INTERPRETATION
of the Atmospheric ν data

What is the role of systematic uncertainties in the PREDICTION of the fluxes ?

Compare DATA and PREDICTION
to study the existence of NEW PHYSICS
beyond the STANDARD MODEL.

What are the "theoretical" uncertainties on the prediction of the atmospheric ν fluxes ? Are they a significant limitation in the interpretation of the data ?

SPECIAL THANKS

Ralph Engel
Tom Gaisser
Todor Stanev
} "Bartol" + Roma

M. Honda
T. Kajita
K. Kasahara
} "Honda"

Giuseppe Battistoni
Alfredo Ferrari
Paola Sala
} "Fluka"

Chris Waltham
Yaroslav Tserkovnyak
} "SNO"

4π Calculations are in progress.

Time scale ~ few months

Plan of this talk

(A) Sources of Uncertainties:

- Primary flux
- Hadronic interactions
- Method of Calculations

(B) Effect on ν fluxes

- Energy spectrum
- Angular distributions
- e/μ ratio

(C) Effects on Interpretation.

- New Physics ??
- Oscillations or other mechanisms
- $\nu_\nu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_{\text{sterile}}$
- Oscillation parameters.

"Disappearance" of μ-like events

$$\Phi_{\nu_\mu}(E_\nu, \cos\theta_z) =$$

$$\Phi_{\nu_\mu}^{\text{standard Model}}(E_\nu, \cos\theta_z) \times$$

$$\left\langle P_{\nu_\mu \to \nu_\mu}(E_\nu, \cos\theta_z) \right\rangle$$

Form of $P_{\nu_\mu \to \nu_\mu}$ ⇒ physical mechanism

Oscillation

. . . . .

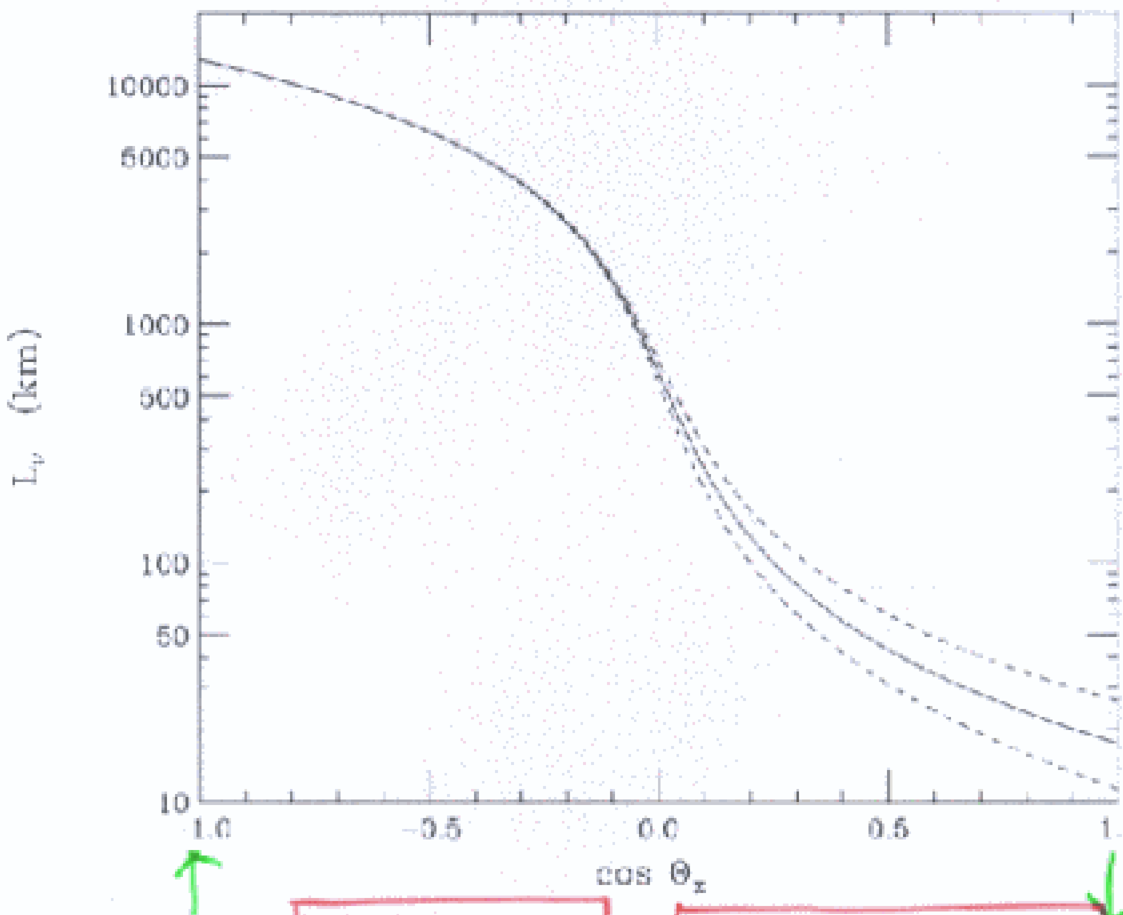
parameters of New Physics

$(\Delta m^2, \sin^2 2\theta)$

Relation Θ_{zenith} L correlation

L dependence of $P(\nu_\mu \to \nu_\mu)$

To a good approximation the Zenith Angle: Θ_z
determines: the ν path-length L_ν
[and the entire "path" (density profile)]



"Horizontal": $\Theta_z = 80°–100°$ $\leftrightarrow$ $L_\nu = 150–2400$ Km

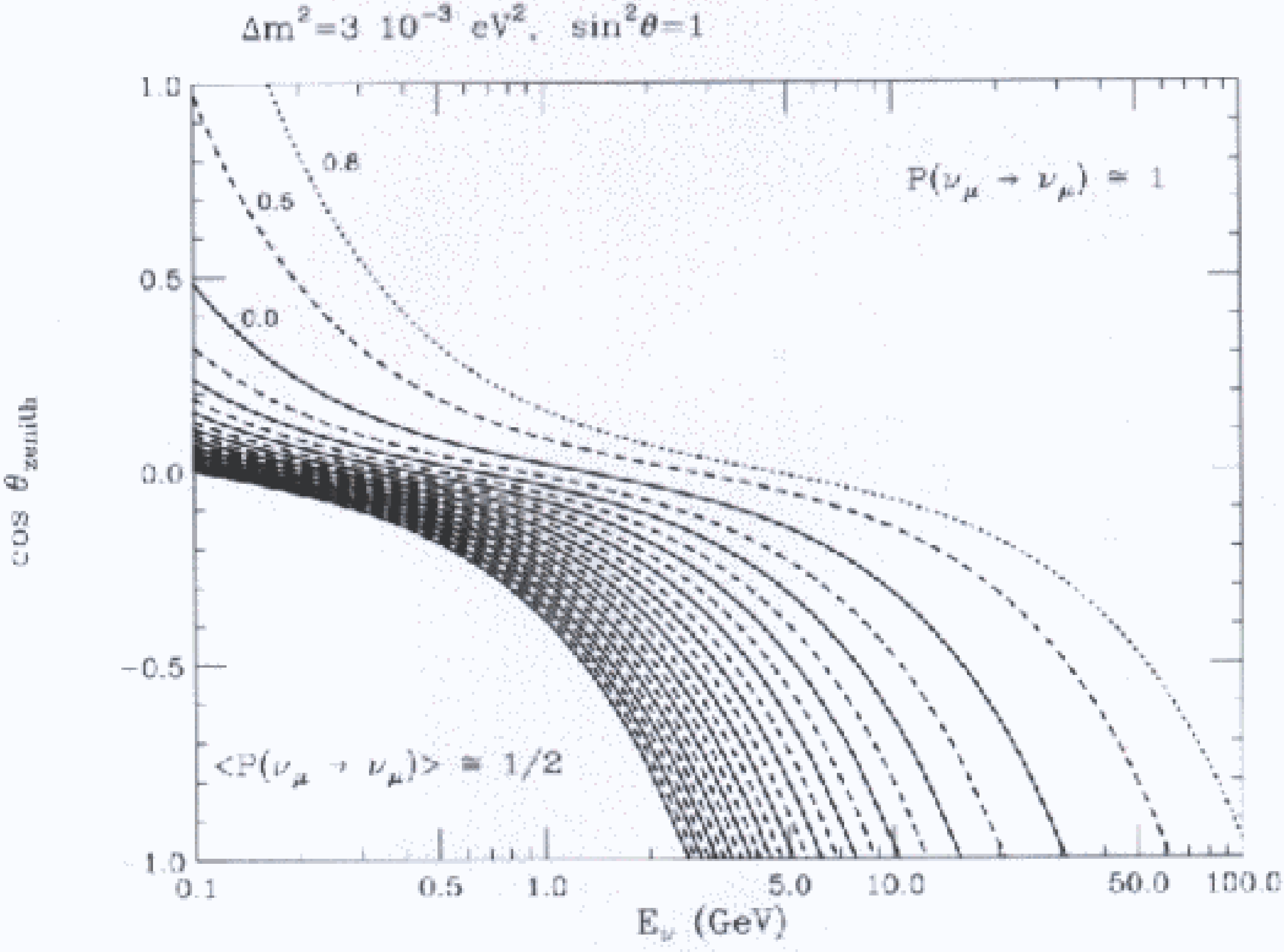
"Down–going": $L_\nu \simeq 10–1500$ Km

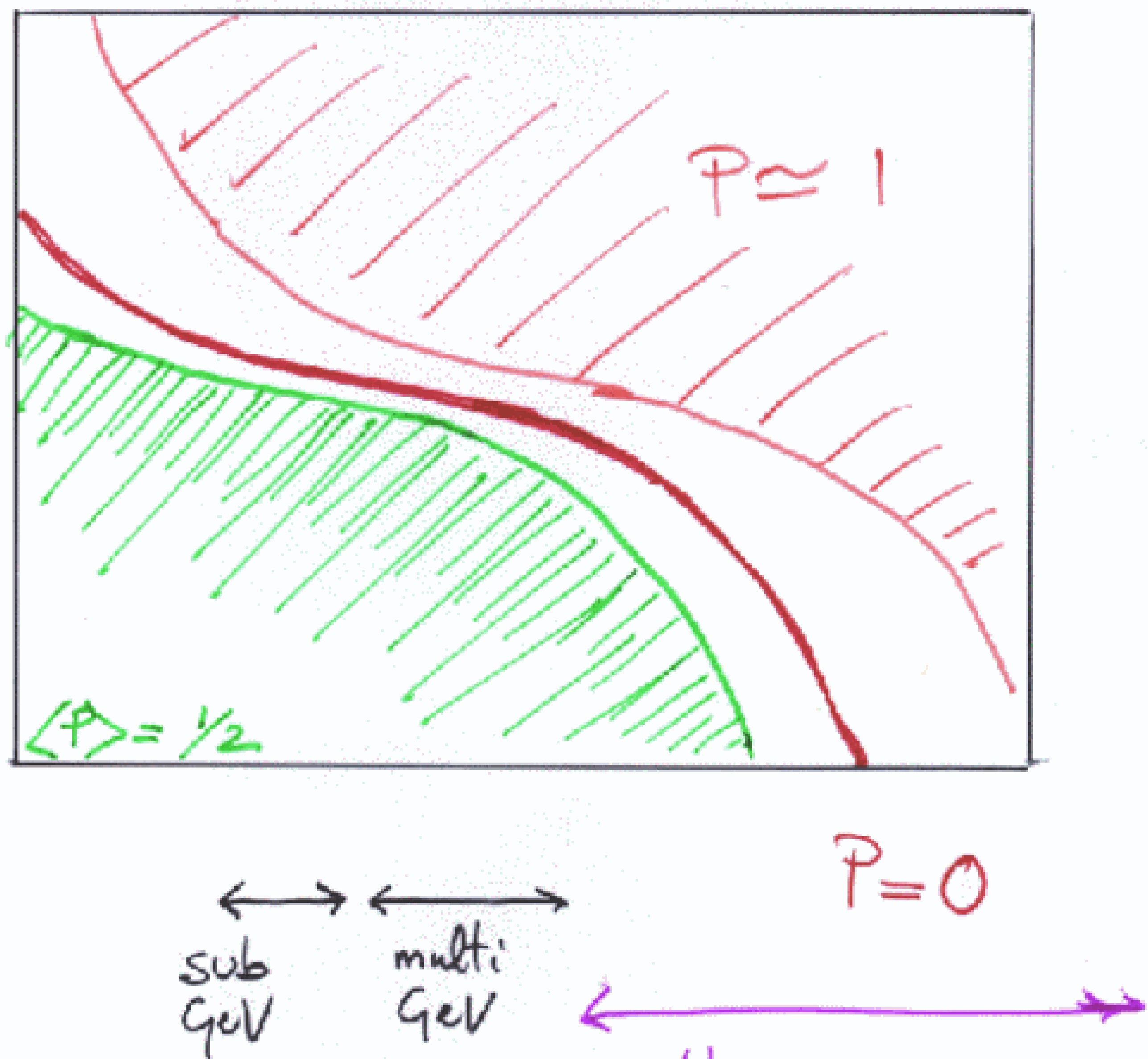
"Up–going": $L_\nu = 2400–12700$ Km

Oscillation Probability $(\nu_\mu \leftrightarrow \nu_\tau)$

$$P_{\nu_\mu \to \nu_\tau} = 1 - \sin^2 2\theta \ \sin^2 \left[\frac{\Delta m^2 \ L}{4 \ E_\nu}\right]$$

Strong correlation between L and $\cos\Theta_\nu$

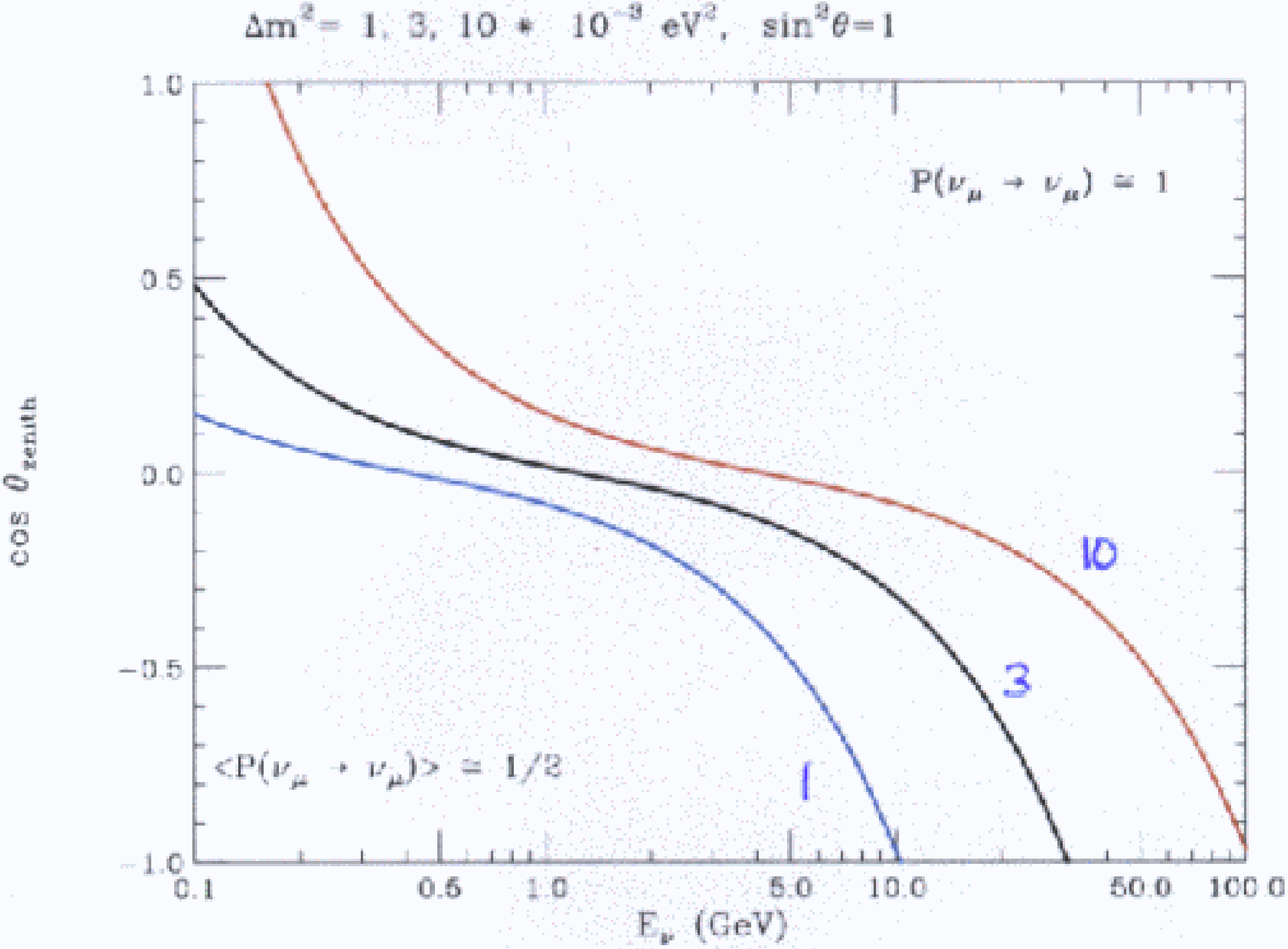




P ≃ 1
⟨P⟩ = 1/2
P = 0
sub GeV
multi GeV
Up-going μ

$\Delta m^2 = 1, 3, 10 * 10\ eV$

DEPENDENCE on Δm^2



High Energy (semi) contained events

μ↑ events

Curves: "First Minimum" of $P_{\nu_\mu \to \nu_\mu}$

Measurement of $\sin^2 2\theta$

$$U = U_0 \left\langle P\left(\frac{L}{E_\nu} \to \text{large}\right)\right\rangle$$

$$\simeq U_0 \left(1 - \frac{1}{2}\sin^2 2\theta\right)$$

Neglect
- backgrounds
- Finite resolution
- Detector asymmetry

$$D = D_0 \left\langle P\left(\frac{L}{E_\nu} \to \text{small}\right)\right\rangle$$

$$\simeq D_0 \times 1$$

$$\sin^2 2\theta \simeq 2\left[1 - \frac{U/D}{U_0/D_0}\right]$$

$$U_0/D_0 \simeq 1$$

Measurement of Δm^2

- "Model independent" method.

  ~ High Energy $E_\nu \sim 20$ GeV

  [Low statistics –
  pattern recognition problems]

  "The MONOLITH Philosophy"

- Average suppression in transition region.

  Some model dependence

  – $(\mu/e)_0$ Ratio

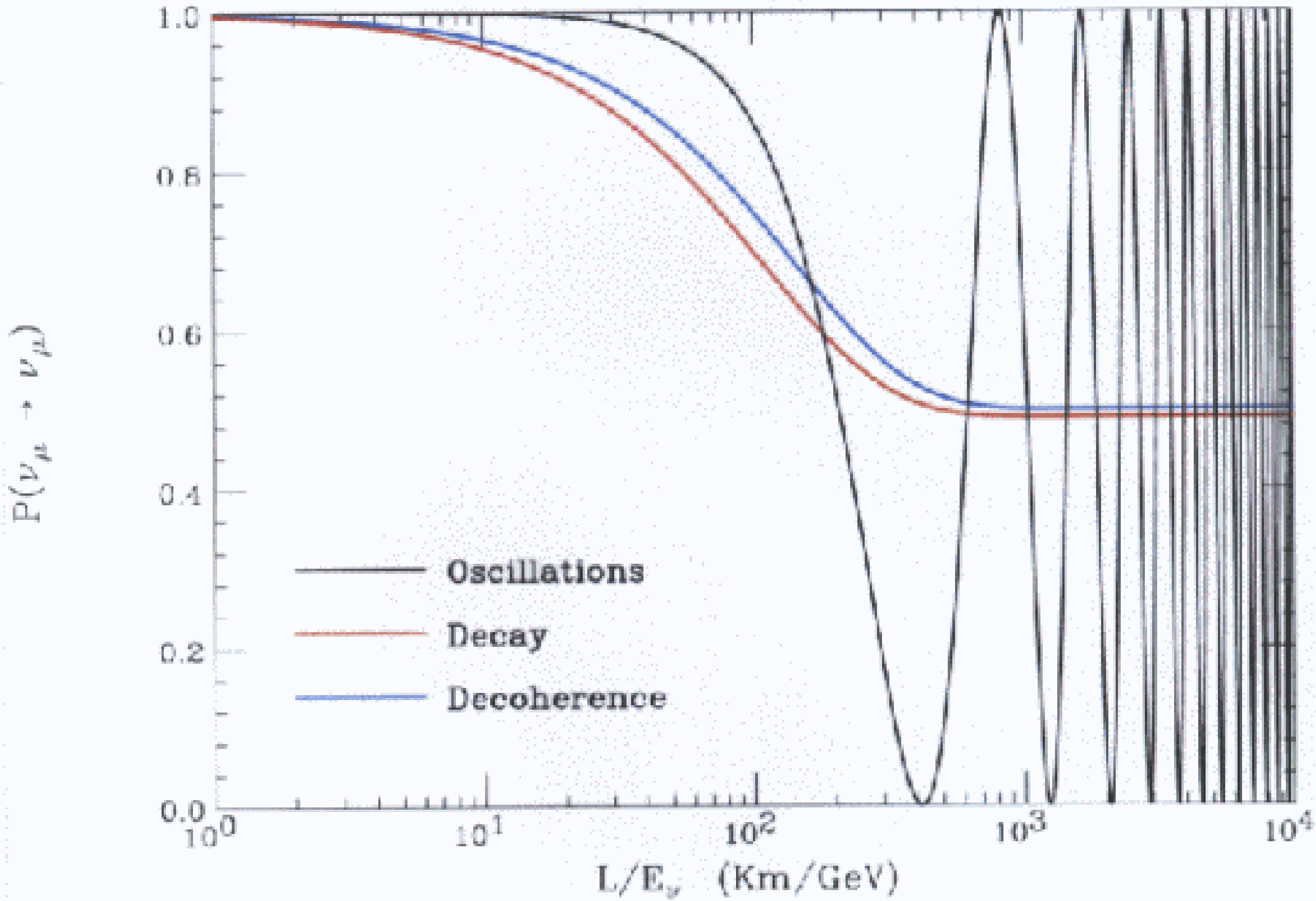
  – Energy Distribution

  – Shape of Zenith angle Distributions

but also SK has interesting potential.

"Smooth Probability models"

Alternative Models for the "disappearance" of the μ–like events.

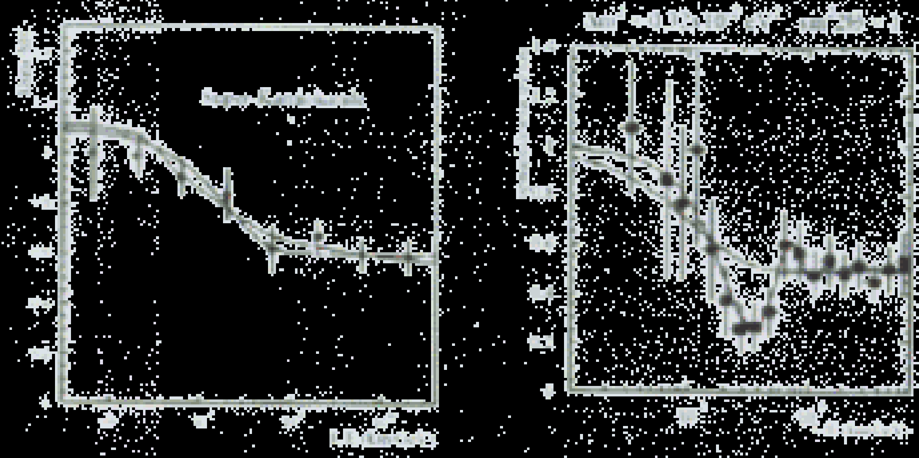


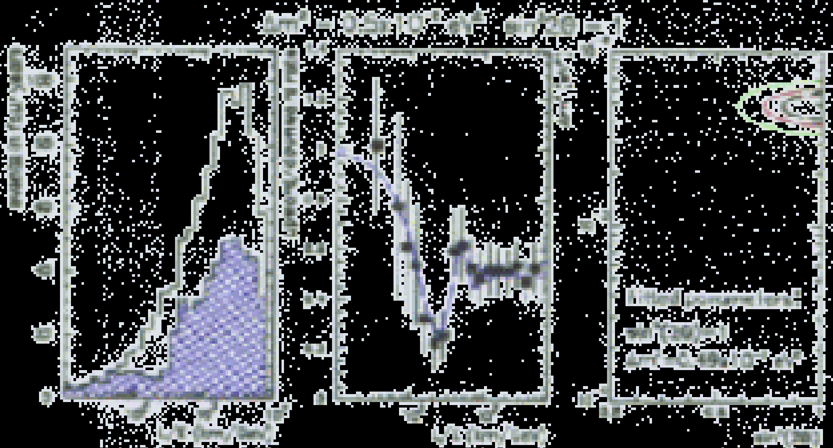
$$P_{\rm decay} = \left\{ \sin^2 2\theta + \cos^2 2\theta \ \exp\left[-\frac{m}{\tau_\nu} \frac{L}{E_\nu} \right] \right\}^2$$

$$P_{\rm decoherence} = \frac{1}{2} \left[1 + \ \exp\left(-\beta \frac{L}{E_\nu} \right) \right]$$

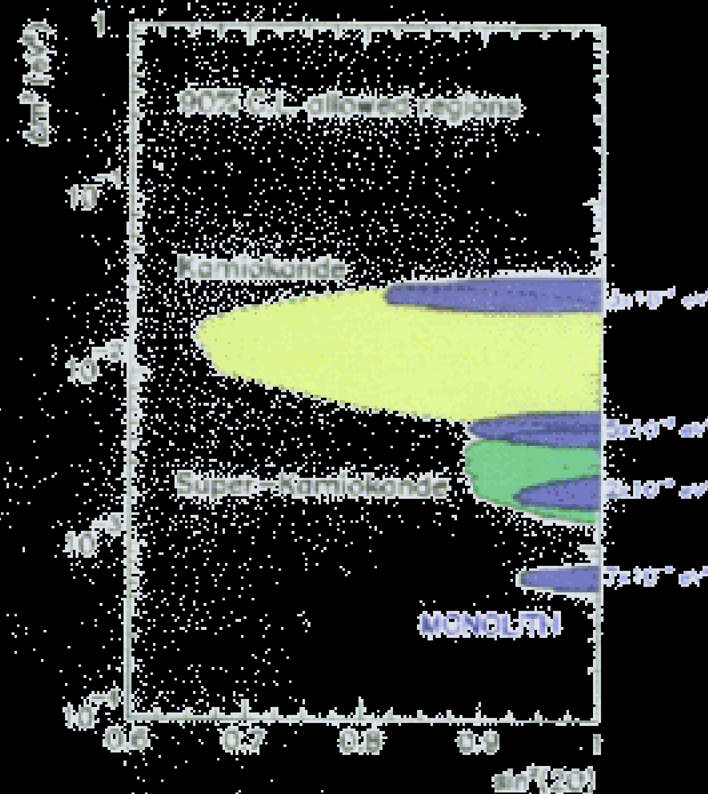
Monolith Sensitivity – 4 years

The oscillation pattern as seen in SuperKamiokande and expected in Monolith for $\Delta m^2 = 3.2\times10^{-3}$ eV^2





L/E distributions and oscillation pattern for $\Delta m^2 = 5\times10^{-3}$ eV^2



The superior *L/E* resolution will allow detection of the first oscillation period. It will also result in a substantial improvement in the measurement of Δm^2.

S. Ragazzi, July 2000

SOURCES of UNCERTAINTIES

1. Primary Flux
2. Hadronic interactions
3. Description of the Earth
   - Atmosphere
   - Mountains
   - Magnetic field
4. Calculation Method

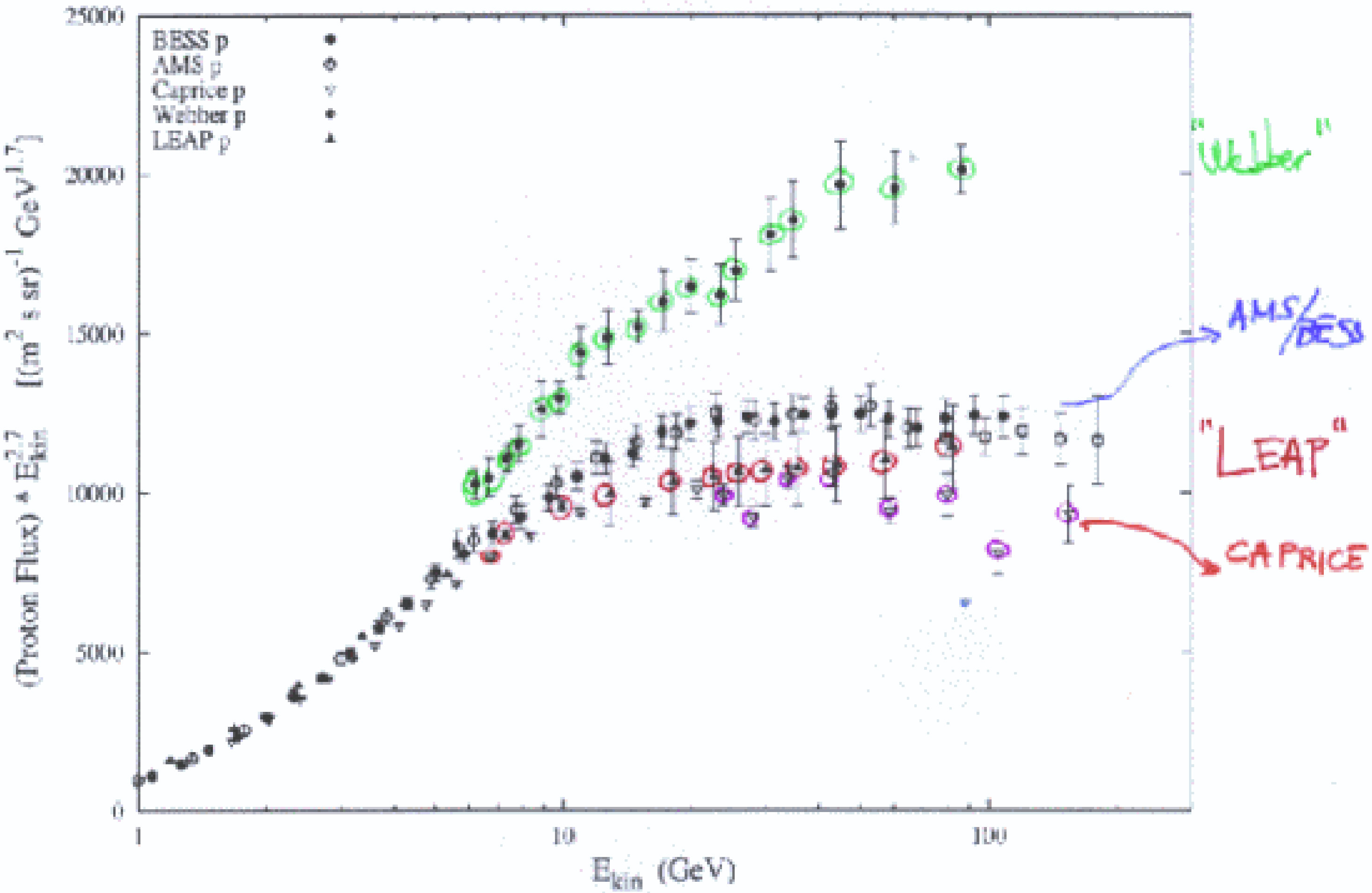
Dramatic improvement over the past

Primary Cosmic Ray Fluxes

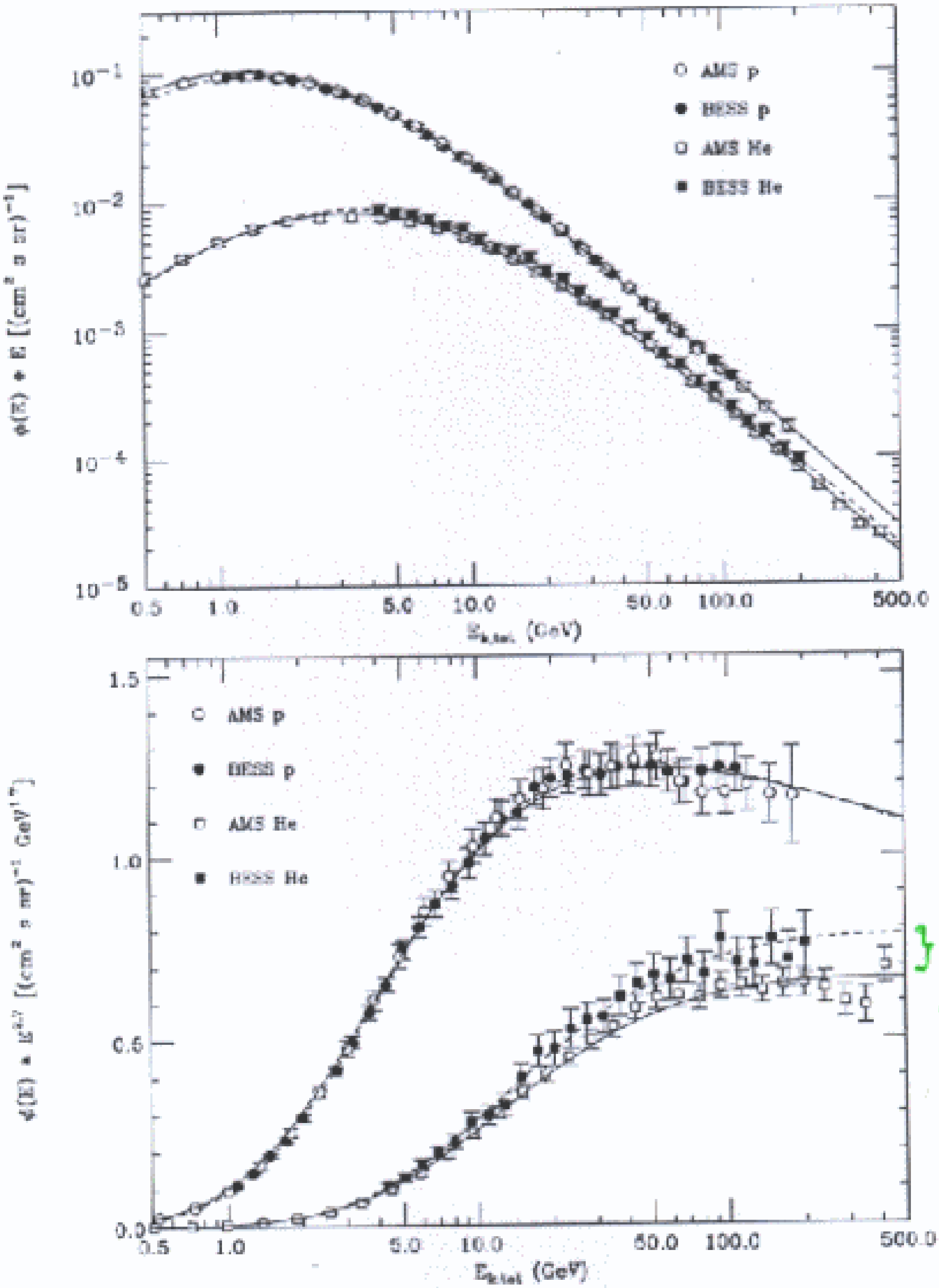
Measurements of BESS and AMS are in ~~good~~ agreement excellent!

Discrepancies with other experiments
Origin of the discrepancy: not–understood systematic effects.

PROTON FLUX



AMS and BESS p, He measurements
(JUNE 1998) [Solar modulation]



} 10% discrepancy

Figure 2: Fits to the AMS and BESS data (Energy per particle)

Energy / Particle

AMS and BESS results. p, He

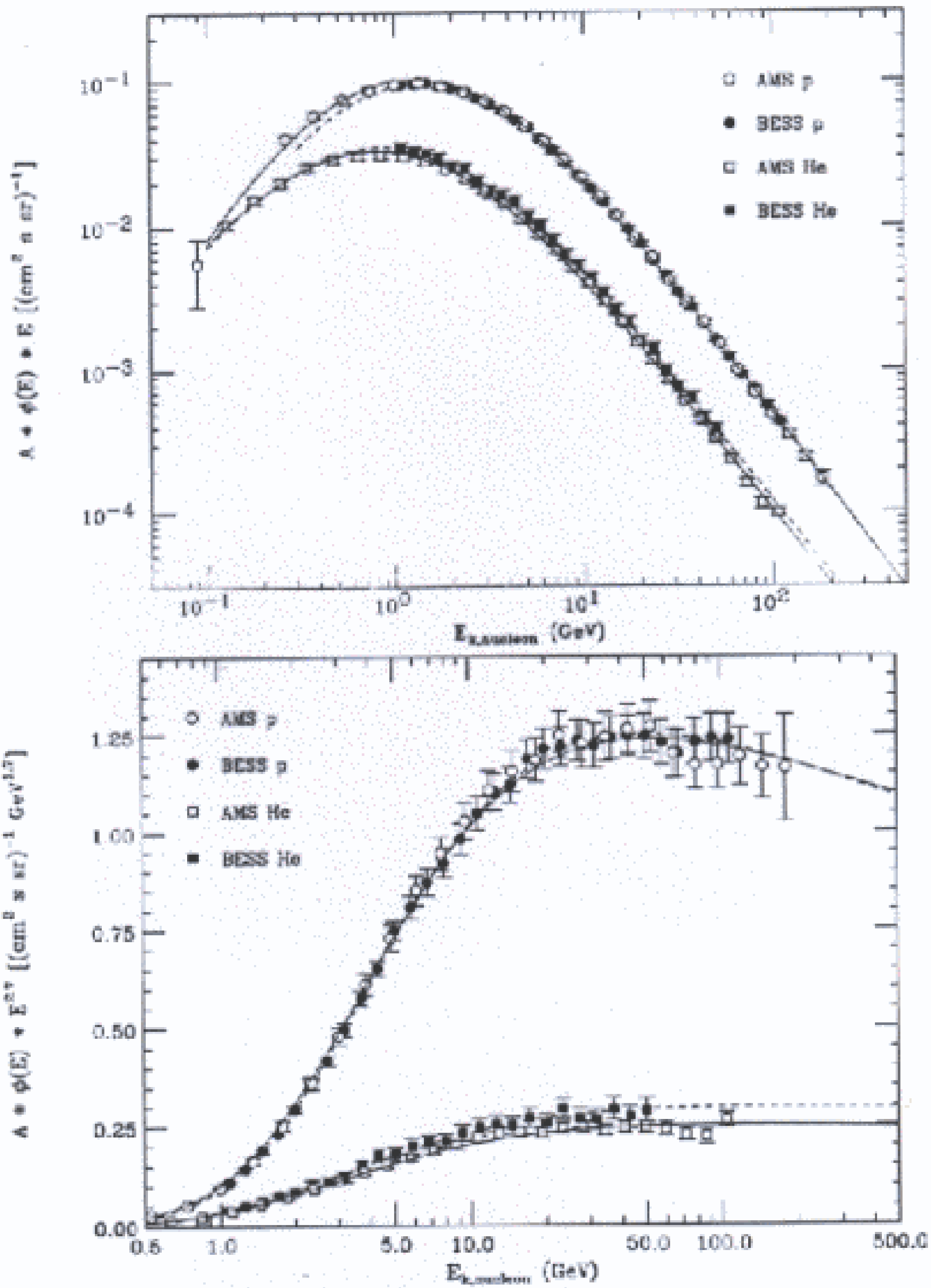
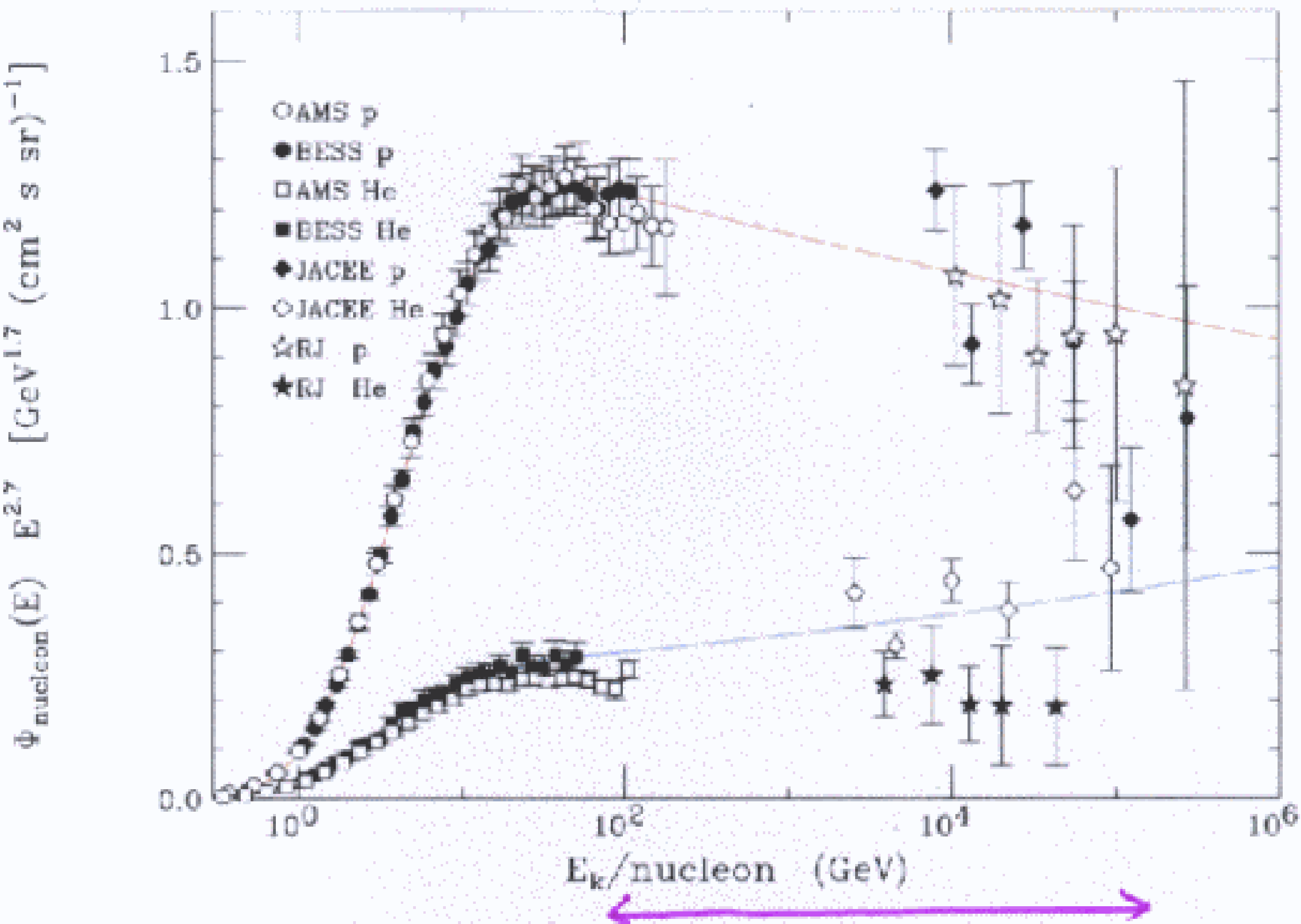


Figure 1: Fits to the AMS and BESS data (Energy per nucleon)

Energy/Nucleon



Ambiguity RUNJOB/JACEE

$\mu\uparrow$

Problem :

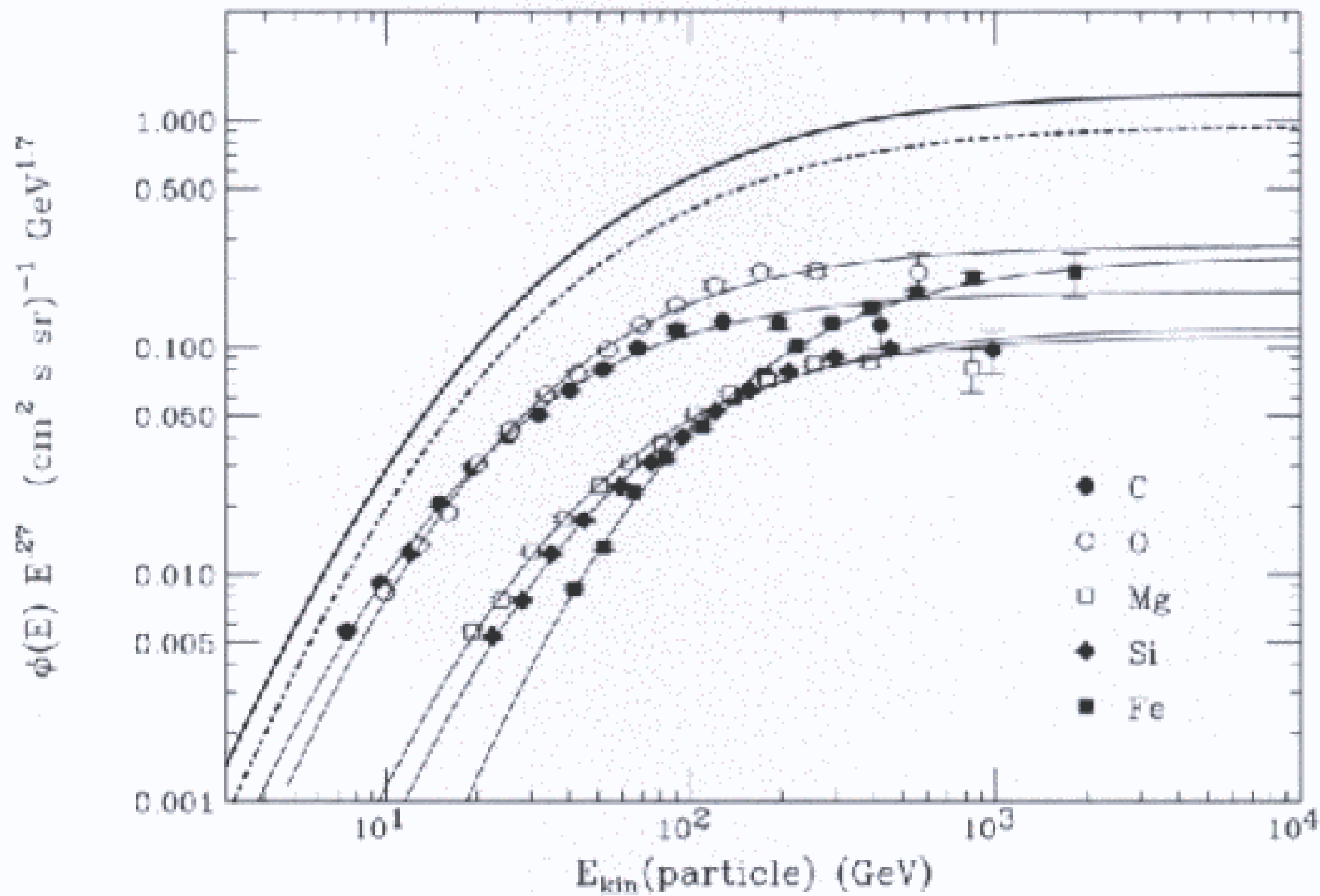
Extrapolation to higher energy

"Heavy Nuclei"

Components with $Z \geq 6$.

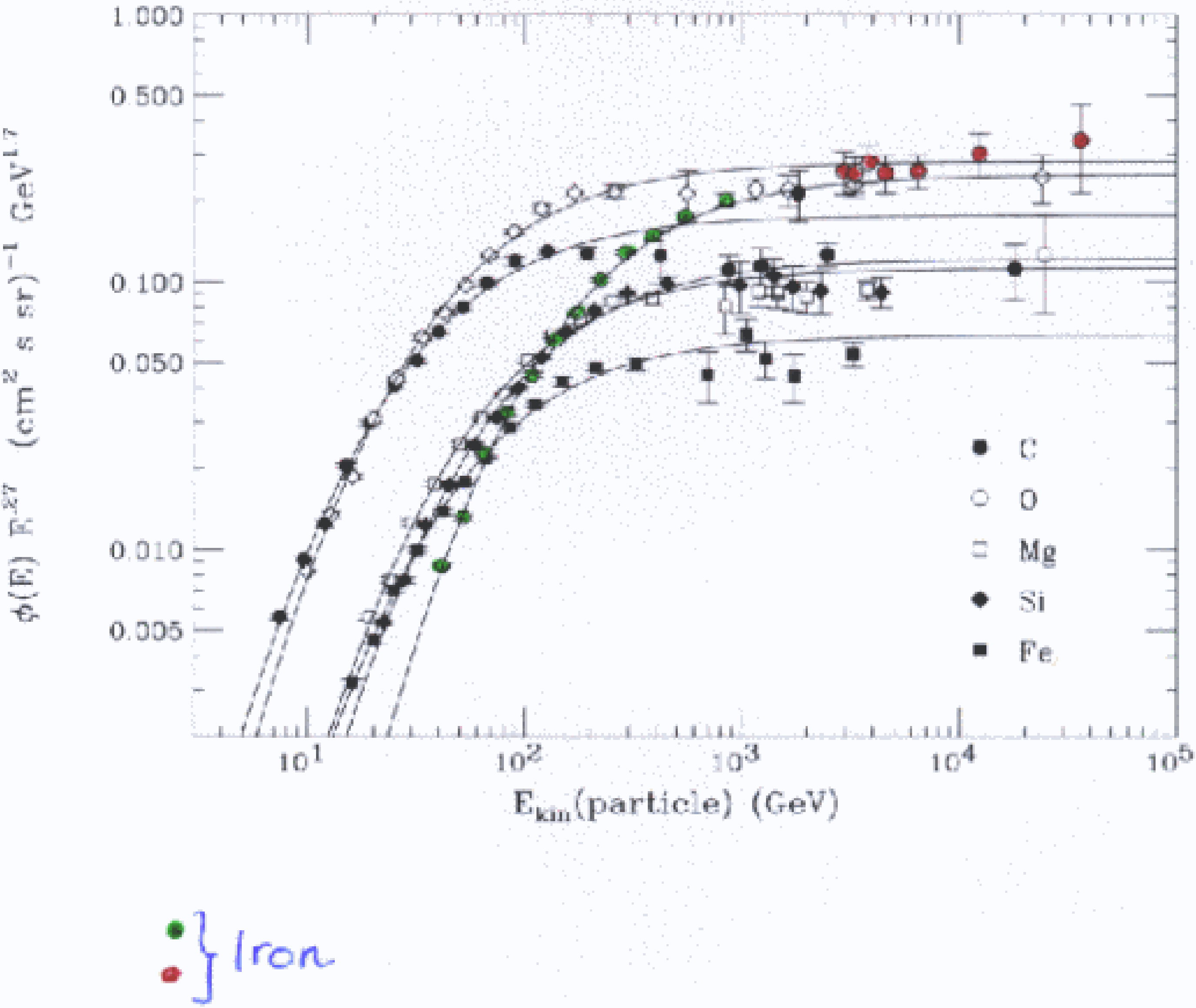
Data of HEAO–3

Engelmann et al. Astr. Astr. 148, 12 (1985)

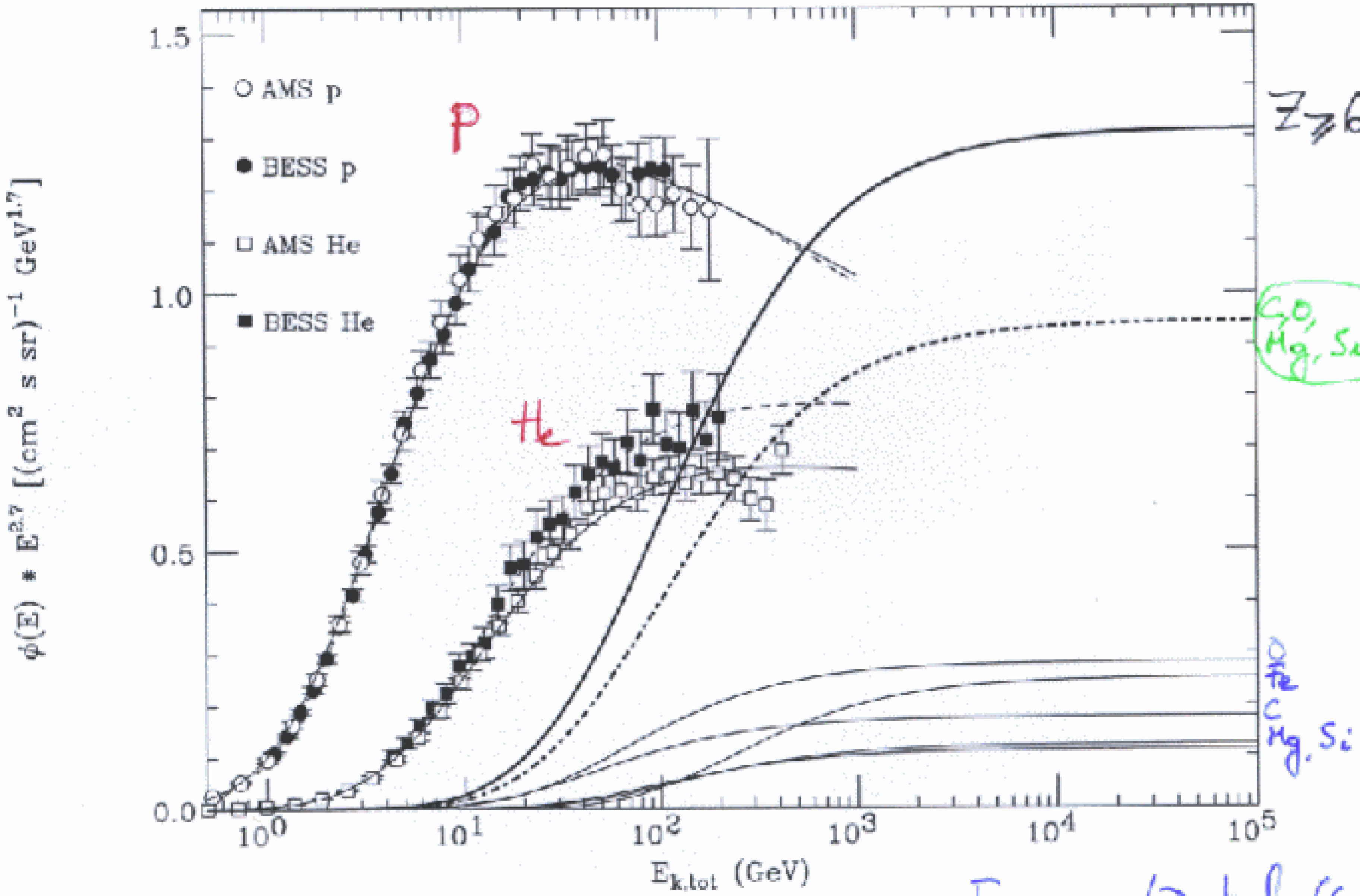


Data of HEAO–3
Engelmann et al. Astr. Astr. 148, 12 (1985)

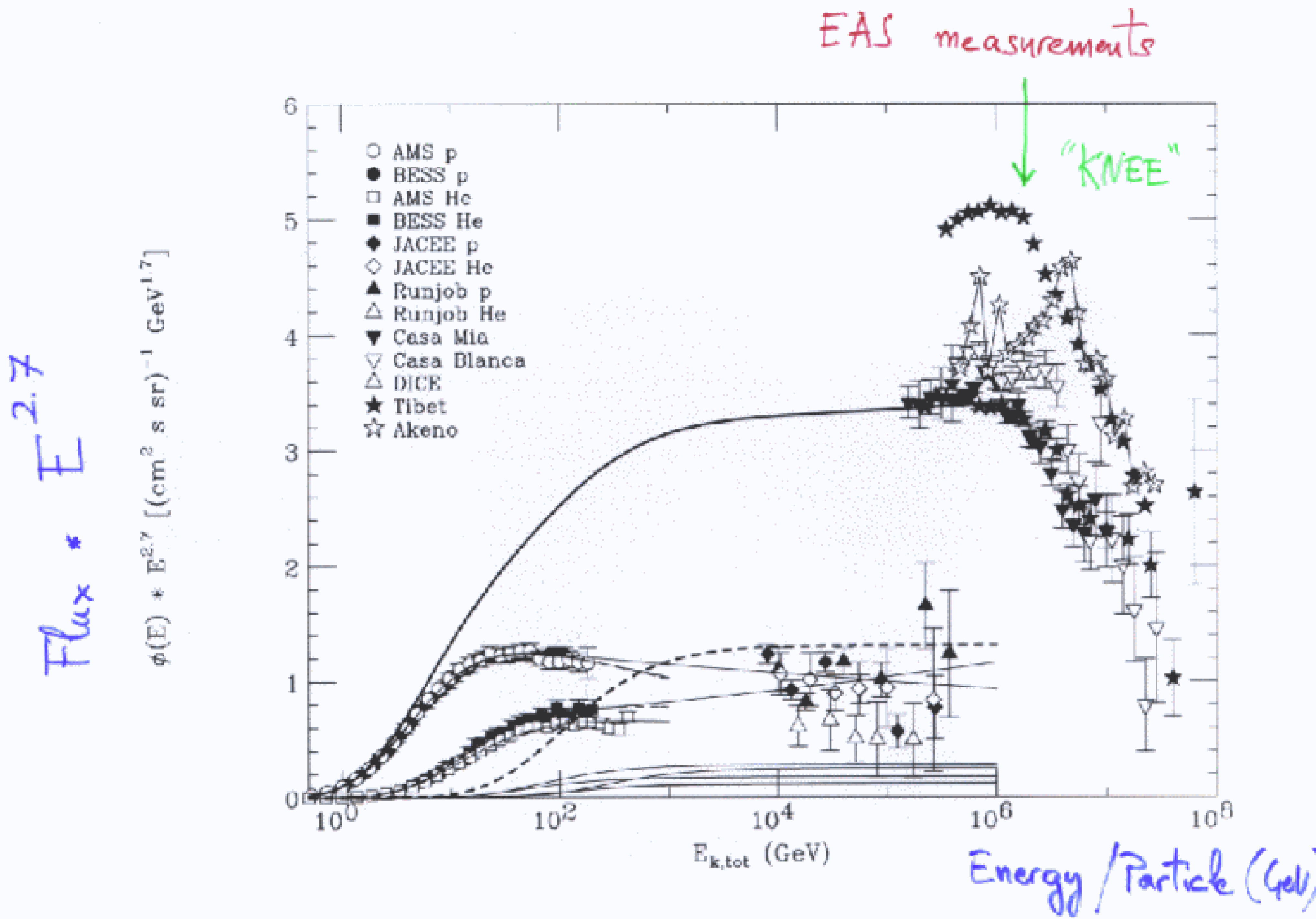
Data of CRN detector (space shuttle)
D.Muller et al, Ap. J 374, 356 (1991).



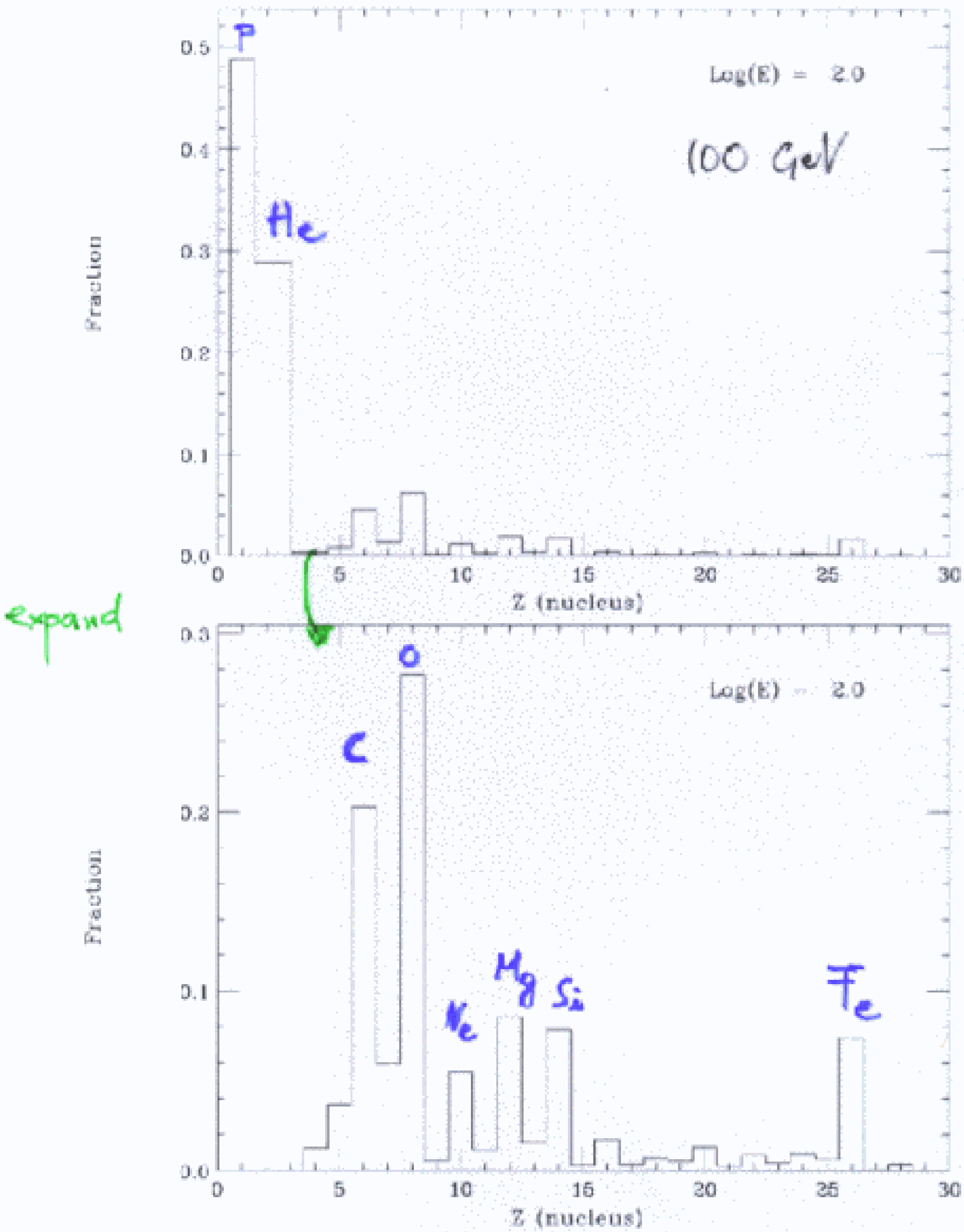
Particle Flux

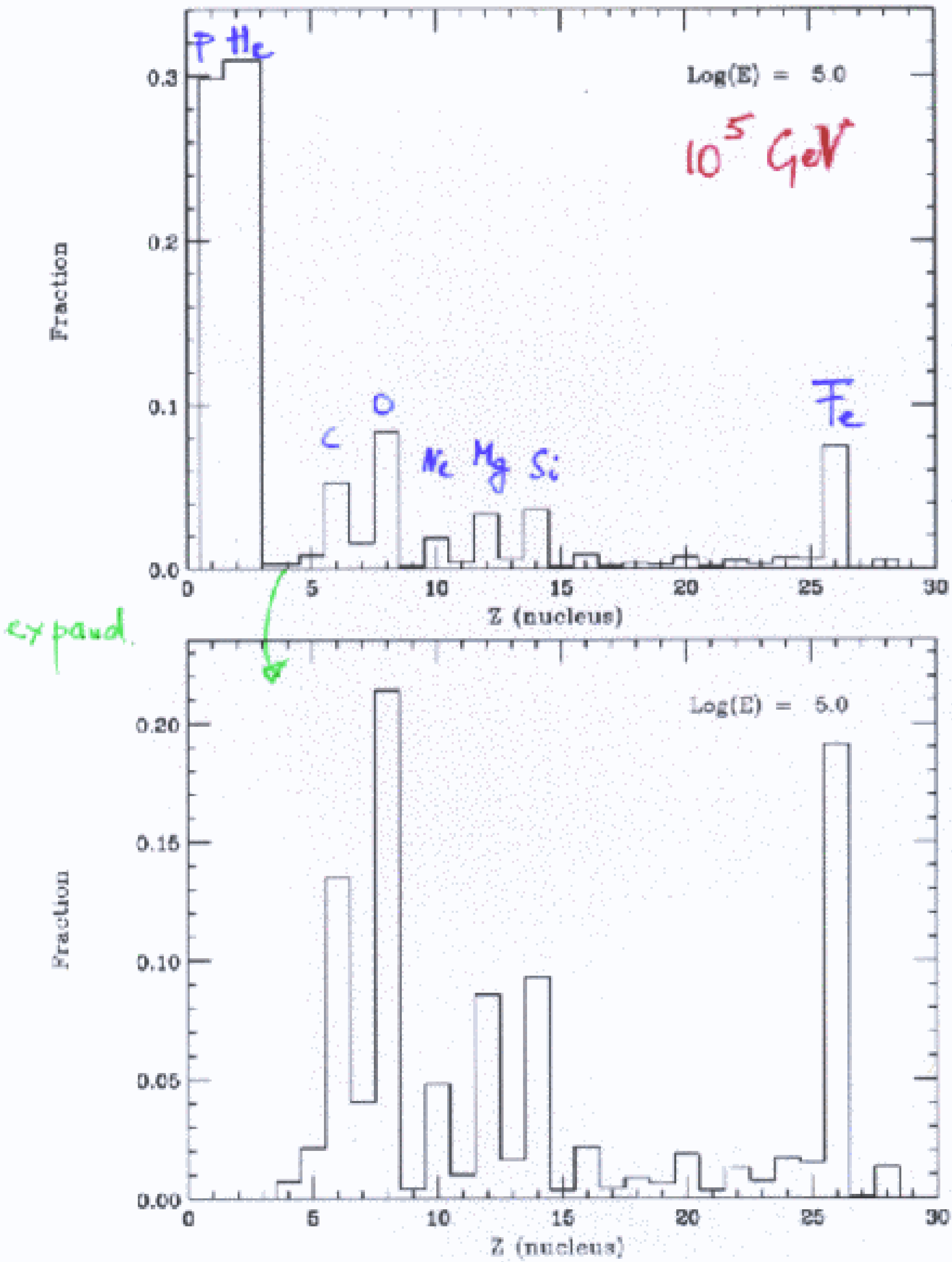


Energy / Particle (Ge


EAS measurements
"KNEE"
Flux * $E^{2.7}$
$\phi(E) * E^{2.7}$ [(cm^2 s sr)$^{-1}$ GeV$^{1.7}$]
AMS p
BESS p
AMS He
BESS He
JACEE p
JACEE He
Runjob p
Runjob He
Casa Mia
Casa Blanca
DICE
Tibet
Akeno
0
1
2
3
4
5
6
10^0
10^2
10^4
10^6
10^8
$E_{k,tot}$ (GeV)
Energy / Particle (GeV)

Composition Fixed L/particle





P He
Log(E) = 5.0
10^5 GeV
Fraction
C
O
Ne
Mg
Si
Fe
Z (nucleus)
expand
Log(E) = 5.0
Fraction
Z (nucleus)

Relation between the Particle flux and the NUCLEON flux

$$E_{particle} \equiv A E_0$$

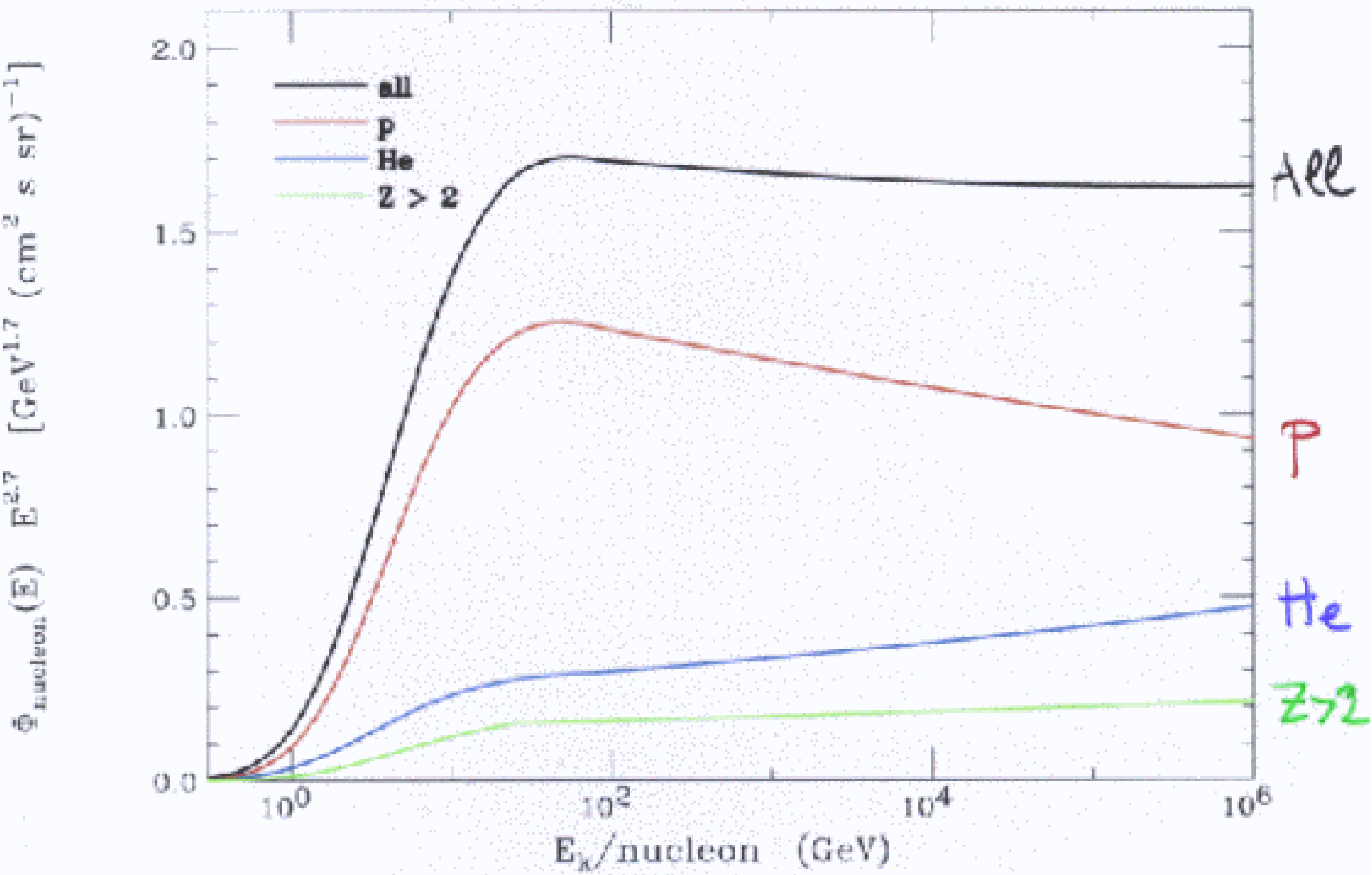
$$\frac{dN_{nucleon}}{dE_0} = A \frac{dE_{particle}}{dE_0} \frac{dN_{particle}}{dE_{particle}}\left(A E_0\right)$$

$$= A^2 K (A E_0)^{-2.7}$$

$$= A^{-0.7} K E_0^{-2.7}$$

Contribution to heavy Nuclei to the Nucleon flux is suppressed.

Nucleon flux

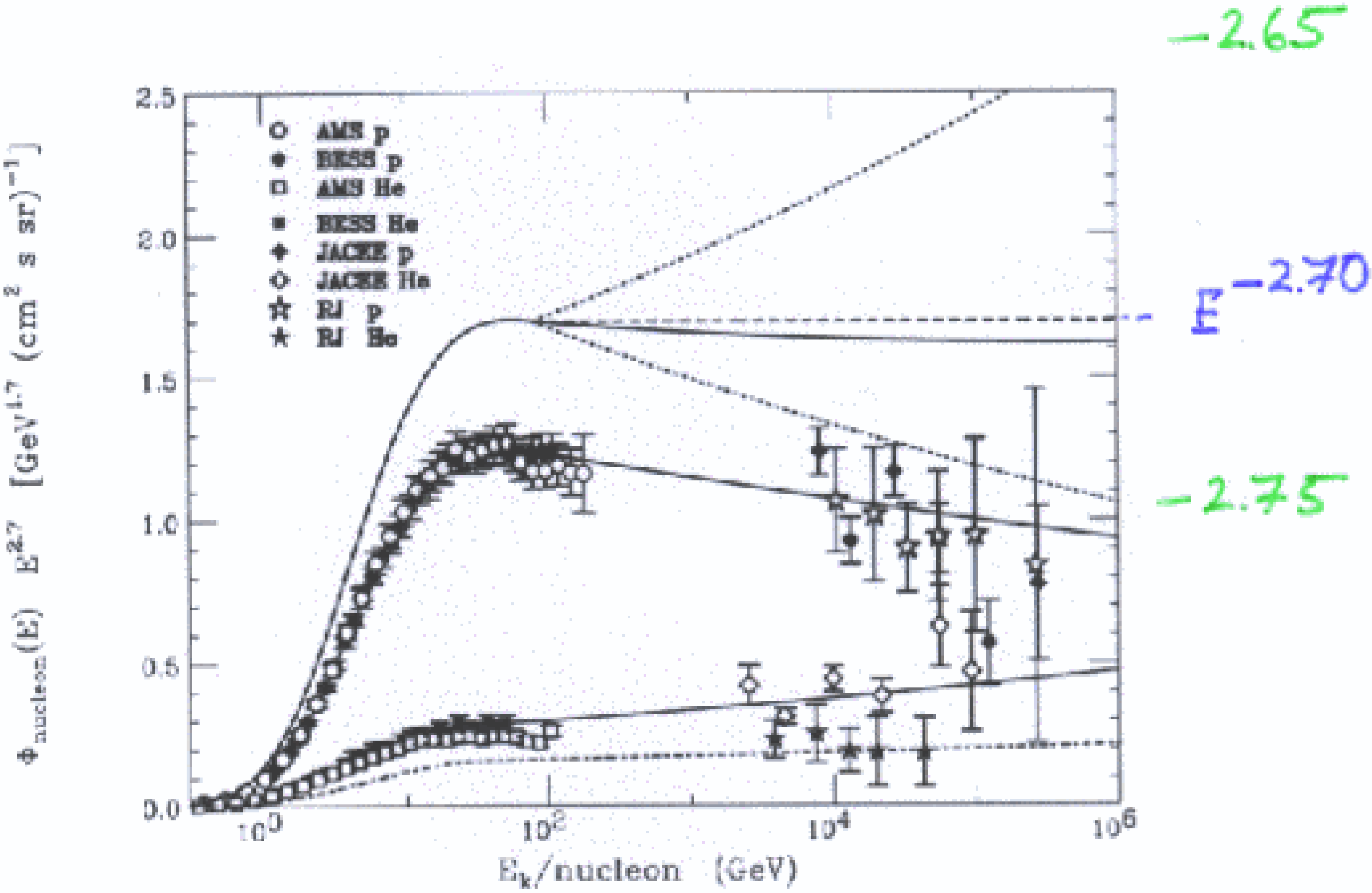


All Nucleon Flux.

- Proton
- Helium
- Heavy contributions

Shape of the primary nucleon flux

in the region $E_0 = 10^2$–10^5 eV
important for $\mu \uparrow$



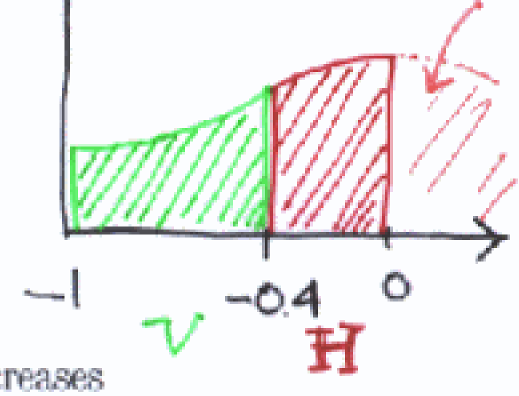
Dashed and dotted lines:

$\phi_{\text{all nucleons}} \propto E^{-\alpha}, \qquad \alpha = 2.70 \pm 0.05.$

Critical Discussion of Uncertainty on "extrapolation" soon ready.

NEW DATA: BESS TeV – p, He – 500 ~ 1000 GeV
AMS-02

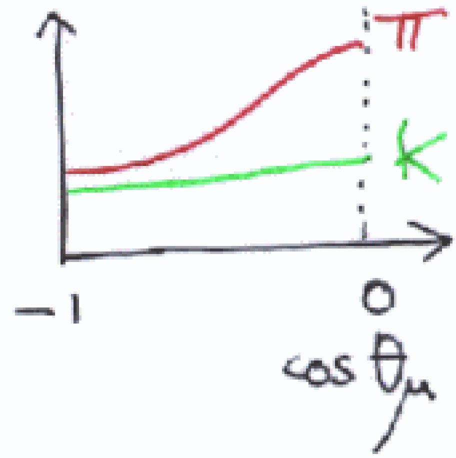
of UP-GOING MUONS



The Shape of the zenith angle distribution is determined by (dominant effects):

- The energy spectrum of primary radiation
  Harder spectrum $\Longleftrightarrow$ Vertical/Horizontal decreases

- The ratio K/π ratio
  Smaller K/π $\Longleftrightarrow$ Vertical/Horizontal decreases.



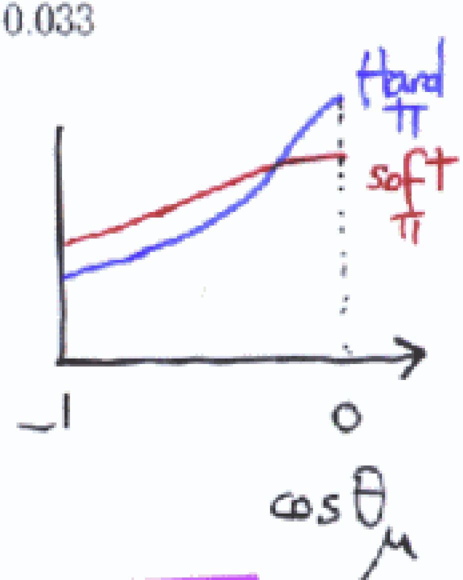
Effect of uncertainty on K/π ratio:

$$\frac{\delta(V/H)}{(V/H)_0} \simeq 0.12\ \frac{\delta(K/\pi)}{(K/\pi)_0}$$

Effect of Cosmic ray spectrum. ($\phi_0 \propto E_0^{-\alpha}$):

$$\frac{\delta(V/H)}{(V/H)_0} \simeq 0.25\ \delta\alpha$$

$$\text{Combined} = [0.12 \times 0.25] \otimes [0.25 \times 0.05] \simeq 0.033$$



$$\delta\left(\frac{\text{Vertical}}{\text{Horizontal}}\right) \sim 3.3\%$$

$\delta(K/\pi) \simeq 25\%$

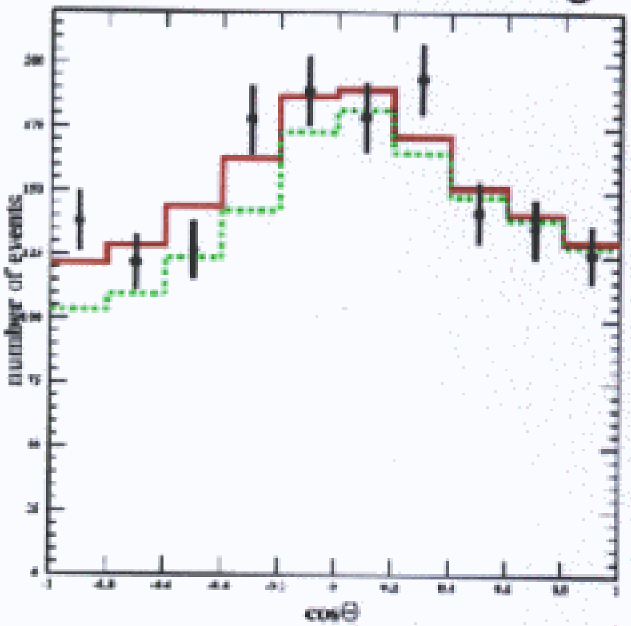
$\delta(\alpha) \simeq 0.05$

$\pi^\pm$ Decay

Evidence that atmospheric ν oscillation is $\nu_\mu - \nu_\tau$ not $\nu_\mu - \nu_{sterile}$

T.Toshito

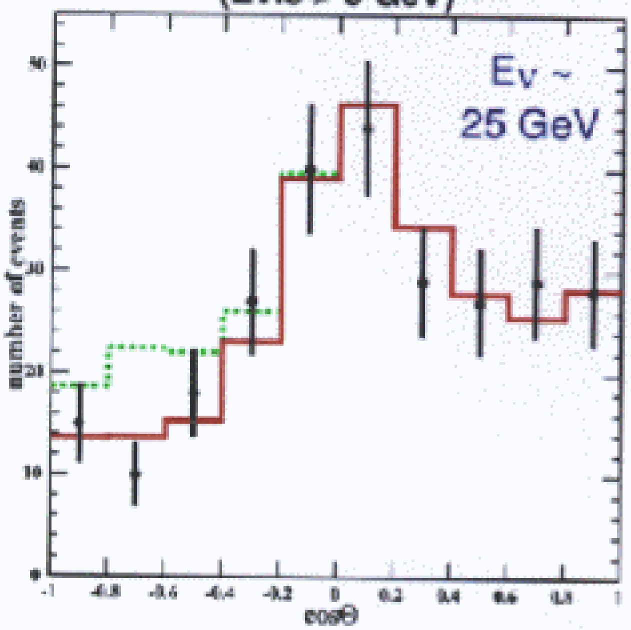
N.C. enriched multi-ring



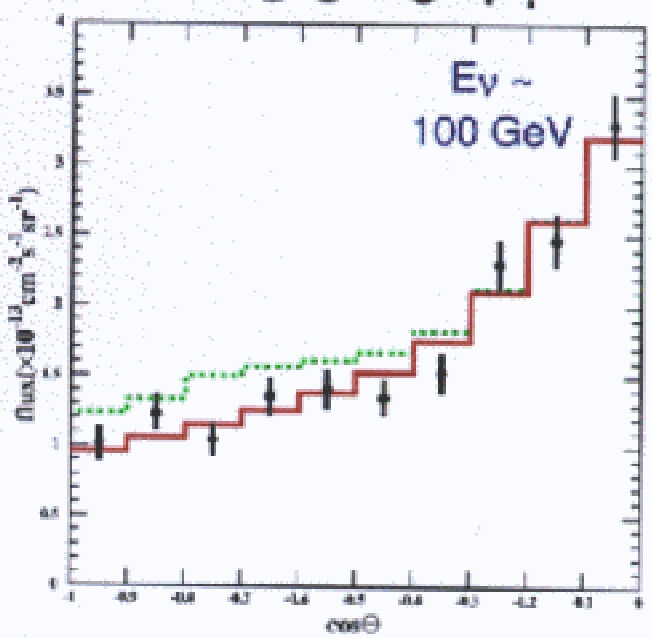
① Neutral Current Enhanced Sample

② High Energy Events with Matter Effects

Partially Contained
(Evis > 5 GeV)



Throughgoing Up-μ



upgoing direction

$\nu_\mu - \nu_\tau$ preferred at 99% C.L.

Form of the **SURVIVAL PROBABILITY**

⇔ Physical Mechanism at the origin.

OSCILLATIONS

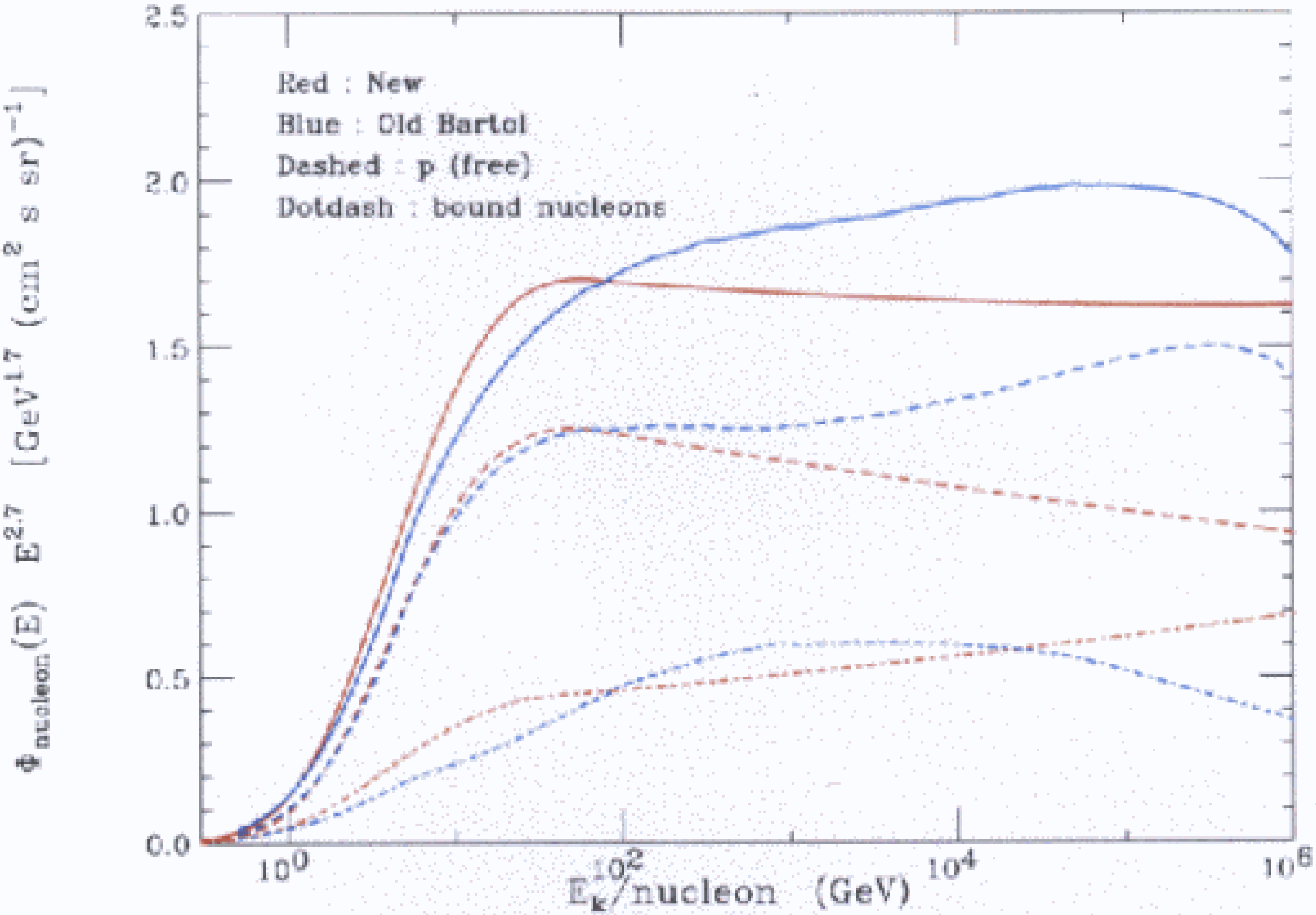
$$P^{osc}(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \, \sin^2\left[\frac{\Delta m^2 \, L}{4 E_\nu}\right]$$

FCNC

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta' \, \sin^2\left[\alpha X\right]$$

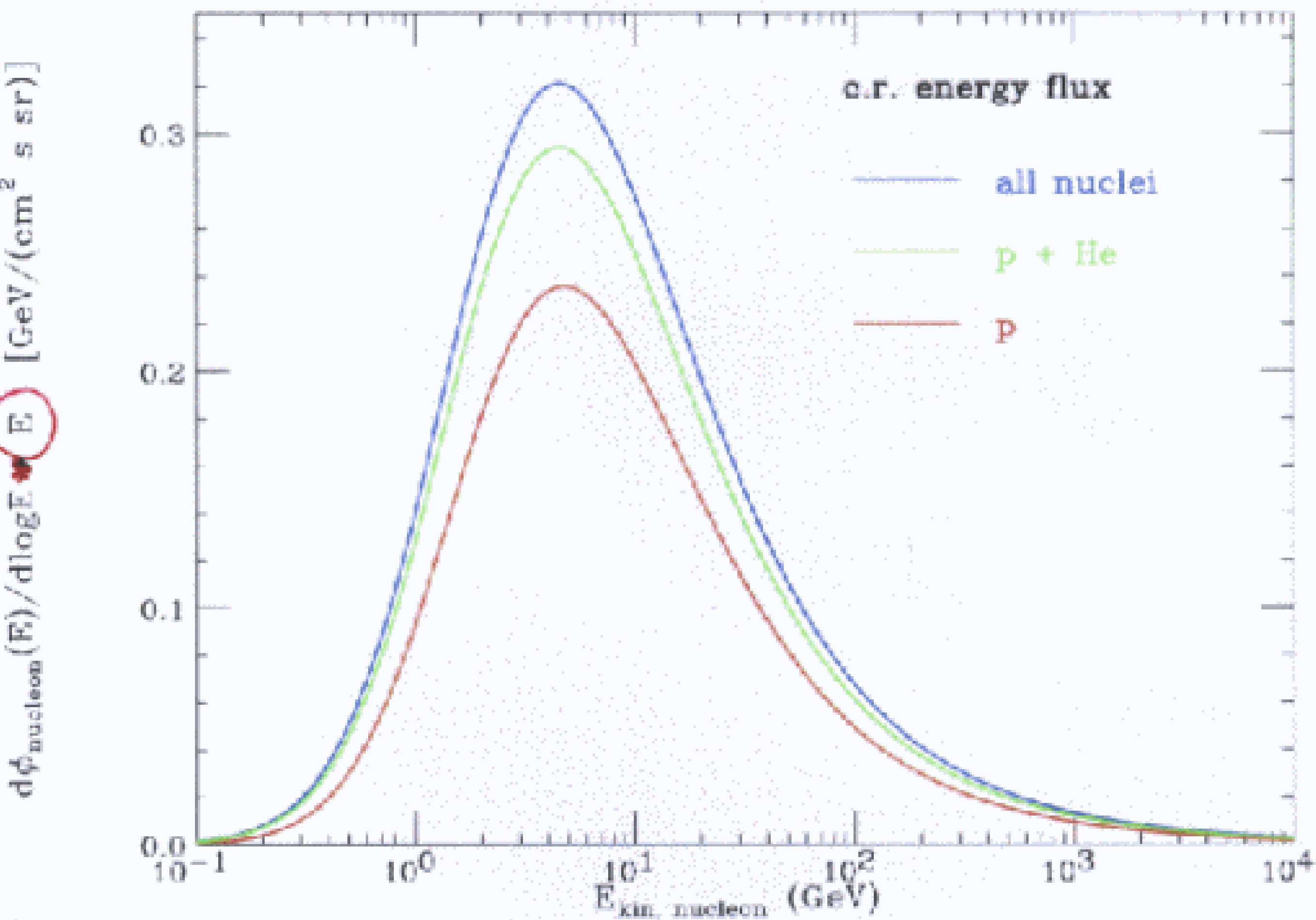
Violation of Equivalence Principle

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta'' \, \sin^2\left[\beta \, L E_\nu\right]$$



Comparison with "Old Bartol"

Cosmic Ray energy flux



$$\int dE\ \phi_0(E) = 1.17\ \frac{\text{GeV}}{\text{cm}^2\text{s sr}}$$ · {Geomagnetic reduction}

$\sim 72\%$ p

$\sim 19\%$ He

$\sim 9\%$ other nuclei (CNO, Ne, Mg, Si, Fe)

Cosmic Ray fluxes ⇒ Hadronic Showers

⇒ ν fluxes

~ 30% of c.r. energy "dissipated" into νs

NEUTRINO EVENT RATE

Qualitative estimate

N target nuclei

$$\text{Rate}_\nu = \frac{M_{\text{det}}}{m_p}\,\Delta t \int d\Omega \int dE_\nu\, \phi_\nu(E_\nu)\, \sigma(E_\nu)$$

$$\simeq 4\pi\,\frac{M_{\text{det}}}{m_p}\,\Delta t \int dE_\nu\, \phi_\nu(E_\nu)\, \sigma_0 \times E_\nu$$

$$\simeq 4\pi\,\frac{M_{\text{det}}}{m_p}\,\Delta t\, \sigma_0\, (\text{Energy flux})_\nu$$

$$\simeq 4\pi\,\frac{M_{\text{det}}}{m_p}\,\Delta t\, \sigma_0\, (\text{Energy flux})_{\text{c.r.}}\, \left\langle \frac{E_\nu}{E_{\text{c.r.}}} \right\rangle$$

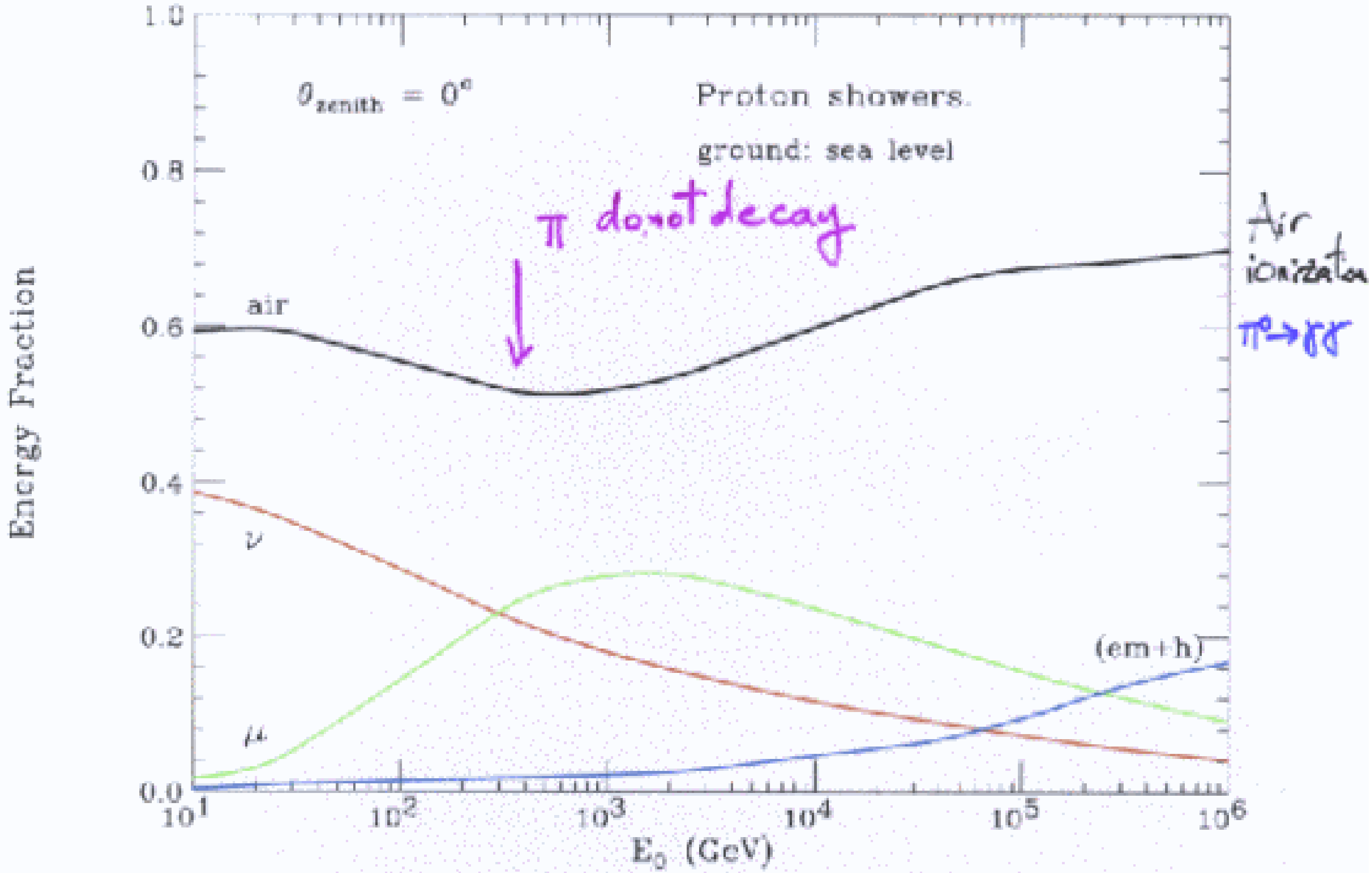
{Fraction of c.r. Er into Neutrinos}

$$\sim 4\pi \left(\frac{6.023\times 10^{32}}{\text{kton}}\right) \left(\frac{3.15\times 10^{7}\ \text{s}}{\text{yr}}\right) \left(0.5\times 10^{-38}\ \frac{\text{cm}^2}{\text{GeV}}\right) \times \left(1.15\ \frac{\text{GeV}}{\text{cm}^2\text{s sr}}\right) 0.3$$

$$\sim 350\ (\text{kton yr})^{-1}$$

(Rate-ν) ∝ Energy flux

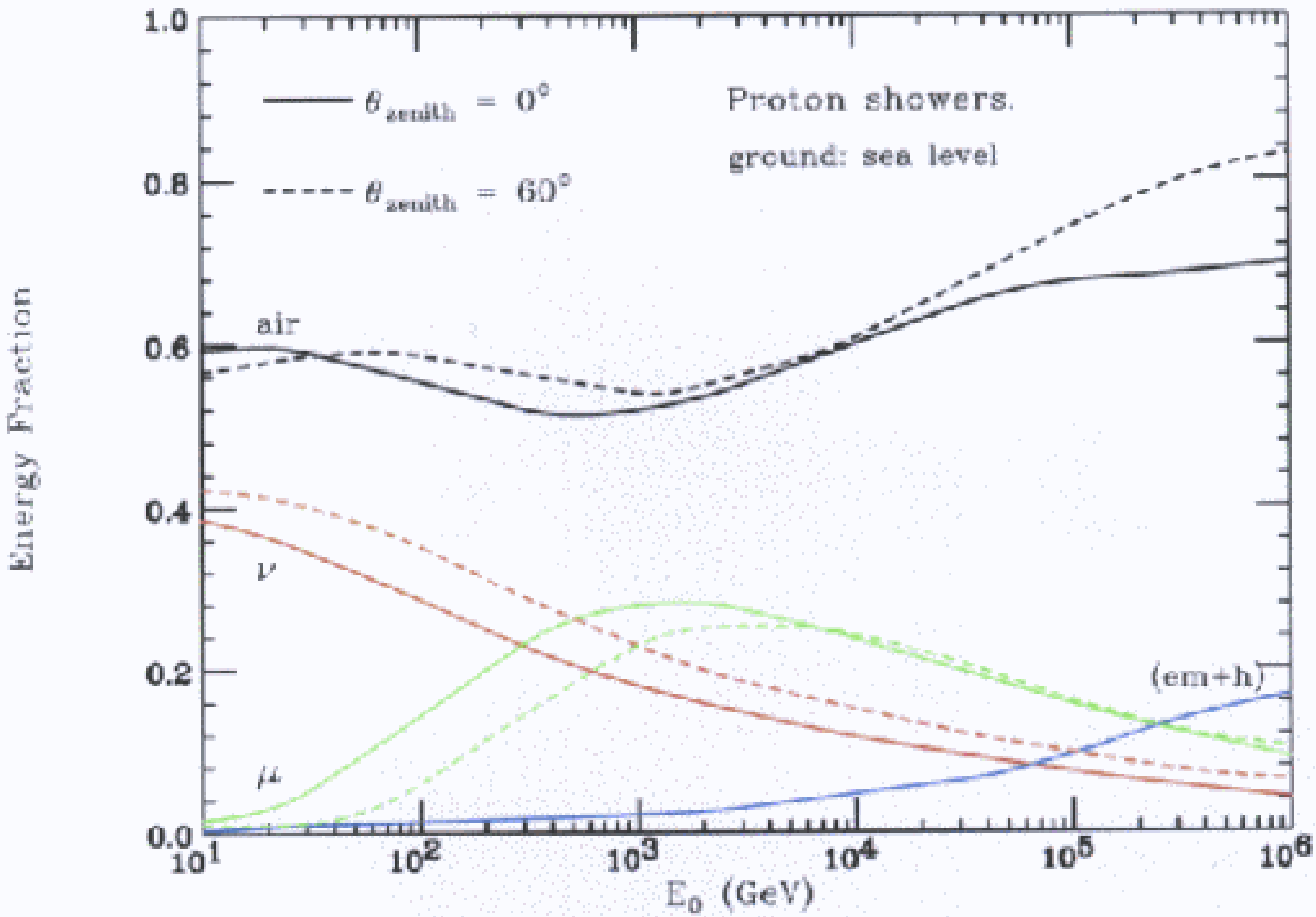
Vertical Showers



Energy balance for proton showers.

1. Energy in ν
2. Energy in μ (that reach ground level)
3. Energy dissipated in air
4. Energy reaching ground level
   - e.m. component
   - hadronic component

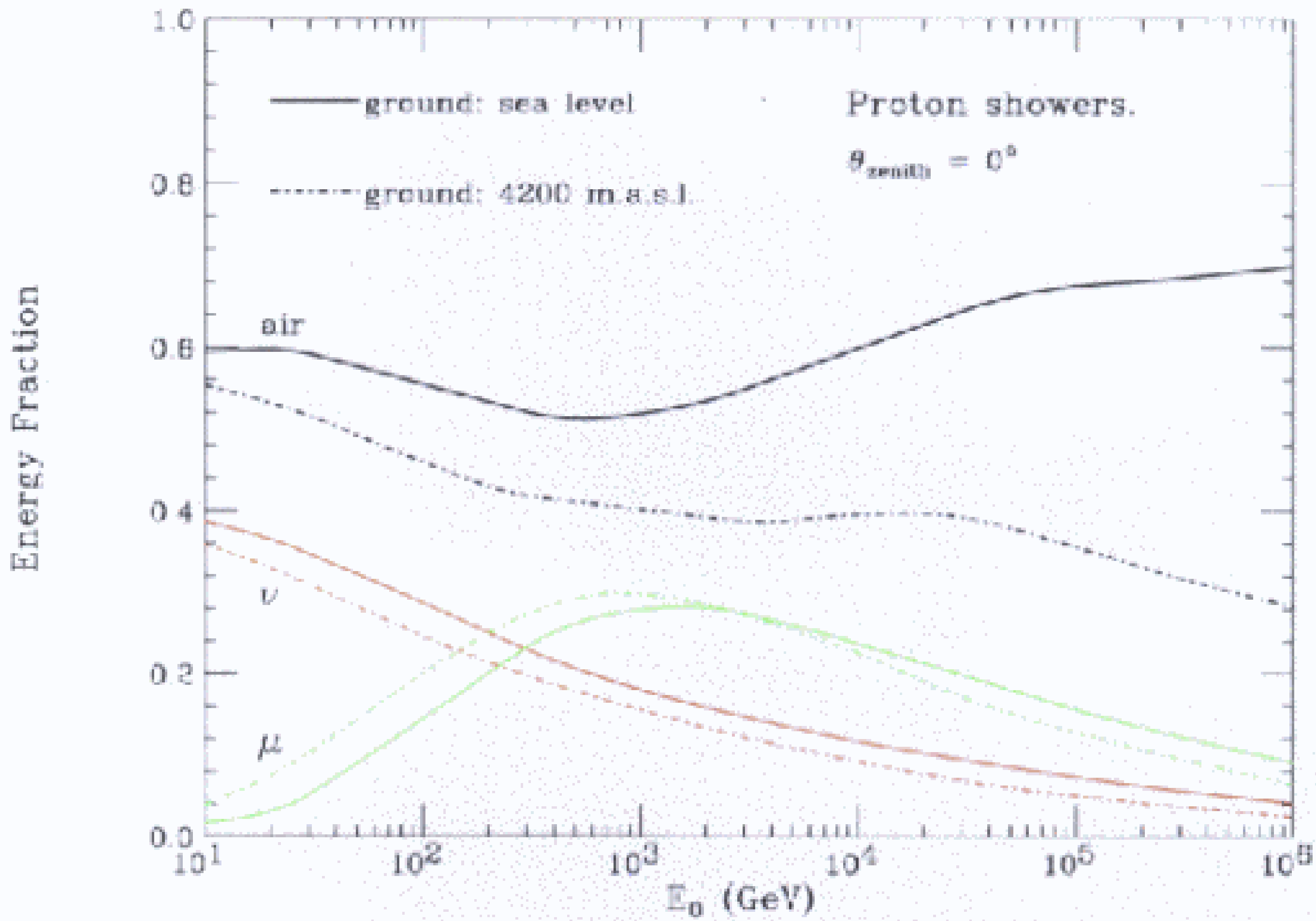
$\theta_{zenith} = 60°$ - - - - -



More inclined shower:

- More neutrinos
- Less muons (longer decay path)

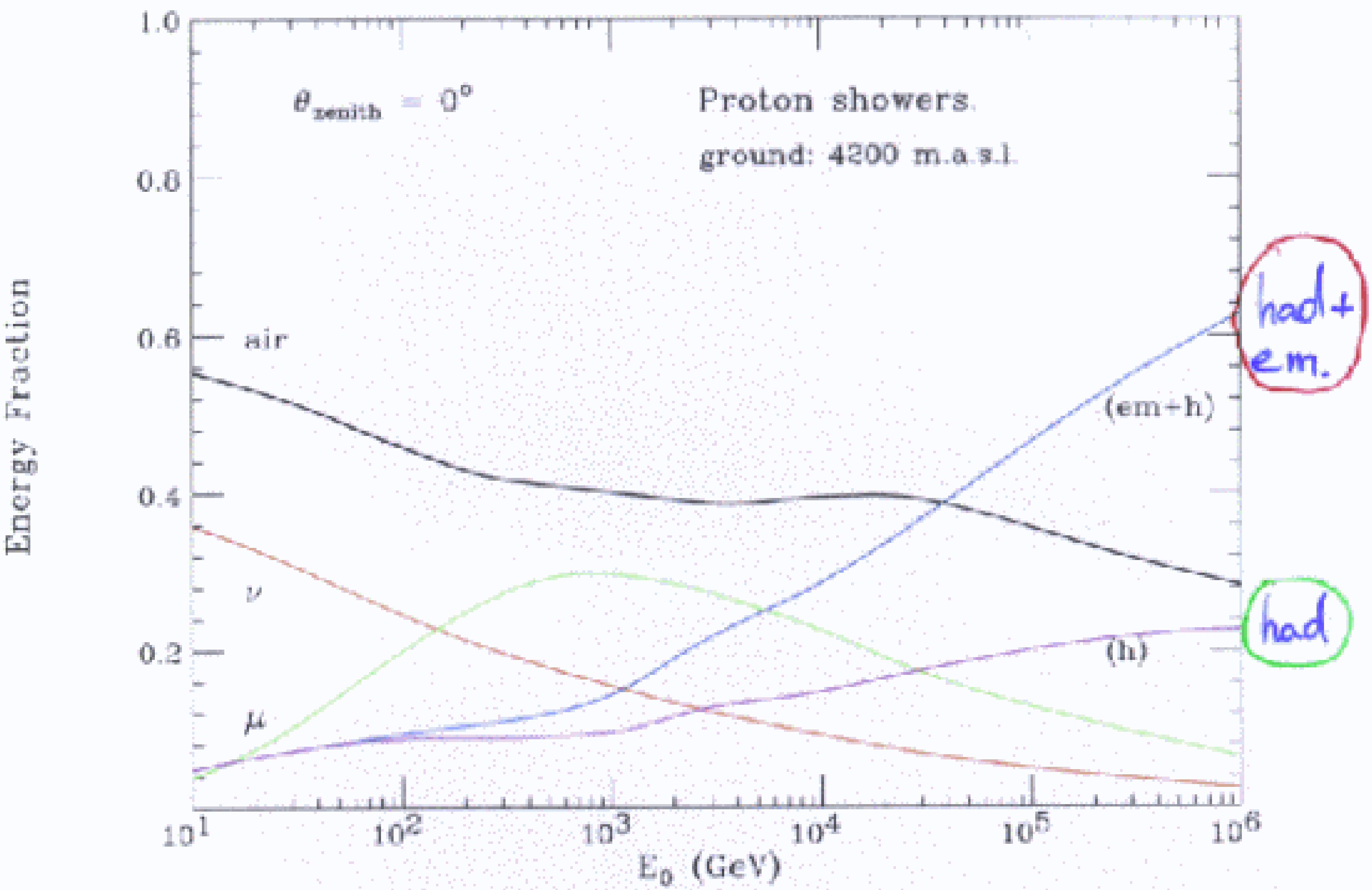
Effect of Ground altitude   Vertical showers.



h = 4200 m.asl. (Tibet)

- Shower Development "interrupted"
- μ decay suppressed
- ν- fluxes suppressed.

High Altitude Detector



Significant fraction of Energy can reach the ground for High Energy ~ vertical showers.

HADRONIC INTERACTIONS

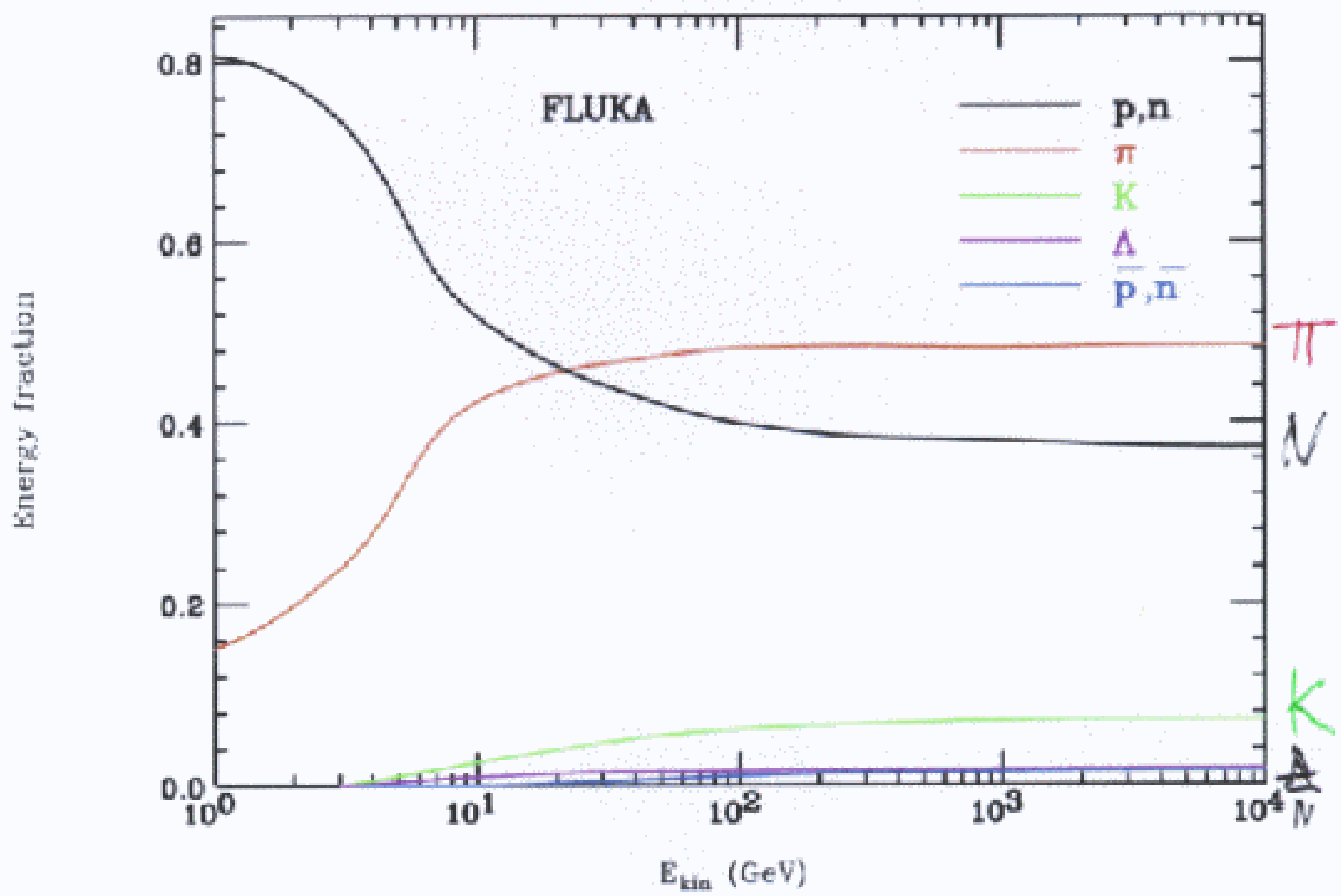
ENERGY BALANCE

in one p–Air interaction.

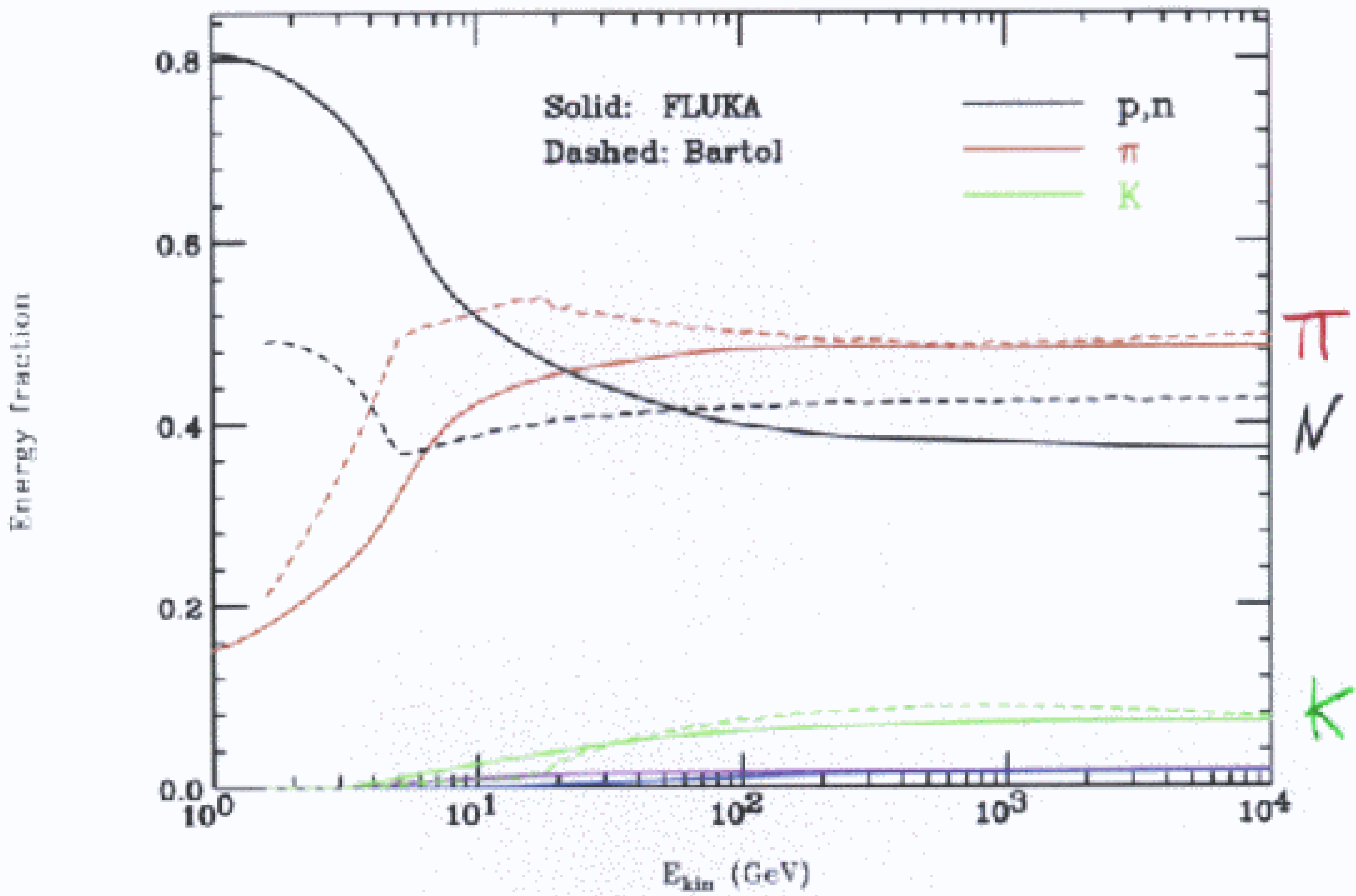
Kinetic energy of projectile channel:

Main channels: $[p, n]$, $[\pi^{\pm}, \pi^{\circ}]$, $[K^{\pm}, K_S, K_L]$, Λ, $[\bar{p}, \bar{n}]$.

$$\Sigma_\pi E_\pi \ , \quad \Sigma_K E_K \ , \quad \Sigma_N E_{\mathrm{kin},N}$$



Comparison FLUKA, BARTOL



BARTOL has MORE energy into π:

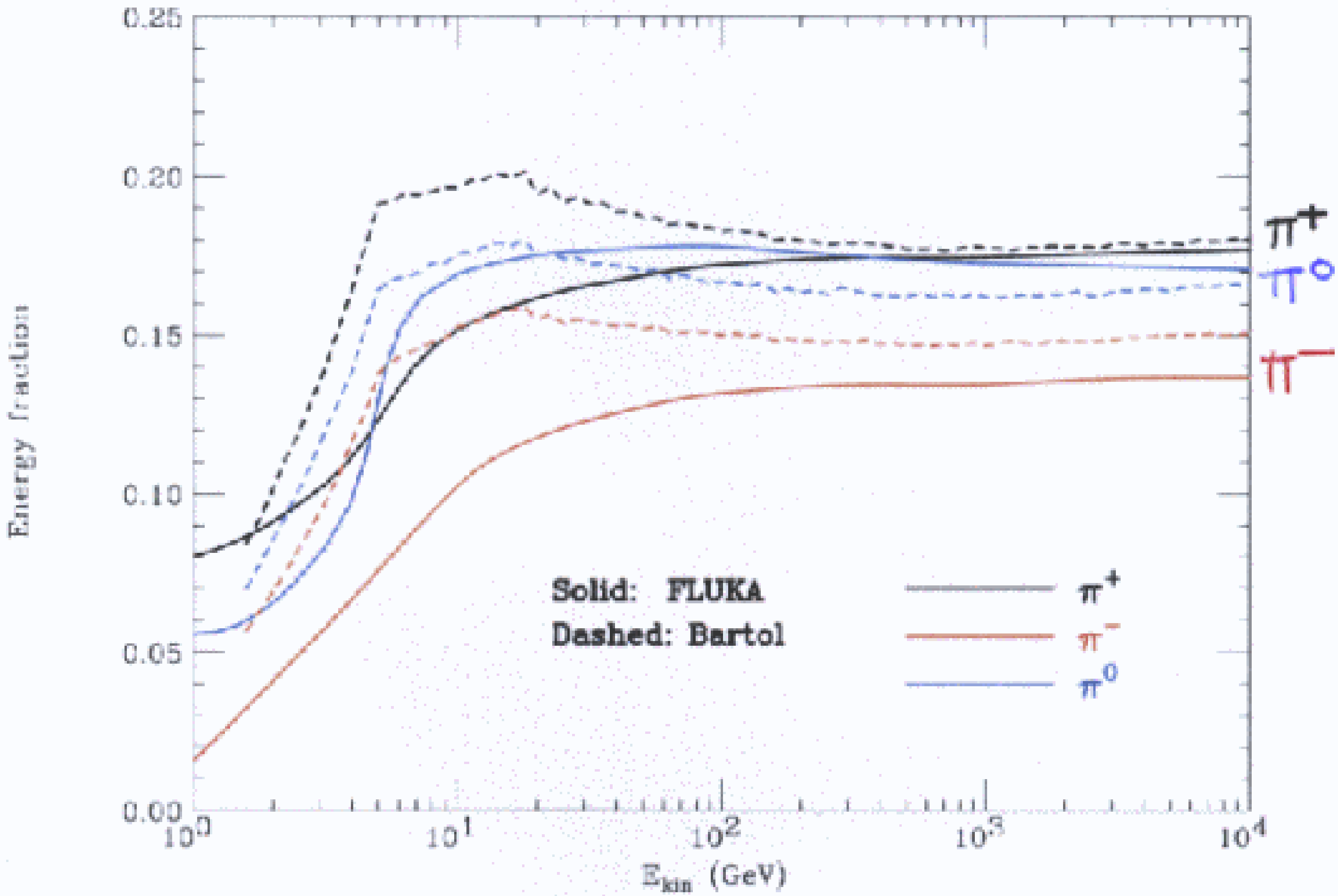
$\Longrightarrow$ higher normalization for ν fluxes

Effect more marked at low E_0

$\Longrightarrow$ softer ν spectra.

Comparison FLUKA–BARTOL
Decomposition of π^+, π^-, π°.

π⁺/π⁻ Ratio



$$\left(\frac{\langle E_{\pi^+}\rangle}{\langle E_{\pi^-}\rangle}\right)^{\text{Fluka}}_{100\text{ GeV}} = 1.31, \quad \left(\frac{\langle E_{\pi^+}\rangle}{\langle E_{\pi^-}\rangle}\right)^{\text{Bartol}}_{100\text{ GeV}} = 1.21$$

$$e = \nu_e \times \sigma_{\nu_e} + \overline{\nu}_e \times \sigma_{\overline{\nu}_e}$$

$$\pi^+ \to \nu_e,\ \pi^- \to \overline{\nu}_e, \qquad \sigma_\nu \sim 2\sigma_{\overline{\nu}}$$

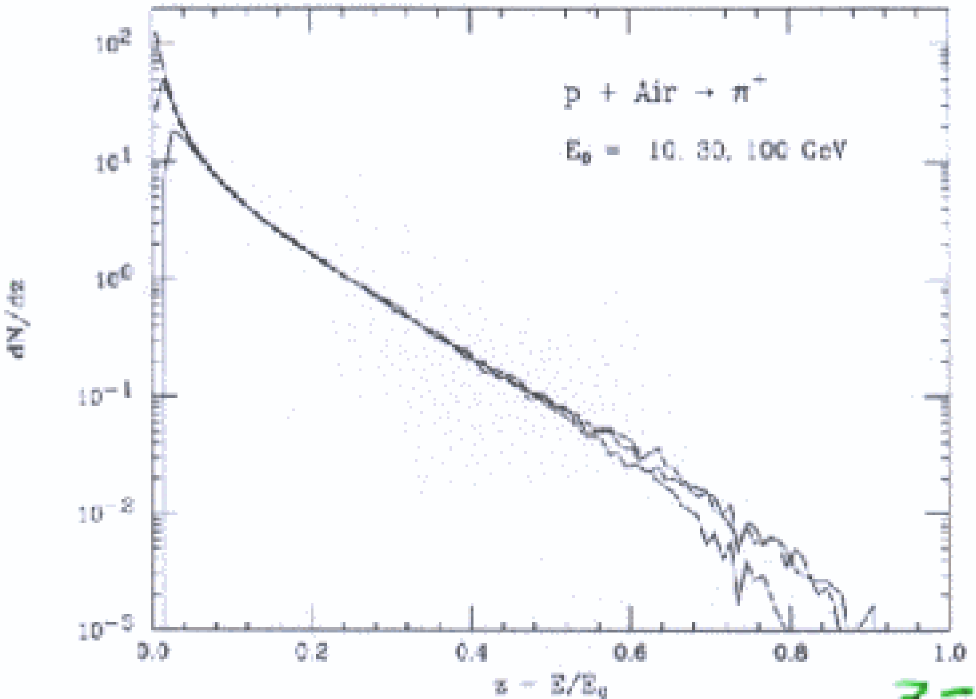
e/μ ratio LARGER in FLUKA.

Note : for FLUKA $\langle E(\pi^0)\rangle > \langle E(\pi^+)\rangle$

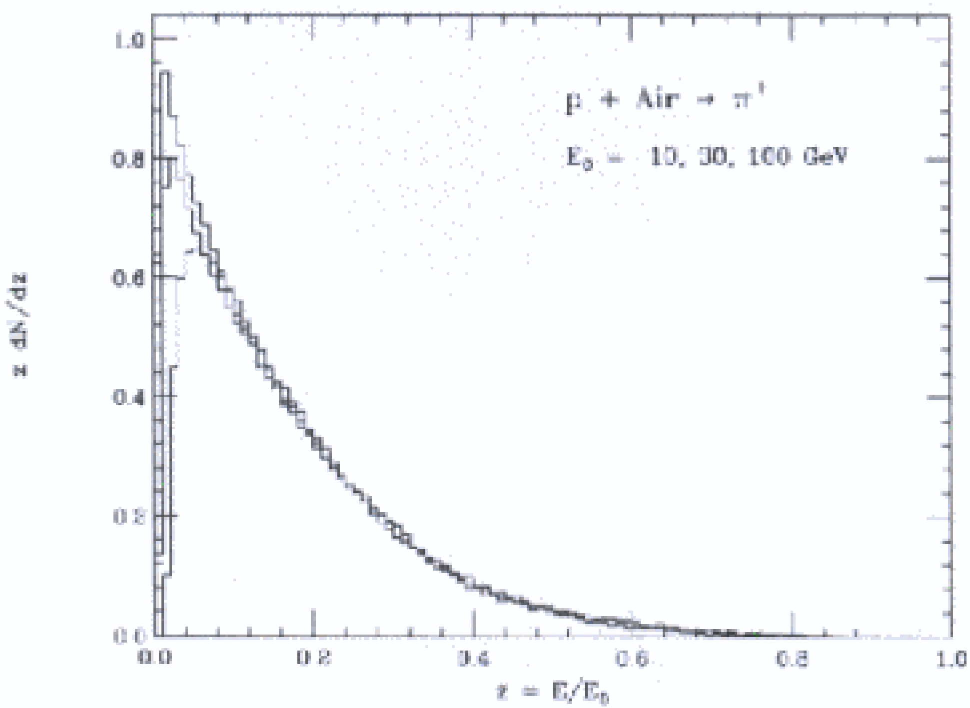
To be verified

$$\frac{dN}{dE_\pi}(E_\pi, E_0) \simeq \frac{1}{E_0} f\left(\frac{E_\pi}{E_0}\right)$$

Example of Fluka inclusive distributions:

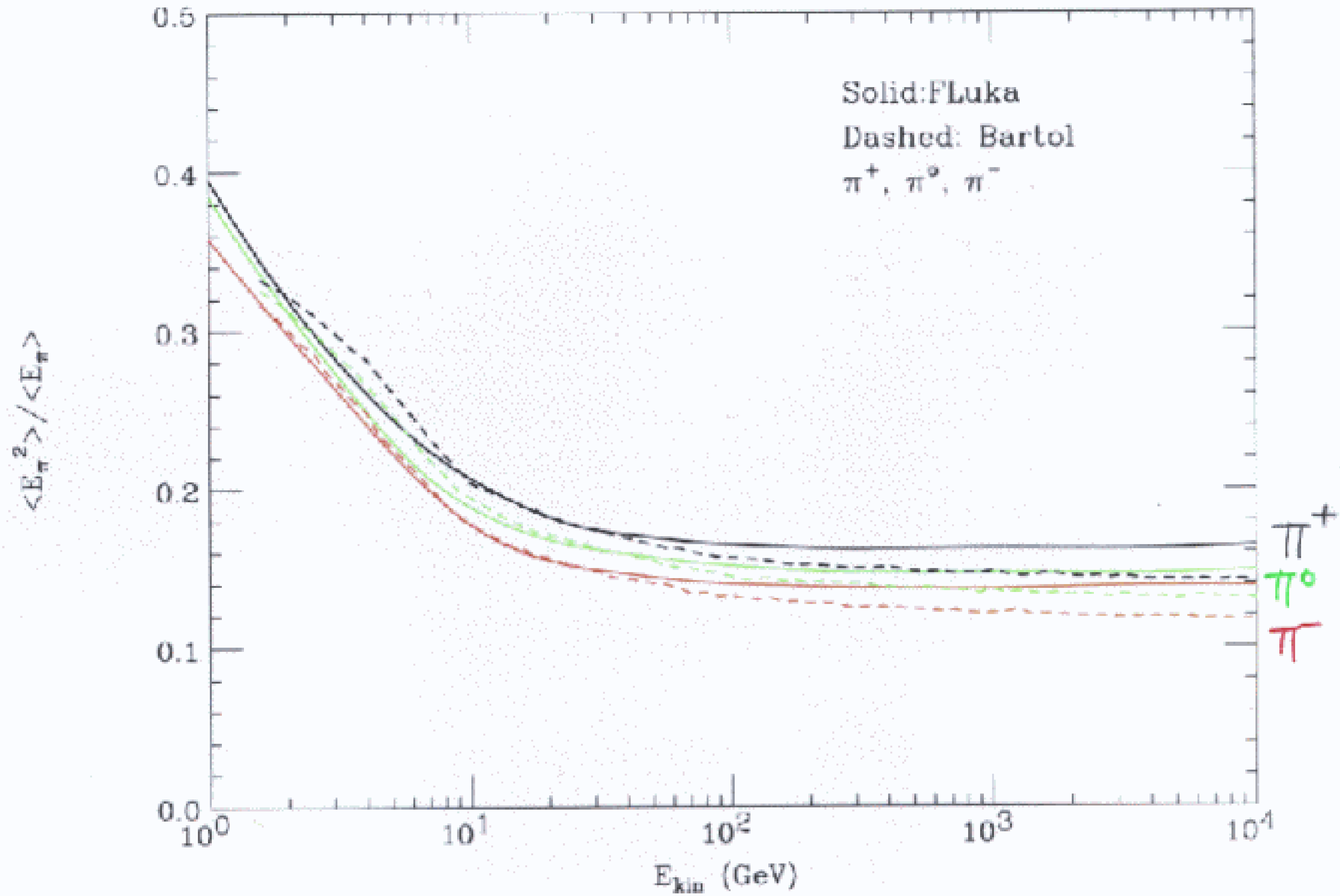


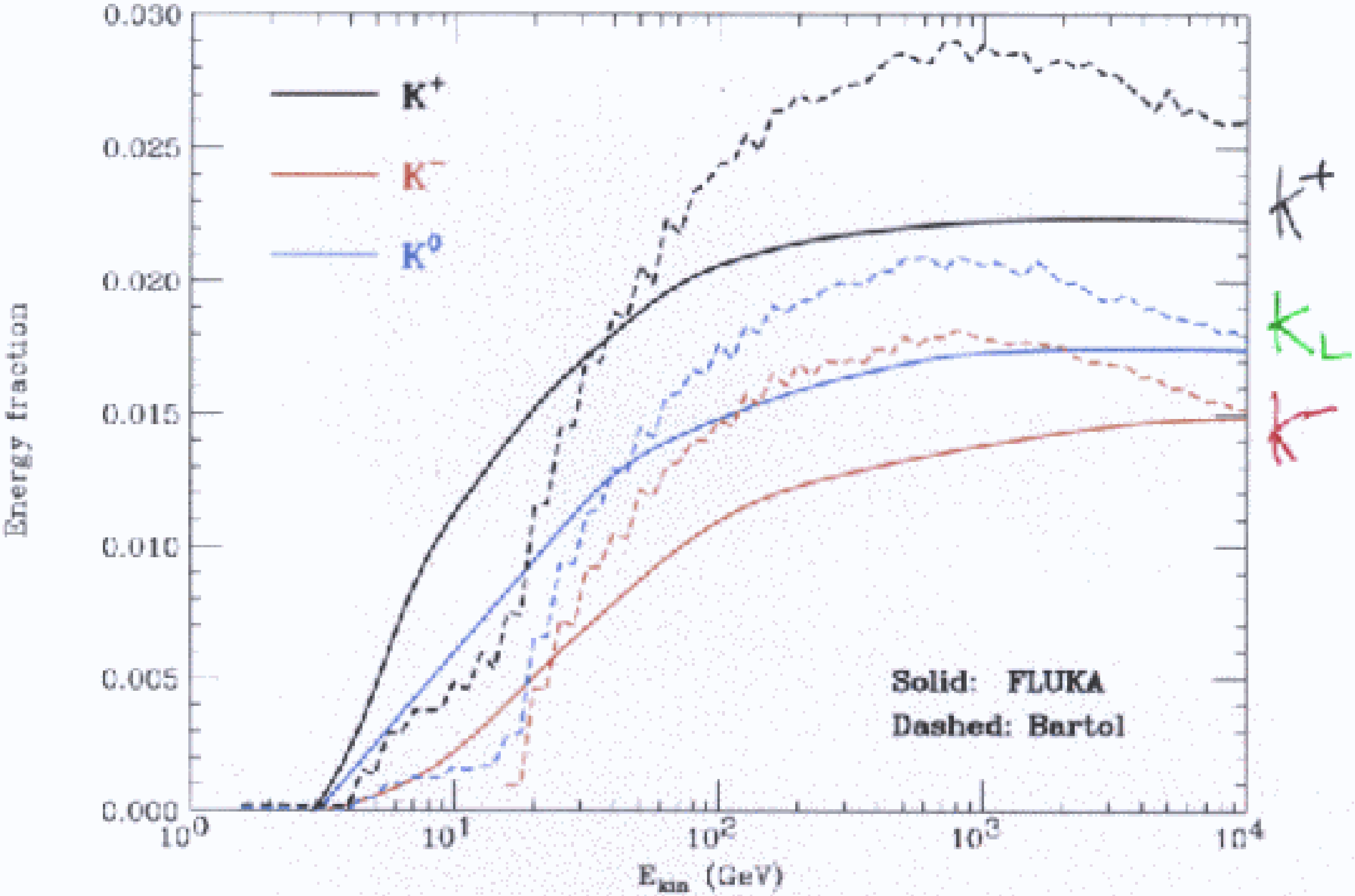
$z = E_\pi / E_0$



Feynman scaling

Shape of Spectra





$K^+ = [u\bar{s}]$

$\Lambda = [uds]$

$K^- = [\bar{u}s]$

Production: $pA \to \Lambda K^+ + \ldots\ldots$

$\Lambda K_L + \ldots\ldots$

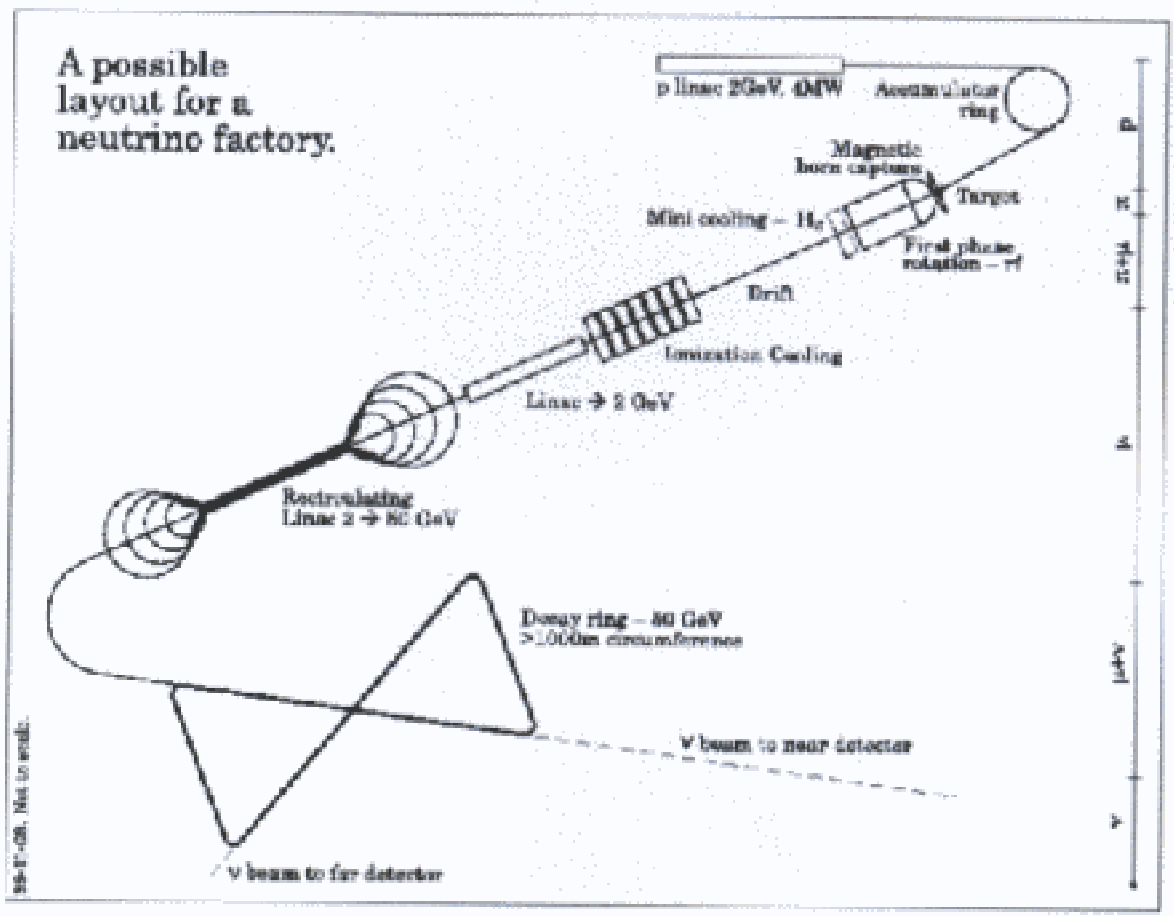
$$\left[K_L = \frac{1}{\sqrt{2}}\left(d\bar{s} + \bar{d}s\right) \right]$$

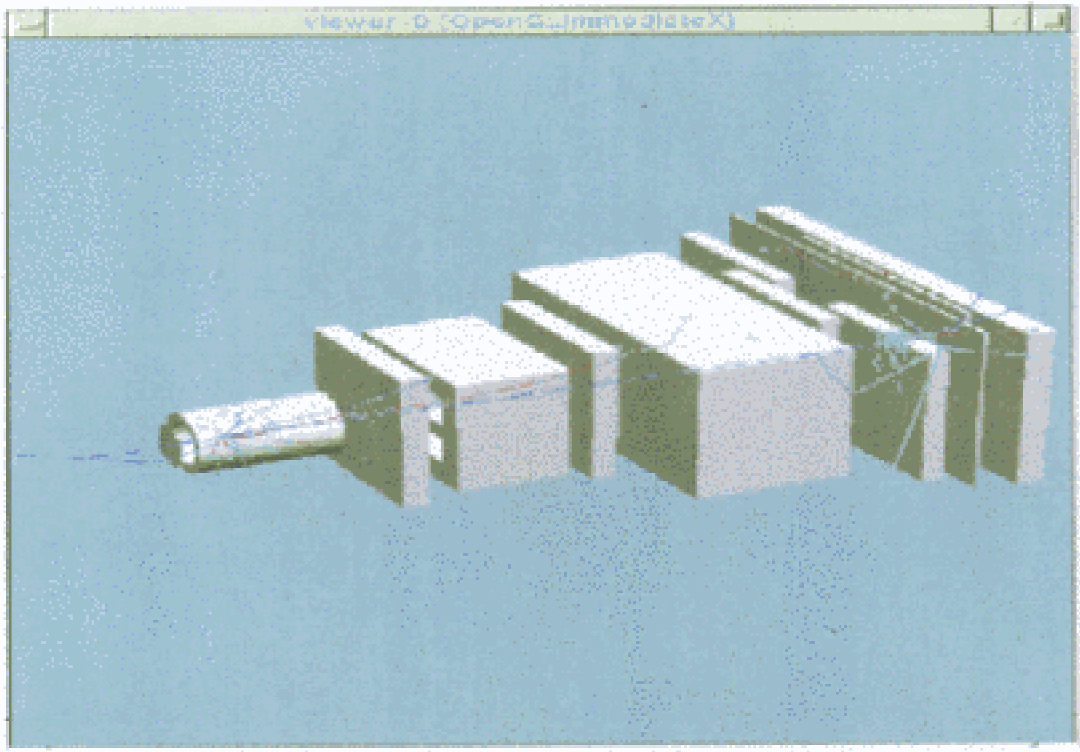
The HARP experiment at CERN

Physics Motivations:

- Detailed study of $\pi^{\pm}$ production for the development of the first stage of a ν–Factory,
- Study of hadronic interactions for the calculation of the Atmospheric ν Fluxes.

LAYOUT of ν–Factory





- Proton and Pion beams in the range $p = 2$–15 GeV.
- Several THIN targets spanning the full range Be, W. (include Oxygen and Nitrogen). Also thick targets.
- Particle Identification.
- Large phase space coverage
- Aim at a $\sim 2\%$ accuracy in the measurement of inclusive cross–sections.

Important Measurements

HARP contribution

- Energy into $\pi^{\pm}$
  (and distribution $\frac{dn}{dx}\Big|_{\pi^{\pm}}$)

- E_0 dependence of $\pi^{\pm}$ production.
  - Energy spectrum of sub-GeV
  - Up/Dow "intrinsic asymmetry

- π^{+}/π^{-} Ratio

- Onset of kaon production

UP/DOWN asymmetry

An elementary GEOMETRY THEOREM. If:

IF:

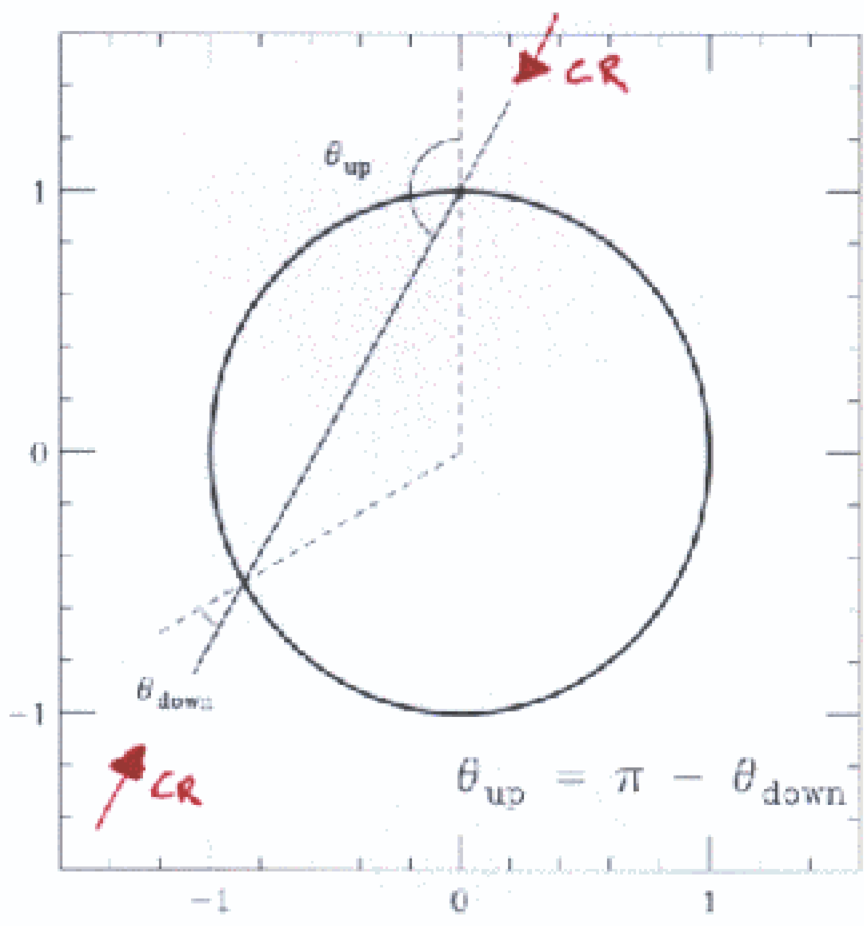
1. The Earth is spherically symmetric
2. The Cosmic Ray fluxes are isotropic

THEN:

Then: The atmospheric neutrino fluxes are Up–Down symmetric.

$U_0 = D_0$

$$\phi_{\nu_\alpha}(E_\nu, \theta) = \phi_{\nu_\alpha}(E_\nu, \pi - \theta)$$



Assumptions not exactly valid:

- Cosmic Ray fluxes not isotropic
  Geomagnetic effects. (cutoffs)

- Earth not exactly a sphere.
  - Atmosphere profile
  - Mountains
  - Sphere → Ellipsoid.

$\Rightarrow U_0/D_0 \neq 1$

but deviations are small.

Primary Cosmic Rays that are the sources of UP-going and DOWN-going ν have different Geomagnetic Cutoffs.

$$\left(P/Z\right)_{min}(\vec{x}, \Omega) = 0 - 20\,\text{GeV}$$

$$(\text{UP})_0 = \int_{E_{min}^{\uparrow}(\Omega)}^{\infty} dE_0 \; \phi_0(E_0) \left\langle n_{p \to \nu}(E_0) \right\rangle$$

$$(\text{DOWN})_0 = \int_{E_{min}^{\downarrow}(\Omega)}^{\infty} dE_0 \; \phi_0(E_0) \left\langle n_{p \to \nu}(E_0) \right\rangle$$

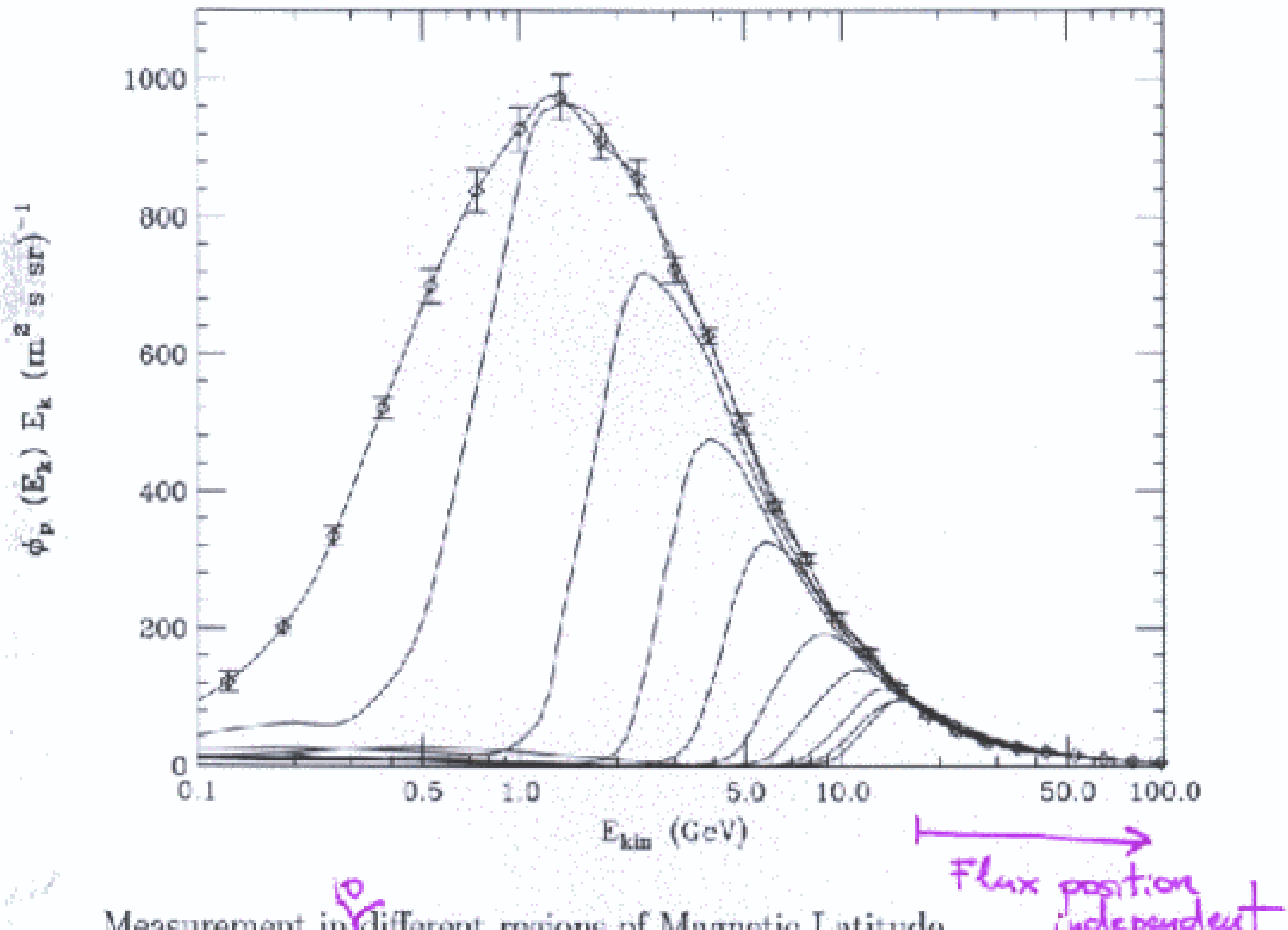
Cutoff

Form of Spectrum

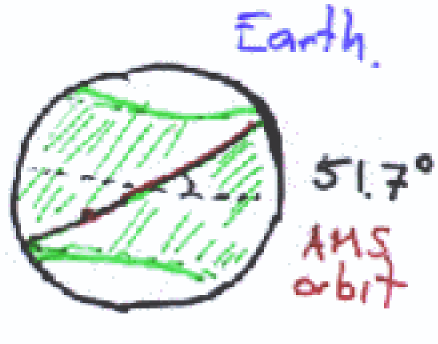
Hadronic interactions

$\theta_{\rm zenith} \lesssim 30^\circ$

AMS measurement of the proton flux



Measurement in different regions of Magnetic Latitude.
$E_{\rm kin}$(threshold) $\simeq 0.3$ GeV.



$$\Phi(\text{polar}) \simeq 2470 \;\; (\mathrm{m}^2\;\mathrm{s}\;\mathrm{sr})^{-1}$$

$$\Phi(\text{equator}) \simeq 140 \;\; (\mathrm{m}^2\;\mathrm{s}\;\mathrm{sr})^{-1}$$

$\Phi_{\rm primary}(\text{equator}) \simeq 100\;(\mathrm{m}^2\,\mathrm{s}\,\mathrm{sr})^{-1}$, $\quad \Phi_{\rm albedo}(\text{equator}) \simeq 40\;(\mathrm{m}^2\,\mathrm{s}\,\mathrm{sr})^{-1}$

The Algorithm used for the primary cosmic ray flux in the atmospheric neutrino calculations:

1. Assume that the primary cosmic ray flux at 1 A.U. from the sun, in the ***absence*** of the Earth is ISOTROPIC: $\phi_p^\infty(E;\ t)$

   small time variability connected with "solar modulation"

2. The flux reaching the atmosphere in the position $\vec{x}$

   - Depends on $\vec{x}$
   - Depends on both zenith and azimuth angle Θ, φ.

   and is:

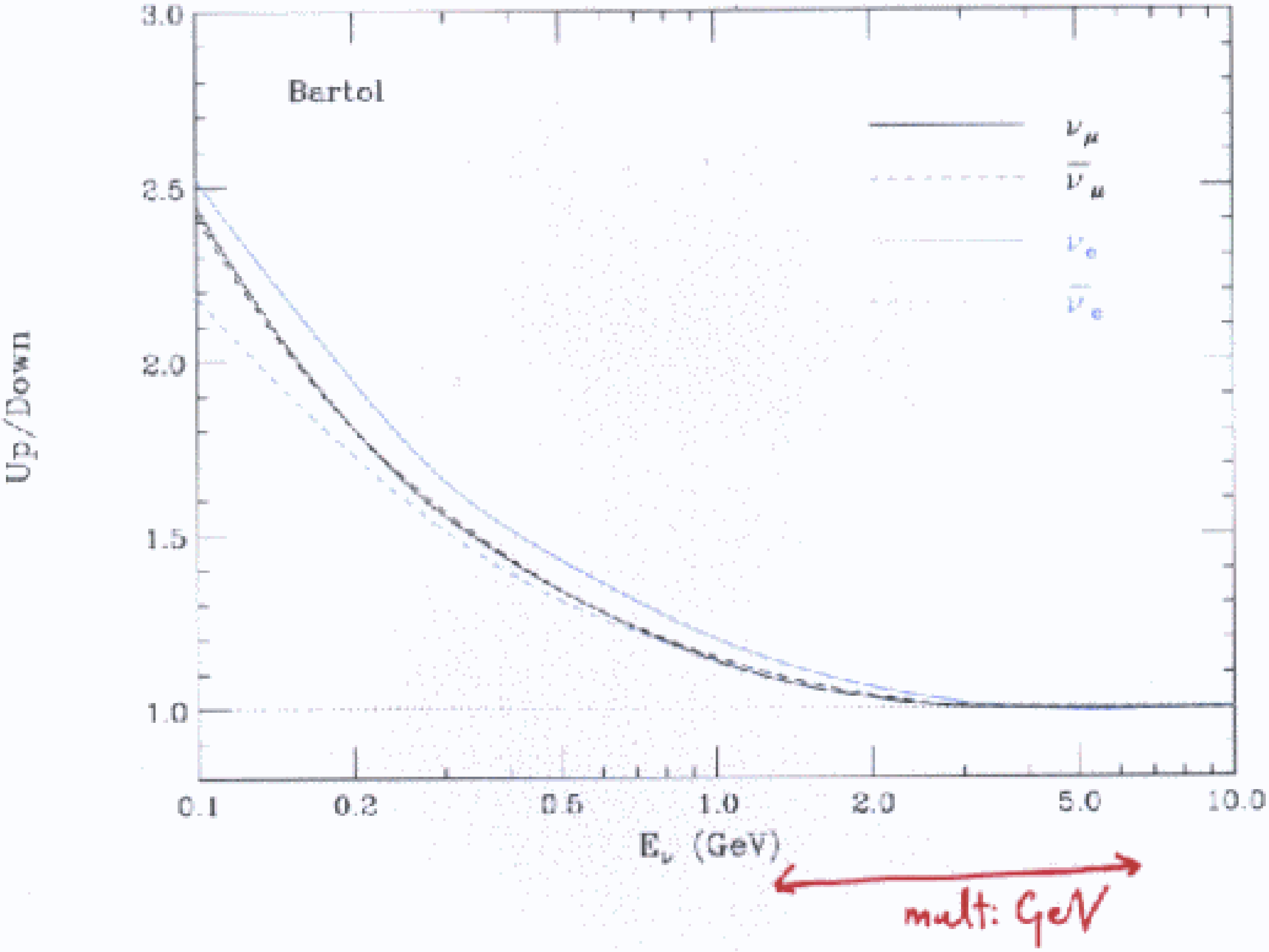
$$\phi_p(E;\ \vec{x}, \Omega) = \begin{cases} \phi_p^\infty(E) & \text{for "allowed" trajectories,} \\ 0 & \text{for "forbidden" trajectories.} \end{cases}$$

This Algorithm is based on the **Liouville theorem** and is rigorously valid for: (i) A primary flux isotropic at large distance from the Earth; (ii) Propagation in a purely static magnetic field

No angular smearing

Kamioka

Up/Down ratio. Bartol

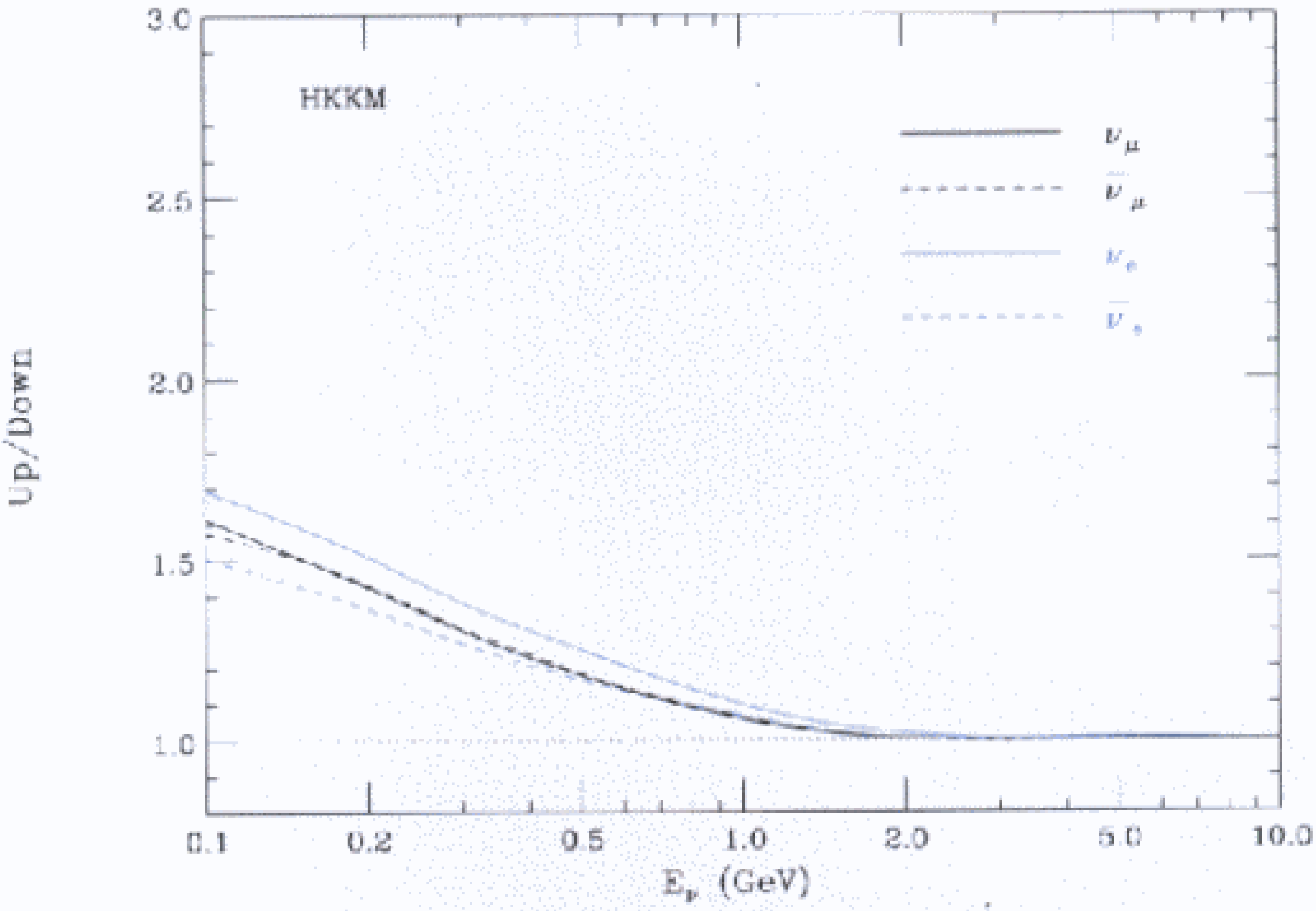


$$\left(\frac{U}{D}\right)_{\nu_e} > \left(\frac{U}{D}\right)_{\nu_\mu} \simeq \left(\frac{U}{D}\right)_{\overline{\nu}_\mu} > \left(\frac{U}{D}\right)_{\overline{\nu}_e}$$

Consequence of importance of π^+ for $x \to 1$.

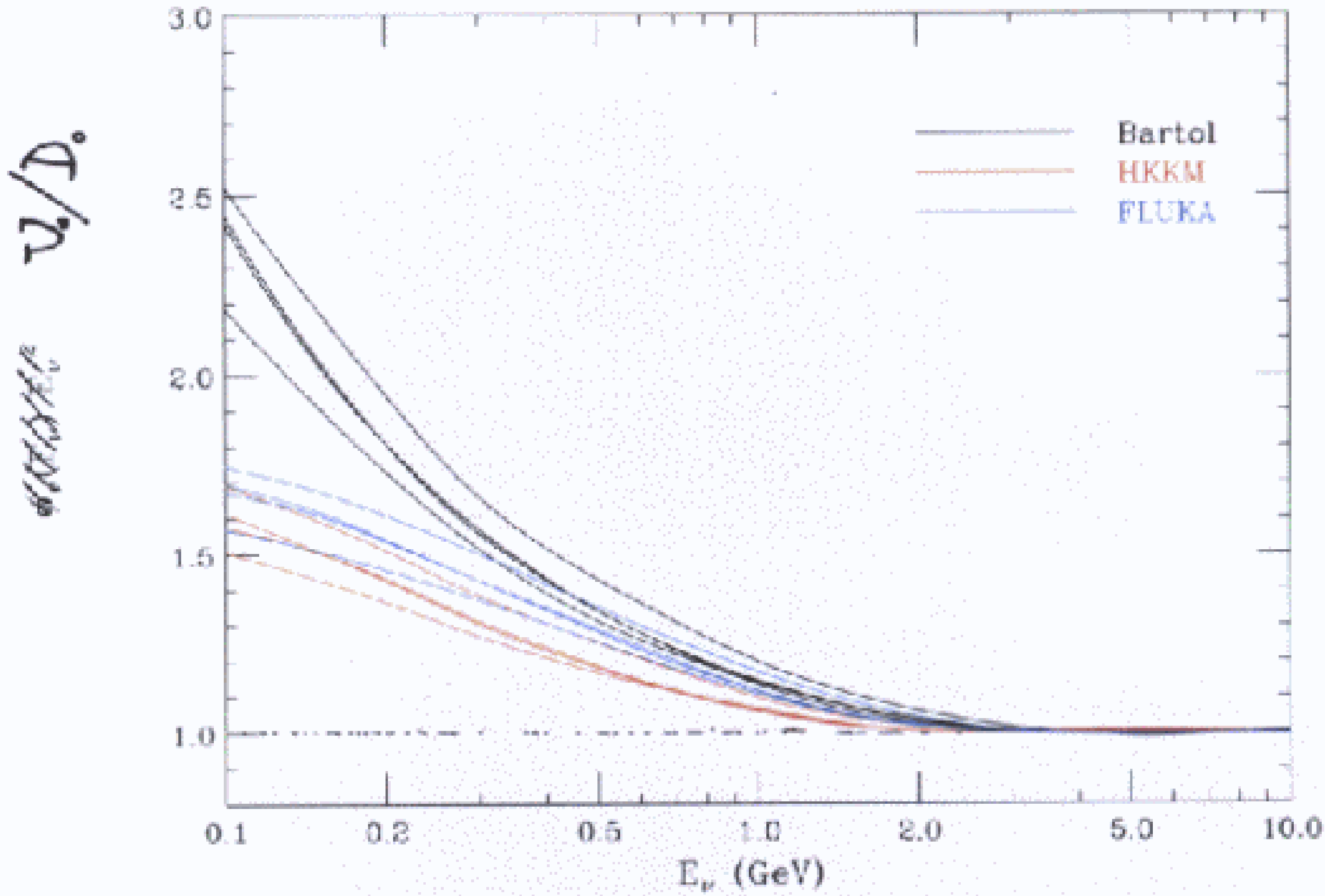
$U = \cos\theta_\nu \in \left[-1, -\frac{1}{3}\right]$

$D = \cos\theta_\nu \in \left[\frac{1}{3}, 1\right]$



Size of effect much smaller for HKKM.

Different Models BARTOL HONDA FLUKA



Effect of difference in $\pi^{\pm}$ production for low energy primary particles.

"Intrinsic asymmetry" different in different calculations.

Particularly important for Soudan-2 detector

DOUBLE RATIO $(\mu/e)/(\mu/e)_0$

measure of the average suppression of the μ–like events:

$$\frac{\mu}{\mu_0} \simeq \frac{\mu/e}{\mu_0/e_0}$$

The value of $(e/\mu)_0$ in the "Standard Model" is a robust prediction.

The NORMALIZATION and ENERGY spectrum of e–like and μ–like event rates are very strongly correlated reflecting the parent distributions.

Sources of uncertainty:

- The π^+/π^- ratio.
- Production of Kaons
- Muon Bending in the Geomagnetic field.

"Standard Argument"

$$\pi^+ \rightarrow \nu_\mu \, \mu^+ \rightarrow \nu_\mu \, \overline{\nu}_\mu \, \nu_e \, e^+$$

$$\pi^- \rightarrow \overline{\nu}_\mu \, \mu^- \rightarrow \overline{\nu}_\mu \, \nu_\mu \, \nu_e \, e^-$$

Each $\pi^\pm$ after chain decay results into
TWO $(\nu_\mu, \overline{\nu}_\mu)$ and **ONE** $(\nu_e, \overline{\nu}_e)$.

and a crucial **kinematical "cancellation"**:
all 3 ν's in a chain decay have $\simeq \langle E_\nu \rangle$

$$\langle E_{\pi^+ \rightarrow \nu_\mu} \rangle = \frac{1}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \simeq 0.213 \; E_\pi$$

$$\langle E_{\pi^+ \rightarrow \mu^+} \rangle = \frac{1}{2} \left(1 + \frac{m_\mu^2}{m_\pi^2} \right) \simeq 0.787 \; E_\pi$$

Taking into account μ polarization effects:

$$\langle E_{\pi^+ \rightarrow \mu^+ \rightarrow \overline{\nu}_\mu} \rangle \simeq 0.265 \; E_\pi$$

$$\langle E_{\pi^+ \rightarrow \mu^+ \rightarrow \nu_e} \rangle \simeq 0.257 \; E_\pi$$

Schematically:

$$\mathbf{e} = \pi^+ \; f_{(\pi^+\to\mu^+\to\nu_e)}\, \sigma_{\nu_e}$$
$$+ \; \pi^- \; f_{(\pi^-\to\mu^-\to\overline{\nu}_e)}\, \sigma_{\overline{\nu}_e}$$

$$\mu = [\pi^+ \, f_{(\pi^+\to\nu_\mu)} + \pi^- \; f_{(\pi^-\to\mu^-\to\nu_\mu)}]\, \sigma_{\nu_\mu}$$
$$[\pi^- \, f_{(\pi^-\to\overline{\nu}_\mu)} + \pi^+ \, f_{(\pi^+\to\mu^+\to\overline{\nu}_\mu)}]\, \sigma_{\overline{\nu}_\mu}$$

For $P_{\rm dec}(\mu) \simeq 1$:

$$f_{(\pi\to\nu_\mu)} \simeq f_{(\pi\to\mu\to\nu_\mu)} \simeq f_{(\pi\to\mu\to\nu_e)}$$

"kinematic factors" approximately equal.

Effect of π^+/π^- Ratio

Estimate of the stadard model prediction

$$\left(\frac{\mu}{e}\right)_0 \simeq \frac{(\pi^+ + \pi^-)\,(\sigma_\nu + \sigma_\nu)}{\pi^+\,\sigma_\nu + \pi^-\,\sigma_{\overline{\nu}}}$$

$$\simeq \frac{[1 + (\pi^+/\pi^-)]\,[1 + (\sigma_\nu/\sigma_{\overline{\nu}})]}{1 + (\pi^+/\pi^-)\,(\sigma_\nu/\sigma_\nu)}$$

Approximating $\sigma_\nu/\sigma_{\overline{\nu}} \simeq 2$

$$\Delta\left(\frac{\mu}{e}\right)_0 \simeq -0.3\,\Delta\left(\frac{\pi^+}{\pi^-}\right)$$

Difference between existing MC models
is of order $\Delta(\pi^+/\pi^-) \simeq 0.1$
Corresponding to $\Delta(\mu/e)_0 \simeq 1.5\%$ $0.03/2$
Major source of difference.

(i) Accelerator data
(ii) Atmospheric μ^+/μ^- data

Contribution of Kaons ($\sim 5\%$) of ν rate

- Kinematical suppression of $[K \to \mu \to e\nu_e\nu_\mu)]$

  $\langle E_\mu \rangle_{(K \to \mu\nu_\mu)} = \frac{1}{2}(1 + m_\mu^2/m_K^2)E_K \simeq 0.523\ E_K$
- Direct channels into ν_e $(K \to \pi e \nu_e)$

  "Opposite sign effects"

Different channels:

$$K^\pm \to \mu^+ + \nu_\mu \qquad (63.5\%)$$
$$K^+ \to \pi^0 + \mu^+ + \nu_\mu \qquad (3.2\%)$$
$$K^+ \to \pi^0 + e^+ + \nu_e \qquad (4.8\%)$$

$$K_L \to \pi^\pm + \mu^\mp + \nu_\mu \qquad (27.2\%)$$
$$K_L \to \pi^\pm + e^\mp + \nu_e \qquad (38.8\%)$$

$$\begin{aligned}(e)_{K,0} = {} & K^+\ f_{K^\pm \to \mu \to \nu_e}\ B_{K^\pm \to \mu\nu_e}\ \sigma_\nu \\ & + K^+\ f_{K^\pm \to \pi\mu \to \nu_\mu}\ B_{K^\pm \to \pi\mu\nu_\mu}\ \sigma_{\overline{\nu}} \\ & + K^-\ f_{K^\pm \to \mu \to \nu_e}\ B_{K^\pm \to \mu\nu_e}\ \sigma_{\overline{\nu}} \\ & + K^-\ f_{K^\pm \to \pi\mu \to \nu_\mu}\ B_{K^\pm \to \pi\mu\nu_\mu}\ \sigma_\nu \\ & + K_L\ f_{K_L \to \pi^\circ \mu\nu}\ B_{K_L \to \pi e \nu_e}(\sigma_\nu + \sigma_{\overline{\nu}})\end{aligned}$$

Cancellation effect:

For only the channel $K^{\pm} \to \mu\nu_{\mu} \to e\nu_e\nu_{\mu}\bar{\nu}_{\mu}$:

$$\left(\frac{\mu}{e}\right)_{0,K} \sim 3.1$$

Including the direct channels $K \to \nu_e$:

$$\left(\frac{\mu}{e}\right)_{0,K} \sim 2.3$$

Need to include uncertainties in K/π ratio, and K^+/K^-, $K_L/K^{\pm}$ ratios

Order of magnitude $\Delta(\mu/e)_K \sim 1\%$.
of Kaon correction:

Uncertainty on correction ~ 30%

The uncertainty on the "Denominator" of the double Ratio

$$\frac{\mu/e}{\mu_0/e_0}$$

for SK-subGeV is currently estimated:

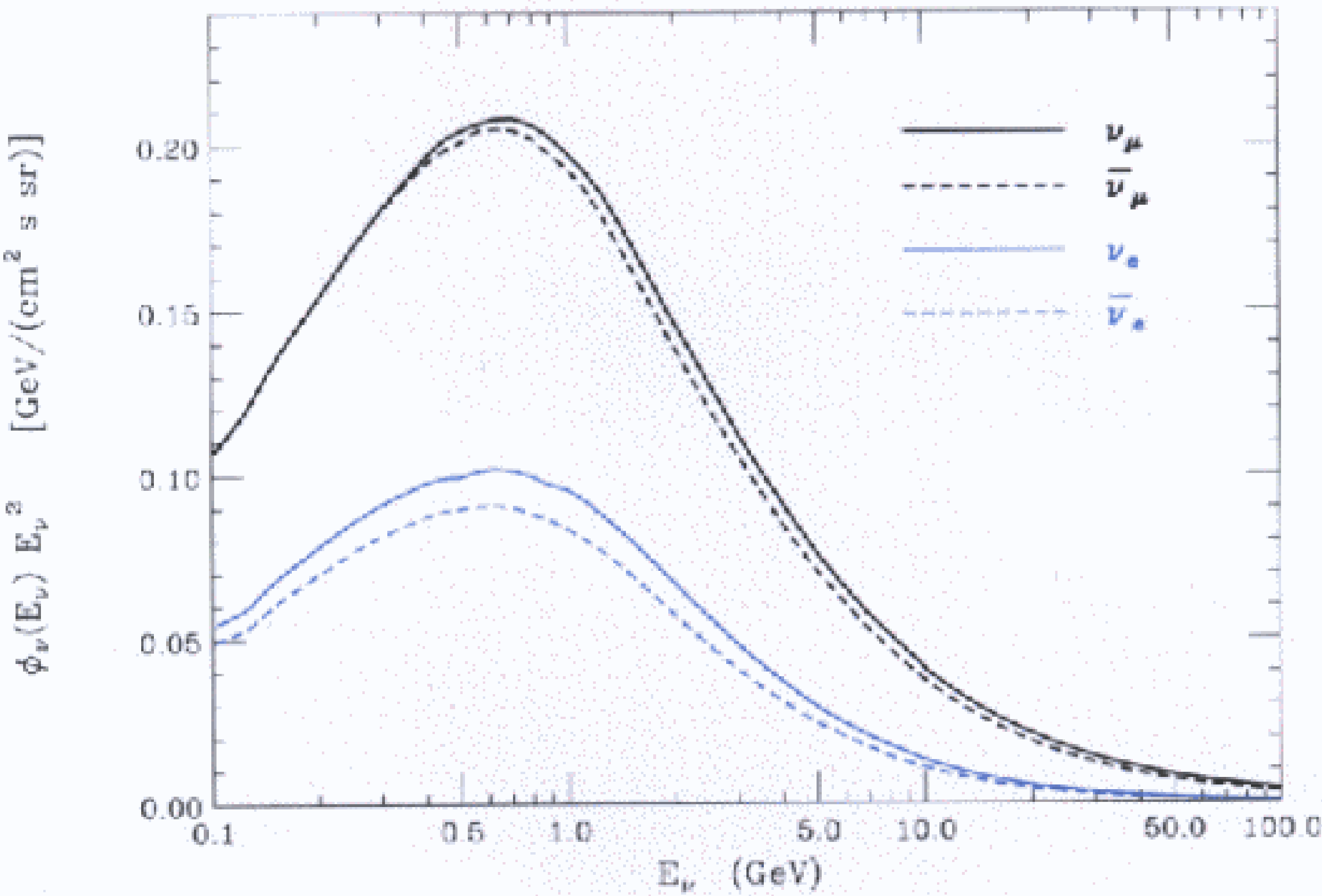
$$\Delta\left(\frac{\mu_0}{e_0}\right) = \text{"Theory"} \oplus \text{"Experiment"}$$

$$= 5\% \oplus 6\% = 8\%$$

Can become significantly smaller.

Prospects for reducing error good. (K2K near detector)

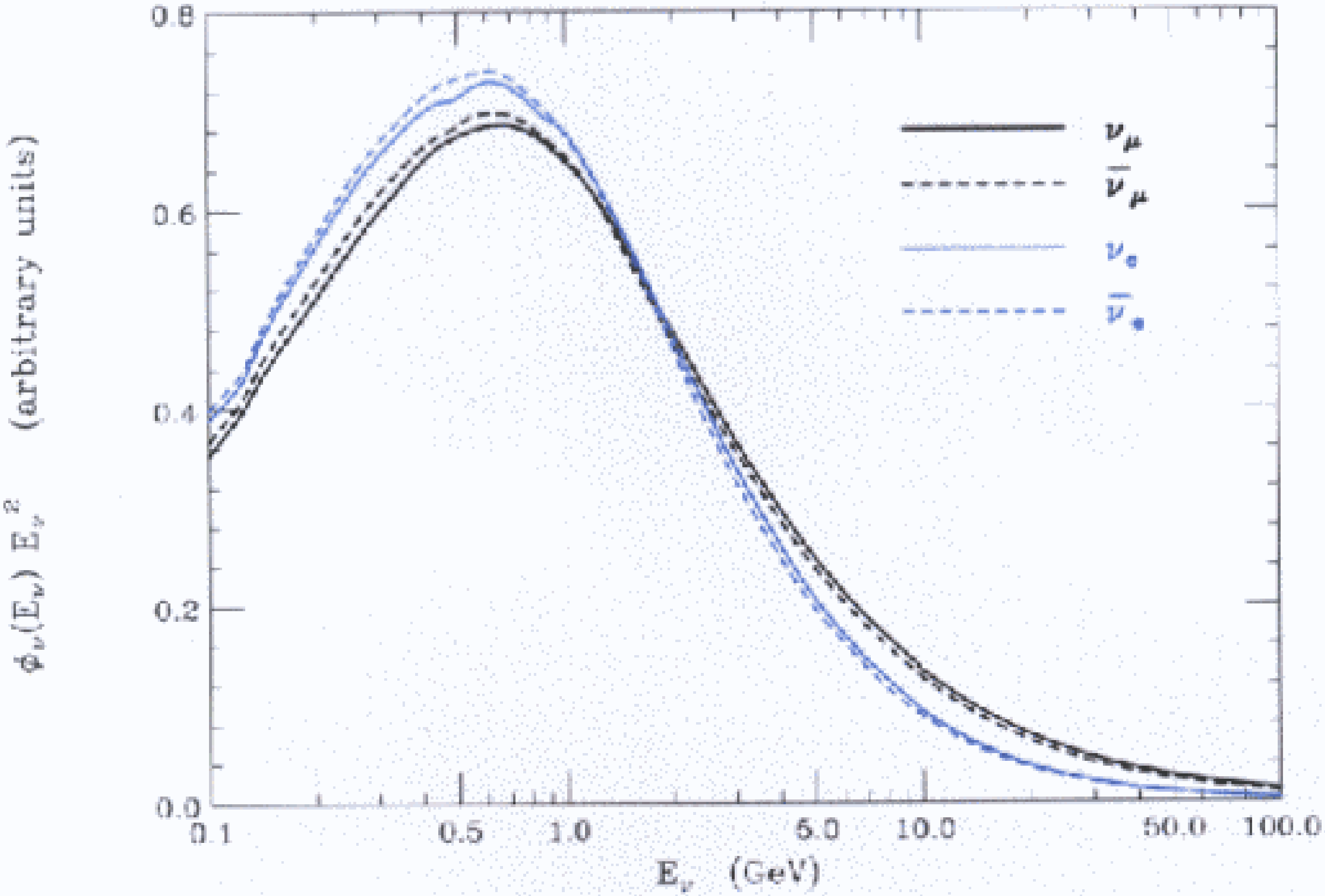
ENERGY SPECTRA



$\sigma_\nu(E_\nu) \propto E_\nu$

$$\phi_\nu(E_\nu)\ E_\nu^2 \simeq \frac{dN_{\rm events}}{d \log E_\nu}$$

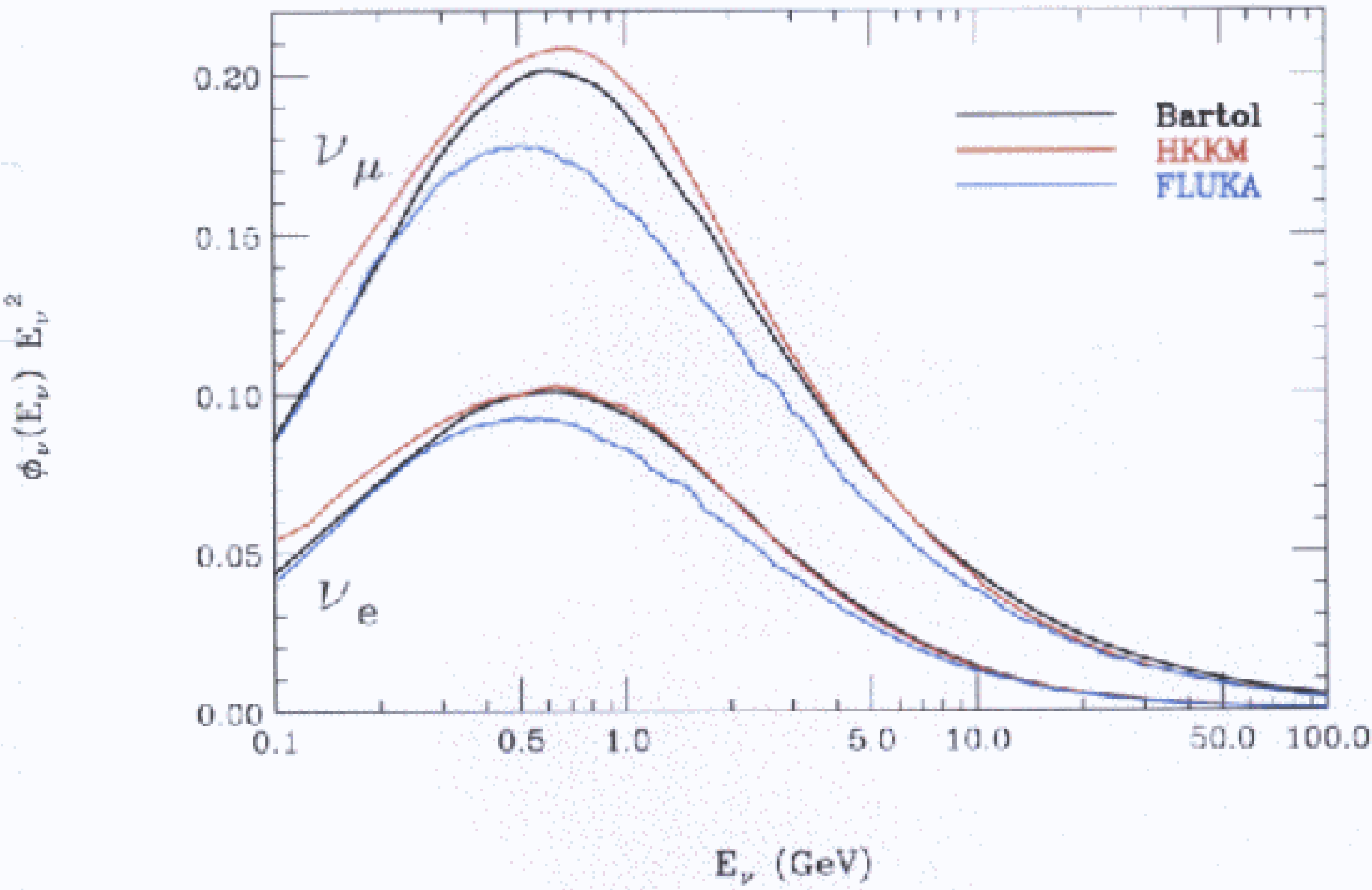
Shape of the spectra



Shapes of ν_μ ($\overline{\nu}_\mu$) and ν_e ($\overline{\nu}_e$) are different
Main effect is a softer ν_e ($\overline{\nu}_e$)
because of μ decay suppression at high energy.

$\overline{\nu}_e$ softer than ν_e
$\overline{\nu}_\mu$ softer than ν_μ
Reflections of π^- softer than π^+.

MODEL dependence of ENERGY spectrum



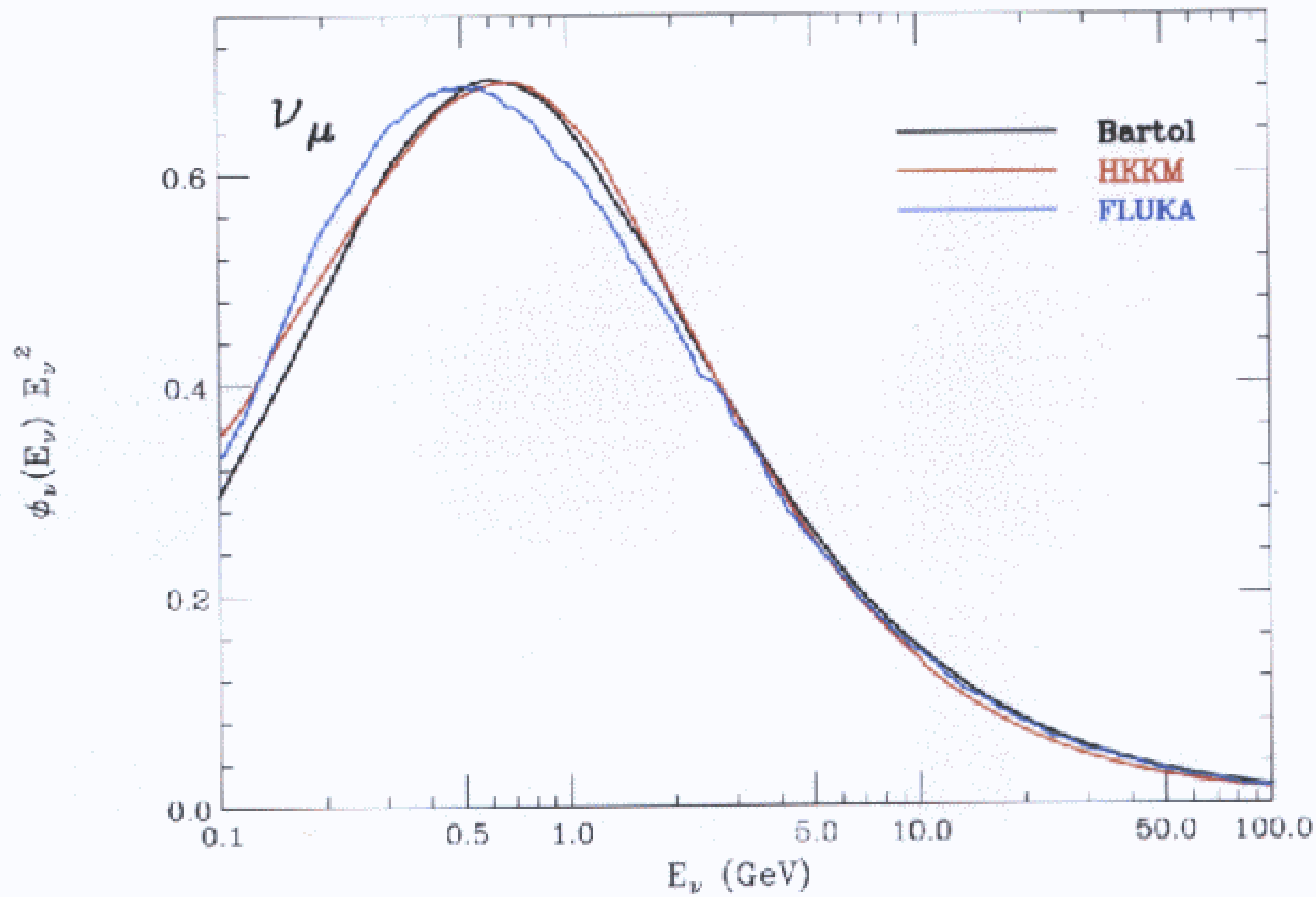
Note strong CORRELATION

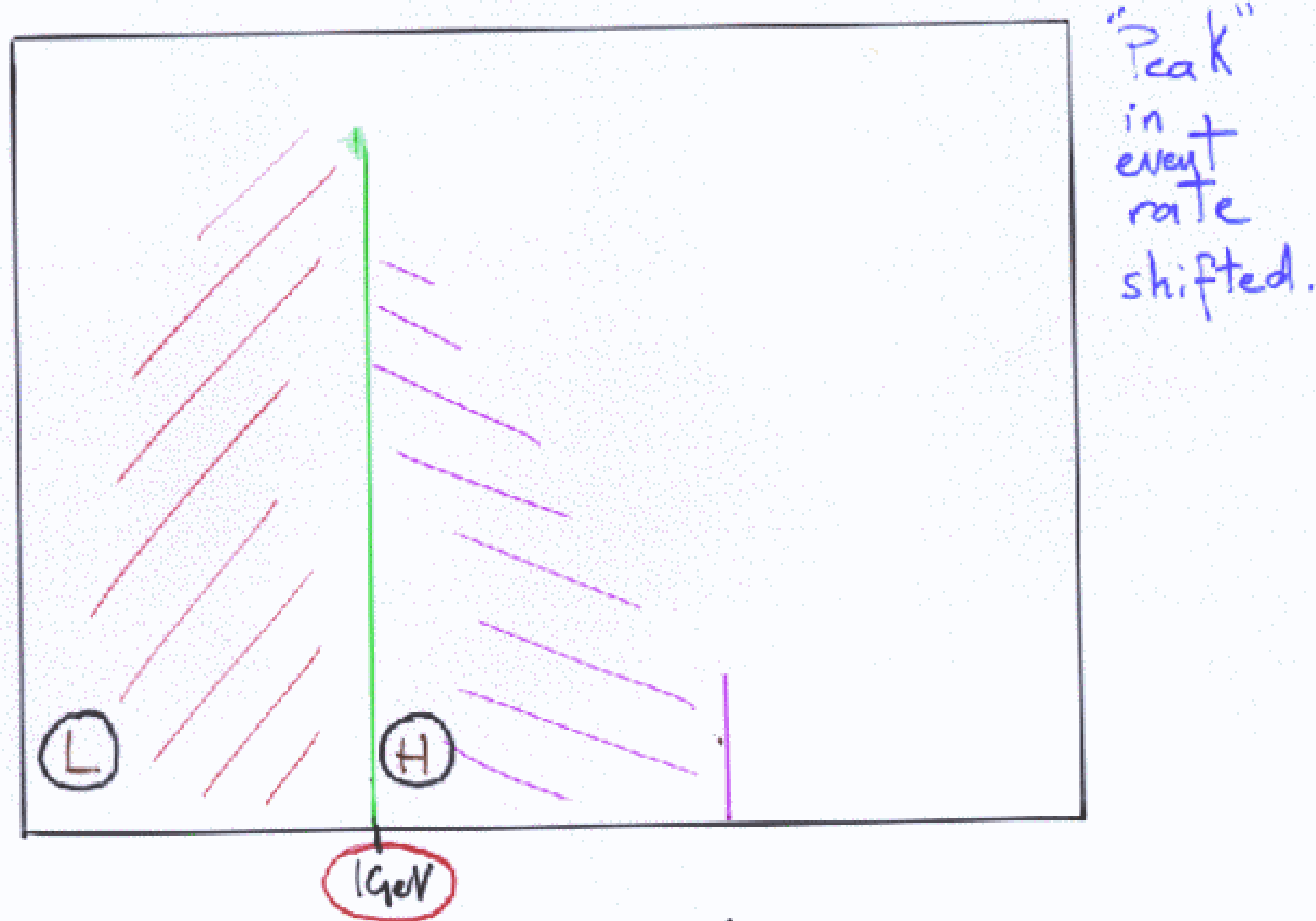
between ν_μ and ν_e spectra

— Normalization

— Shape

Shape of Energy Spectrum (Angle averaged)





10% difference in ratios of event Rates.

ZENITH ANGLE DISTRIBUTIONS

$$\mathrm{UP} = \int_{-1}^{-\frac{1}{3}} d\cos\theta_z \; \phi_\nu(\cos\theta_z)$$

$$\mathrm{HORI} = \int_{-\frac{1}{3}}^{+\frac{1}{3}} d\cos\theta_z \; \phi_\nu(\cos\theta_z)$$

$$\mathrm{DOWN} = \int_{-\frac{1}{3}}^{1} d\cos\theta_z \; \phi_\nu(\cos\theta_z)$$

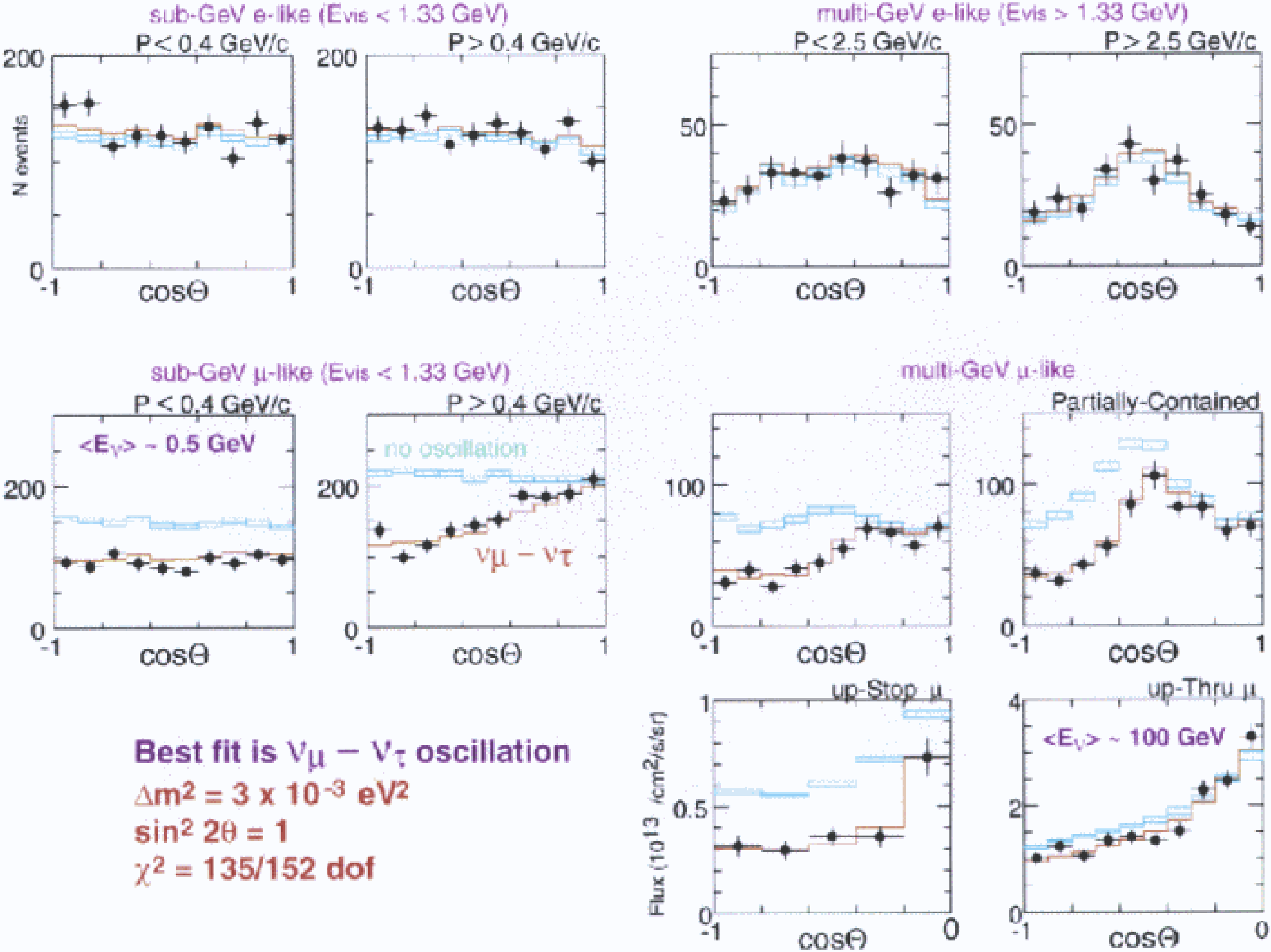
Up/Down Ratio

$$\frac{\mathrm{UP}}{\mathrm{DOWN}}$$

Shape of the distribution

$$\frac{\mathrm{HORIZONTAL}}{\mathrm{VERTICAL}}$$

Fit to Super-K Atmospheric Neutrinos over 3 Decades in Energy



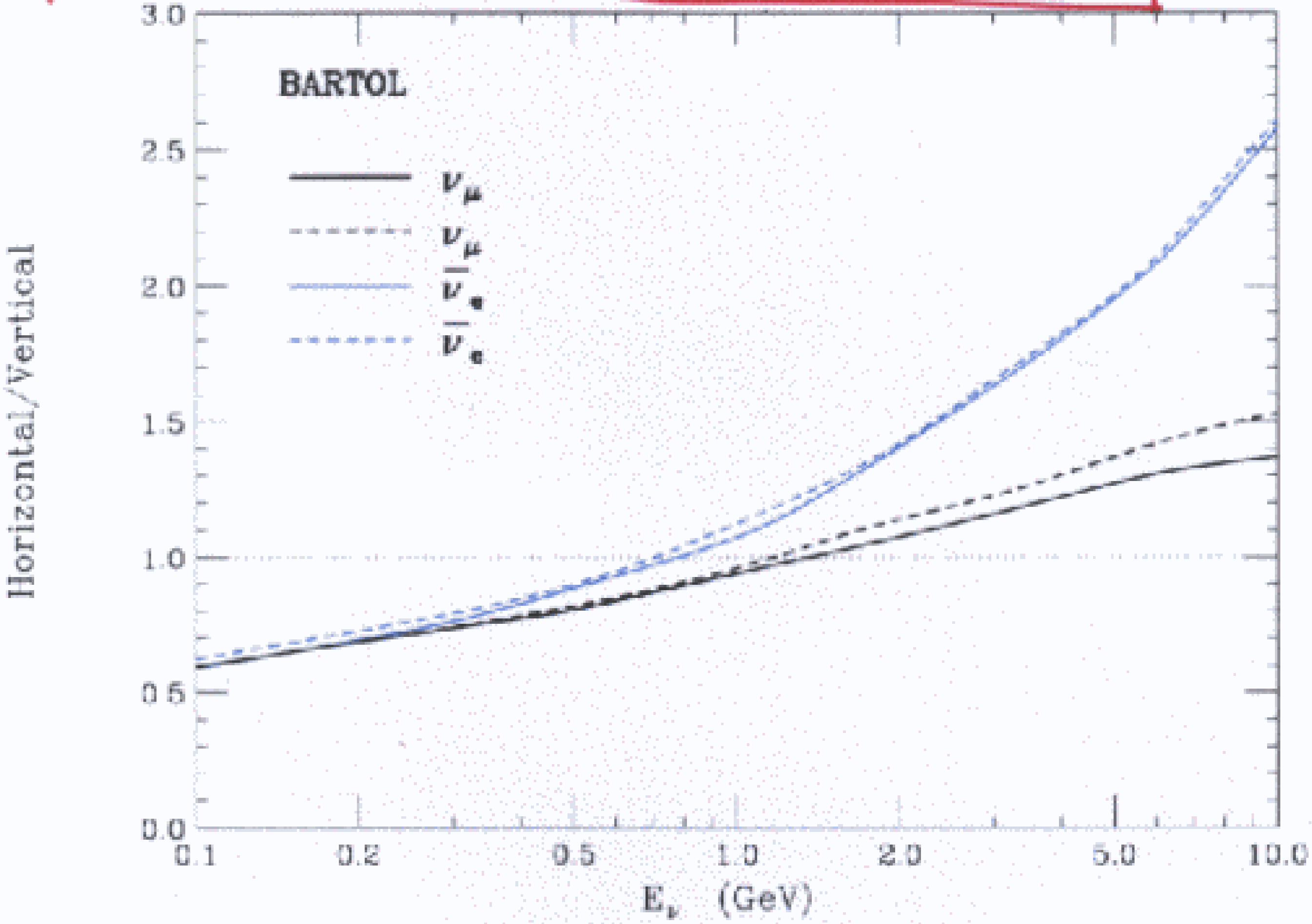
Best fit is $\nu_\mu - \nu_\tau$ oscillation

$\Delta m^2 = 3 \times 10^{-3}$ eV^2

$\sin^2 2\theta = 1$

$\chi^2 = 135/152$ dof

SHAPE of the θ_{zenith} distributions

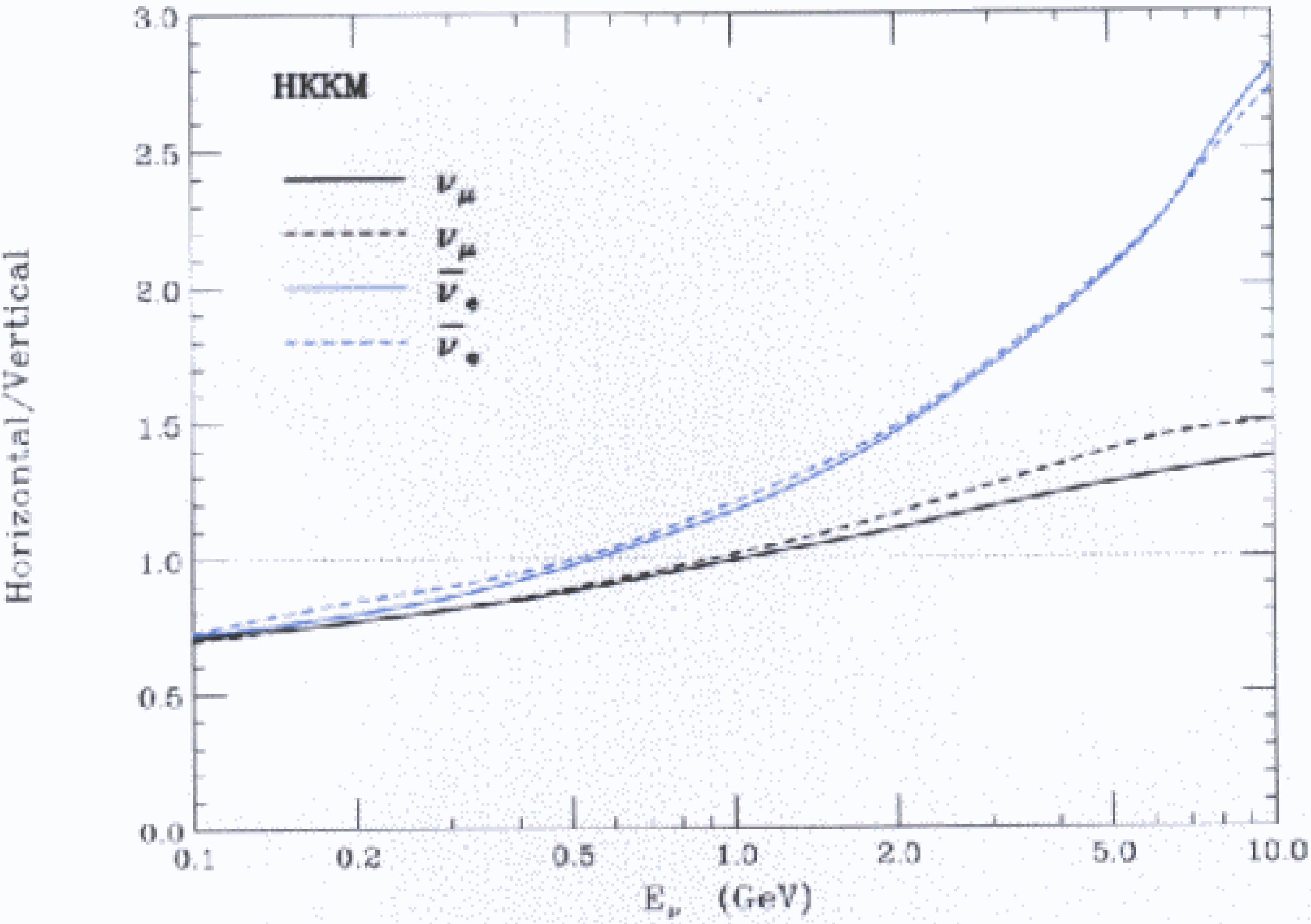


1–D calculation

Horizontal/Vertical ratio: controled by μ–decay:

$$\ell_\mu = \frac{c\tau_\mu p_\mu}{m_\mu} \simeq 6.27 \, \frac{p_\mu}{\text{GeV}} \text{ Km}$$

- Energy dependence
- Effect larger for ν_e, $\overline{\nu}_e$ ($\sim 100\% \; \mu \to \nu$)

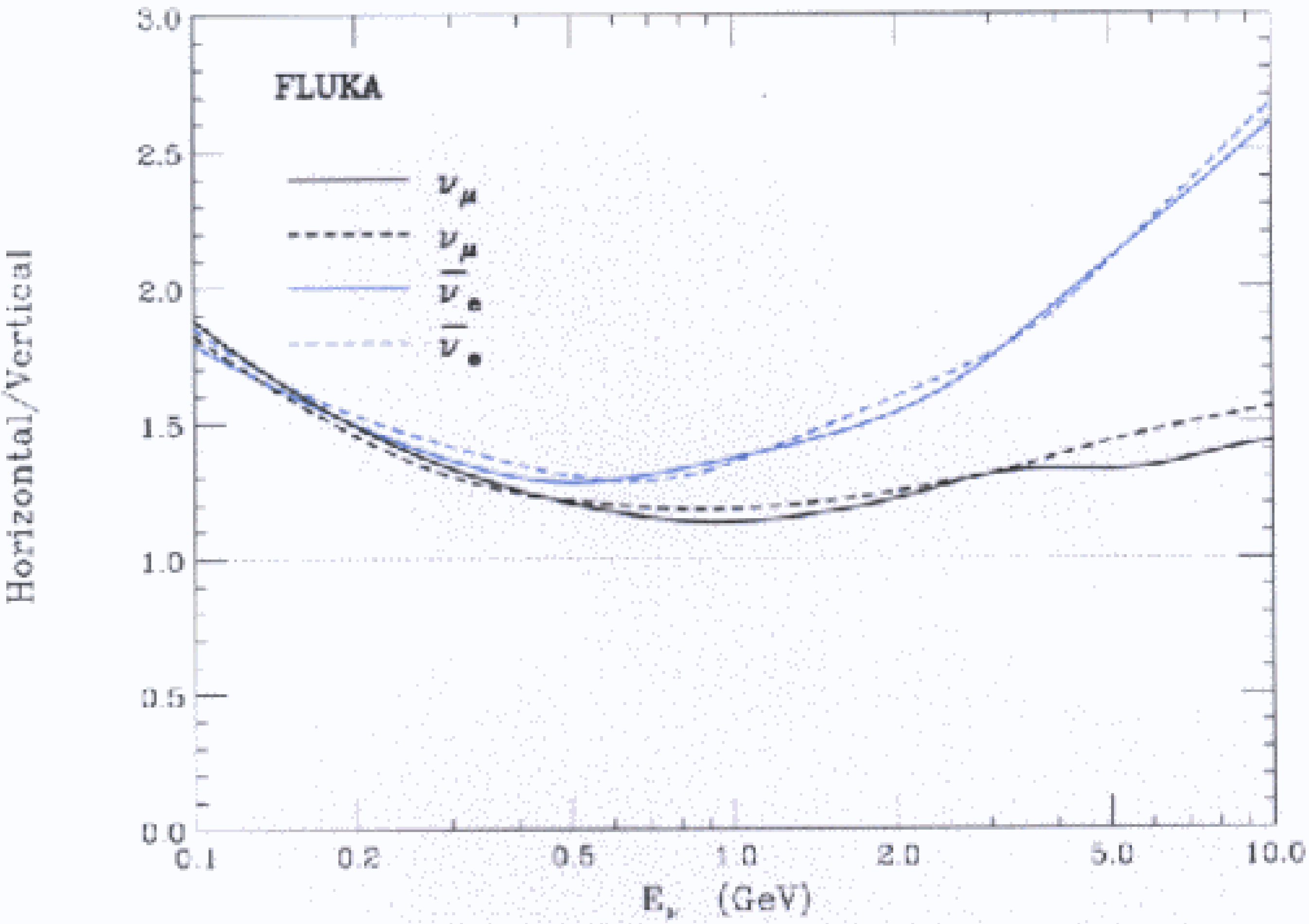


Small difference between ν_μ and $\overline{\nu}_\mu$

$$\nu_\nu \ : \pi^+ \rightarrow \nu_\mu \qquad \pi^- \rightarrow \mu^- \rightarrow \nu_\mu$$

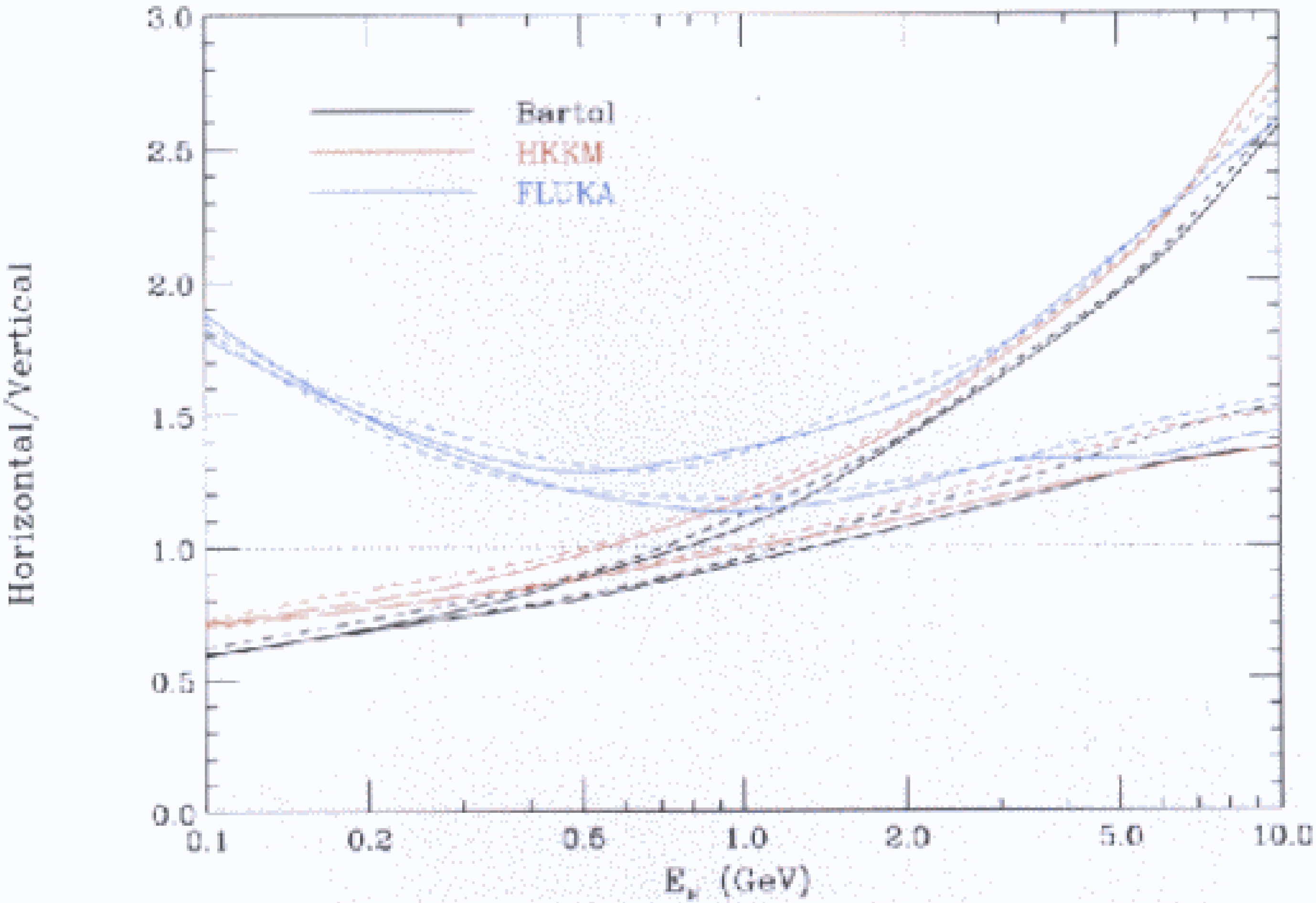
$$\overline{\nu}_\nu \ : \pi^- \rightarrow \overline{\nu}_\mu \qquad \pi^+ \rightarrow \mu^+ \rightarrow \overline{\nu}_\mu$$

Spectrum of π^+ harder π^-.



“3–Dimensional effect

enhancement of Horizontal plane


Bartol
HKKM
FLUKA
Horizontal/Vertical
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.1
0.2
0.5
1.0
2.0
5.0
10.0
E_ν (GeV)

3–DIMENSIONAL EFFECTS

"First generation" One–Dimensional:
ν's collinear with primary,

Neglect:

- Tranverse momentum $p_\perp$
- Multiple scattering
- Bending in geomagnetic field $\vec{B}$

WHY this approximation ??

CPU time ! More efficient MC calculation

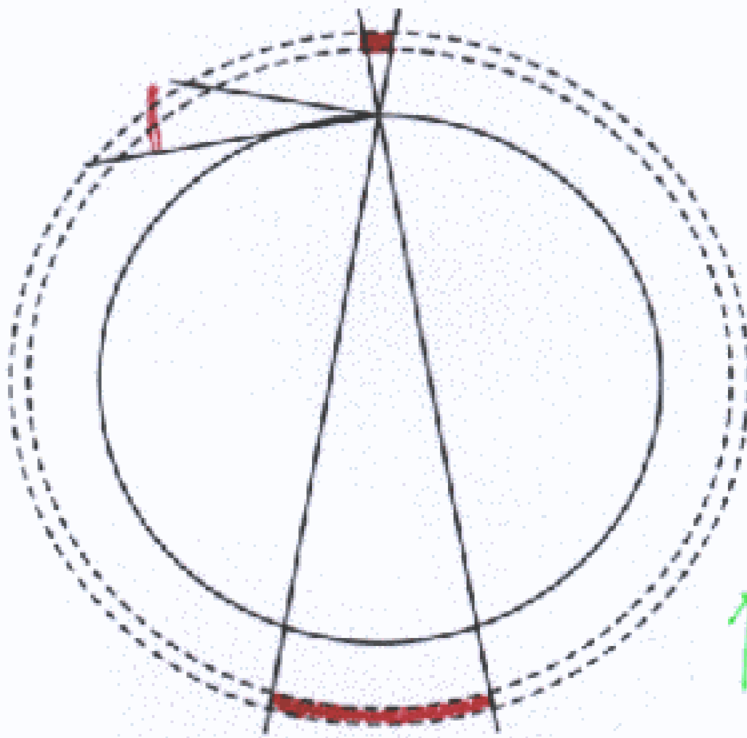
New 3–Dimensional calculations:
New effect seen:

GEOMETRIC ENHANCEMENT
On the HORIZONTAL PLANE

"Solid Angle" argument.

Geometry of Neutrino Production

Source Volume of atmospheric neutrinos



$\downarrow \phi_{down}$

$\uparrow \phi_{up}$

$$V_{\nu\ \mathrm{source}}(\theta + \delta\theta) \propto \frac{\ell^2(\theta)}{\cos\theta_{\mathrm{emission}}}$$

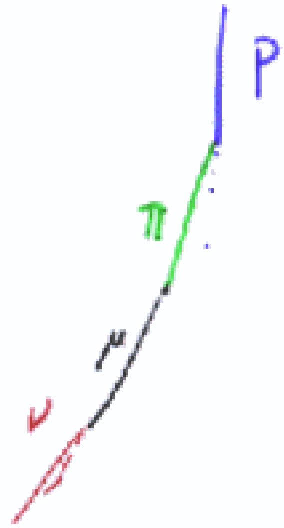
"Gauss theorem" implies Up-Down symmetry.
but **NOT** isotropy.

$$\phi_{up} = \frac{\mathrm{Source}(\uparrow)}{\ell^2(\uparrow)}, \qquad \phi_{down} = \frac{\mathrm{Source}(\downarrow)}{\ell^2(\downarrow)}$$

Sources of Neutrino–Primary angle

$$\theta_{\nu p} = \theta_\pi \oplus \theta_{\pi\nu}$$

$$= \theta_\pi \oplus \theta_{\pi\mu} \oplus \theta_{\mu B} \oplus \theta_{\mu\nu}$$



$$\theta_\pi \sim \frac{p_{\perp\pi}}{p_\pi} \sim \frac{350\ \text{MeV}}{4 p_\nu} \sim \boxed{\frac{5^\circ}{p_\nu(\text{GeV})}} \sim \frac{1}{P}$$

$$\theta_{\pi\nu} \sim \frac{1.5^\circ}{p_\nu}$$

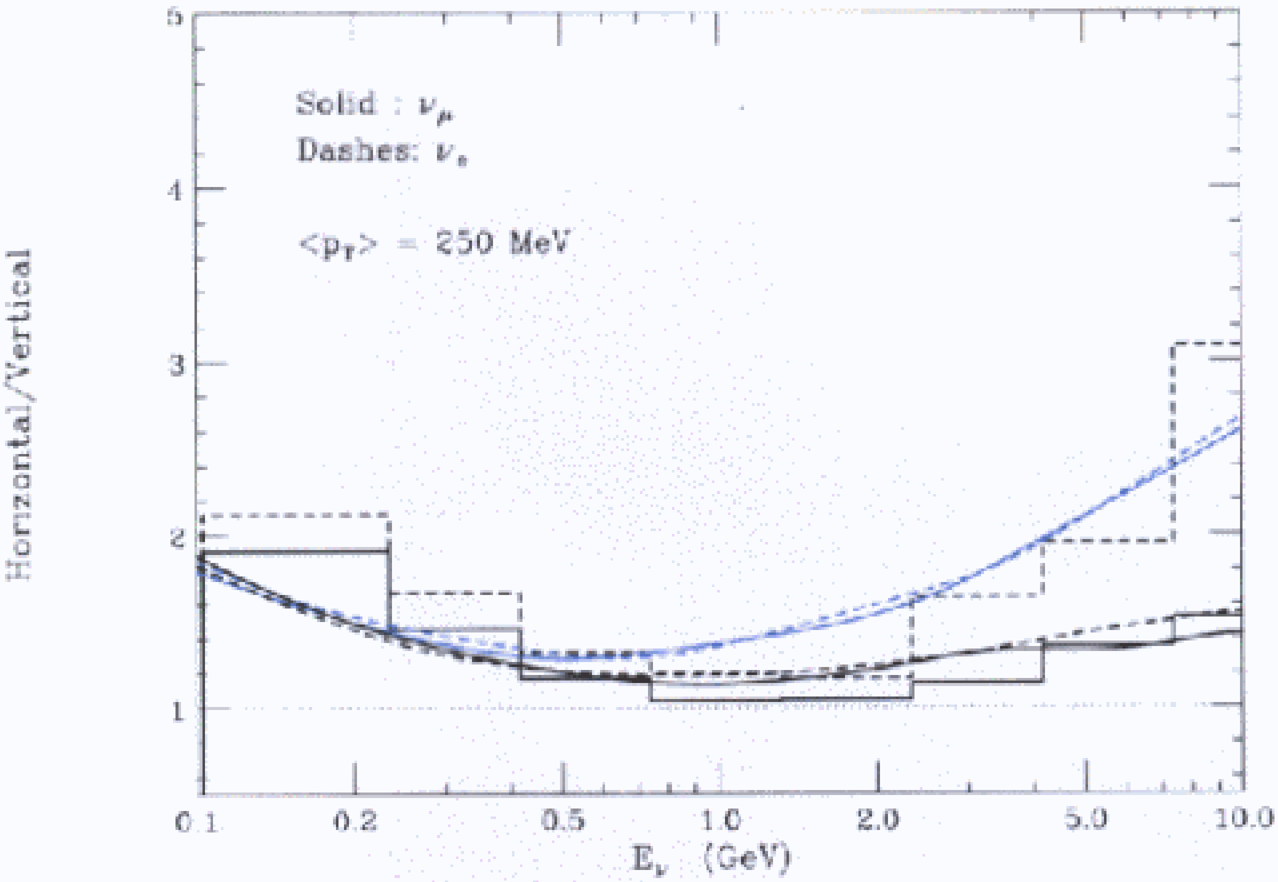
$$\theta_{\pi\mu} \sim \frac{0.5^\circ}{p_\nu}, \quad \theta_{\mu\nu} \sim \frac{2^\circ}{p_\nu}$$

$$\theta_{\mu B} = \frac{L_\mu}{R_{\text{curv.}}(\mu)} = \frac{c\,\tau_\mu\, p_\mu / m_\mu}{c\, p_\perp / B} \sim \boxed{10.7^\circ\ B(\text{Gauss})}$$

Momentum independent

Higher p : Higher rigidity
Longer path.

P.L.
Astrop. Phys. 14 (2000)
hep-ph/0002282



Inclusion of bending in the geomagnetic field.

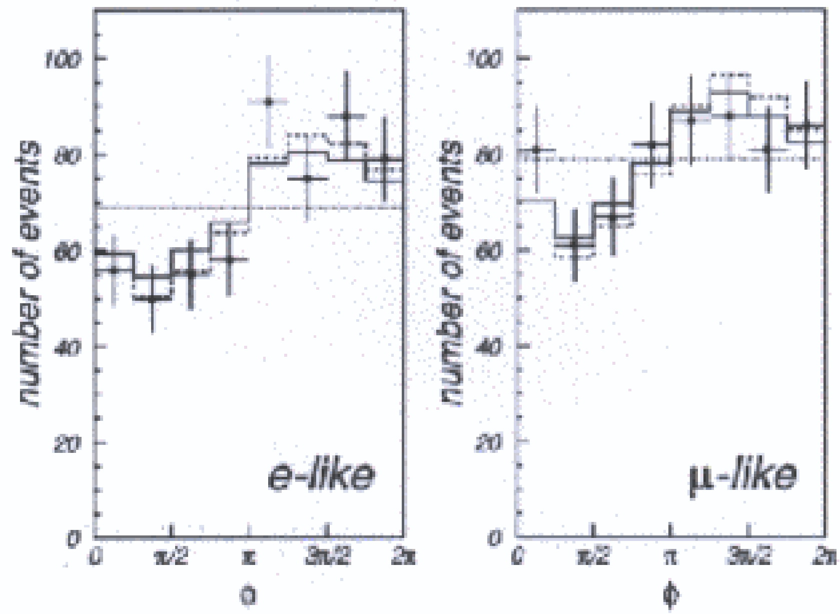
$\mu^\pm$

- Larger enhancement of horizontal plane
- Different enhancements for μ-like (~50% μ dec) and e-like (~100% from μ decay) events

RESULTS need confirmation.

[Test East-West effect for ν]

The EAST–WEST effect in Super–Kamiokande



$p_{e,\mu}$ = 0.4–3 GeV; $\cos\Theta_z = [-0.5, -0.5]$

Hint of a discrepancy with the data.

$$A_{EW} = \frac{E - W}{E + W}$$

$$A_e^{\rm SK} = 0.21 \pm 0.04, \qquad A_\mu^{\rm SK} = 0.08 \pm 0.04$$

$$A_e^{\rm HKKM} = 0.13 \pm 0.04, \qquad A_\mu^{\rm HKKM} = 0.11 \pm 0.04$$

$$A_e^{\rm Bartol} = 0.17 \pm 0.04, \qquad A_\mu^{\rm Bartol} = 0.15 \pm 0.04$$

Enhancement of East–West effect for neutrinos from
$\mu^+ \to \nu \qquad (\nu_e \overline{\nu}_\mu)$

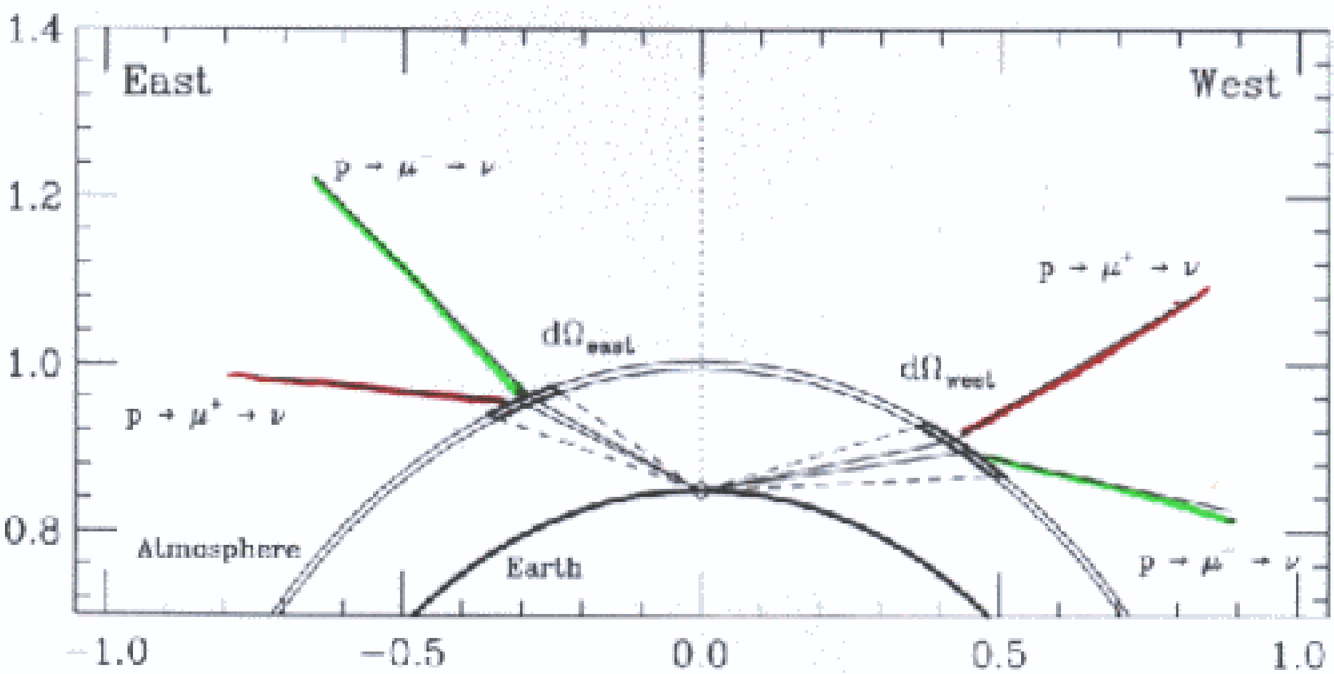
Suppression of East–West effect for neutrinos from
$\mu^- \to \nu \qquad (\overline{\nu}_e \nu_\mu)$

Since:

$\phi(\nu_\mu)/\phi(\overline{\nu}_\mu) \simeq 1;$
$\phi(\nu_e)/\phi(\overline{\nu}_e) \simeq \pi^+/\pi^- \simeq 1.2;$

Enhancement for e–like events

Suppression for μ–like



the Neutrino

Cross

Section

Important source of uncertainty.

K2K near detectors

LBL projects

CONCLUSIONS

EXISTENCE
of
NEW PHYSICS
is ROBUST

1. Oscillations are an excellent Fit.
2. Significant discrimination against alternative mechanisms. Some model dependence (analysis of up–going muons).
3. Some model dependence in the determination of Δm^2 (expect small effect).
4. "Second generation calculations" are in progress. Time scale of few months.

   Expect small differences for important quantities

   Systematic errors will be smaller.