Neutrino Masses: Shedding Light On Unification & Our Origin

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3) JCP – Review KEK Talk – ph/0407220.

I. Introduction

A) Why Neutrinos are special?

Since the discoveries/confirmations of Atmospheric & Solar ν-Oscillations, ν's have emerged as being among the most efficient probes into the nature of Higher Unification.

→ simply because of their (non-zero but) tiny masses

$$(m_{\nu_e}/m_e) \lesssim 10^{-6} \quad , \quad \boxed{m_{\nu_3}/m_{top} \sim 10^{-11}}$$

→ A subtle clue to some of the deepest Laws of Nature pertaining to

- The Unification-Scale
- Nature of the Unification Symmetry

→ In this sense, ν's provide us with a rare window to view physics at truly short distances ($\sim 10^{-30}$ cm)

→ Furthermore, it seems most likely that tiny ν- masses are also at the root of the

→ Origin of matter - Anti-matter Asymmetry

→ Thus ν's may also be crucial to our own origin!

IB) Why ν-Masses Suggest Physics Beyond SM

SM: $SU(2)_L \times U(1)_Y \times SU(3)^c$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \; u_R, \; d_R, \; \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \; e_R, \; \cancel{\nu_R}$$

ν_L can acquire Majorana Mass Using Quantum gravity :

$$\lambda_L \, LLHH / M_{Planck} \; + h.c \; (|\Delta L| =$$

$$\rightarrow m(\nu_L) \sim \lambda_L \, (250\,GeV)^2 / 10^{19}\,GeV$$

$$\sim \lambda_L \, (0.6) \times 10^{-5}\,eV$$

$$\updownarrow$$

$$\text{SuperK} \Rightarrow \sqrt{\Delta m^2_{23}} \approx 1/20\ eV$$

Can argue Need New Physics at an eff. scale $\sim 10^{15}$ GeV $\longleftrightarrow$ Can Link it to Scale of meeting of 3 gauge Couplin $\sim 2 \times 10^{16}$ GeV

$\rightarrow$ HINT AT A LINK BETWEEN ν-OSCill & Grand Unification !

II) Purpose is to Present A Unified Picture of

- Fermion Masses & Mixings
- Neutrino Oscillations
- CP and Flavor Violations
- Baryogenesis Via Leptogenesis

in accord with observations, Within a single predictive framework based on the Symmetry $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ or SO(10) with SUSY.

Have proposed such a framework that successfully describes fermion masses & ν-oscillations with 7 predictions (m_b, $\sqrt{\Delta m^2(\nu_2-\nu_3)}$, V_{cb}, $\Theta(\nu_2-\nu_3)$, V_{us}, V_{ub} & m_d), all in good accord with the data (Babu, Pati & Wilczek 98-2000)

→ But this work ignored CP phases for simplicity.

Now, address the issues of CP & Flavor violations ($\mu \to e\gamma$, $b \to s\gamma$ etc.) within the same framework $\leftrightarrow$ Intimately Linked With ν-(

(Babu, Pati & Parul Rastogi).

Question: Can observed CP & Flavor Viols emerge consistently within the SUSY G(224)/SO(10) - Framework, while preservin its successes wrt predictions for fermion masses & Neutrino Oscillations?

→ A Non-Trivial Challenge

→ Further Tests

→ Study Baryogenesis within the same framework.

IC) Purpose is to present a Unified Picture of

- Fermion Masses & Mixings
- Neutrino Oscillations
- CP Non-Conservation
- Flavor Violations (quark & lepton Sectors)
- Baryogenesis Via Leptogenesis

Main Theme: To exhibit how these diverse pieces fit together neatly, in accord with observations, within a Single Predictive framework based on eff. Symmetry in 4D

Either, $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$

or

$SO(10)$

$\oplus$ Low Energy Supersymmetry.

The Need For SU(4)-Color ⊂ G(224) ⊂ SO(

$$F^{e}_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R}$$

⇒ (i) Exist. of ν_R $\longleftrightarrow$ Needed For Seesaw & Leptogenesis

(ii) B-L $\longleftrightarrow$ Protects ν_R from Planck Scale Contrib & Sets

(iii) $m(\nu^{\tau}_{Dirac}) \approx m_{top}(M_X)$ $\longrightarrow$ Need for Seesaw

$m_b(M_X) \approx m_\tau$ $\longrightarrow$ Works

These 3 ingredients crucial to understanding ν masses via Seesaw & implementing baryogenesis via Leptogenesis

IB) Insight From SuperK Result: $\sqrt{\Delta m^2_{23}} \approx 1/20$ eV

SeeSaw

ignore mixing for a moment

$$\boxed{m(\nu_L^\tau) \approx \frac{m(\nu^\tau_{Dirac})^2}{M(\nu_R^\tau)}}$$

(a) $\boxed{m(\nu^\tau_{Dirac}) \approx m_t(M_X) \approx 120\,\text{GeV}} \Leftarrow$ SU(4) - Color
SU(5), [SU(3)]

$\boxed{m_b^0 \approx m_\tau^0}$

(b) Get $M(\nu_R^\tau)$ from SUSY Unif. Scale: $M_X \approx 2\times 10^{16}$ Ge

$$\underset{(\approx 1)}{f_{33}} \frac{16_3 16_3 \langle \overline{16}_H\rangle\langle\overline{16}_H\rangle}{M \rightarrow \sim 10^{18}\,\text{GeV}} \Rightarrow M(\nu_R^\tau) \sim \frac{(2\times 10^{16}\,\text{GeV})^2}{10^{18}\,\text{GeV}}$$

$$\approx \boxed{4\times 10^{14}\,\text{GeV}\,(1/2 - 2)}$$

$\Downarrow$

$$\boxed{m(\nu_L^\tau) \sim \frac{(120\,\text{GeV})^2}{4\times 10^{14}\,\text{GeV}} \approx \left(\frac{1}{30}\,\text{eV}\right)\left(\frac{1}{2} \text{ to } 2\right)}$$

Also get $m(\nu_L^\mu) \sim \frac{m(\nu_L^\tau)}{10} \Rightarrow \sqrt{\Delta m^2(\nu_2\nu_3)}_{Th} \approx \left(\frac{1}{30}\,\text{eV}\right)\left(\frac{1}{2}\text{-}\right.$

Thus SuperK result brings to light the existence of ν_R // reinforces the ideas of a) SeeSaw // (b) SU(4) Color // & (c) SUSY Unif.

In short just this single piece of informatio

$\left(\sqrt{\Delta m^2(\nu_\mu \nu_\tau)} \sim 1/20 \text{ eV}\right)$

Disfavors

- ~~$SU(5)$, $[SU(3)]^3$~~
- ~~Intermediate Scale ($\lesssim 10^{13}$ GeV) Breaking of B-L~~
- ~~Large Extra Dim (ν_R in Bulk)~~

Favors

- $SU(2)_L \times SU(2)_R \times SU(4)$ SO(10) Route To Higher Unification With SUSY
- Single Step Breaking of G(224) // SO(10) TO SM Symmetry at $M_X \sim 2 \times 10^{16}$ GeV

II) Fermion Masses & Mixings in SO(10)/G(224)

Babu, Pati, Wilczek (98-200

Minimal Higgs For SO(10) - Breaking

$\boxed{45_H, 16_H, \overline{16}_H, 10_H}$, $\underbrace{126_H, 120_H, 54_H}$

Allowed by String Solns

Too large GUT-Scale Threshold Corrections

$$\begin{array}{c} SO(10) \\ \langle 45_H \rangle \ \ \Big\downarrow \sim \langle 16_H \rangle \sim M_{GUT} \\ \propto B-L \qquad \qquad \\ SU(2)_L \times U(1)_Y \times SU(3)^c \\ \langle 10_H \rangle \ \Big\downarrow \ \langle 16_H^d \rangle \sim M_{EW} \\ U(1)_{em} \times SU(3)^c \end{array}$$

$16_i 16_j 10_H \rightarrow$ B-L Indep // $V_{CKM} = \mathbb{1}$

$16_i 16_j 10_H \cdot 45_H/M \rightarrow \{$ B-L dep // $V_{CKM} = \mathbb{1}$; Family Antisym.

$16_i 16_j 16_H \cdot 16_H^d/M \rightarrow \{ V_{CKM} \neq \mathbb{1}$

Assume Flavor Symmetries(*) Such that

- 3rd Family gets the dominant contribution $\rightarrow h_{33}\, 16_3\, 16_3\, 10_H$

  The lighter Families get their masses primarily Through off-diagonal mixings with the heavier families :

"33" $\gg$ "23" $\gg$ "22" $\gg$ "12" $\gg$ "11" etc.

	16_3	16_2	16_1	10_H	16_H	$\overline{16}_H$	45_H	S
$U(1)\rightarrow$	a	$a+1$	$a+2$	$-2a$	$-a-\frac{1}{2}$	$-a$	0	-1

$$h_{33}\, 16_3\, 16_3\, 10_H + \tilde{h}_{23}\, 16_2\, 16_3\, 10_H \boxed{(S/M)} \quad \boxed{\sim 1/}$$

$$+ \tilde{h}_{22}\, 16_2\, 16_2\, 10_H \langle S/M \rangle^2 + \cdots\cdots$$

"a" gets fixed from LMA Soln $\rightarrow \nu_L^e \nu_L^\mu$ Mas.

$16_1 16_2 16_H 16_H 10_H 10_H / M^3$ Leading if $\boxed{a=-}$

(*) String solns generically do yield Flavor Symmetries

A Concrete Example: Minimal Higgs: $\{45_H, 16_H, \overline{16}_H, 10_H\}$ + "S"

1st consider only μ & τ Families (Flavor sym: $\mu \neq \tau$)

$$\mathcal{L}_{mass} = h_{33}\, 16_3\, 16_3 \langle 10_H \rangle \longrightarrow \text{3rd Family}: m_b^0 = m_\tau^0$$

$$\propto 1 \quad \propto \sigma \quad + \tilde{h}_{23}\, 16_2\, 16_3 \langle 10_H \rangle \langle S/M \rangle \longrightarrow \text{2nd Family}$$

ANTISYMMETRIC, $\propto B-L$

$$\propto \epsilon \quad + a_{23}\, 16_2\, 16_3 \frac{\langle 10_H \rangle \langle 45_H \rangle}{M} \rightarrow m_\mu^0 \neq m_s^0$$

$$\propto \hat{\eta} \quad + g_{23}\, 16_2\, 16_3 \langle 16_H \rangle \langle \overline{16}_H^D \rangle_{EW} / M$$

$\hookrightarrow$ CKM $\neq 1$

$10_H \rightarrow (2 \leftrightarrow 2) \ll (2 \leftrightarrow 3) \ll (3,3)$ Flavor sym

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$$U = \begin{array}{cc} c & t \\ \left(\begin{array}{cc} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{array}\right) \end{array} m_U^0 \qquad D = \begin{array}{cc} s & b \\ \left(\begin{array}{cc} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{array}\right) \end{array} m_D^0$$

$$N_{Dirac} = \begin{array}{cc} \nu_\mu & \nu_\tau \\ \left(\begin{array}{cc} 0 & -3\epsilon+\sigma \\ +3\epsilon+\sigma & 1 \end{array}\right) \end{array} m_U^0 \qquad L = \begin{array}{cc} \mu & \tau \\ \left(\begin{array}{cc} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{array}\right) \end{array} m_D^0$$

Note $q-\ell$ correlation (SU(4)-color)

up-down " $(SU(2)_L \times SU(2)_R)$

$$\boxed{\eta \equiv \hat{\eta} + \sigma}$$

Dirac Masses (3 Families)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & \epsilon+\eta & 1 \end{pmatrix} m_D$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; L = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & +3\epsilon+\eta & 1 \end{pmatrix}$$

$\times$

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2 New param (ϵ', η'), but 5 new observable just (q, l) System $\Rightarrow$ 3 New predictions for (q, l) // With $\epsilon' = 0$ $\to m_u \to 0$

ν Majorana Masses

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

Saw before. $M_R \approx 10^{15}$ GeV

Expect $y \sim 1/10$

$$f_{ij}\, 16_i\, 16_j \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle / M_{st}$$

$$\downarrow$$

$$f_{ij}\, \nu_{iR}^T\, \bar{C}'\, \nu_{jR}\, \langle \overline{16}_H \rangle^2 / M_{st}$$

$$(M_R^\nu)_{ij} = f_{ij} \langle \overline{16}_H \rangle^2 / M_{st}$$

Note Same hier. pattern as in Diracs

Including $m_U^0 \to$ 7 param $(\eta, \epsilon, \sigma, \eta', \epsilon', m_U^0, m_D^0)$ [1]
describing $9 \times 4 = 36$ entries $\longrightarrow$ Will it work ?

Input: Assume all param real for a moment

$m_t^{phys} = 174$ GeV ; $m_c(m_c) = 1.37$ GeV,
$m_s(1\,\text{GeV}) = 116$ MeV, m_μ, m_τ, $m_u(M_X) = 1.5$ MeV, m

$\Downarrow$

$\sigma \simeq 0.110$, $\eta \simeq 0.151$, $\epsilon \simeq -0.095$,
$\epsilon' = \sqrt{m_u/m_c}\,(m_c/m_t) \simeq 2\times10^{-4}$; $\eta' \simeq \sqrt{m_e/m_\mu}\left(\frac{m_c}{m_t}\right) \simeq 4\times10^{-3}$
$m_U^0 = m_t(M_X) \simeq 110$ GeV ; $m_D^0 \simeq 1.5$ GeV

Majorana Mass of ν_R's : $f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M_{pl}$

$\Rightarrow f_{ij}\, (\nu_R^{iT} C^{-1} \nu_R^j)\, \langle 16_H \rangle^2 / M$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} \boxed{M_R}$$

determined by $m_{\nu_2}/m_{\nu_3} \approx 1/6$

calculated $\approx 5\times10^{14}$ GeV

6 New observables.

Summary on Fermion Masses & Mixings (FPW) 1

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,GeV$	$\approx 4.2\,GeV$
$m(\nu_\tau) \sim (1/24\,eV)(\frac{1}{2}-2)$	$\approx (1/15 - 1/25)\,eV$ ⊛
$V_{cb} \approx 0.043$	≈ 0.04
$\sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx \underbrace{\boxed{0.92}}_{SMA} \leftrightarrow \underbrace{\boxed{0.99}}_{LMA}$	$\approx 0.92 \leftrightarrow 1$
$V_{us} \approx 0.22$	≈ 0.21
$\lvert V_{ub}\rvert \simeq 0.0032$	$\approx 0.003 - 0.001$
$m_d(1\,GeV) \simeq 8\,MeV$	$\approx 8-10\,MeV$

$m(\nu_\mu) \approx (2-10)\times 10^{-3}\,eV \longleftrightarrow \begin{cases} SMA \sim 3\times 10^{-3}\,eV \\ \boxed{LMA \approx 7\times 10^{-3}\,eV} \end{cases}$

$m(\nu_e) \sim (1 \text{ to few}) \times 10^{-3}\,eV$

Consistent with the framework

$M(\nu_R^\tau, \nu_R^\mu, \boxed{\nu_R^e}) \approx (10^{15}, 2\times 10^{12}, (\frac{1}{3}-3)\times 10^{10})$ GeV

Just right for Lepto/Baryo gene

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ng -i pu

$$M_{3R} \approx M_R \approx 10^{15}\,\mathrm{GeV}\,(1/2 - 1)$$

$$M_{2R} \simeq |y^2|\, M_{3R} \approx 2.5 \times 10^{12}\,\mathrm{GeV}\,(1/2 - 1)$$

$$M_{1R} \simeq |x - z^2|\, M_{3R} \approx (1/2 - 2) \times 10^{-5}\, M_{3R}$$

$$\approx 10^{10}\,\mathrm{GeV}\,(1/4 - 2)$$

Predictions

Writing only for 2×2 (for simplicity)

$$U = \begin{array}{c} \quad c \qquad t \\ \begin{pmatrix} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \end{array} \qquad D = \begin{array}{c} \quad s \qquad b \\ \begin{pmatrix} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{pmatrix} m_D^0 \end{array}$$

$$N = \begin{pmatrix} 0 & -3\epsilon+\sigma \\ 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \qquad L = \begin{array}{c} \quad \mu \qquad \tau \\ \begin{pmatrix} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{pmatrix} \end{array}$$

$$\Downarrow$$

$$m_b^0 \simeq m_\tau^0 (1-8\epsilon^2) \Rightarrow \boxed{m_b(m_b) \simeq 4.7}$$

$$V_{cb} = \left| \underbrace{\sqrt{\frac{m_s}{m_b}}}_{(0.156)} \underbrace{\left(\frac{\eta+\epsilon}{\eta-\epsilon}\right)^{1/2}}_{(1/2.2)} - \sqrt{\frac{m_c}{m_t}} \left(\frac{\sigma+\epsilon}{\sigma-\epsilon}\right)^{1/2} \right| = |0.$$

$$\boxed{=0.04}$$

Suppressed

ENHANCED $\simeq 1.8$

$$\theta^{osc}_{\nu_\mu\nu_\tau} = |\theta^{\ell}_{\mu\tau} - \theta^{\nu}_{\mu\tau}| \approx \left| \sqrt{\frac{m_\mu}{m_\tau}} \left(\frac{\eta-3\epsilon}{\eta+3\epsilon}\right)^{1/2} + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \right.$$

$$= 0.437 + \sqrt{m_{\nu_2}/m_{\nu_3}} \qquad \boxed{\approx 0.3}$$

Expt

$$\Rightarrow \boxed{\sin^2 2\theta^{osc}_{\nu_\mu\nu_\tau} = 0.92 \leftrightarrow 0.99} \qquad .92 \leftrightarrow 1$$

$$m_{\nu_2}/m_{\nu_3} = (1/15) \longleftrightarrow (1/7)$$ LMA.

SMA or LMA ?

$\xrightarrow{} f_{ij}\ 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

Just with Standard See-saw ν_L-masses, SMA rather generic

$m(\nu_L^e) \sim 2\times10^{-5} - 2\times10^{-6}\,eV$ // $m(\nu_L^\mu) \sim 3\times10^{-3}\,eV$

$\Theta^{osc}_{\nu_e\nu_\mu} = \Theta^{\ell}_{e\mu} - \Theta^{\nu}_{e\mu} \approx 0.05$

Situation alters once allow for direct Maj masses of ν_L's — Most likely to arise through Higher Dim. op. involving GUT & EW VEV's – Through tiny $\sim 10^{-3}$ eV entries → IMPORTANT For $(\nu_e - \nu_\mu)$

$$W \supset g_{12}\ \underbrace{16_1\ 16_2}\ 16_H\ 16_H\ \underbrace{10_H\ 10_H} / M^3_{GUT}$$

$$g_{12}\,(\nu_L^e \nu_L^\mu)\left(\langle\tilde{\nu}_{RH}\rangle / M_{GUT}\right)^2 (v_u^2)$$

$(\sim 175\ GeV)^2$

$$\boxed{\sim g_{12}(\nu_L^e \nu_L^\mu)\,(1.5-6)\times10^{-3}\,eV} \qquad (\langle 16_H\rangle \approx (1-2) M_G$$

$$\begin{array}{cc} \nu_L^e & \nu_L^\mu \\ \end{array}$$
$$\begin{bmatrix} \approx 0 & (3-4) \\ (3-4) & 7 \end{bmatrix} \times 10^{-3}\,eV \Longrightarrow \begin{bmatrix} \Theta^{\nu}_{\nu_e\nu_\mu} \approx 1/2 \\ \sin^2 2\Theta^{osc}_{\nu_e\nu_\mu} \sim 0.7 \end{bmatrix}$$ Quite plausi

Thus LMA not strictly a prediction, but perfectly plausible within the framework.

$$ {}^{e}_{L}\nu^{\tau}_{L})_{\text{Non-seesaw}} \sim (2-6)\times 10^{-3}\,\text{eV} $$

$$ 3 \sim \frac{(2\text{-}6)\times 10^{-3}\,\text{eV}}{5\times 10^{-2}\,\text{eV}} $$

$$ \sim .03 - .1 $$

s 2β decay : $\Delta L = \pm 2$

$$ = \left|\sum_i m_i U_{ei}^2\right| $$

ew $\times 10^{-3}$ eV, $m_2 \approx (6\text{-}8)\times 10^{-3}$ eV

5×10^{-2} eV

$1/2$, $\theta_{13} \sim 0.03 - 0.1$

$$ \Downarrow $$

$(1 \text{ to } 6)\times 10^{-3}$ eV

Summary on Fermion Masses & Mixings in the $G(224)/SO(10)$ Framework

Given the bizarre pattern of masses & mixings of quarks, charged leptons and neutrinos, it seems remarkable that the simple pattern of fermion mass matrices*, motivated in large part by the group th of $G(224)/SO(10)$ and the assumption of minimality of the Higgs system makes 7 predictions in agreement with observation.

→ Study Proton Decay // Leptogenesis // $\cancel{CP}$ within this framework.

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* Need to understand the origin of flavor symmetry
→ Hierarchical entries.

III 3 Recent Works Within The Same SUSY SO(10) // G(224) - Framework

① CP & Flavor Viols in quark-Sector

Babu, Pati, Parul Rastogi (ph/0410200

$\boxed{\Delta m_K, \epsilon_K, \Delta m_{B_d}, S(B_d \to J/\psi K_s)}$

Predictions for

ϵ'/ϵ, Δm_{B_s}, $S(B_d \to \phi K_s)$, $(edm)_n$,

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② LEPTON FLAVOR Viol ($\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu N \to e$
Pol. $\mu^+ \to e^+\gamma$)

Babu, Pati, Rastogi (ph/0502152)

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③ Leptogenesis ⟶ Baryogenesis

JCP. ph/0209160 (Phys. Rev 2003)

VI CP & Flavor Violations Within The Same G(224)/SO(10) - Framework

Babu, Pati & Parul Rastogi (ph/0410200

EXPTL. FACTS

(a) $\boxed{\Delta m_K, \ \epsilon_K, \ \Delta m_{B_d}, \ S(B_d \to J/\psi K_s)}$

All Four in good agreement with SM-CKM
(Allowing for ~15% uncertainty in ME)

$$\Rightarrow \boxed{(\bar{\rho}_W)_{SM} = 0.178 \pm .046, \ (\bar{\eta}_W)_{SM} = 0.34 \pm \ldots}$$

This agreement poses a challenge for Phys Beyond SM, especially SUSY GUT Models

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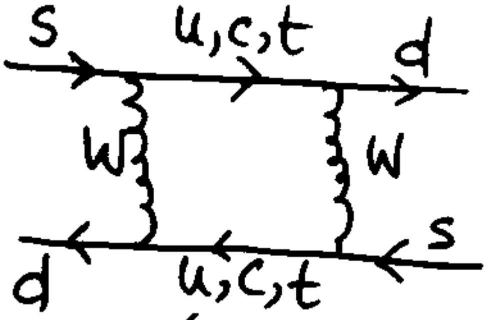
(b) Limits

LFV $\rightarrow B(\mu \to e\gamma) < 1.2 \times 10^{-11}$; $B(\tau \to \mu\gamma) < 5 \times 10^{-7}$

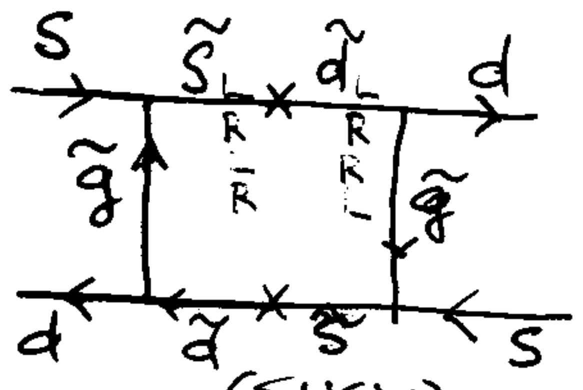
$$\cancel{CP} \begin{cases} d_n < 6 \times 10^{-26} \text{ e cm} \\ d_e < 4.3 \times 10^{-27} \text{ e cm} \end{cases}$$

$K^0 - \bar{K}^0$ $(s + s \to d + d)$

(ii



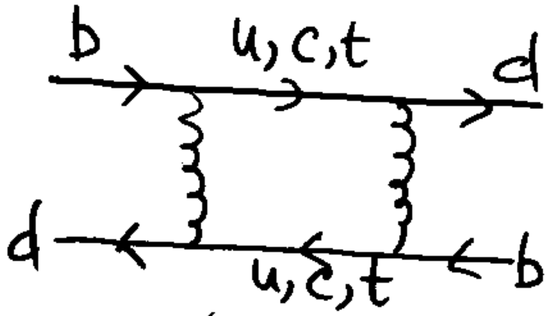
(SM)



(SUSY)

$B_d - \bar{B}_d$ $(bb \to dd)$

SUSY



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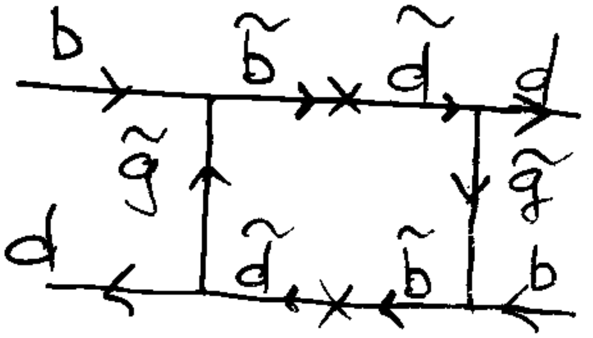
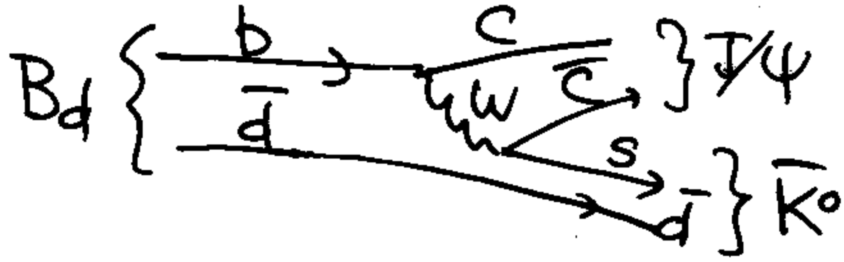
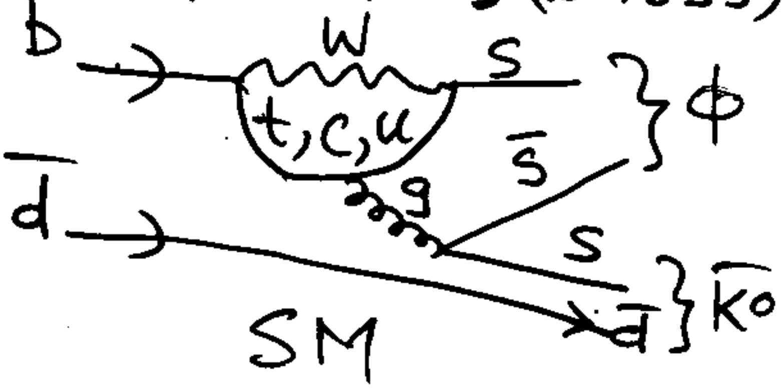
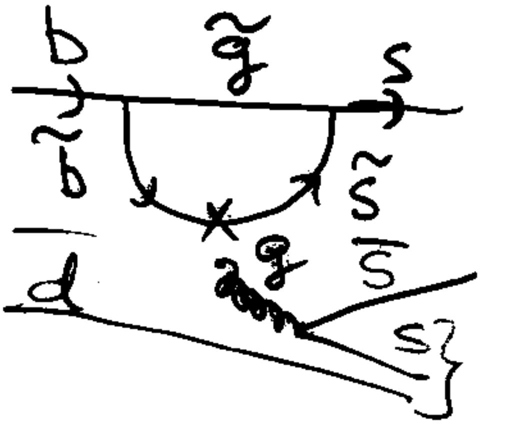


$B_d \to J/\psi \, K_s$ $(b \to c\bar{c}s)$



$B_d \to \phi K_s$ $(b \to s\bar{s}s)$





III CP & Flavor Viols in SUSY SO(10) // G(224

① SUSY BREAKING :

Assume Flavor-Universal, Chirality-Preserving

at $\boxed{M^{*} \gtrsim M_{GUT}}$ → mSUGRA // Gaugino Med

Extreme Case (CMSSM) → $\boxed{m_0, m_{1/2}, A_0, \tan\beta, \mathrm{Sgn}\,\mu}$

(Gaugino Med

② $\cancel{CP}$: Phases in Fermion Mass Matrices

↕

Complex VEV's and/or Complex Yukawas

③ $\cancel{\text{Flavor}}$: (i) Flavor-Dep Yukawa Couplings

⊕

(ii) RGE ($M^{*} \rightarrow M_{GUT} \longrightarrow M_{EW}$)

Extremely important, but commonly neglected

Completely determined, bringing no new parameters, barring few flavor-universal SUSY-Parameters (m_0, $m_{1/2}$, M^{*}, ..)

LINK $\cancel{CP}$ & $\cancel{\text{Flavor}}$ ⟷ Fermion Masses & ν OSC.

SUSY Flavor Violation : 3 Sources

① RG Running of Scalar Masses From $M^* \to M_{GUT}$

$$h_t \; 16_3 \; 16_3 \; 10_H$$

Heavy color triplets (SO(10)) + Heavy Doublets which couple to fermions owing to $10_H \leftrightarrow 16_H$ (Both SO(10) & G(224))

$$\Delta = \Delta \hat{m}^2_{\tilde{b}_L} = \Delta \hat{m}^2_{\tilde{b}_R} \approx -\frac{30 m_0^2}{16\pi^2} h_t^2 \ln(M^*/M_{GUT})$$

Fermion Mass Matrix Wolfenstein basis $\Big\}$ $\boxed{M^{diag}_{d,u,\ell} = X_L^{d,u,\ell\,\dagger}\, M^{(0)(GUT)}_{d,u,\ell}\, X_R^{d,u,\ell}}$

Squarks SUSY Basis $\Big\}$ $X_L^{d\,\dagger}\left[\begin{pmatrix}1 & & \\ & 1 & \\ & & 1-\Delta\end{pmatrix} m_0^2\right] X_L^d \Rightarrow$ FLAVOR VIOLATION

$$\Downarrow$$

$$\boxed{\begin{array}{c} \tilde{q}^i_L \to\!\!\times\!\!\to \tilde{q}^j_L \quad (i \neq j) \\ \text{Similarly} \quad \tilde{q}^i_R \to\!\!\times\!\!\to \tilde{q}^j_R \quad (i \neq j) \end{array}}$$

Completely determined in the SO(10)/G(224) Framework by the Fermion mass matrices & m_0, $\ln(M^*/M_{GUT})$.

② Flavor Violation Through RG Running From $M_{GUT} \Rightarrow m_W$

$$h_t \, \tilde{b}_L \, \tilde{t}_R \, H_u \Longrightarrow \Delta m_L'^2 \, \tilde{b}_L^{*\prime} \, \tilde{b}_L'$$

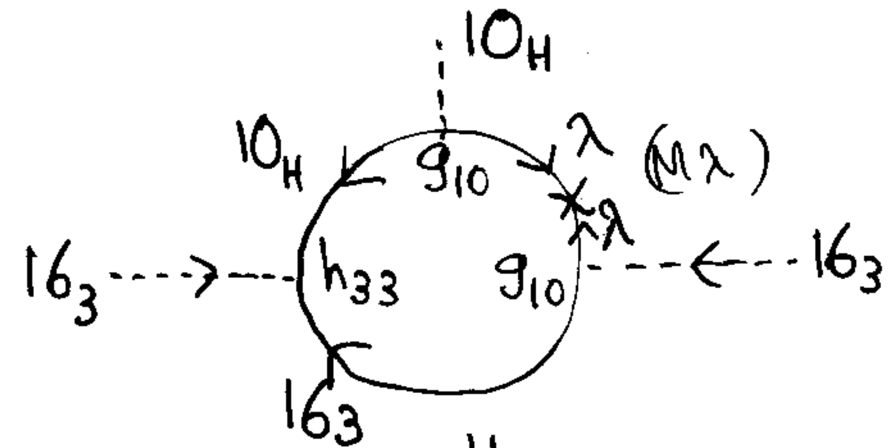
$$\left(\delta m_{LL}'^2\right)^{(12,13,23)} = \left(\Delta m_L'^2\right)\left(V_{td}^* V_{ts},\ V_{td}^* V_{tb},\ V_{ts}^* V_{tb}\right)$$

$$\Delta m_L'^2 = -3/2\, m_0^2 \left(h_{top}/h_{fix}\right) + m_{1/2}^2 \left(3\eta_t^2 - 7\eta_t\right)$$

$$m_{\tilde{g}} \approx 3\, m_{1/2} \quad ; \left(m_{sq}^2\right)_{1,2} \approx m_0^2 + 7.2\, m_{1/2}^2$$

③ A term Induced by RGE from $M^* \to M_{GU}$

e.g.



SO(10)

$$\Downarrow$$

$$\boxed{A_{33}^{(h_{33})} = \left(\frac{63}{2}\right)\left(1/8\pi^2\right) h_{33}\, g_{10}^2\, M_\lambda \ln\left(M^*/M_{GU}\right)}$$

SUSY Basis $\quad A^{(d)} = \left(X_L^d\right)^\dagger A^{(d)}(GUT)\, X_R^d \quad$ (Completely determined

Important for ϵ'/ϵ, edm, Lepton Flavor Violation

(b) .1?

(4) Question: Can observed CP & Flavor Viol
emerge consistently within the SUSY
G(224)/SO(10)-Framework, while preservin
its successes wrt fermion masses & ν-oscill

→ A Non-trivial Challenge (Many SO(10)-models do not satisfy both Sets of Constraints).

(5) New Results (Babu, Pati, Rastogi; ph/04102
find for natural phases (~10-40%) in the Dirac mass-parameters ($\sigma, \eta, \epsilon', \cdots$), of the same G(224)/SO(10)-framewor
Can get observed CP & Flavor Viols while preserving the successes in fermion masses & ν-oscillations!

(a) For phases in a natural range, get

$$\boxed{\hat{\eta}_W \approx 0.30-0.37\,;\ \hat{\rho}_W \approx 0.15-0.18}$$ BPR-Values

$\updownarrow$

Close To SM CKM} $\bar{\eta}_W \approx 0.34 \quad \bar{\rho}_W \approx 0.18$ SM CK Values

BPW ($\cancel{CP}$ Phases $=0$)	BPR (with Phases - A specific F
$\eta = 0.15$	$\eta = 0.12 - 0.0464i$
$\sigma = 0.11$	$\sigma = 0.1 - 0.012i$
$\epsilon = -0.095$	$\epsilon = -0.095$
$\epsilon' = 2\times10^{-4}$	$\epsilon' = 2.37\times10^{-4}\, e^{-i\,69^\circ}$
$\eta' = 4.4\times10^{-3}$	$\eta' = 2.4\times10^{-3}$
$\zeta_{22}^{d} \lesssim (1/3)\times10^{-2}$	$\zeta_{22}^{d} = 9.8\times10^{-3}\, e^{-i\,149^\circ}$
$\rho_{22}^{u} \lesssim 1/3\times10^{-2}$	$\rho_{22} = 4.8\times10^{-3}\, e^{i\,10.}$

$\hat{\eta}_W = 0.32$; $\hat{\rho}_W = 0.17$ "observed"

ALL IN GOOD AGREEMENT

		2004 CKM Fit
$V_{us} = 0.224$	$\longleftrightarrow$	$0.2250 \pm .0020$
$V_{cb} = 0.043$	$\longleftrightarrow$	0.0414 ± 0.001
$\lvert V_{ub}\rvert = 0.0036$	$\longleftrightarrow$	0.0033 ± 0.0005
$\lvert V_{td}\rvert = 0.0087$	$\longleftrightarrow$	0.0078 ± 0.000

$(m_c, m_s)_{1\,GeV} = (1.3\,GeV, 109\,MeV)$

barring "11" entry $\rightarrow (m_u^0, m_d^0)_{1\,GeV} = (9.9, 3.6)$ MeV

RESULTS

e.g. Take $m_{sq} \approx (0.8-1)$ TeV, $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2 \approx 0.5-$

$(\Delta m_K)_{\text{short dist}} \approx 3 \times 10^{-15}$ GeV + Long dist

$\boxed{\epsilon_K \approx (2 \text{ to } 2.4) \times 10^{-3} \leftrightarrow \text{expt} \approx 2.27 \times 10^{-3}}$

Im $\left(\frac{SUSY}{SM}\right) \approx -20$ to -30%

$\Delta m_{B_d} \approx (3 \text{ to } 3.3) \times 10^{-13}$ GeV $\leftrightarrow$ expt $\approx 3.3 \times 10^{-13}$ GeV

$S(B_d \to J/\psi K_S) \approx 0.67 - 0.74 \leftrightarrow \text{expt} = 0.734 \pm .05$

For $\hat{B}_K = 0.87$, $f_{B_d}\sqrt{\hat{B}_{B_d}} = 215$ MeV

All Four in good agreement with the data (to within 10 %)!

Thus see that the SUSY G(224)/SO(10) - Framework has met the challenges so far in being able to reproduce the observed features of CP & quark flavor Viol: while preserving its successes wrt predictions for fermion masses & ν - oscillations

$(m_0, m_{1/2})$ GeV	(800, 250)		(600, 300)	
$M^*/M_{GUT} = 3$	(a) SO(10)	(b) G(224)	(c) SO(10)	(d) G(224)
$\Delta m_K^{s.d}$ (GeV)	2.9×10^{-15} →	same →	same →	same
ϵ_K (SM)	2.8×10^{-3}	2.8×10^{-3}	2.8×10^{-3}	2.8×10^{-3}
ϵ_K (Tot)	1.30×10^{-3}	2.32×10^{-3}	2.01×10^{-3}	2.56×10^{-3}
Δm_{B_d} (Tot) GeV	3.62×10^{-13}	3.56×10^{-13}	3.6×10^{-13}	3.55×10^{-13}
$S(B_d \to J/\psi K_s)$	0.740	0.728	0.732	0.726

Expt. $\Delta m_K = 3.5\times10^{-15}$ GeV ; $\epsilon_K = 2.27\times10^{-3}$

$\Delta m_{B_d} = 3.3\times10^{-13}$ GeV , $S(B_d \to J/\psi K_s) = 0.734 \pm 0.0$

Have used central values: $\hat{B}_K = 0.86 \pm 0.13$, $f_K = 159$ MeV ,

$\eta_1 = 1.38 \pm 0.20$, $\eta_2 = 0.57 \pm 0.04$, $\eta_3 = 0.47 \pm 0.$

Unquenched lattice → $f_{B_d}\sqrt{\hat{B}_{B_d}} = 215(11)(^{+0}_{-23})(15)$ MeV

NOTE Can use ϵ_K to distinguish between SUSY SO(10) / SUSY G(224) & / SM(CKM) / once SUSY Param determined & improvement in lattice results

Further Tests

$S(B_d \to \phi K_s)\Big|_{\text{THEORY}} \approx 0.65 - 0.73 \to$ Close to SM-CKM Prediction

BABAR $(0.45 \pm .43 \pm .07)$ $\to$ $0.50 \pm 0.25^{+.}_{-.}$

BELLE $(-0.96 \pm .50\ ^{+.09}_{-.07})$ $\to$ $+0.06 \pm 0.33 \pm 0.09$

(ii) $\Delta m(B_s)_{Th} \approx 17.3\ ps^{-1} \left(\dfrac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{245\ \text{MeV}} \right)^2$

Many other processes under study
$B_s \to J/\psi\,\phi$, $B \to K\pi$, $K \to \pi\nu\bar{\nu}$ ---

---

INDUCED A Term MOST IMPORTANT

(iii) $B(\mu \to e\gamma) \approx 10^{-13} - 10^{-8}$, expt $< 1.2 \times 10$

$B(\tau \to \mu\gamma) \approx (10^{-7} - 3\times 10^{-9})$, expt $< 5 \times 10$

$m_{\tilde{q}} = 700\,\text{GeV}$, $m_{\tilde{g}} = 500\,\text{GeV}$

(iv) $d_n \approx (6 \text{ to } 1) \times 10^{-26}\ ecm$, expt $< 6.3 \times 10^{-2}$ ecm

$\tan\beta = 5 - 10$, Only A-Term Contrib $\to$ $d_e \approx \dfrac{2.67 \times 10^{-28}\ ecm}{\tan\beta}$, expt $< 4.3 \times 10^{-2}$ ecm.

The framework is presently most successful & thoroughly testable in near future.

Further Tests

(i) $S(B_d \to \phi K_S)\Big|_{\text{Theory}} = 0.65 - 0.73 \to$ Close to CKM Prediction

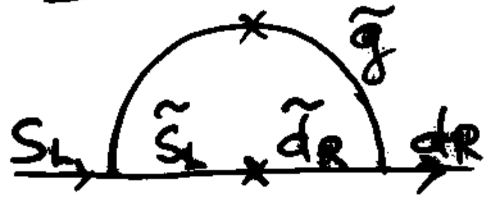
BaBar $\to (0.45 \pm 0.43 \pm .07)$ $\to 0.50 \pm 0.25 \,{}^{+.07}_{-.04}$
BELLE $\to (-0.96 \pm 0.50 \,{}^{+.09}_{-.07})$ $\to +0.06 \pm 0.33 \pm 0.0$

(ii) $\Delta m(B_s)_{Th} \approx 17.3\, ps^{-1} \left[\dfrac{f_{B_s}\sqrt{B_{B_s}}}{245\ \text{MeV}}\right]^2$

EXPT. $\gtrsim$ $14.4\, ps^{-1}$

Many other Processes under study
$B_s \to J/\psi\, \phi$, $B \to K\pi$, $K \to \pi \nu \bar{\nu}$

(iii) ϵ'_K/ϵ_K



$Q_{\tilde{g}} \propto (\bar{s}_L \sigma^{\mu\nu} t^a d_R - \bar{s}_R \sigma^{\mu\nu} t^a d_L) G^a_{\mu\nu}$
A-Term

$\text{Re}(\epsilon'/\epsilon)_{\tilde{g}} \approx 91\, B_G \left(\dfrac{110\ \text{MeV}}{m_s + m_d}\right)\left(\dfrac{500\ \text{GeV}}{m_{\tilde{g}}}\right) X$

$X_{21} \equiv \text{Im}\left[(\delta^d_{LR})_{21} - (\delta^d_{LR})^*_{12}\right] \approx \left[\dfrac{2.1\times 10^{-5}}{\tan\beta}\right]$

Determined in model (+Ve)

$(m_0, m_{1/2}) = (600, 300)$ G

B_G (Estimated) $\approx +(1\text{-}4)$ (Buras et al ph/9908371

$$\Rightarrow \boxed{\mathrm{Re}\,(\epsilon'/\epsilon)_{\tilde{q}} \approx +(8.8\times 10^{-4})(B_G/4)(5/\tan\beta)}$$

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{SM} \rightarrow [(-4 \pm 2.3) \longleftrightarrow (3\text{-}13)] \times 10^{-4}$$

$\boxed{\text{Need Reliable Calculation of SM \& } B_G}$

Blum et al. (RBC)

Partial Quenching & staggered fermions (Bhattacharya et al.)

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{obs} = (17 \pm 2)\times 10^{-4}$$

(iv) <u>edm</u> (Completely determined given $m_0, m_{1/2}$, $\tan\beta$)

$$(m_0, m_{1/2}) = (600, 300)\text{ GeV}$$

$$(d_n)_{A\,Ind} = (1.6, 1.08)\times 10^{-26}\text{ ecm}$$

$(\tan\beta = 5, 10)$

$$(d_e)_{A\,Ind} = (1.1\times 10^{-28}/\tan\beta)\text{ ecm}$$

<u>Expt</u> $\boxed{d_n < 6.3\times 10^{-26}\text{ ecm}}$, $d_e < 4.3\times 10^{-27}$ ecm

Should be seen with improvement in current limit by factor 10.

VI B) ν Masses $\longleftrightarrow$ LFV

$$\tilde{l}_{jL} \xrightarrow[\tilde{\mu}]{} \times \xrightarrow[\tilde{e}]{} \tilde{l}_{iL}$$

$$(\delta m^2_{\tilde{l}})_{ij} \approx -\frac{1}{8\pi^2}(3m_0^2+a_0^2)(Y_\nu^\dagger Y_\nu)_{ij} \ln\frac{M}{M_{N_i}}$$

Dirac ν Mass $\Rightarrow$ Basis Y_l & $Y(\nu_R)$ Diagon.

$$B(\mu\to e\gamma) = C\left(\alpha^3/G_F^2\right)\frac{|(\delta m^2_{\tilde{l}})_{12}|^2}{m_S^8}\tan^2\beta$$

$O(1) \to$ can be $(1/10 - 5)$

$|\tan\beta = $

$(m_0, m_{1/2}, \mu_{\ldots}$

$$\boxed{B(\mu\to e\gamma) \approx 10^{-12} - 10^{-14}} \left(\begin{matrix}\text{BPW}\\ \text{BPR}\end{matrix}\right) \to \begin{matrix}\text{Expt}\\ < 1.2\times 1\ldots\end{matrix}$$

Can distinguish between SO(10)-Models

BPW	Albright & Barr (lopsi...
$(M_\nu^D) \propto \begin{array}{cc} \nu_{\mu R} & \nu_{\tau R} \\ \end{array} \begin{bmatrix} 0 & \sigma-3\epsilon \\ \sigma+3\epsilon & 1 \end{bmatrix}$	$M_\nu^D \propto \begin{bmatrix} 0 & \tilde{\epsilon} \\ -\tilde{\epsilon} & 1 \end{bmatrix}$
$M_l \propto \begin{array}{cc} \mu_R & \tau_R \\ \end{array} \begin{bmatrix} 0 & \eta-3\epsilon \\ \eta+3\epsilon & 1 \end{bmatrix}$	$M_l \propto \begin{bmatrix} 0 & \tilde{\sigma}+\tilde{\epsilon} \\ -\tilde{\epsilon} & 1 \end{bmatrix}$, $\tilde{\sigma}\approx 1$

$\Rightarrow (Y_\nu)_{23}/(Y_\nu)_{33} = \sigma - 3\epsilon \longrightarrow \tilde{\epsilon}$

$(Y_l)_{23}/(Y_l)_{33} = \eta - 3\epsilon \longrightarrow \tilde{\sigma}+\tilde{\epsilon}$

Diff $= \underset{.15}{\eta} - \underset{.11}{\sigma} \approx 0.04 \longrightarrow \approx 1$

$$\boxed{B(\mu\to e\gamma)_{\text{Albright, Barr}}/B(\mu\to e\gamma)_{\text{BPW}} \approx 625}$$

SUSY LFV in G(224)/SO(10)

Babu, Pati, Rastogi (ph/05021

Completely Determined in the Model, brings no new parameters (Barring the few flavor-universal SUSY parameters)

(1) RG RUNNING FROM $M^* \longrightarrow M_{GUT}$

(a) Scalar Masses: $h_{33}\, 16_3\, 16_3\, 10_H$

$$\Delta \hat{m}^2_{\tilde{\tau}_{L,R}} \approx -(30/16\pi^2)\, h^2_{top} \ln (M^*/M_{GUT})$$

For G(224) $(30 \to 12)$

(b) Induced A-Term $(M^* \to M_{GUT})$

(2) RG Running of Scalar Masses From $M_{GUT} \to M_{R_i}$ (RH ν's)

$$\left(\delta^{\ell}_{LL}\right)^{RHN}_{ij} = \frac{-(3m_0^2 + A_0^2)}{8\pi^2} \sum_{k=1}^{3} (Y_N)_{ik} (Y_N^*)_{jk} \ln \frac{M_{GUT}}{M_{R_k}}$$

$\boxed{1}$ From POST-GUT PHYSICS Dominates over $\boxed{2}$

SO(10)

① $(m_0, m_{1/2}) // \tan\beta$	$B(\mu \to e\gamma)$ $\mu > 0$	$\mu < 0$
$(600, 300) // 10$	3.3×10^{-12}	9.8×10^{-12}
$(450, 300) // 10$	2.7×10^{-11}	4.6×10^{-11}
$(500, 250) // 10$	5.9×10^{-12}	1.9×10^{-11}
$(100, 400) // 10$	1.02×10^{-8}	1.02×10^{-8}
$(1000, 250) // 10$	1.6×10^{-13}	5.6×10^{-12}
$(400, 300) // 20$	9.5×10^{-12}	3.8×10^{-11}

For G(224) Rates are smaller by (4 to 6)

Expt : $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$

Should be seen with improvement by 10-100

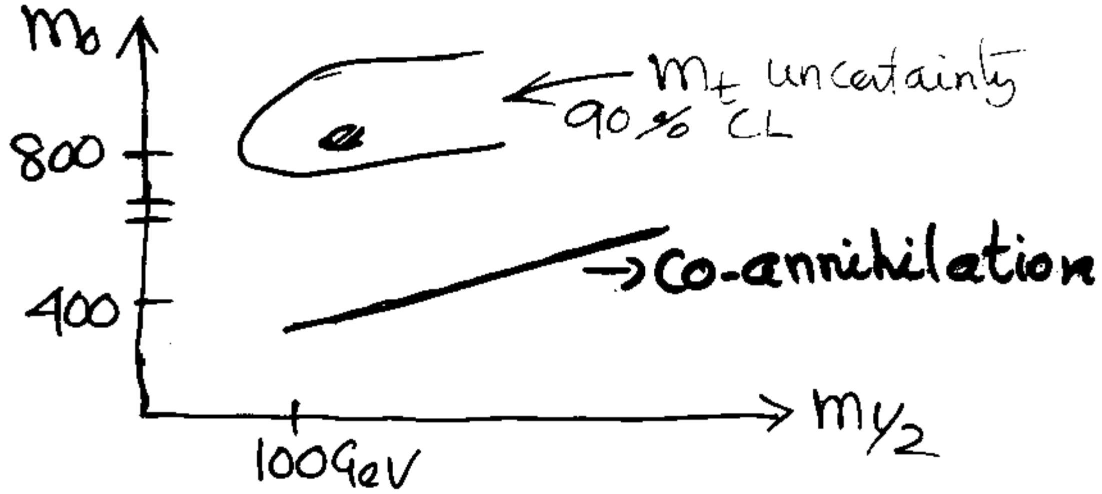
Being Planned.

② Similarly $(\mu N \to e N)$

③ $B(\tau \to \mu\gamma)_{Th} \approx (2 \times 10^{-8}$ to $1 \times 10^{-9})$ LHC Super B

④ $B(\tau \to e\gamma) \approx 10^{-12}$ to 2×10^{-11} → Inaccessible at Present

WMAP CONSTRAINT WITH NEUTRALINO CDM



(∃ other Alternatives : ① Higgs - squark - slepton Non-universality // ② Axion CDM ($\cancel{R}$))

$\underline{\tan\beta = 10}$

$(m_0, m_{1/2})$	$B(\mu \to e\gamma)$ $\mu > 0$	$\mu < 0$
$(800, 250)$ → $m_{sq} \approx 1$ TeV, $x \approx 0.5$	1×10^{-12}	5.5×10^{-12}
$(100, 440)$ → $m_{sq} \approx 1.17$ TeV, $x \approx 1.2$	1.4×10^{-8}	1.4×10^{-8}
$(500, 250)$ $m_{sq} \approx 830$ GeV, $x \approx .8$	5.9×10^{-12}	1.9×10^{-11}

VI ν Masses $\leftrightarrow$ Leptogenesis Within The Same G(224) // SO(10) - Framework

JCP (hep-ph/2002, Phy Rev 2003)

Idea: (Fukugita, Yanagida // Sphaleron → Kuzmin Rubakov & Shaposhnikov)

Inflation ⟶ Reheat ⟶ Superheavy ν_R's (N_1)
From Thermal Bath // Or Non-Thermal Inflaton Decay

$\nu_R \to l + H$ & $\bar{l} + \bar{H}$ (& SUSY Analogs)

LEPTON ASYM Parameter
$$\varepsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \mathrm{Im}\left[(M_D^\dagger M_D)_{j1}\right]^2 f\left(M_j^2/M_1^2\right)$$

$M_D = M^\nu_{Dirac}$ in a basis where M^ν_R is diagonal

$$\rightarrow \quad \varepsilon_1 \approx \left(\frac{1}{8\pi}\right)\left(\frac{m_t^0}{v_{EW}}\right)^2 \left|(\sigma + 3\epsilon) - y\right|^2 \left(\sin 2\phi_{21}\right)(-3)\left(\frac{M_1}{M_2}\right)$$

ϕ_{21} = Eff. Phase From (Entries in) Dirac & Majorana Neutrino Mass Matrices

Note ε_1 depends crucially on Both M^ν_{Dirac} & $M^{(\nu_R)}_{Majorana}$. which are Known within Based inv

$$\boxed{Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} = \kappa\, \epsilon_1 / g^*} \quad (g^*_{MSSM} \approx 228$$

Wash out Factor } $\kappa \approx 10^{-4} \times (\tilde{m}_1/eV)^{-1} \longrightarrow$ Thermal (Buchmuller et al, Giudice et al)

$\approx 1 \longrightarrow$ Non-Thermal

$$\tilde{m}_1 \equiv (m_D^\dagger m_D)_{11}/M_1 \sim (8-3) \times 10^{-3}\, eV$$

Lepton Excess $\xrightarrow[\text{Sphalerons}]{EW}$ $\boxed{Y_B = C\, Y_L \approx -Y_L/3}$

$\Downarrow$

$$\left[Y_B / \sin 2\phi_{21}\right]_{Theory} \begin{cases} \xrightarrow[T_{RH} \sim \text{few} \times 10^{9}\, GeV]{\text{THERMAL}} (10-30) \times 10^{-1\cdot} \\ \xrightarrow[T_{RH} \sim 10^{7}-10^{8}\, GeV]{\text{Non-Thermal}} (10-100) \times 10^{-1} \end{cases}$$

Goes Very Well with

$$(Y_B)_{WMAP} \approx (8.7 \pm 0.4) \times 10^{-11} \quad !$$

for $\boxed{\sin 2\phi_{21}} \approx (1 - 1/3)$ (Thermal) // $\approx (1/10 - 1)$ (Non-Thermal)

in accord with Gravitino Constraint.

→ A Unified Description of fermion Masses, ν-Oscillati[ons], CP & Flavor-Violations, & Baryogenesis (via leptogenesis) Within a single predictive Framew[ork]

CONCLUSION

SUSY Grand Unification based on Eff. Symmetry

G(224) or SO(10) Most Promising :

Evidence

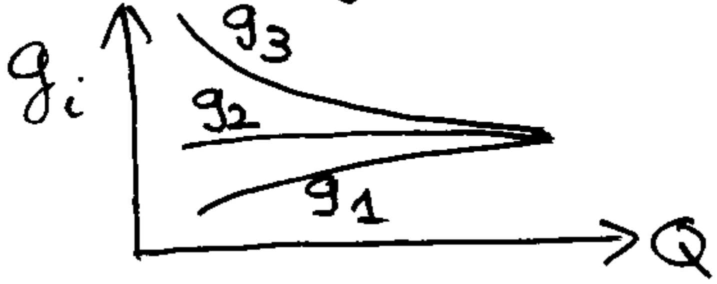
(i) Quantum NOS. of Members in a family

(ii) Quantization of Q_{em} // $\boxed{Q_{e^-} = -Q_{proton}}$

(iii) ν oscillations // $\sqrt{\Delta m^2(\nu_\mu \nu_\tau)} \approx 1/20\ eV$

(iv) $\boxed{\Theta_{\nu_\mu \nu_\tau} \text{ maximal} \approx \pi/4,\ V_{cb} \text{ minimal} \approx 0.04}$

(V) Gauge Coupling Unification



(Vi) Baryogenesis Via Leptogenesis

These agree quantitatively !
Hard to believe, this could be a Coincidence.

Conclusion (Contd.)

The tiny mass of the neutrino ($m_\nu < 1\,eV$) holds the key to:

(i) THE UNIFICATION SCALE
$M_X \approx 2\times10^{16}$ GeV
dist $\approx 10^{-30}$ cm
$\rightarrow M_R^\nu \sim 10^{15}$ GeV

(ii) NATURE OF THE UNIFICATION SYMMETRY
SU(4) - Color
$m(\nu^\tau)_{Dirac} = m_{top}^{(M_X)}$
$\rightarrow$ G(224)/SO(10) Route, ~~SU(5)~~

(iii) ORIGIN OF AN EXCESS OF MATTER OVER ANTIMATTER $\rightarrow$ OUR OWN ORIGIN!

MANY TESTS in $\not{C}\not{P}$ & Flavor $\{B_d \to \phi K_S,\ \mu \to e\gamma$, edm

Proton Decay and Supersymmetry remain The Two Missing Links.