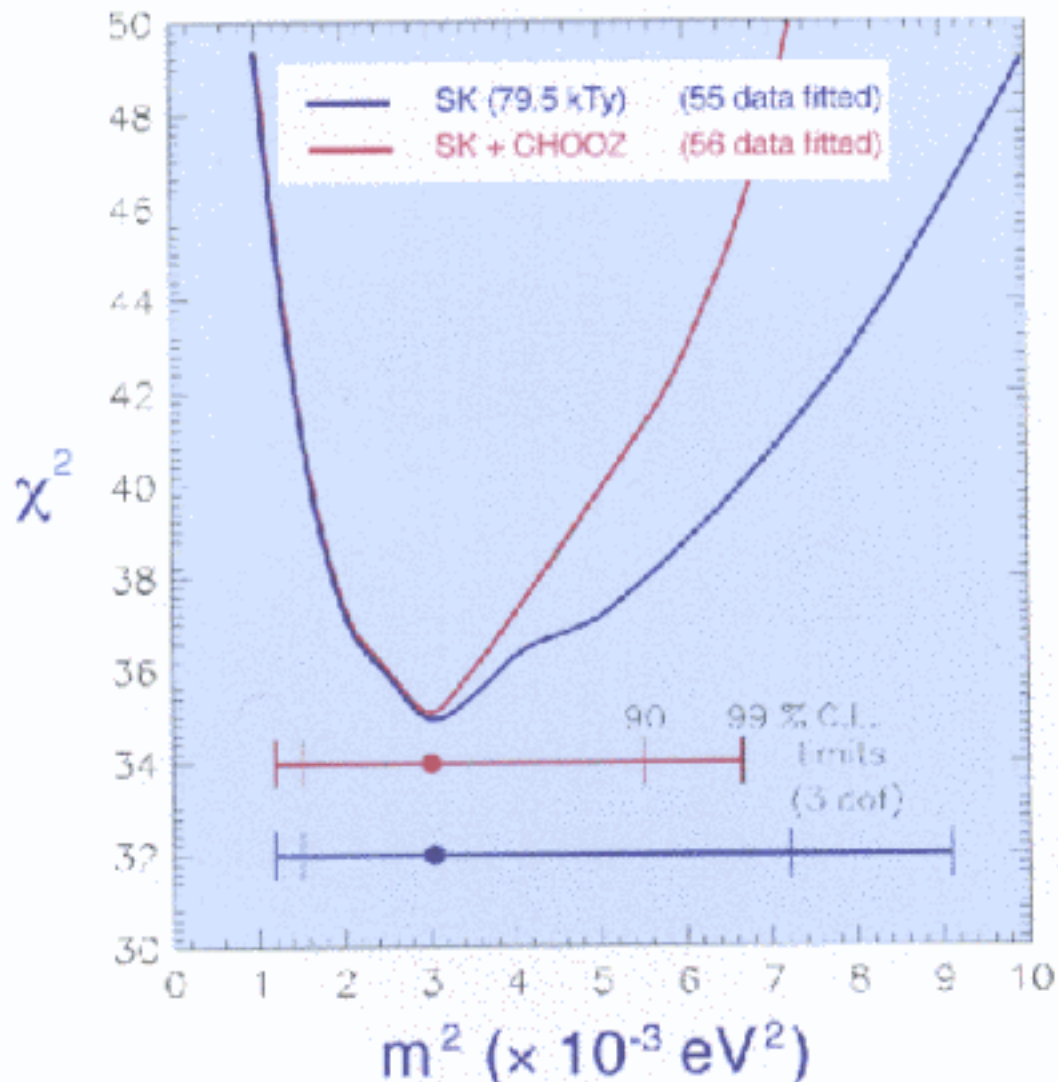
--- 99% C.L.



scenarios with large  $\nu_e$  mixing excluded, e.g. threefold maximal mixing

# Bounds on $m^2$ for unconstrained 3v mixing

(79.5 kTy SK data)



● best-fit @  $m^2 = 3.0 \times 10^{-3} \text{ eV}^2$

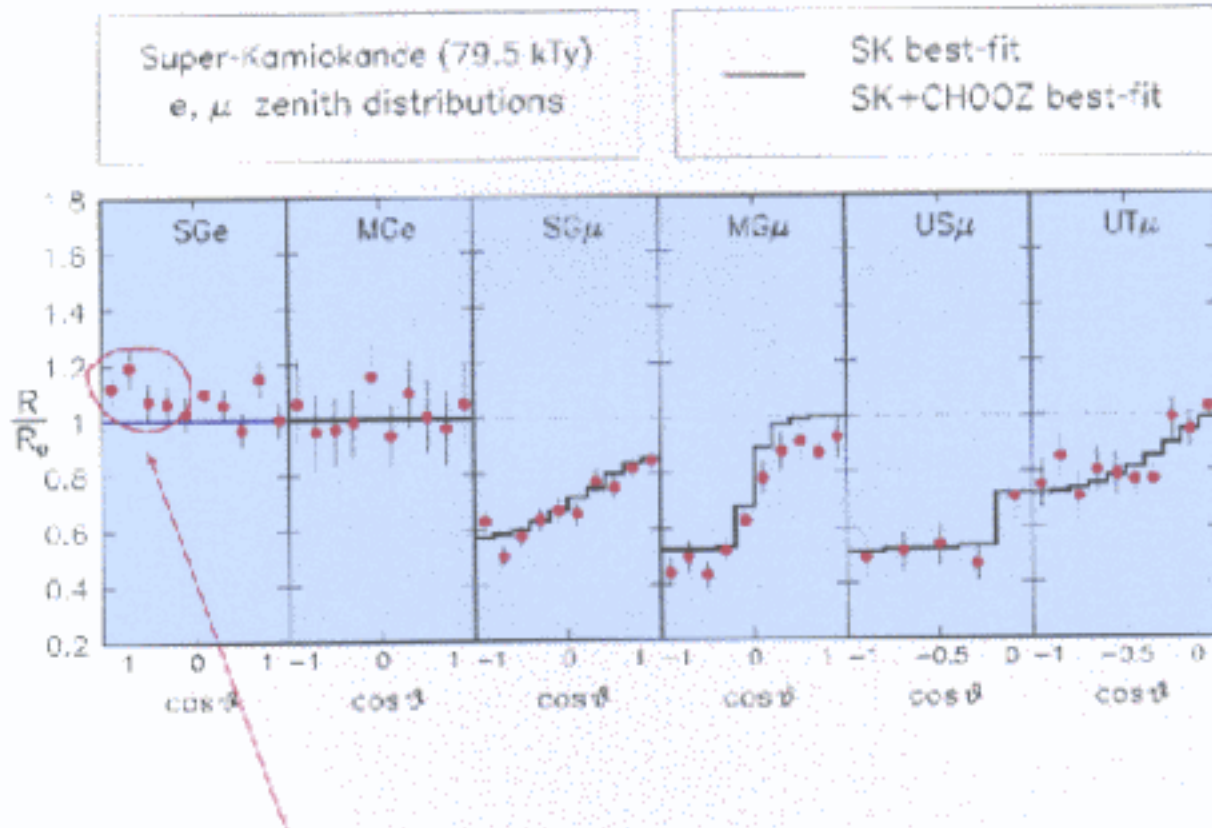
●  $m^2_\nu \uparrow$   $\begin{cases} \text{---} \nu_2 \\ \text{---} \nu_1 \\ \text{---} \nu_3 \end{cases}$  The fit for  $-m^2$  is very similar: SK does not distinguish the two cases. In the limit of pure  $\nu_\mu \leftrightarrow \nu_\tau$ , i.e.  $\varphi = 0$ ,

$$P_{\text{osc}}(m^2) = P_{\text{osc}}^{\text{vac}}(m^2) = P_{\text{osc}}^{\text{vac}}(-m^2)$$

For small  $\varphi$  there are small differences due to matter effects, unobserved at present

# Best-fit distributions

SK alone  
SK+CHOOZ  $\left. \begin{array}{l} \\ \end{array} \right\} U_{e\theta}^2$  consistent with zero  
(pure  $\nu_\mu \leftrightarrow \nu_e$  favoured)



Possible deviations from flat due to  $\nu_\mu \leftrightarrow \nu_e$  oscillations are typically smaller than these

Sources of possible deviations:

- $\varphi \neq 0$
- subleading  $\delta m^2$  effects if  $\delta m^2 \sim 10^{-4} \text{ eV}^2$

Both effects may alter the SGeV/MGeV  $\nu_e$  distributions:  
much higher SK **statistics** needed to see such effects !!



## Status of $(m^2, \psi, \varphi)$ constraints

- 1  $m^2 = 3 \times 10^{-3} \text{ eV}^2$  within a factor of 2 ( $1.5 \div 6.0 \times 10^{-3} \text{ eV}^2$ )
- 2  $s_{\psi}^2 \approx 0.5 \pm 0.17$
- 3  $s_{\varphi}^2 \lesssim \text{few percent}$

## Prospects

- SK will steadily narrow the range of  $(m^2, s_{\psi}^2)$  until systematics will dominate. Next major improvement will be provided by LBL experiments.

- Signals of  $s_{\varphi}^2 \neq 0$  are, and will be, more difficult to observe:

SK  $\Rightarrow$  Typical signals of  $s_{\varphi}^2 \neq 0$  are smaller than present  $1\sigma$  statistical uncertainties. To establish them at the  $2\sigma$  level, more than 4 years are required... (much more if systematics are included ...)

LBL  $\Rightarrow$  Signals of  $s_{\varphi}^2 \neq 0$  should be searched in the  $\nu_{\mu} \leftrightarrow \nu_e$  channel, (with  $P_{e\mu} \propto s_{\varphi}^2$ ). However, CHOOZ implies that  $S/B \lesssim 1$  in LBL. So, the **e-flavor background** should be known precisely. This is difficult but important:  $\varphi \neq 0$  is the only chance to observe MSW effect with terrestrial expts. (apart from the "exotic"  $\nu_{\mu} \leftrightarrow \nu_{\tau}$  case)

$\nu$  factories  $\Rightarrow$  Of course, they may provide the real option to observe  $s_{\varphi}^2 \neq 0$  in a "relatively far" future

reactors  $\Rightarrow$  Next logical step to increase the sensitivity to  $s_{\varphi}^2$  is to place a **near detector**. There is a proposal of Krasnoyarsk.

## 4. $3\nu$ oscillations: solar neutrinos

$$\begin{aligned} \nu_e &= U_{e1} \nu_1 + U_{e2} \nu_2 + U_{e3} \nu_3 \\ &= s_\phi \nu_3 + c_\phi (s_\omega \nu_1 + c_\omega \nu_2) \end{aligned}$$

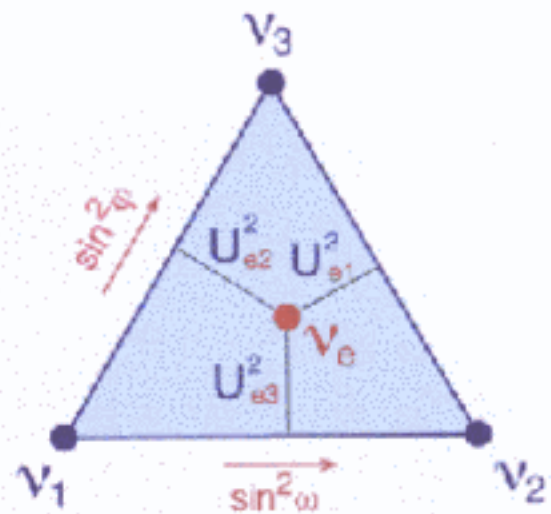
@  $\delta m^2$  fixed

being

$$U_{e3}^2 = s_\phi^2$$

$$U_{e2}^2 = c_\phi^2 c_\omega^2$$

$$U_{e1}^2 = c_\phi^2 s_\omega^2$$



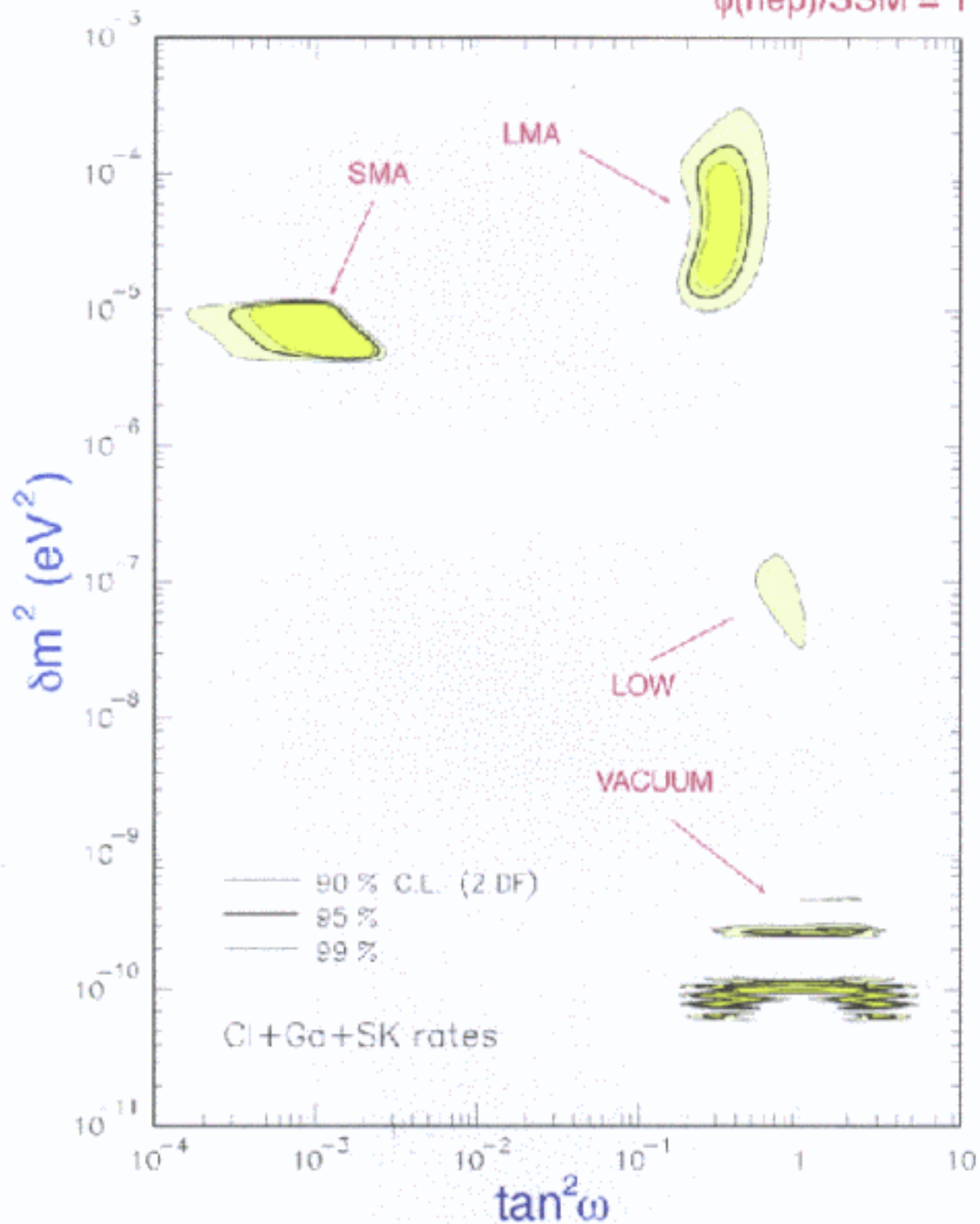
The analysis includes:

- 4 total rates:
  - Cl = Homestake
  - Ga = GALLEX+SAGE
  - SK = SuperKamiokande (data presented Dec., @ 2000)
- 18 SK energy bins (-1 free renormalization factor)
- day-night effect from SK including separately Sp(D) & Sp(N)

In the following we will assume: SSM = BP 2000  
 $\phi(\text{hep}) = \text{SSM value}$

# 2ν oscillations ( $\varphi = 0$ ): total rates

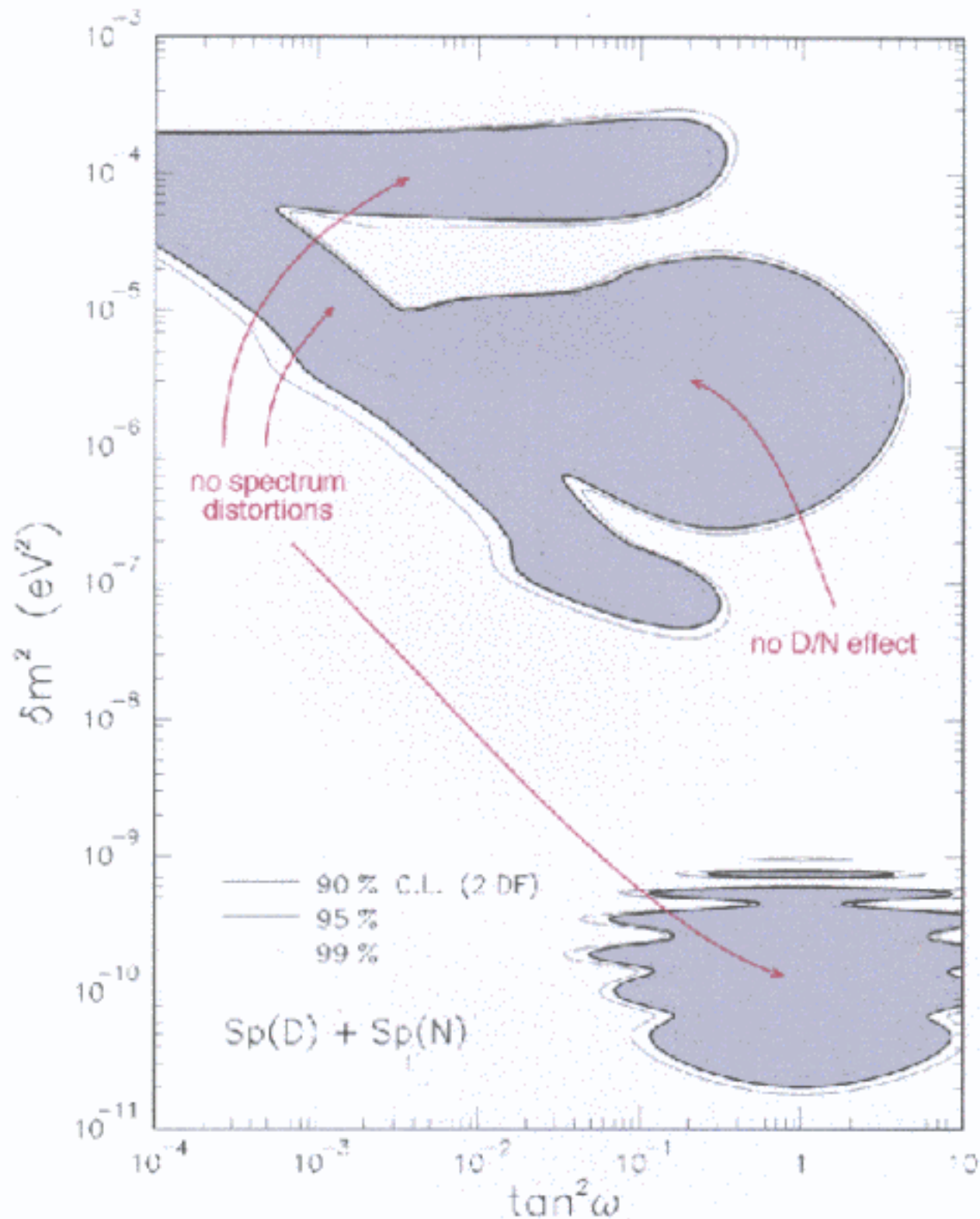
$\phi(\text{hep})/\text{SSM} = 1$



SMA:	$\chi^2 \sim 0.25$	best fit (2 dof)
VACUUM:	$\chi^2 \sim 0.26$	
LMA:	$\chi^2 \sim 3.55$	
LOW:	$\chi^2 \sim 8.05$	

# 2ν oscillations ( $\varphi = 0$ ): SK spectrum

regions excluded by  $Sp(D) + Sp(N)$  (hep/SSM=1)

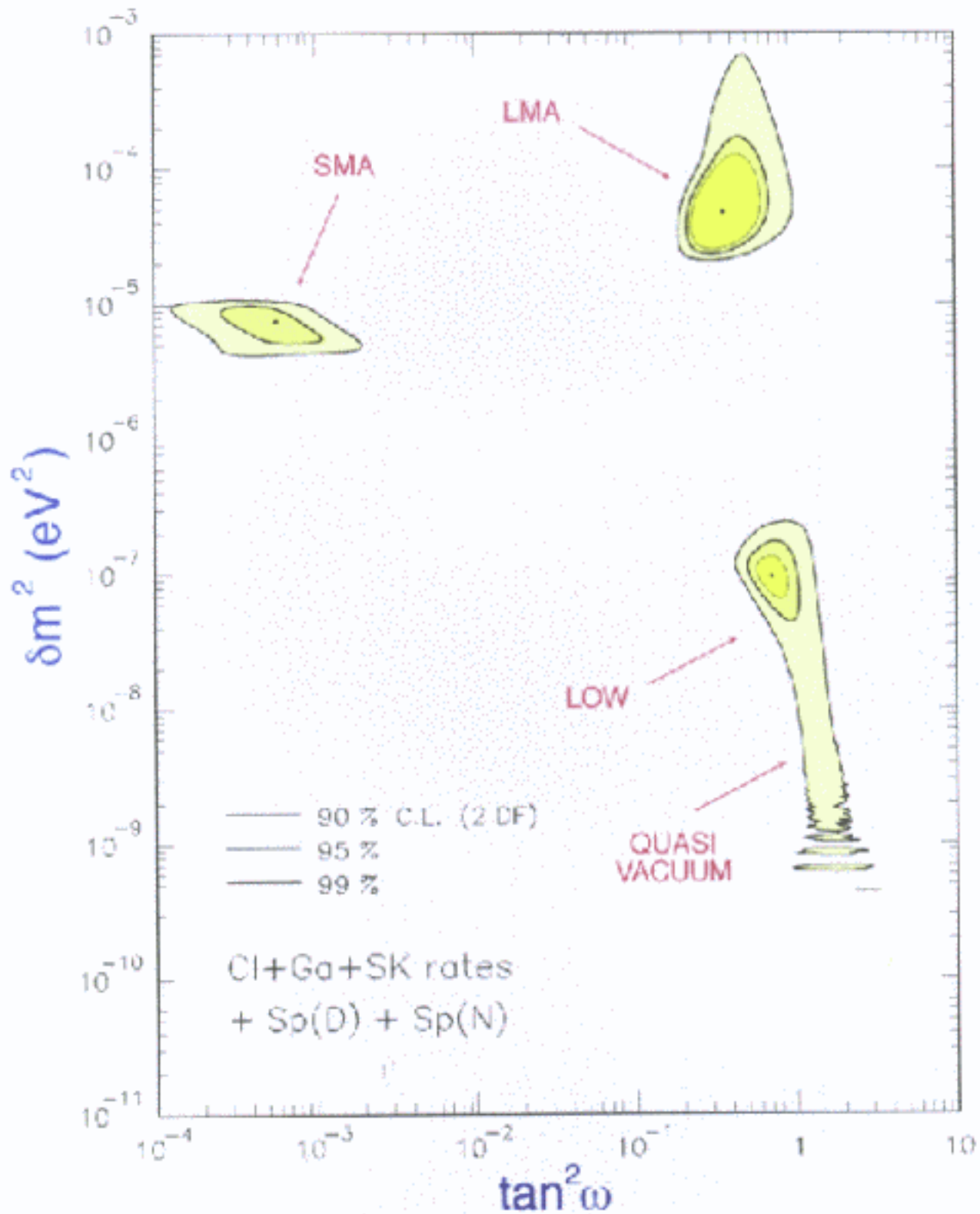


Comment:

- Poor overlap between regions allowed by spectrum and by rates (see the previous figure)

## 2ν oscillations ( $\varphi = 0$ )

total rates and SK spectrum ( $N_{\text{cor}} = 36$ )



LMA:  $\chi^2 \sim 35.1$

LOW:  $\chi^2 \sim 39.0$

SMA:  $\chi^2 \sim 40.8$

More spectral data needed to prefer  
or to exclude one of the solutions



## Towards a 3ν analysis

$$\begin{array}{ccc} \textcircled{2\nu} & & \textcircled{3\nu} \\ (\delta m^2, \omega) & \xrightarrow{m^2 \rightarrow \infty} & (\delta m^2, \omega, \varphi) \\ P_{2\nu}(v_e \rightarrow v_e) & \xrightarrow{m^2 \rightarrow \infty} & P_{3\nu} = c_\varphi^4 P_{2\nu} \Big|_{N_e = c_\varphi^2 N_e} + s_\varphi^4 \end{array}$$

$\varphi$  small (CHOOZ) implies that  $P_{3\nu} \sim P_{2\nu}$ , so why we study the case of unconstrained  $\varphi$  ?

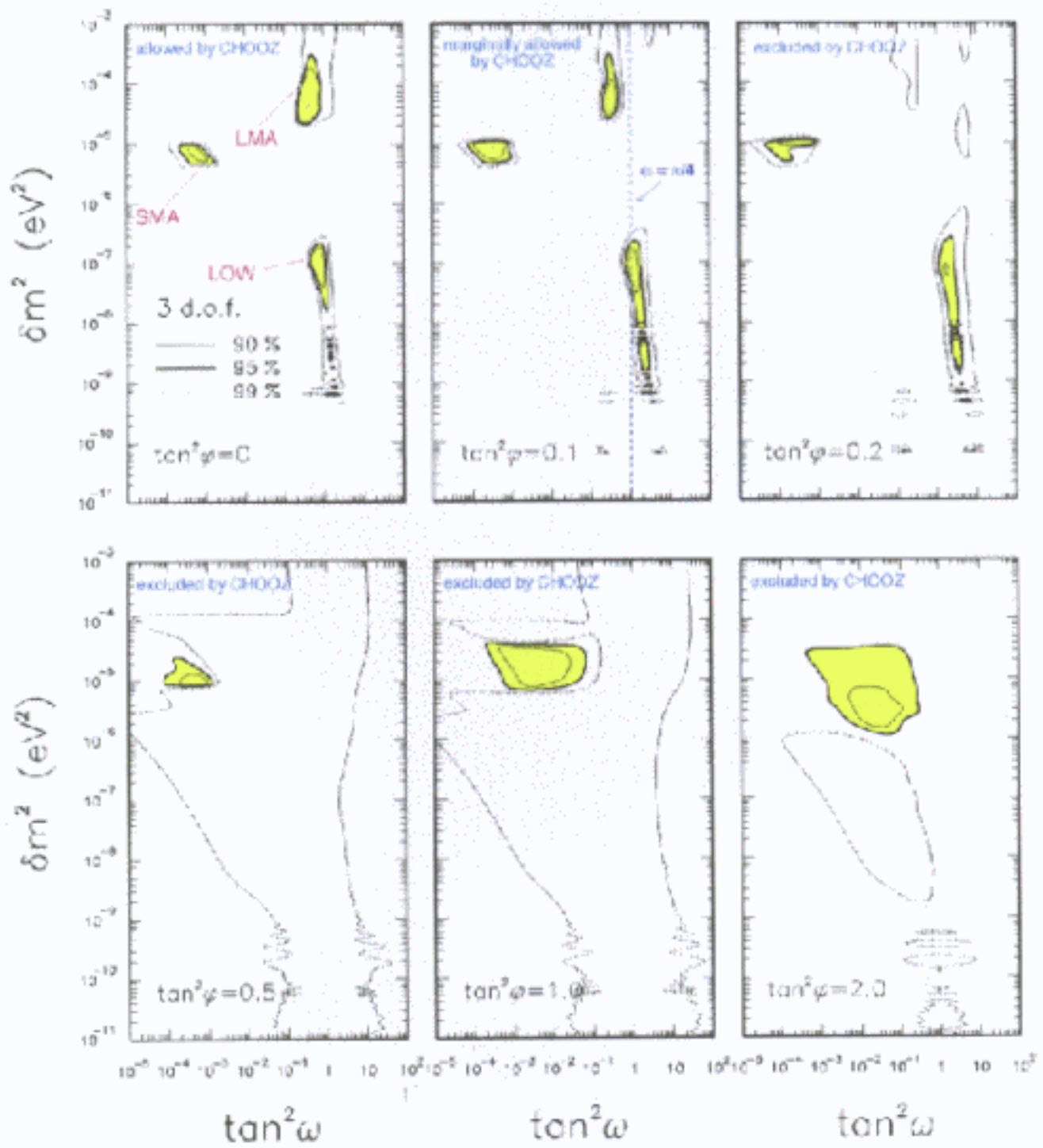
Two reasons:

- Investigate if **solar ν data** alone (without CHOOZ) prefer small  $\varphi$ , in the same way as atmospheric data alone
- Study the behaviour of the usual 2ν solutions, in particular **SMA**, **LMA** and **LOW**, under small  $\varphi$  perturbations

In the following  $\phi(\text{hep}) = \text{SSM value}$  is assumed.

# 3ν solutions (hep/SSM = 1)

total rates with constraints from Sp(D)+Sp(N)

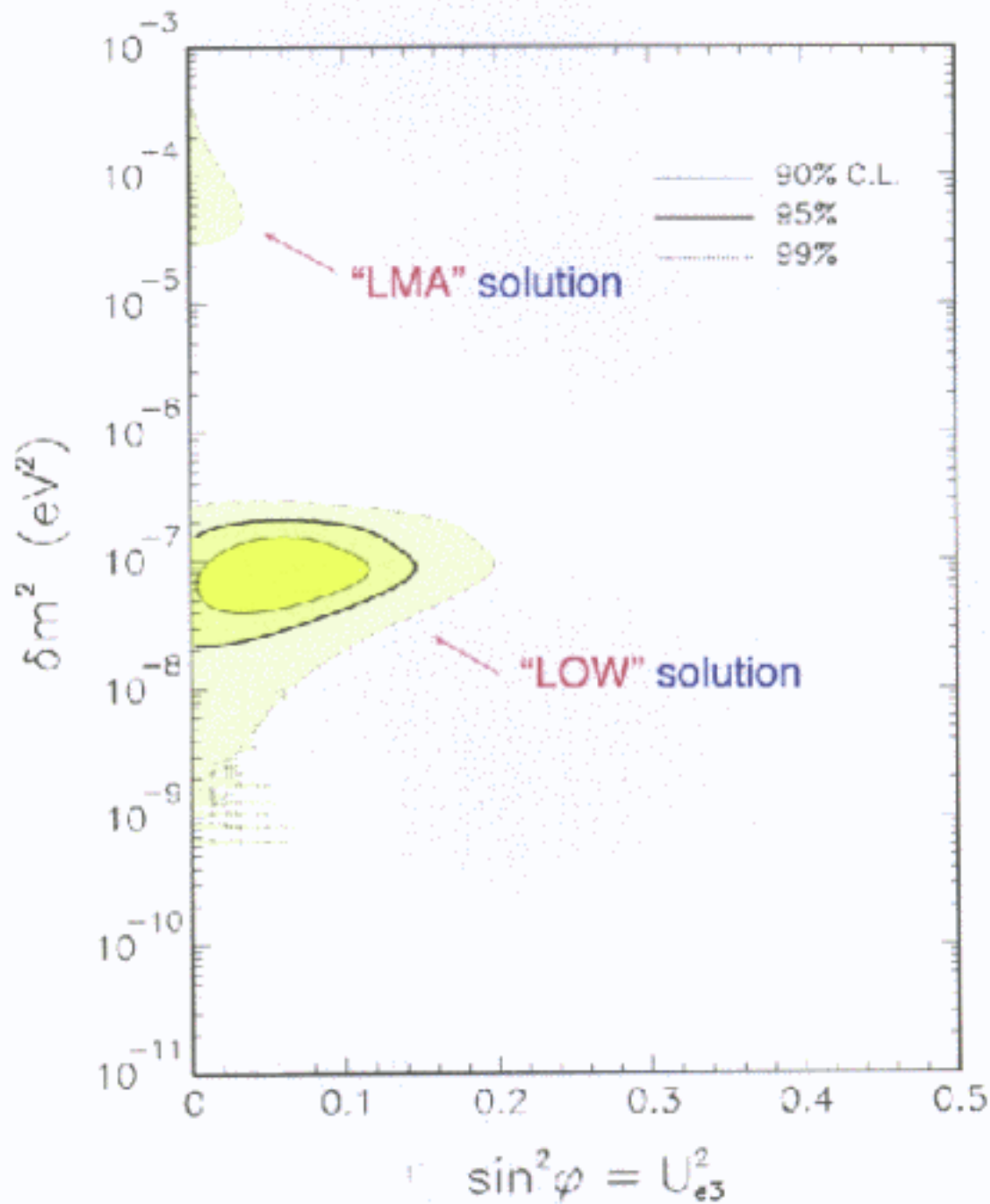


Comments:

- Still loose bounds ( $s_{12}^2 < 0.7$ ) with  $s_{12}^2 \sim 0.1$  preferred
- For small  $\varphi$ , maximal mixing solutions are allowed! The LOW solution migrates toward  $\omega = \pi/4$ , and one can even have solutions for  $\omega = \pi/4 + \epsilon$  (completely missed if one uses  $\sin^2 2\omega$  as variable)

## 3ν solutions @ maximal mixing

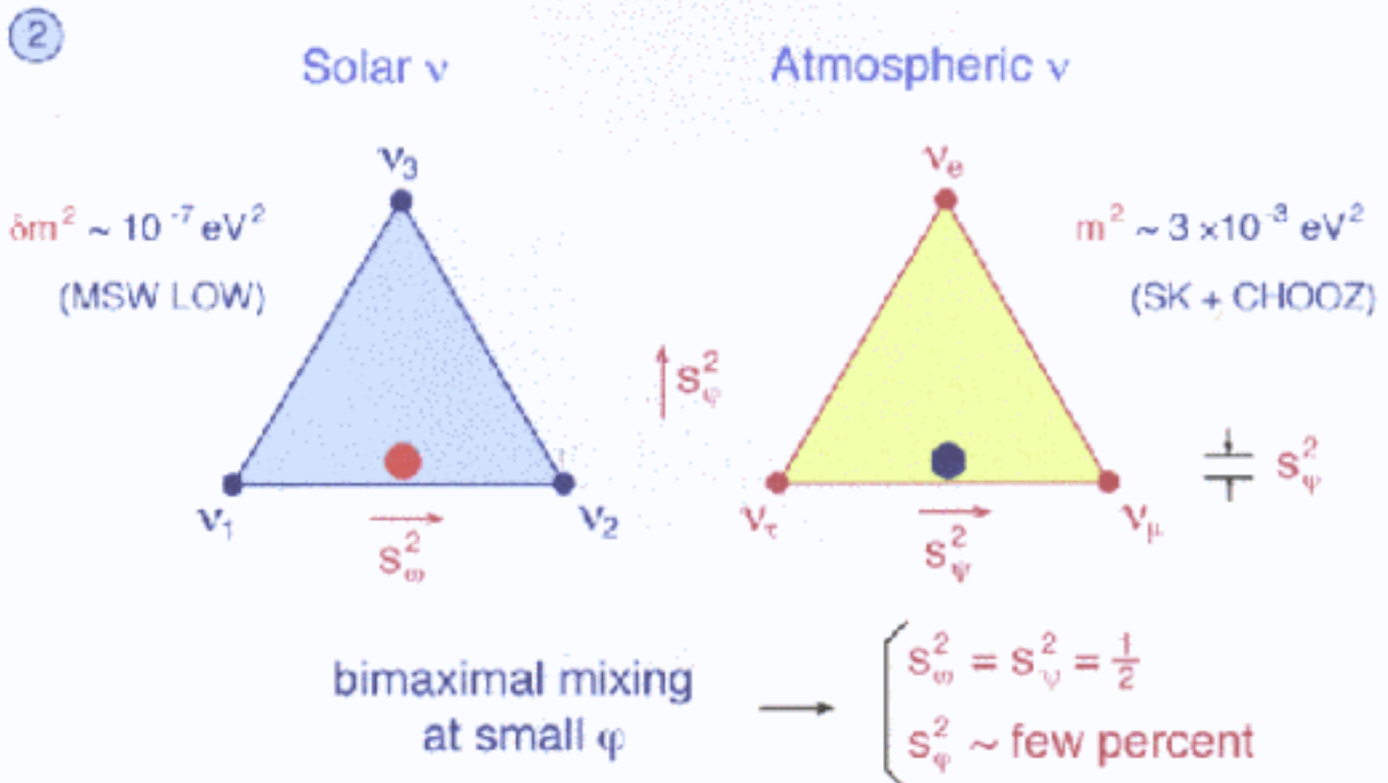
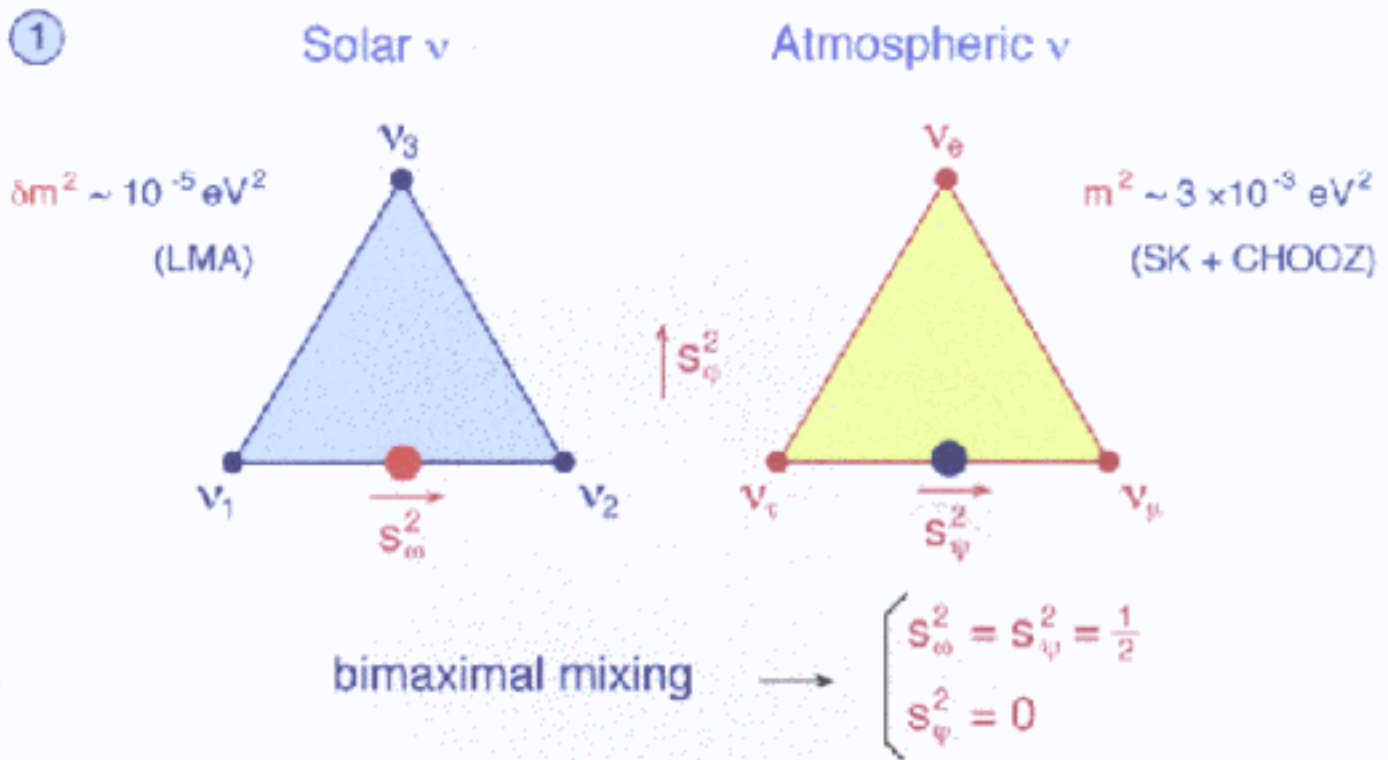
Comparison between "LOW" and "LMA" solutions assuming maximal  $\nu_{1,2}$  mixing:  $U_{e1}^2 = U_{e2}^2$  ( $\theta = \pi/4$ )



At maximal ( $\nu_1, \nu_2$ ) mixing:

- the **LOW** solution is enhanced for  $U_{e3}^2 \sim 0.05$
- $\chi_{\min}^2 = 39.9$  (for  $\delta m^2 = 7.8 \times 10^{-8} \text{ eV}^2$  and  $\sin^2 \varphi = 6 \times 10^{-2}$ )

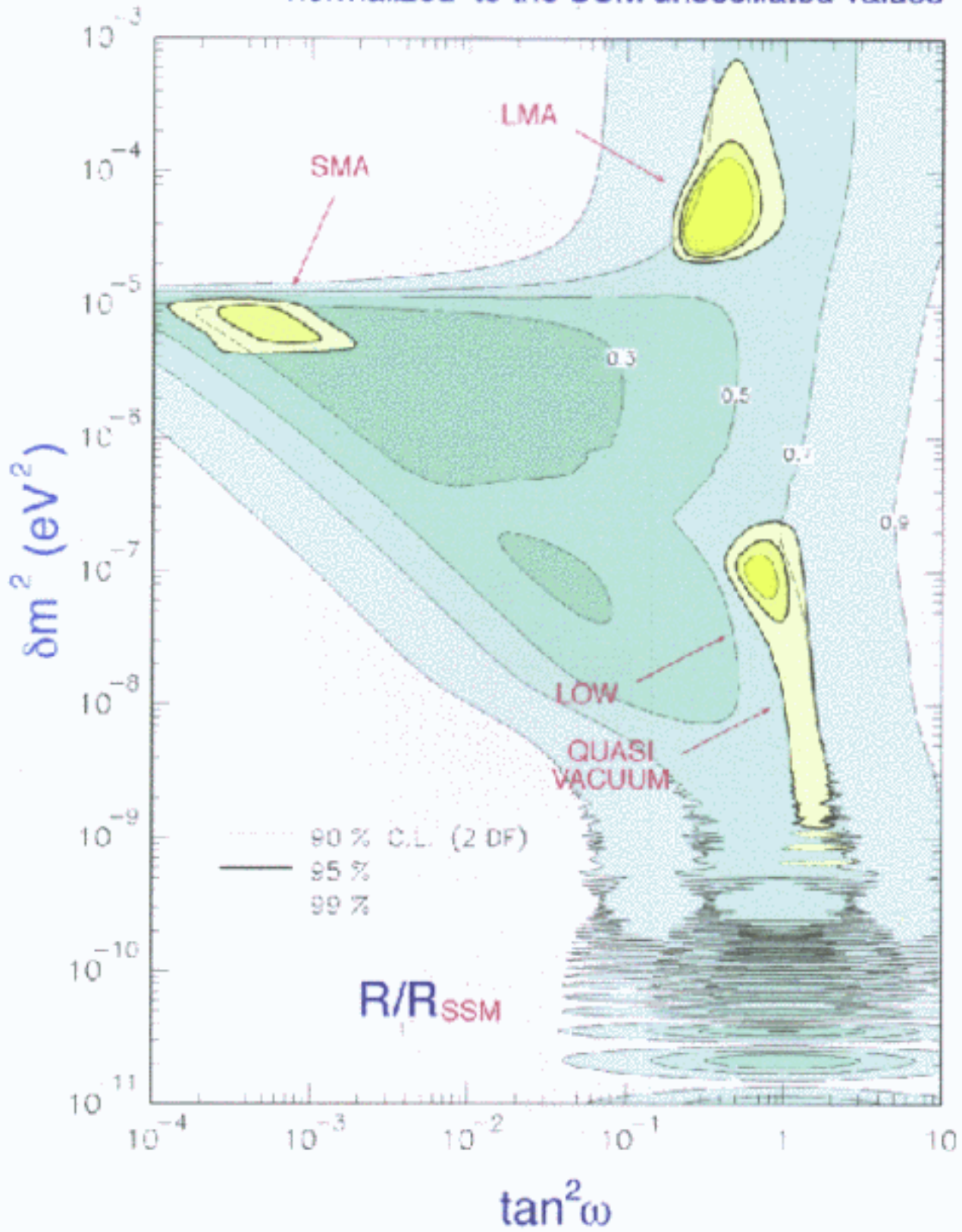
The previous result is interesting for model building: **bimaximal mixing** can be reached not only with the **LMA** solution, but also with the **LOW** solution, but as a **bimaximal mixing** at small  $\varphi$ :





# Borexino total rates compared with the SMA, LMA and LOW solutions

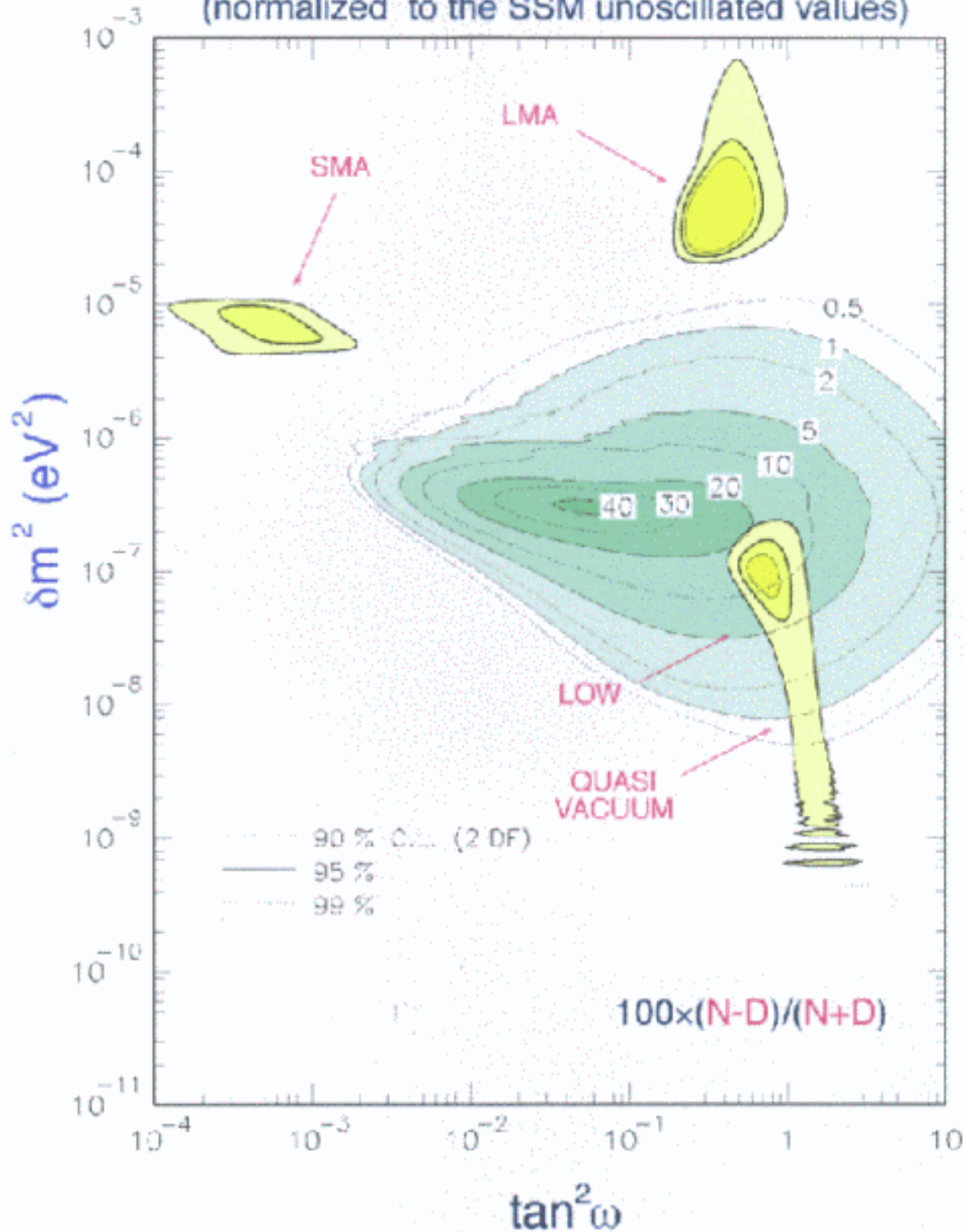
yearly-averaged total rates (N+D)  
normalized to the SSM unoscillated values



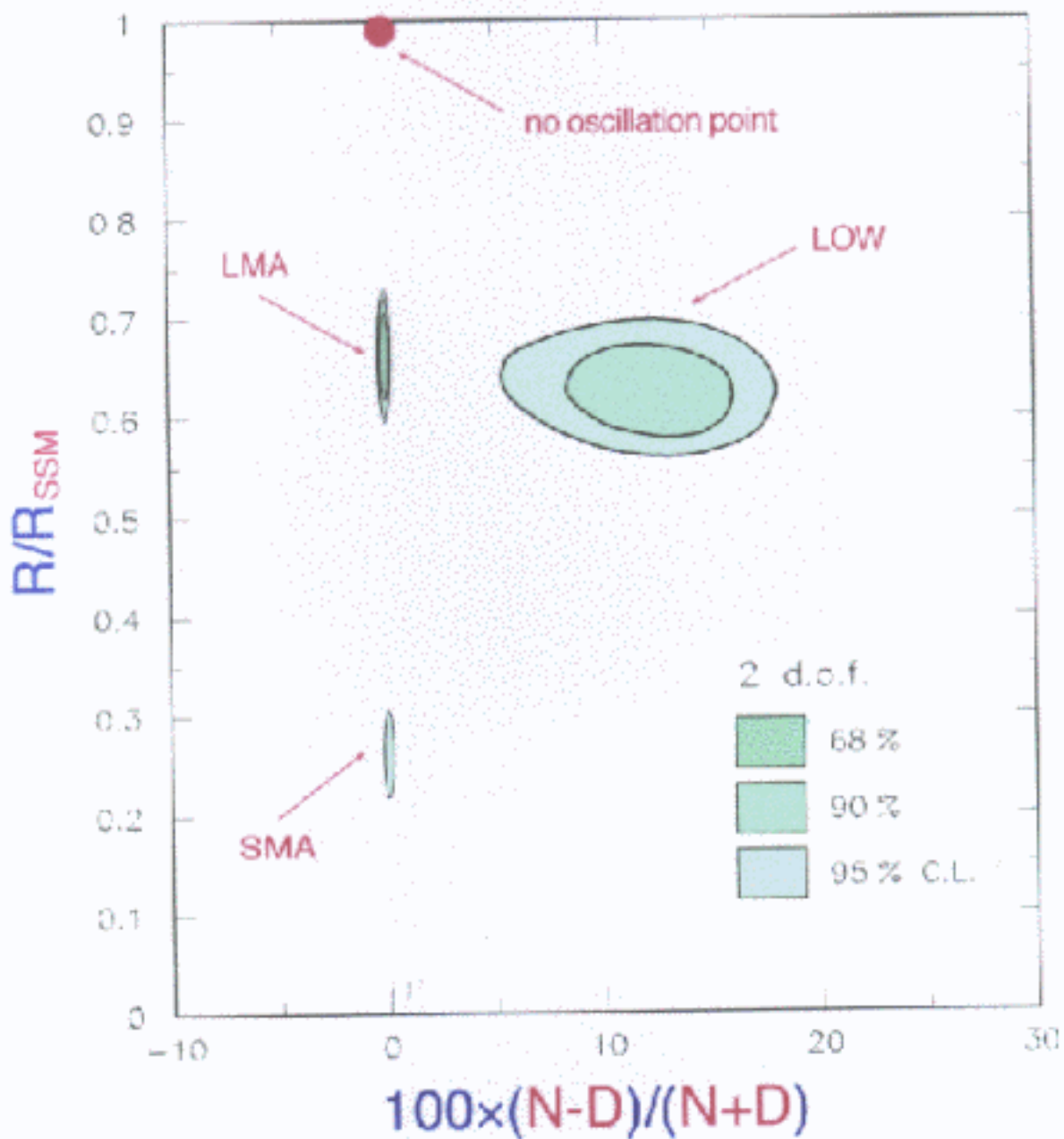


# Borexino N-D asymmetry compared with the SMA, LMA and LOW solutions

yearly-averaged nighttime and daytime rates  
(normalized to the SSM unoscillated values)

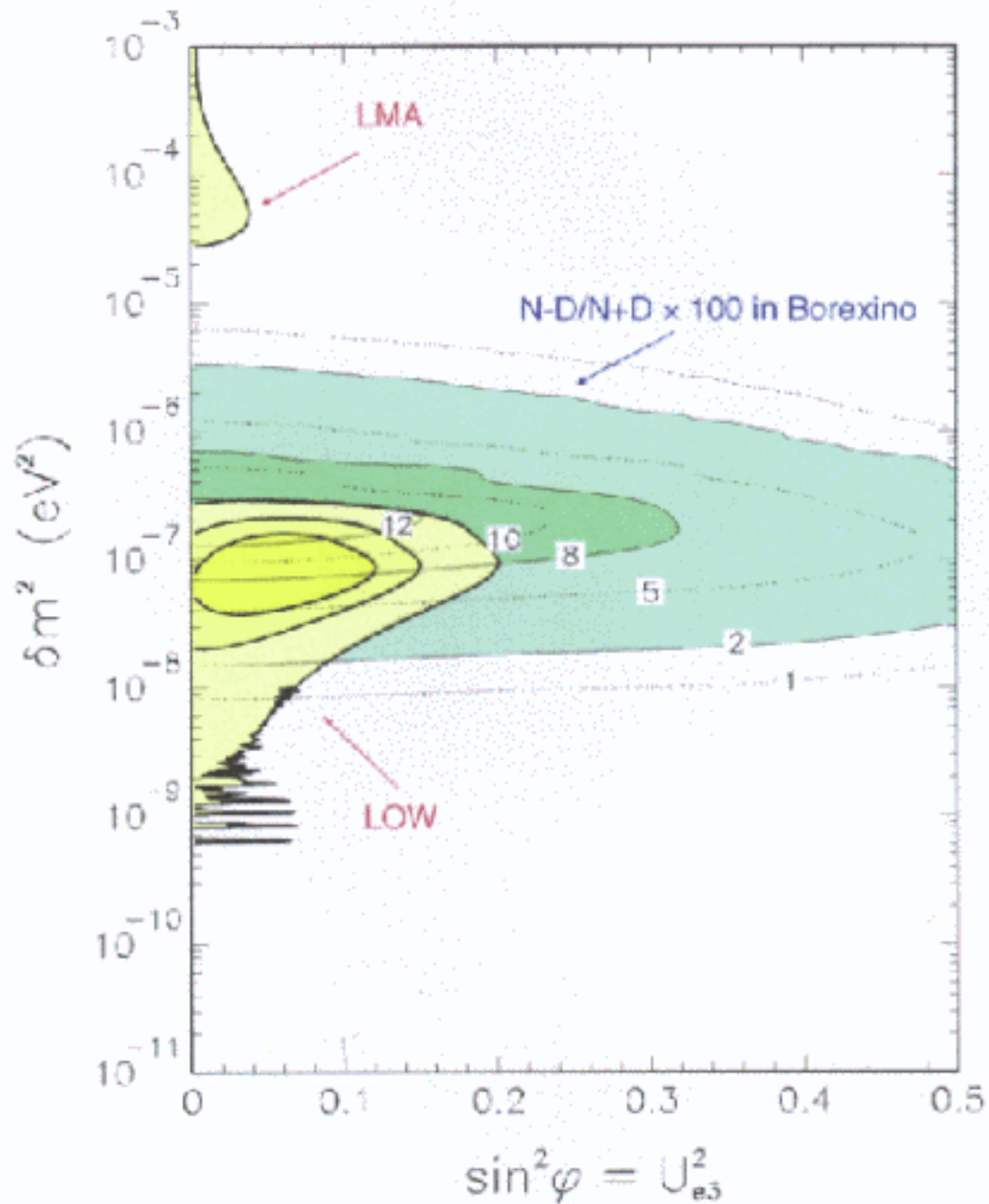


# Borexino discovery potential compared with the SMA, LMA and LOW solutions



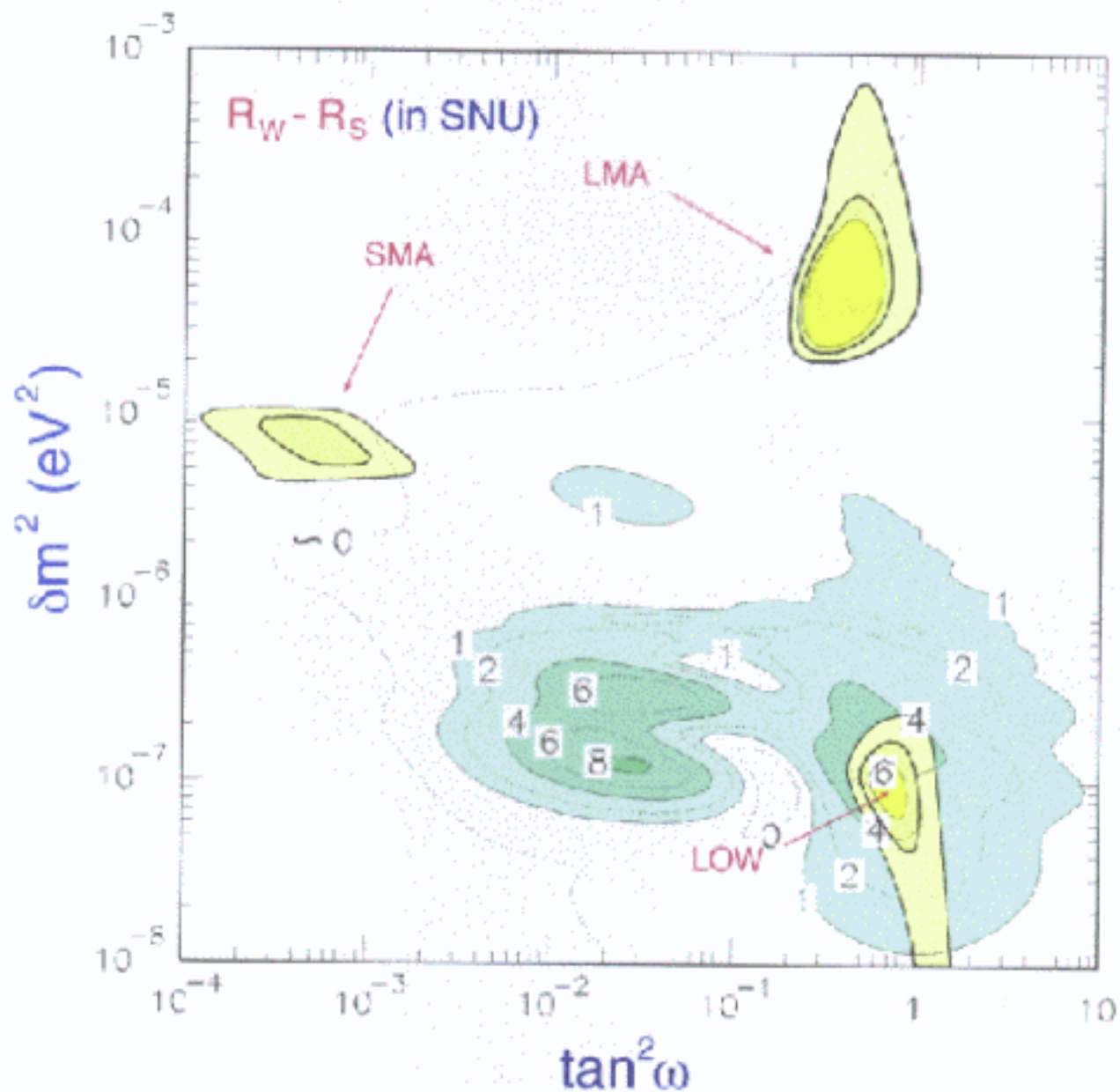
Borexino N-D asymmetry compared with the **LMA** and **LOW** solutions at maximal mixing ( $\omega = \pi/4$ )

yearly-averaged **nighttime** and **daytime** rates  
(normalized to the SSM unoscillated values)



# Gallium Neutrino Observatory

potential discovery of GNO compared with the **SMA**, **LMA** and **LOW** solutions

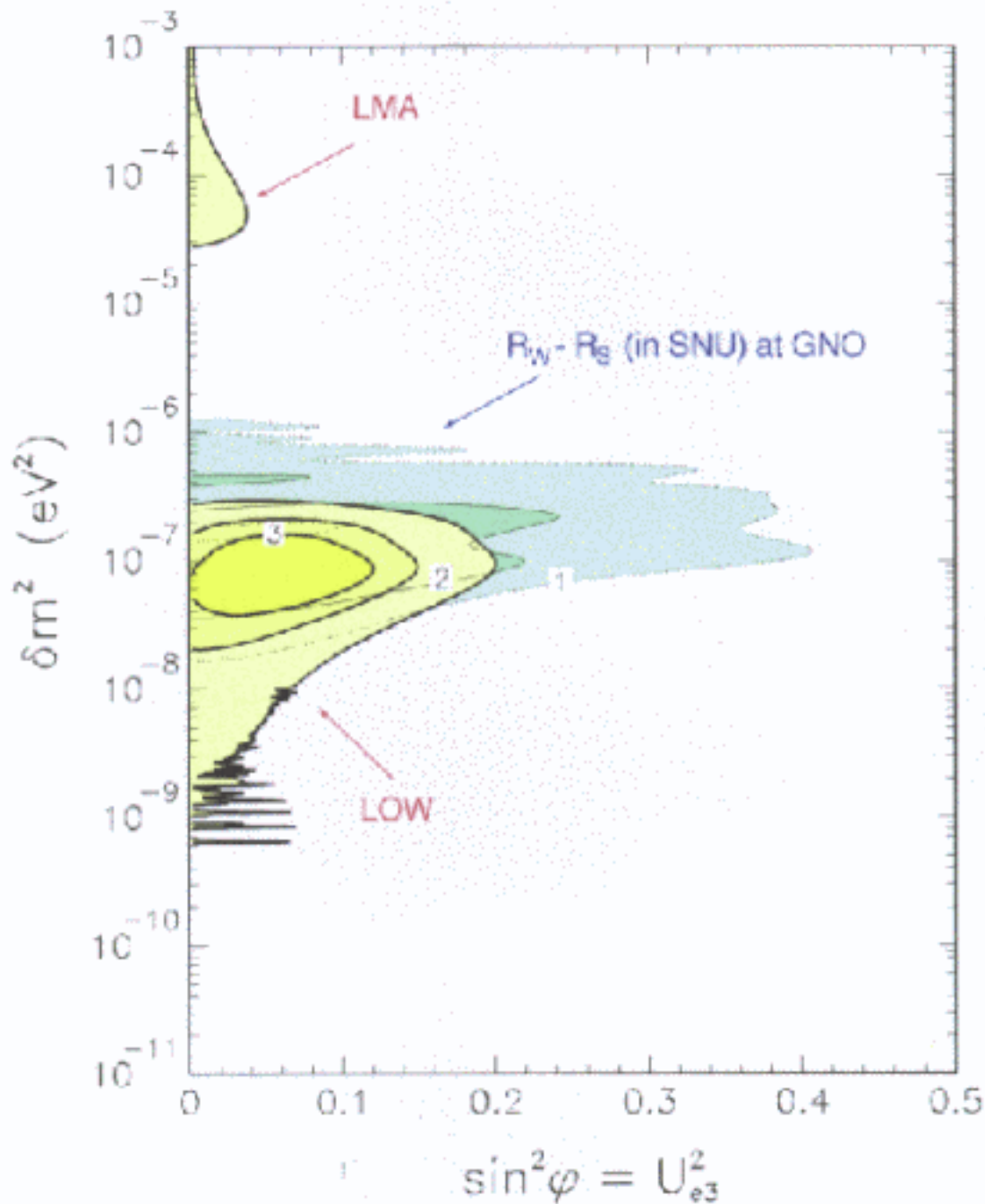


- expected **gallium absorption rates** (in SNU) averaged over "winter" and "summer" (eccentricity effects removed)
- MSW-induced **seasonal** variations of the order  $\sim 4$ - $6$  SNU are expected in a Ga experiment within the **LOW** solution (mainly from **pp** neutrinos)
- such variations might well be **observed at GNO**, the expected **statistical error** after one solar cycle being  $\sim 2$  SNU (or less)



# Gallium Neutrino Observatory

Comparison with the **LMA** and **LOW** solutions at maximal mixing ( $\omega = \pi/4$ )



- expected **gallium absorption rates** (in SNU) averaged over "winter" and "summer" (eccentricity effects removed)
- MSW-induced **seasonal** variations of the order  $\sim 4\text{-}6$  SNU are expected in a Ga experiment within the **LOW** solution (mainly from **pp** neutrinos)
- such variations might well be **observed at GNO**, the expected **statistical error** after one solar cycle being  $\sim 2$  SNU (or less)



# Beyond the "one dominant mass scale approximation"

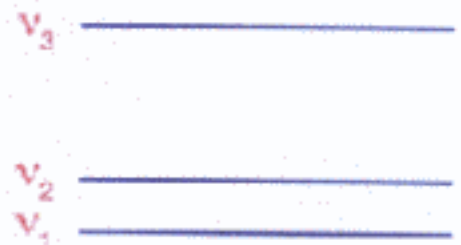


## CHOOZ limits to the solar neutrino problem

① CHOOZ excludes large  $\nu_e$  mixing between any two states separated by  $\Delta m^2 \geq 10^{-3} \text{ eV}^2$

② so, concerning  $m^2$

since  $m^2 \geq 10^{-3} \text{ eV}^2$   
(from atmospheric  $\nu$  data)



$\nu_3$  must have small mixing with  $\nu_e \Rightarrow$  small  $\langle \nu_3 | \nu_e \rangle = U_{e3} = s_\varphi$

③ and concerning  $\delta m^2$

either  $\nu_1$  or  $\nu_2$  or both must be mixed with  $\nu_e$   
(from solar  $\nu$  data)



$\delta m^2$  must be smaller than  $\sim 10^{-3} \text{ eV}^2$

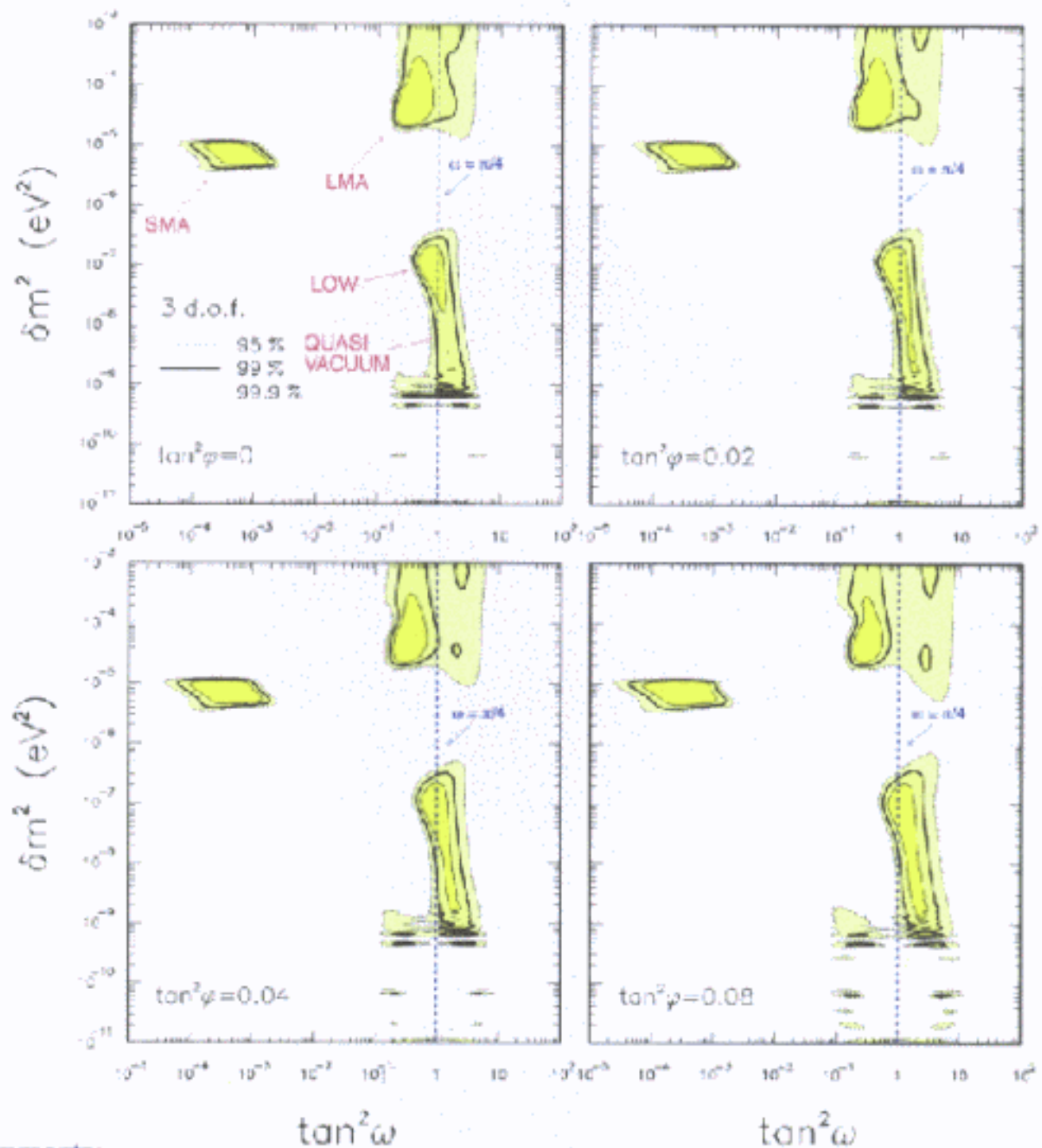
CHOOZ puts upper limits on

$$\sin^2 \varphi \quad (\leq \text{few } \%)$$

$$\delta m^2 \quad (\leq 10^{-3} \text{ eV}^2)$$

# 3ν solar solutions for finite $m^2$ ( $m^2 = 1.5 \times 10^{-3} \text{eV}^2$ )

total rates with constraints from Sp(D)+Sp(N)

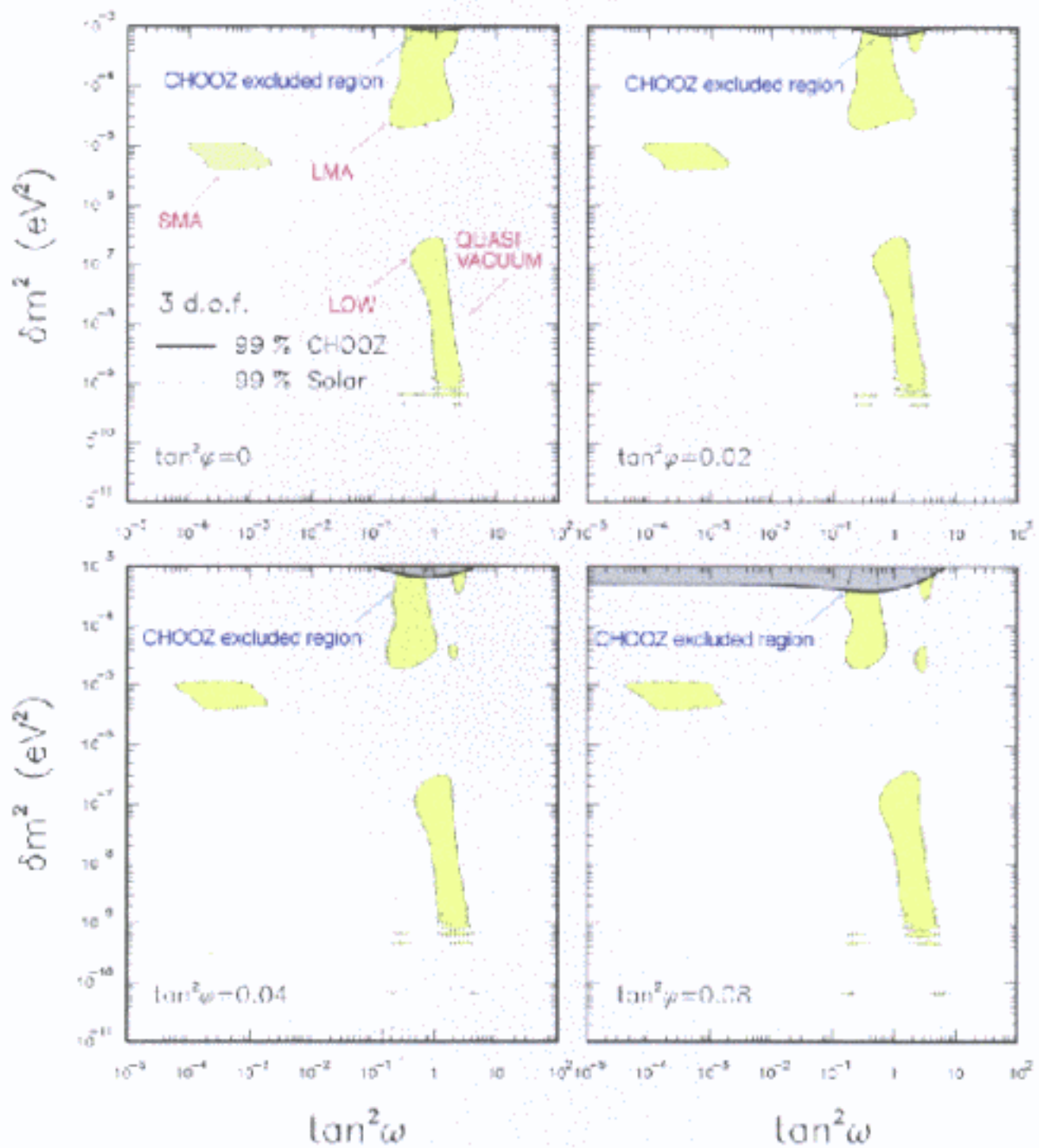


Comments:

- Removal of the  $m^2 = \infty$  approximation in the solar  $\nu$  analysis would not practically change the solar  $\nu$  solutions
- In particular, no upper bound on  $\delta m^2$  at 99% C.L. from solar  $\nu$  data alone

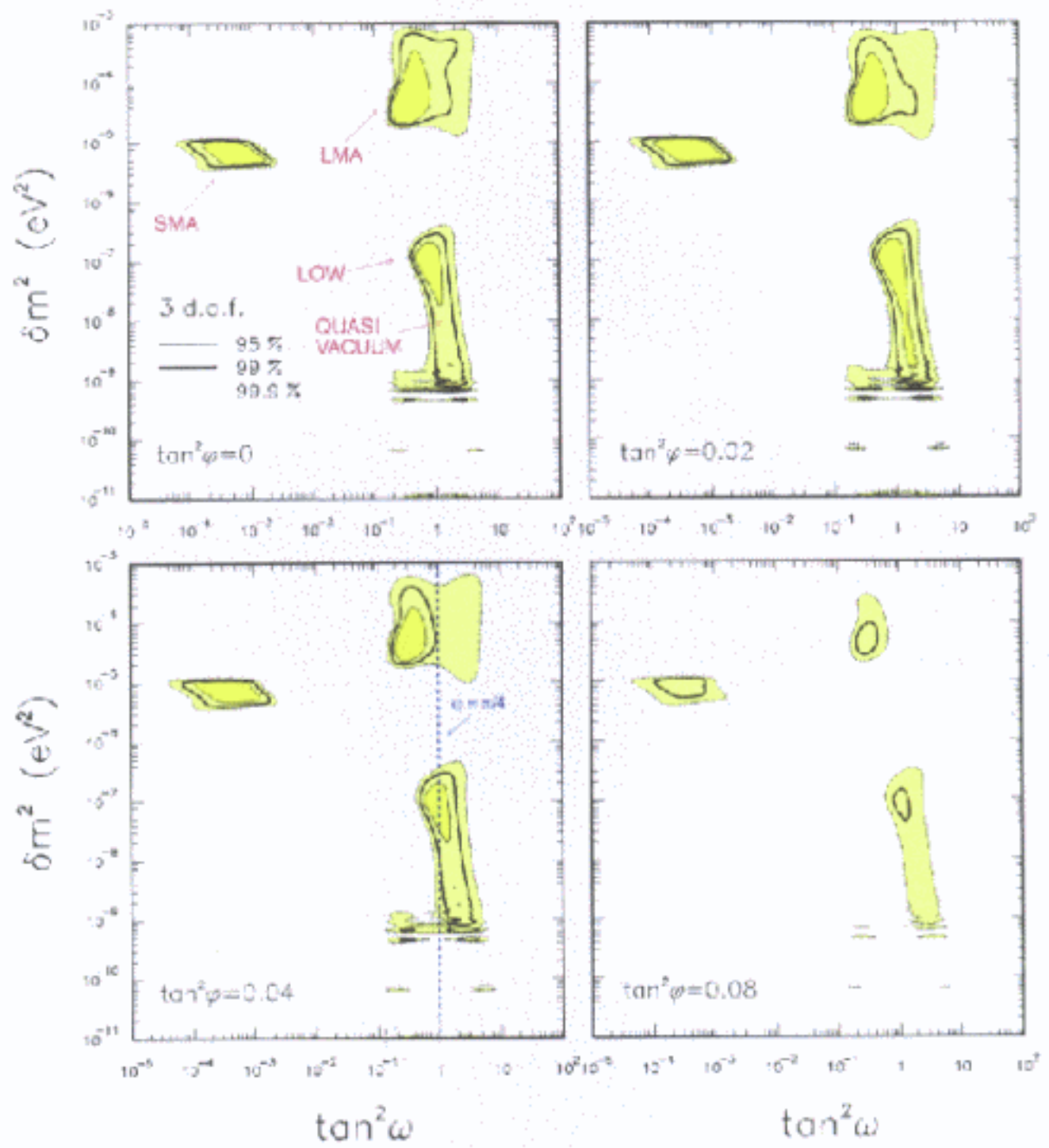
# CHOOZ excluded region compared with the $3\nu$ solar solutions

$(m^2 = 1.5 \times 10^{-3} \text{eV}^2)$



solar + CHOOZ data

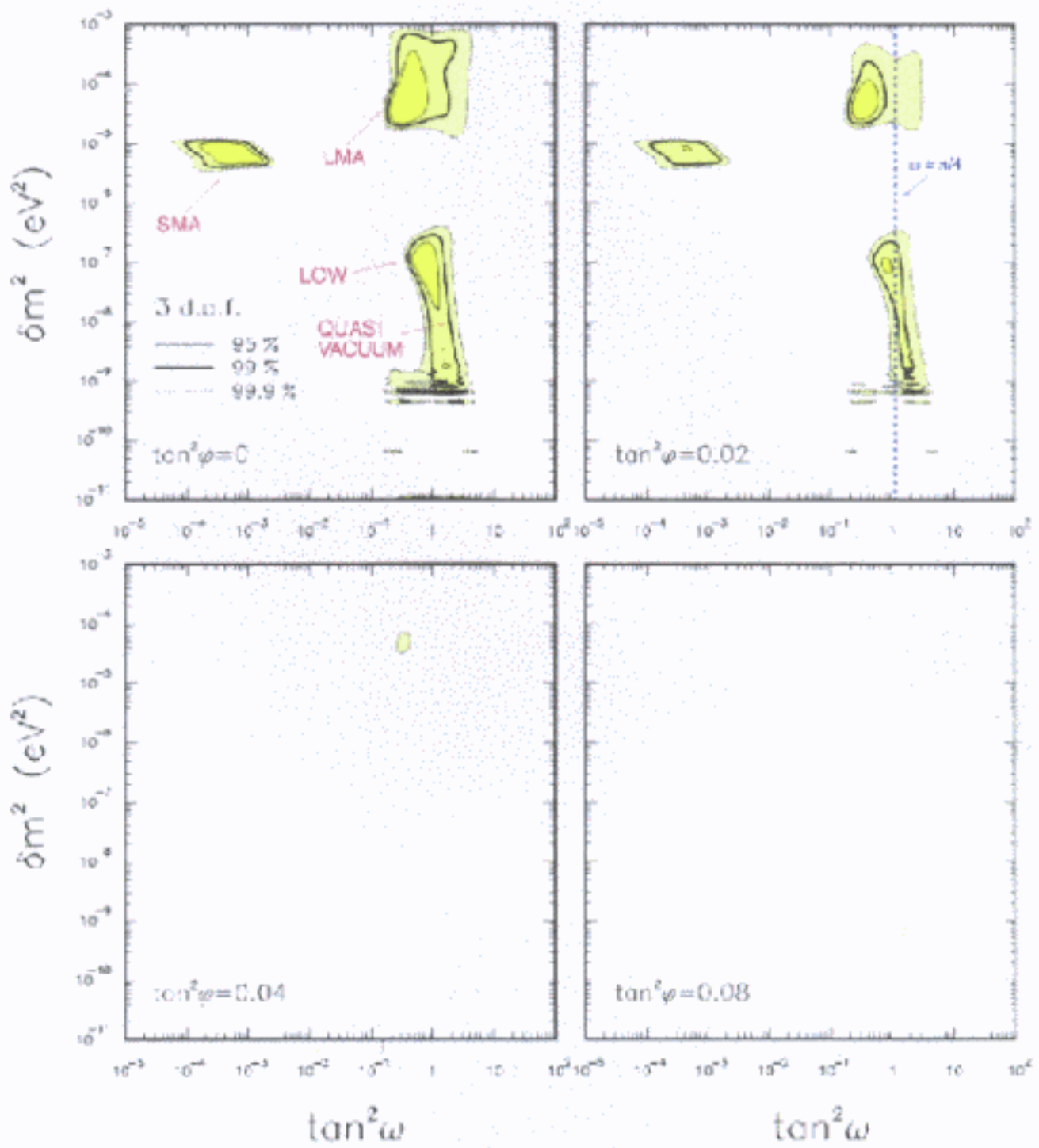
$(m^2 = 1.5 \times 10^{-3} \text{eV}^2)$



⇒ Strong constraints on both  $\delta m^2$  and  $\tan^2 \phi$  when solar + CHOOZ data are taken into account!

solar + CHOOZ data

$(m^2 = 3.0 \times 10^{-3} \text{ eV}^2)$

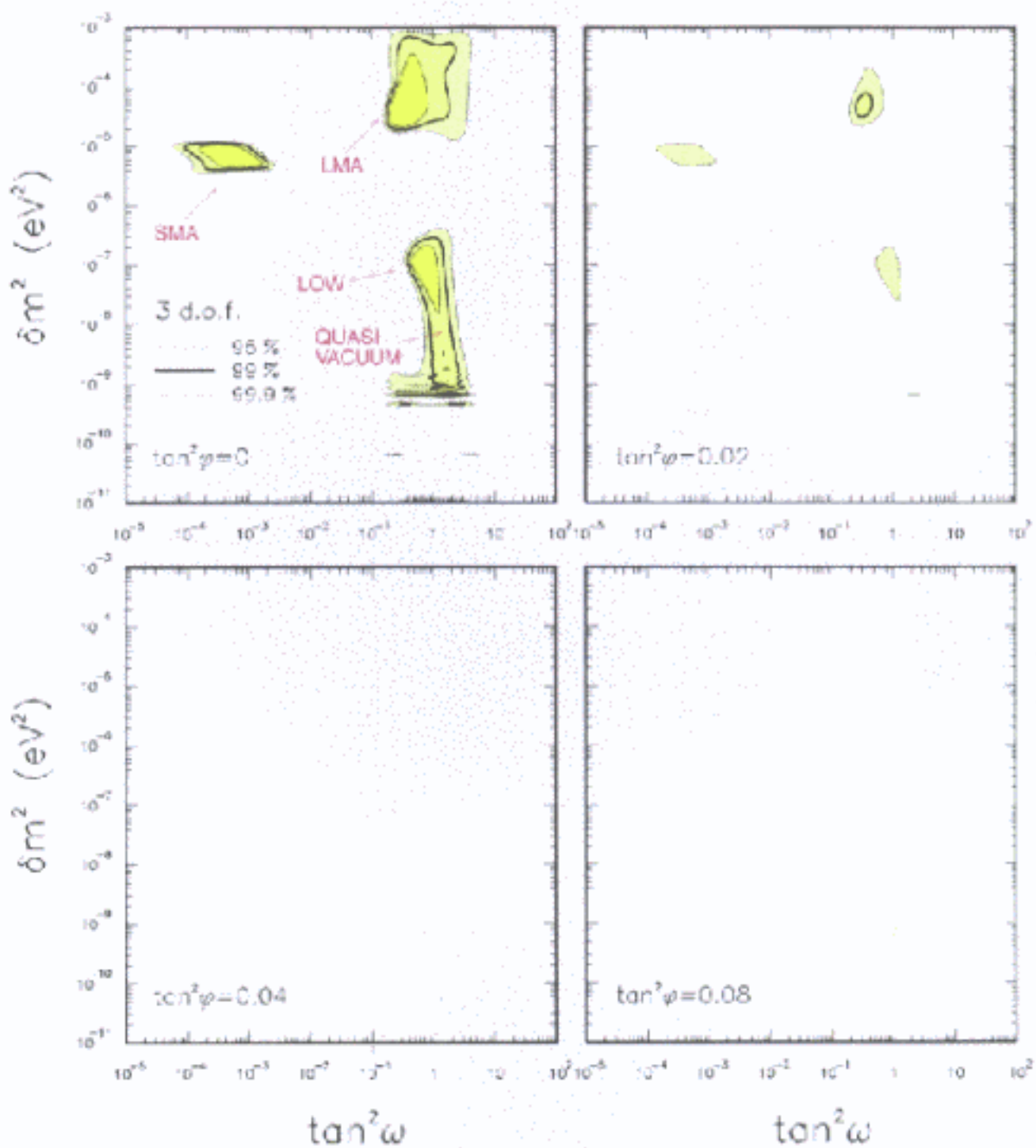


⇒ More stringent constraints on  $\tan^2 \varphi$  for increasing  $m^2$



# solar + CHOOZ data

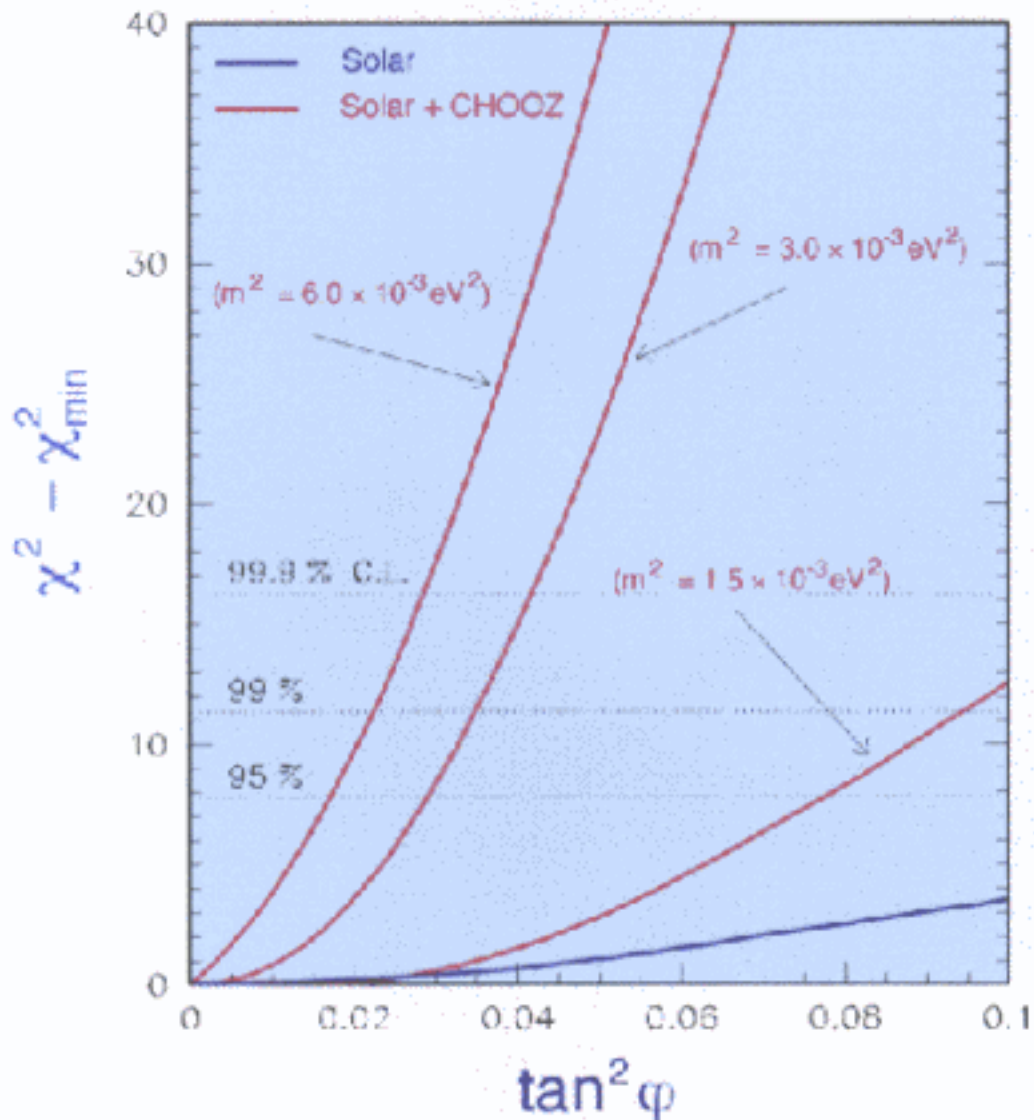
$$(m^2 = 6.0 \times 10^{-3} \text{ eV}^2)$$



⇒ The solar solutions in the limit  $\varphi = 0$  are independent of  $m^2$  (they correspond to a pure  $2\nu$  case)

## solar + CHOOZ data

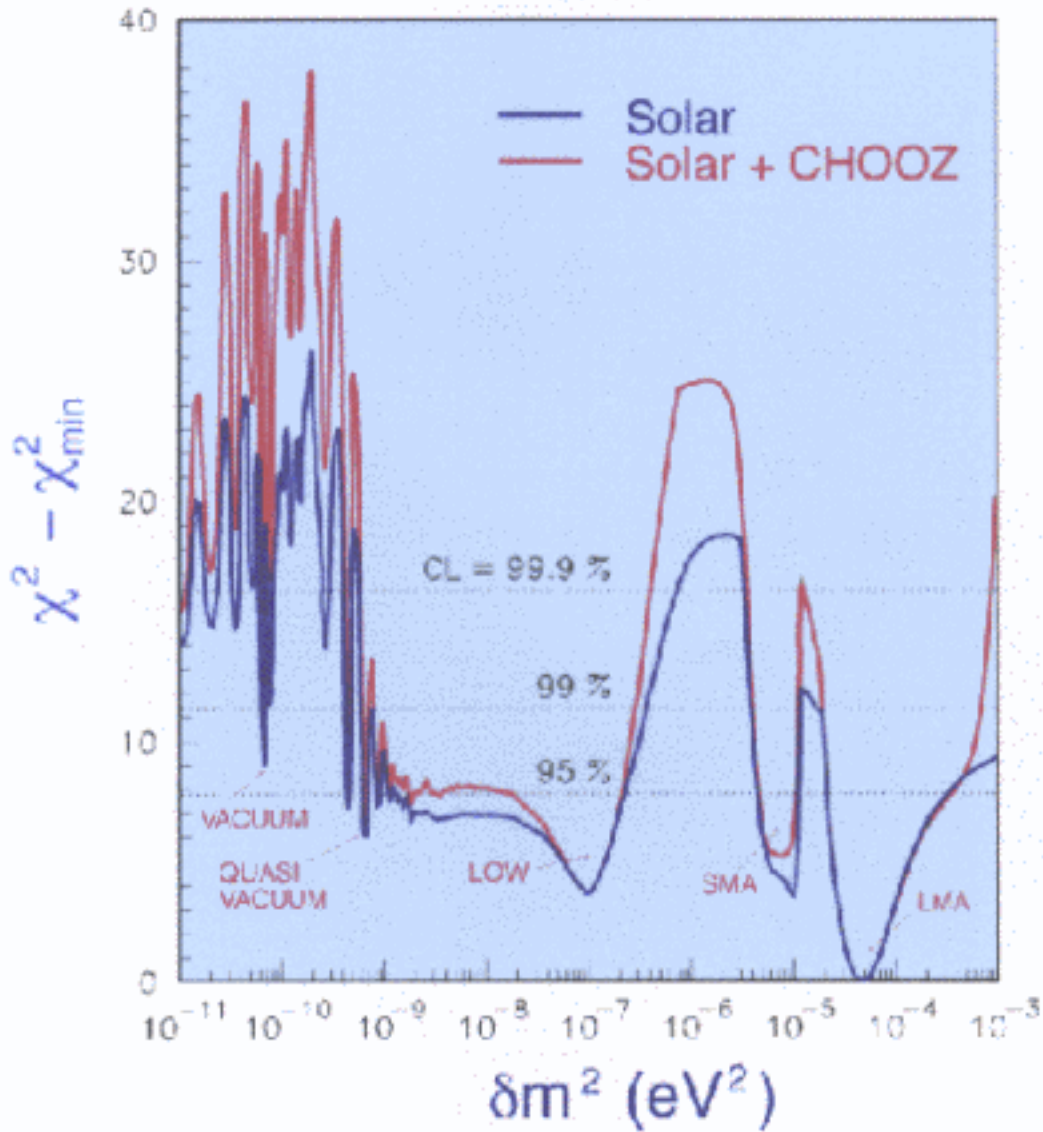
$\chi^2$  behaviour for different values of  $m^2$



- Strong constraints on  $\tan^2 \varphi$  when solar + CHOOZ data are taken into account
- More stringent constraints on  $\tan^2 \varphi$  for increasing  $m^2$

# solar + CHOOZ data

$\chi^2$  behaviour in terms of  $\delta m^2$  ( $\tan^2 \varphi$   ~~$\neq 0$~~  <sup>unconstrained</sup>)



- More stringent constraints on  $\delta m^2$  from solar + CHOOZ data

## Summary on solar $\nu$ (in 6 points)

- ① Still multiplicity of solar neutrino solutions:  
**MSW:** LMA preferred, **LOW** promising, **SMA** less favored  
**VAC:** serious problems for **VAC**, but interest for **quasi-VAC**  
(in the range  $10^{-8} \div 10^{-9} \text{ eV}^2$ )
- ② The **LOW** solution is acceptable and can provide  
**maximal  $\nu_1 \leftrightarrow \nu_2$  mixing** ( $\omega = \pi/4$ ) for **nonzero  $\varphi$**
- ③ **Bounds on  $\varphi$**  from solar  $\nu$  data alone are loose (even more than for atmospheric data). However, their preference for  
 $s_{\varphi}^2 \sim 0$   
is a good sign of consistency with CHOOZ.
- ④ CHOOZ provides upper bounds on  $s_{\varphi}^2$  and  $\delta m^2$
- ⑤ Unambiguous selection of **ONE** solution will not be possible in **SK** for several years. Much higher statistics needed (e.g., in the spectrum).
- ⑥ We absolutely need **more solar  $\nu$  data** ! Eagerly waiting for **SNO, GNO, Borexino** results ..., as well as for direct tests of LMA with **KAMLAND**, to improve discrimination of solar  $\nu$  solutions.



## 3ν summary

Within the "standard" 3ν interpretation of neutrino oscillation data, we have a lot of experiments or projects constraining the neutrino parameters:

masses

mixing

$$\delta m^2$$

$$U_{e1}^2$$

$$U_{e2}^2$$

$$U_{e3}^2$$

$$U_{\mu 1}^2$$

$$U_{\mu 2}^2$$

$$U_{\mu 3}^2$$

$$m^2$$

$$U_{\tau 1}^2$$

$$U_{\tau 2}^2$$

$$U_{\tau 3}^2$$

probed by **solar** neutrino experiments

probed by **"terrestrial"** neutrino experiments

- At present, most stable constraints on  $m^2$  and  $U_{\mu 3}^2, U_{\tau 3}^2$ .
- CHOOZ tells us that  $U_{e3}^2$  must be small, and atmospheric and solar data also prefer small  $\varphi$ , but there is no reason for it to be zero !
- Constraints on  $\delta m^2, U_{e1}^2, U_{e2}^2$  depend on which solar solution is picked up.
- CHOOZ + solar data  $\longrightarrow$  upper bound on  $\delta m^2$
- Most important tasks for the next years:
  - $\Rightarrow$  measure or constrain further  $U_{e3}^2$  (reactors & LBL)
  - $\Rightarrow$  reduce the multiplicity of **solar**  $\nu$  solutions
  - $\Rightarrow$  check and (dis)prove **non-standard interpretations**