On Triangles

Nicola Cabibbo

Lepton mixing and quark mixing

Two different kinds of firmly established neutrino oscillations seem to confirm Brono Pontecorvo's hypothesis of a lepton mixing fully analogous to quark mixing.

Pushing the analogy one would then expect the lepton mixing matrix **U** to have complex matrix elements which would then lead to CP and T breaking in neutrino oscillations, while one would not expect a violation of CPT,

$$P(\nu_a \Rightarrow \nu_b) \neq P(\bar{\nu}_a \Rightarrow \bar{\nu}_b) \qquad CP \text{ violation}$$

$$P(\nu_a \Rightarrow \nu_b) \neq P(\nu_b \Rightarrow \nu_a) \qquad T \text{ violation}$$

$$P(\nu_a \Rightarrow \nu_b) \neq P(\bar{\nu}_b \Rightarrow \bar{\nu}_a) \qquad CPT \text{ violation}$$

We have discussed strategies for progressing in the study of neutrino oscillations and lepton mixing. The question I will face in this talk is a different one: how healthy is quark mixing itself?

Summary

- Quark Mixing.
- The V_{us} problem: Hyperon vs. Kl3.
- The Unitarity Triangle, Present and Future

Quark Mixing - the CKM matrix

The quark mixing matrix **V** can be expressed in terms of four parameters:

$$\mathbf{V} = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix} = \begin{vmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{vmatrix} + O(\lambda^4)$$

CP violation arises from the presence of phase factors in some of the V's, i.e. from a non–vanishing value of η.

Unitarity of the CKM matrix implies relations such as

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad \cdots \text{value of } V_{us}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \qquad \cdots \text{the Unitarity Triangle.}$$

Each of these relations corresponds to areas that have seen substantial progress, and more is expected in the next few years

V_{us} and Unitarity.

There has been a long standing discrepancy between the requirement of unitarity and the experimental value uf V_{us},

$$\text{from CKM Unitarity and } |V_{ud}| \quad \rightarrow \quad |V_{us}| = 0.229 \pm 0.0034$$

$$\text{PDG value, from } K_{l3} \quad \rightarrow \quad |V_{us}| = 0.2196 \pm 0.0026$$

Why not use hyperon data? The vector parts of hyperon beta decay — the f_1 form factor — and the f_+ form factor for K_{l3} decays are protected by the Ademollo-Gatto theorem from large corrections due to SU(3) symmetry breaking. They both are suitable for a precise determination of V_{us}.

In 2001 with R. Winston and E. Swallow we decided to revisit hyperon decays and were pleasantly surprised: the bad reputation of Hyperon beta decays, of suffering large SU(3) breaking effects, turned out to be unfounded.

NC, E. Swallow , R. Winston PRL **92** (2004)

Determination of V_{us} from hyperon decays

The principle: for each decay we have (apart from smaller and well known corrections)

$$\Gamma = (\text{Kin. Factors})\,[V_{us} f_1(0)]^2 \left(1 + \frac{g_1^2}{f_1^2}\right)$$

The Axial/Vector ratio g_1/f_1 can be measured directly, so each decay separately yields a determination of V_{us}. If we neglect flavor-SU(3) breaking (confiding in the Ademollo-Gatto theorem) we see a very consistent picture. The result agrees well with the unitarity requirement.

Decay	g_1/f_1	V_{us}
$\Lambda \to p e^- \overline{\nu}$	$0.718(15)$	0.2224 ± 0.0034
$\Sigma^- \to n e^- \overline{\nu}$	$-0.340(17)$	0.2282 ± 0.0049
$\Xi^- \to \Lambda e^- \overline{\nu}$	$0.25(5)$	0.2367 ± 0.0099
$\Xi^0 \to \Sigma^+ e^- \overline{\nu}$	$1.32^{+.22}_{-.18}$	0.209 ± 0.027
Combined	—	0.2250 ± 0.0027

SU(3) breaking in Hyperon decays

First order SU(3) symmetry breaking effects are expected to manifest themselves in g_1/f_1. The recently measured decay $\Xi^0 \to \Sigma^+ e^- \bar{\nu}$ provides a direct test because it is predicted to have the same form factor ratio as the well-measured neutron beta decay: $n \to p\, e^- \bar{\nu}$. The KTeV results are consistent with this prediction, but the errors are currently rather large.

One can fit the data of the 5 semileptonic decays for the linear combinations $F+D$ and $F-D$ which will then have essentially uncorrelated errors. This fit yields

$$F + D = 1.2670 \pm 0.0035; \quad F - D = -0.341 \pm 0.016; \quad \chi^2 = 2.96/3 d.f.$$

Surprisingly, even with today's improved measurements, no clear evidence of SU(3) symmetry breaking effects emerges. They appear to be much smaller than expected!

There have been many attempts to compute SU(3) breaking effects for the vector form factor $f_1(0)$, with results ranging from positive to negative corrections. I expect the final word to come from Lattice QCD, and that the corrections will be small.

On the experimental side, a new Hyperon Run in NA48, possibly in 2006, would provide a significant improvement on $\Xi^0 \to \Sigma^+ e^- \bar{\nu}$ and $\Lambda^0 \to P\, e^- \bar{\nu}$.

V_{us} from K_{l3} decays.

from CKM Unitarity and $|V_{ud}|$ $\rightarrow$ $|V_{us}| = 0.229 \pm 0.0034$

from K_{l3} (PDG) $\rightarrow$ $|V_{us}| = 0.2196 \pm 0.0026$

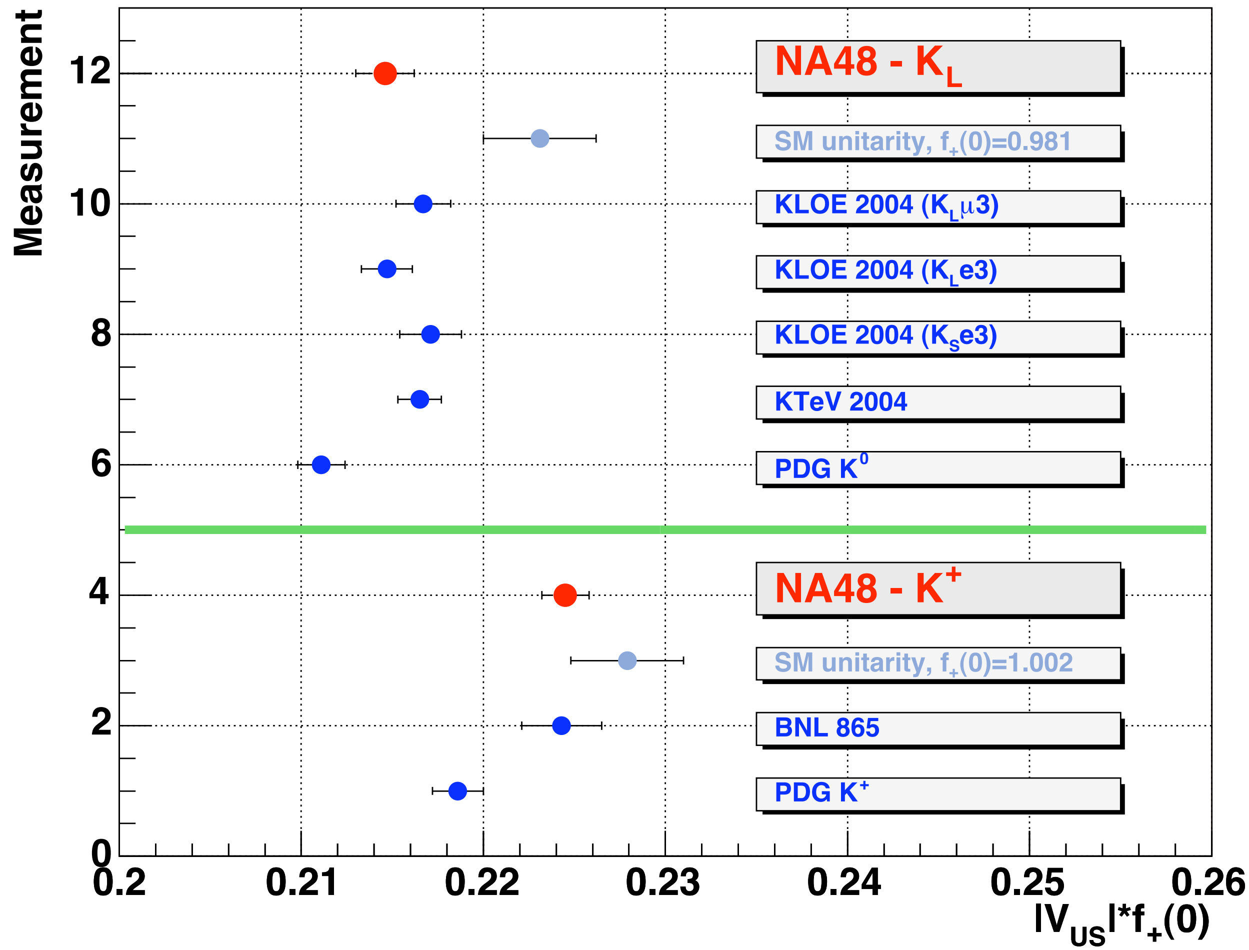
In 2003 the E865 collaboration in Brookhaven published a new result for $K^+ \rightarrow \pi^0 e^+ \nu$, leading to

$$|V_{us}| = 0.2272(\pm 0.0022_{rate} \pm 0.0007_{\lambda_+} \pm 0.0018_{f_+(0)})$$

The discrepancy with the unitarity relation started crumbling, with new results from KTeV, KLOE and NA48 all showing higher branching fractions than listed on PDG, in better agreement with the unitarity requirement.

Discrepancies remain, especially between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ results. I suspect that these reveal a theoretical problem in the evaluation of SU(3) breaking corrections more than an experimental problem, and that a solution is forthcoming.

$K^+ \rightarrow \pi^+ \nu\bar{\nu}$ vs. $K_L \rightarrow \pi^0 \nu\bar{\nu}$ decays.



Lattice QCD and SU(3) breaking corrections.

A first attempt to evaluate SU(3) breaking corrections in lattice QCD has shown that the method is very promising and capable, at least in the K_{l3} case, of attaining the needed precision for V_{us} work. The paper uses the “quenched approximation” (no quark loops), cleverly mixed with Chiral Perturbation Theory. An improved computation without the quenched approximation is under way.

Lattice QCD is becoming a very mature tool.

SU(3) breaking corrections to Hyperon vector transitions will be done next.

Becirevic et al hep-ph/0403217

Conclusion

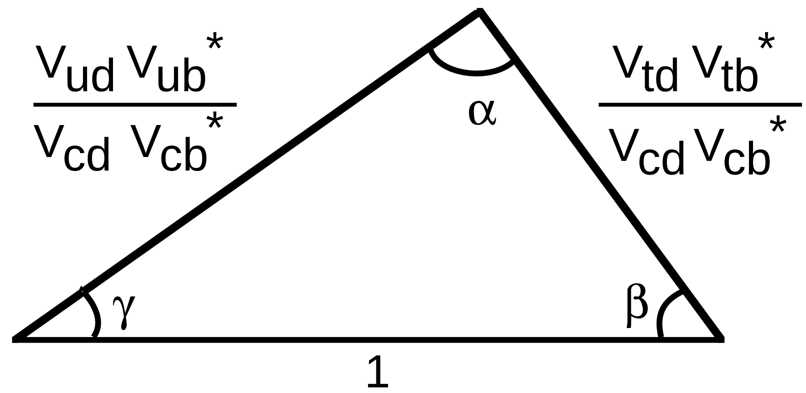
The unitarity puzzle in the $V_{us} \div V_{ud}$ sector is on the way out.

The Unitarity Triangle

Unitarity implies the ortogonality of two rows or columns of the matrix,

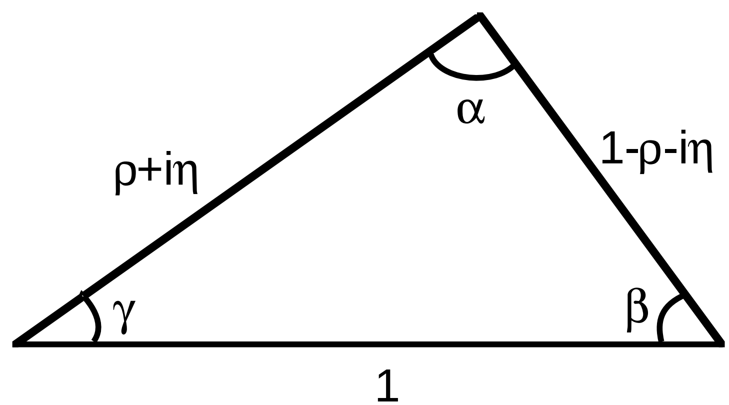
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

This can be represented as a triangle in the complex plane:



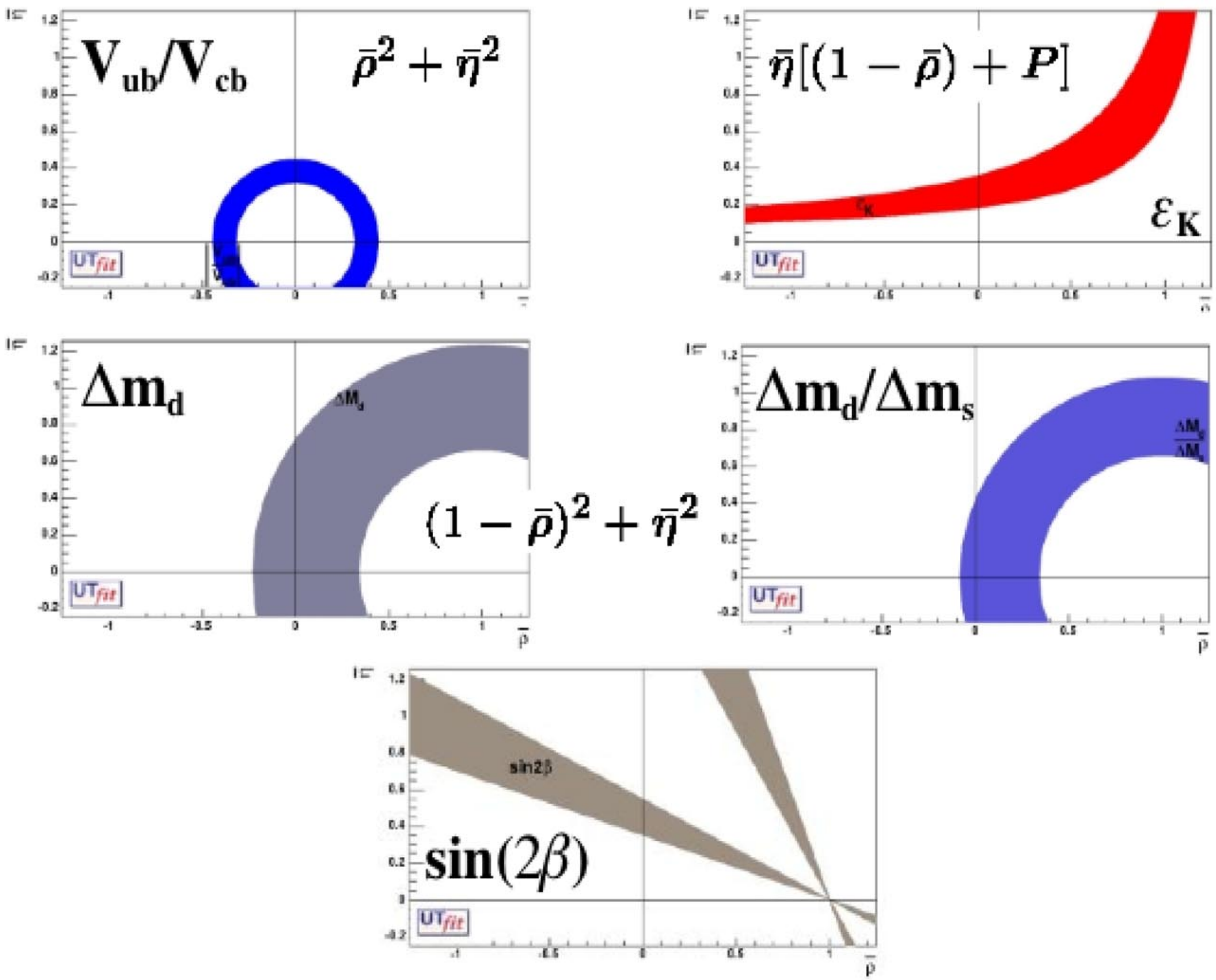
β is the phase of V_{td},
γ is the phase of V_{ub}^*

This relation is trivially satified in the ρ/η parametrization, where it reduces to:



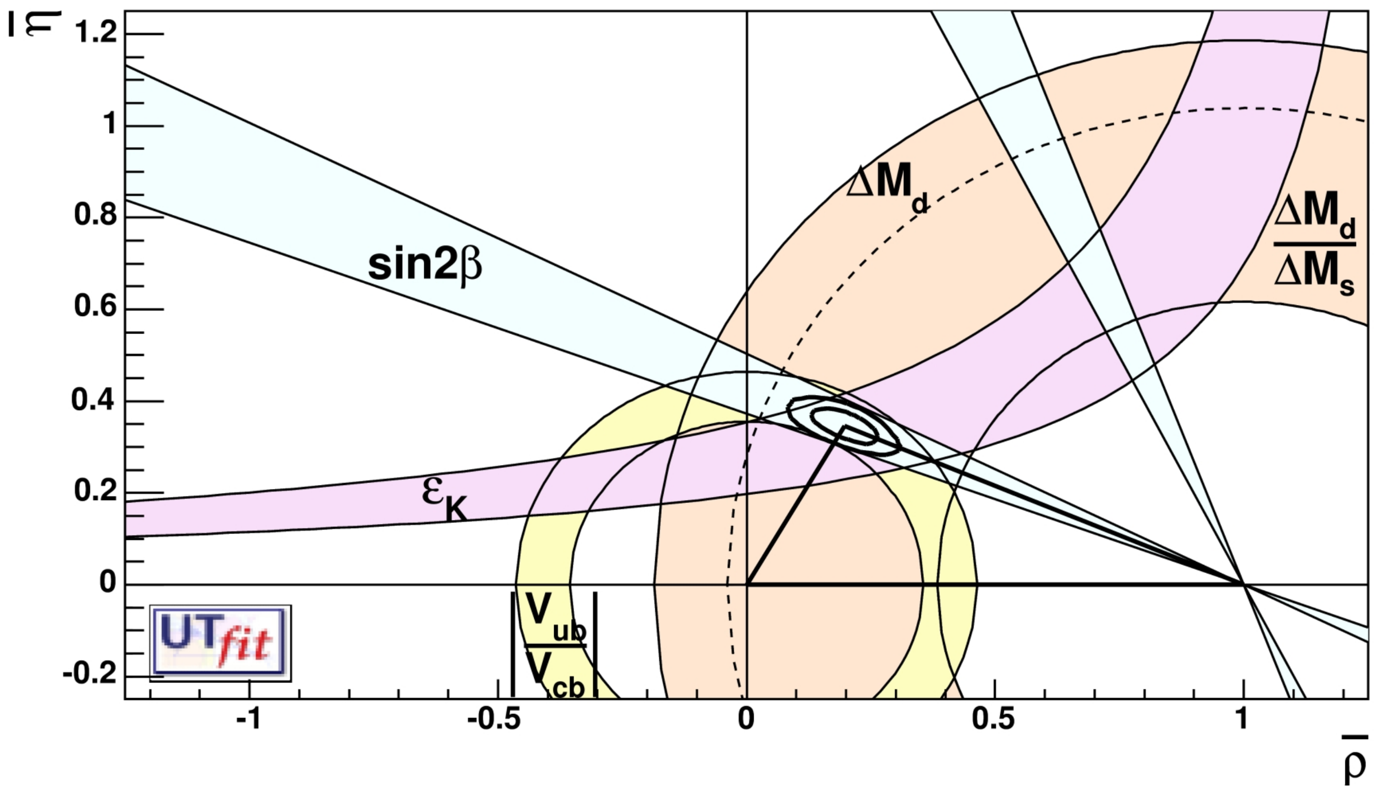
The area of the triangle, $\eta/2$, is a measure of CP violation.

The Magnificent 5:
Determinations of the UT



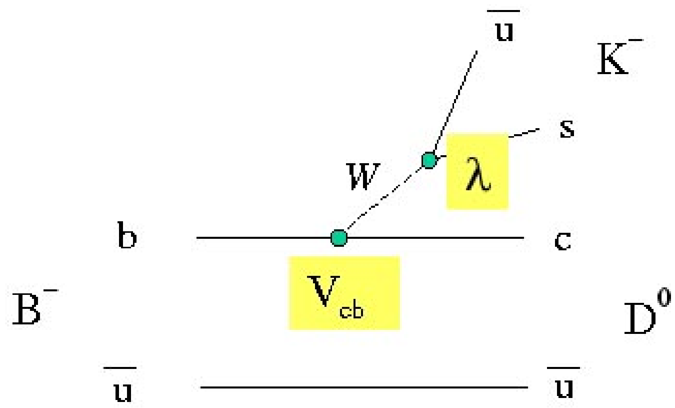
http://www.utfit.org/

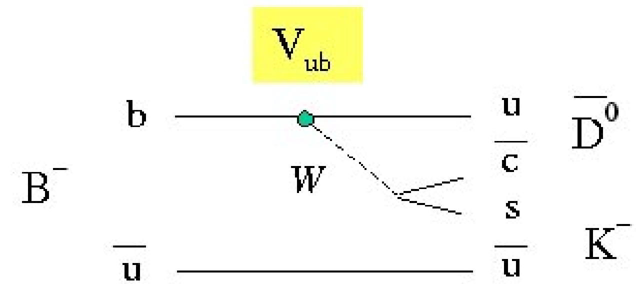
Putting all Together



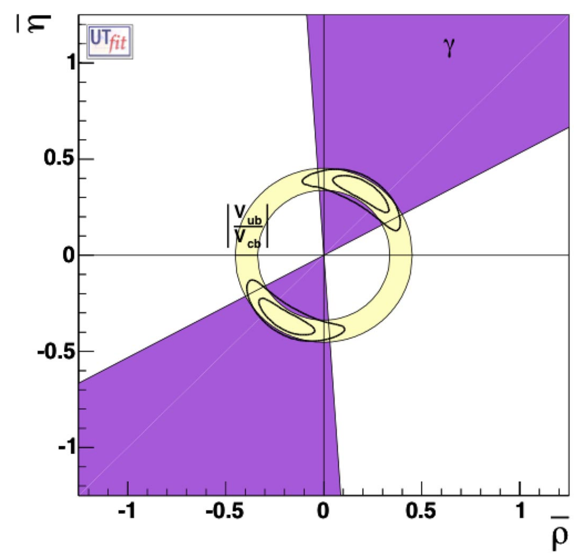
New Babar/Belle contributions

Not yet as accurate as the now classic $B^0 \to J/\Psi + K_S$, but in general agreement.
An example: Determination of γ from $B^\pm \to D(D^*) + K(K^*)$

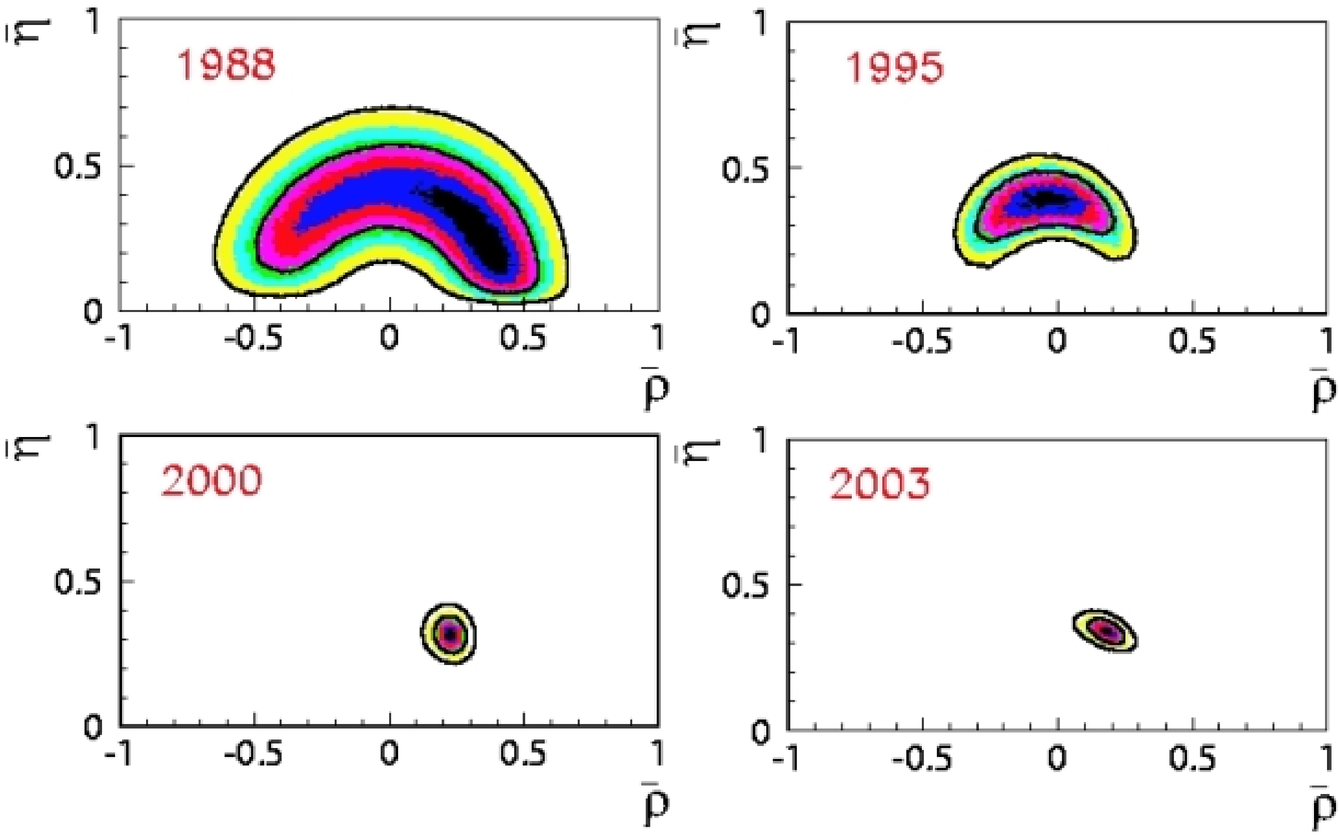




The interference of the two graphs is sensitive to γ, the phase of V_{ub}.

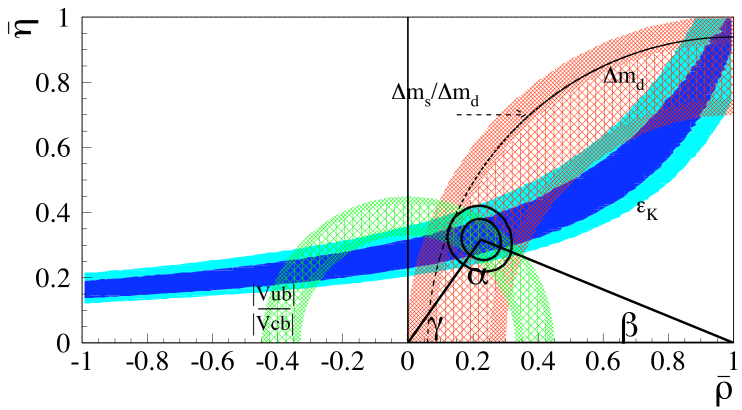


The History of the Unitarity Triangle



The role of Lattice Gauge Thery

Lattice QCD has played a fundamental role in determining the UT parameters. Three of the five determinations depend in a critical way from Lattice QCD results.

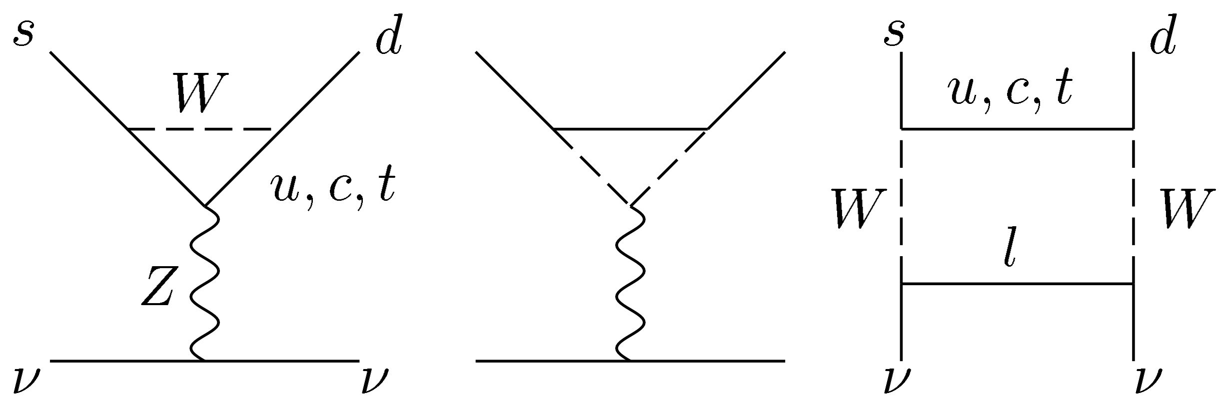


Measurement	$V_{CKM}\times$ other	Constraint
$b\to u/b\to c$	$\|V_{ub}/V_{cb}\|^2$	$\bar{\rho}^2+\bar{\eta}^2$
Δm_d	$\|V_{td}\|^2 f_{B_d}^2 B_{B_d} f(m_t)$	$(1-\bar{\rho})^2+\bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\frac{\|V_{td}\|^2}{\|V_{ts}\|^2}\frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 B_{B_s}}$	$(1-\bar{\rho})^2+\bar{\eta}^2$
ε_K	$f(A,\bar{\eta},\bar{\rho},B_K)$	$\propto \bar{\eta}(1-\bar{\rho})$

A. Stocchi, from analysis by M. Ciuchini et al.

We would like measurements that are as far as possible independent from details of the hadron physics. The answer: $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$.

$K^+ \to \pi^+ \nu\bar{\nu}$ & $K_L \to \pi^0 \nu\bar{\nu}$: The Future of the Unitarity Triangle



$s \to d\nu\bar{\nu}$ is a short distance process dominated by the t quark, with a smaller c quark contribution (absent in $K_L \to \pi^0 \nu\bar{\nu}$) and is described by an effective Fermi interaction,

$$\mathcal{H}_{eff} = \frac{G_l^{(L,+)}}{\sqrt{2}} \sum_{l=e,\mu,\tau} (\bar{s}\gamma^\mu(1-\gamma_5)d)(\bar{\nu}_l\gamma_\mu(1-\gamma_5)\nu_l)$$

There is a small difference between the couplings for ν_τ and $\nu_{e,\mu}$.

Taking for the $G_l^{(L,+)}$ the average value implies a negligible (0.2%) error on the rates.

$K^+ \to \pi^+ \nu\bar{\nu}$ & $K_L \to \pi^0 \nu\bar{\nu}$ – continued

Given $G_l^{(L,+)}$, the branching ratios are directly related by isospin to that of the K^+_{e3} decay,

$$B(K^+ \to \pi^+ \bar{\nu}\nu) = 6\, r_{K^+} B(K^+ \to \pi^0 e^+ \nu) \frac{|G_l^+|^2}{G_F^2 |V_{us}|^2} \quad (1)$$

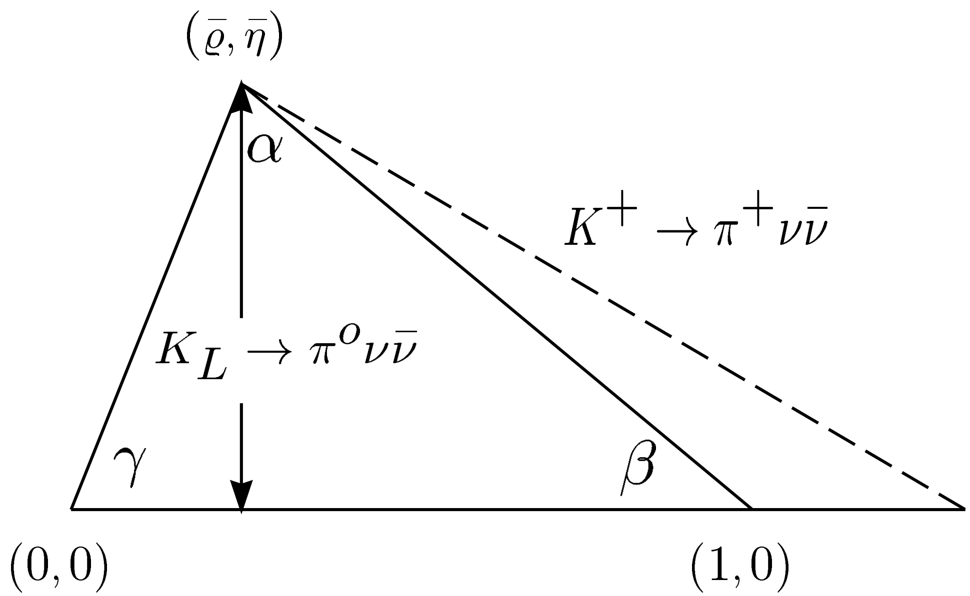
$$B(K_L \to \pi^0 \bar{\nu}\nu) = 6 \frac{\tau_{K_L}}{\tau_{K^+}} r_{K_L} B(K^+ \to \pi^0 e^+ \nu) \frac{(\mathrm{Im}\, G_l^L)^2}{G_F^2 |V_{us}|^2} \quad (2)$$

$r_{K^+} = 0.901$ and $r_{K_L} = 0.944$ are isospin breaking corrections (W.J. Marciano and Z. Parsa, – 1996) that include phase space and QED effects.

$K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ are "Golden Channels"

These decays are sensitive to high energy ($> M_t$) phenomena and to New Physics.

$K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, and the Unitarity Triangle



Theoretical errors in $K^+ \to \pi^+ \nu\bar{\nu}$ are $\sim 5 \div 7\%$. This makes the $K^+ \to \pi^+ \nu\bar{\nu}$ one of the most attractive tools for the exploration of the unitarity triangle, a member of a very short list of theoretically clean processes. A combination of $K^+ \to \pi^+ \nu\bar{\nu}$ and the $\sin(2\beta)$ measurement in $B^0 \to \Psi K_S$ would determine completely the unitarity triangle without any recourse to lattice gauge theory.

The uncertainties are even less for $K_L \to \pi^0 \nu\bar{\nu}$, whose measurement offers a direct determination of the area $\eta/2$ of the unitarity triangle, and a beautiful test of the Standard Model and its short-distance behavior.

Quark Mass Matrix and the CKM Matrix:

$$\mathscr{L}_M = \left[\bar{u}_R M u_L + \bar{d}_R M' d_L \right] + h.c.$$

Diagonal Masses:

$$U_R M U_L^\dagger = D = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

$$U'_R M' U_L'^\dagger = D' = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$$V = U_L U_L'^\dagger = \text{The CKM Matrix}$$

CP Violation is encoded in the mass matrix

Cecilia Jarslog's relation:

$$\det[M, M'] = i\,FF'J$$

Where:

$$F = (m_t - m_u)(m_t - m_c)(m_c - m_u)$$

$$F' = (m_b - m_s)(m_b - m_d)(m_s - m_d)$$

$$J \propto \text{Area of the Unitarity Triangle}$$

The mass matrix must contain complex numbers!.

...but in a gauge theory mass arises from the Higgs Mechanis...

Quark Mixing in the Standard Model

The Higgs Boson and Symmetry Breaking (single Higgs):

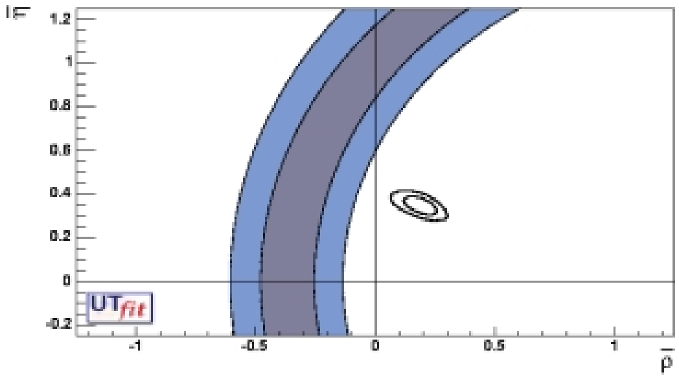
$$\langle 0|\phi|0\rangle = v \quad \text{(we can choose } v \text{ to be real)}$$

Quark Masses and the Higgs Boson Couplings:

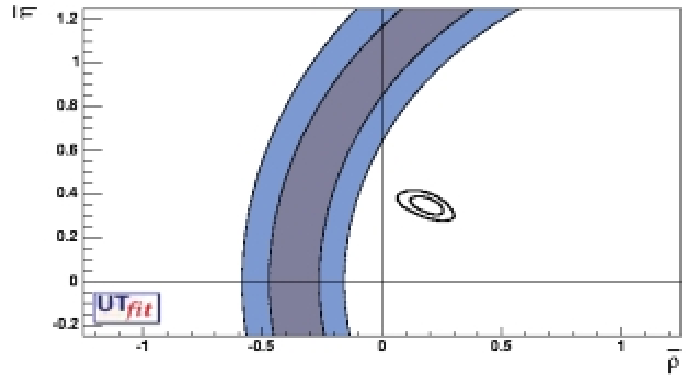
$$\mathscr{L}_M = \frac{\phi}{v}\left[\bar{u}_R M u_L + \bar{d}_R M' d_L\right] + h.c.$$

In the Standard model we need complex Higgs coupling constants: the Higgs couplings directly break CP. More elegant alternatives to this simplest scheme — e.g. spontaneous breaking of CPsymmetry would directly impact FCNC (Flavour Changing Neutral Currents), and the $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ decays. At a visible level?

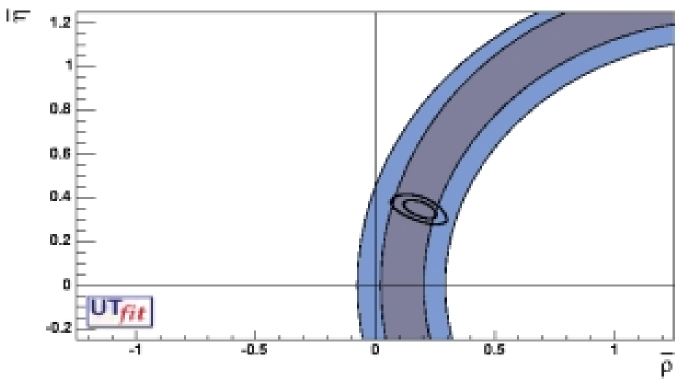
Impact of 100 event $K^+ \to \pi^+ \nu \bar{\nu}$ experiment
NA48-3 to run in 2009-2010



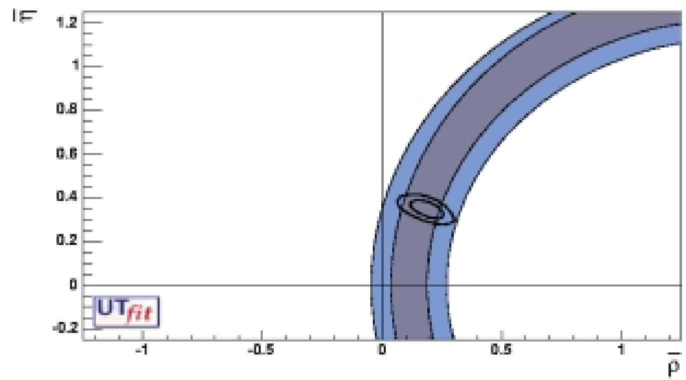
Center value = BNL result



1/2 Theoretical error



Standard Model prediction



1/2 Theoretical error

A 100-event experiment will compete in precision with B-factories, and will be more sensitive to New Physics. The 50% reduction in the theoretical uncertainty would offer a limited improvement, but would be essential in a 500 or 1000-event experiment.