Phenomenology of neutrino oscillations

Part II: future long-baseline experiments

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Outline

- Introduction
  Overview of upcoming LBL exps. and ideas for the future
- General comments on the LBL osc. probability
- Leading atmospheric parameters
- The measurement of θ_{13}
  reactor vs accelerator experiments
- The problem of degeneracies
  combined determination θ_{13}, δ_{CP}, the octant of θ_{23} and the neutrino mass hierarchy

LBL experiments in the next ten years

Label	L	$\langle E_\nu \rangle$	mass	channel
Conventional beam experiments:				
MINOS	$735\,\text{km}$	$3\,\text{GeV}$	$5.4\,\text{kt}$	$\nu_\mu \to \nu_\mu, \nu_e$
OPERA	$732\,\text{km}$	$17\,\text{GeV}$	$1.65\,\text{kt}$	$\nu_\mu \to \nu_e, \nu_\mu, \nu_\tau$
Reactor experiments with near and far detectors:				
D-Chooz	$1.05\,\text{km}$	$\sim 4\,\text{MeV}$	$\sim 10\,\text{kt}$	$\bar\nu_e \to \bar\nu_e$
Reactor-II	$1.70\,\text{km}$	$\sim 4\,\text{MeV}$	$\sim 200\,\text{kt}$	$\bar\nu_e \to \bar\nu_e$
Off-axis superbeams:				
T2K	$295\,\text{km}$	$0.76\,\text{GeV}$	$22.5\,\text{kt}$	$\nu_\mu \to \nu_e, \nu_\mu$
NOνA	$812\,\text{km}$	$2.22\,\text{GeV}$	$50\,\text{kt}$	$\nu_\mu \to \nu_e, \nu_\mu$

T2K: Tokai (JPARC) to Kamioka (SK) 22.5 kt water Cherenkov

NOνA: 50 kt TASD detector, off-axis angle of $0.72°$

LBL experiments beyond ten years

- superbeam upgardes $(\nu_\mu \to \nu_e, \nu_\mu) + (\bar\nu_\mu \to \bar\nu_e, \bar\nu_\mu)$
  T2HK: beam 0.77 $\to$ 4 MW, SK (22.5 kt) $\to$ HK (500 kt)
  NOνA: proton driver, second detector
- new superbeams $(\nu_\mu \to \nu_e, \nu_\mu) + (\bar\nu_\mu \to \bar\nu_e, \bar\nu_\mu)$
  SPL: CERN to $\sim$Mt water Cerencov at Frejus (130 km)
  BNL: broadband beam, $E_\nu \sim \text{GeV}, L \gtrsim 1000\,\text{km}$
- beta beams (**βB**) $(\nu_e \to \nu_\mu) + (\bar\nu_e \to \bar\nu_\mu)$
  low γ **βB** z.B. CERN-Frejus ($E_\nu \sim 0.4\,\text{GeV}$) or
  high γ **βB** (longer BL)
- neutrino factory (**NuFact**) $(\nu_e, \nu_\mu \to \nu_\mu) + (\bar\nu_e, \bar\nu_\mu \to \bar\nu_\mu)$
  $E_\nu \sim 20 - 50\,\text{GeV},\ 1000\,\text{km} \lesssim L \lesssim 7000\,\text{km}$

Simulation of future experiments

Calculation of event rates for given experiment:

$$N_i(\boldsymbol{\theta}) = \Phi \cdot \sigma \cdot R \cdot \epsilon \cdot P(\boldsymbol{\theta})$$

Φ: neutrino flux
σ: detection cross section
R: energy resoltion
ϵ: efficiencies

$P(\boldsymbol{\theta})$: 3-flavour osc. prob., $\boldsymbol{\theta} = (\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{23}, \theta_{13}, \delta)$

simulate data for "true values" $\hat{\boldsymbol{\theta}}$: $\hat{N}_i = N_i(\hat{\boldsymbol{\theta}})$

$\chi^2(\boldsymbol{\theta}; \hat{\boldsymbol{\theta}}) \quad \rightarrow$ allowed regions for $\boldsymbol{\theta}$

including systematical errors, correlations, degeneracies

GLoBES: http://www.ph.tum.de/~globes/

P. Huber, M. Lindner, W. Winter, hep-ph/0407333

General comments on LBL oscillation probabilities

LBL probabilities

2-flavour osc probability in constant matter:

$$P = \frac{\sin^2 2\theta}{C^2} \sin^2 C\Delta\,, \quad C \equiv \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

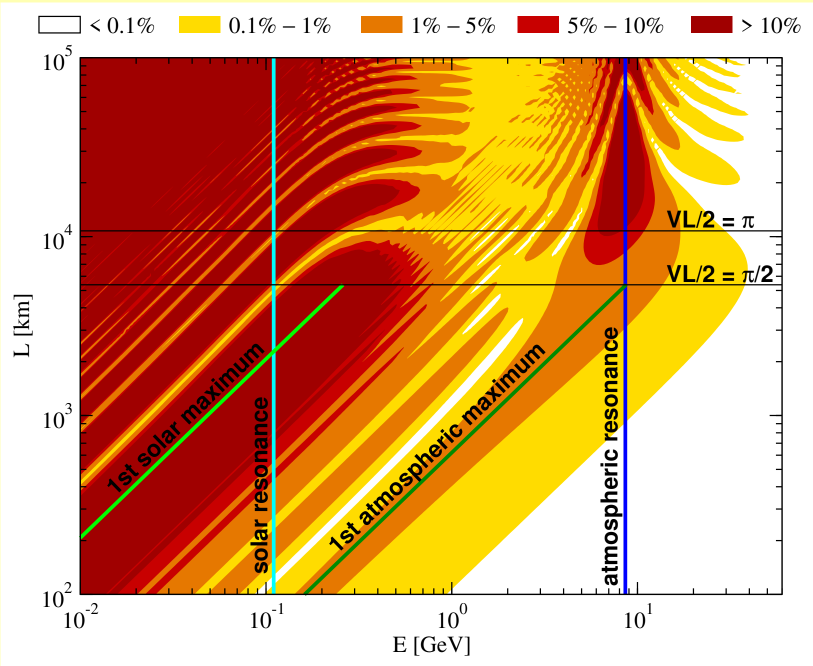
with $\Delta \equiv \frac{\Delta m^2 L}{4E}$, $A \equiv \frac{2EV}{\Delta m^2}$

vacuum: $V = 0, A = 0, C = 1, P_{vac} = \sin^2 2\theta \sin^2 \Delta$

$\Delta = (n + \frac{1}{2})\pi$: oscillation maximum in vacuum

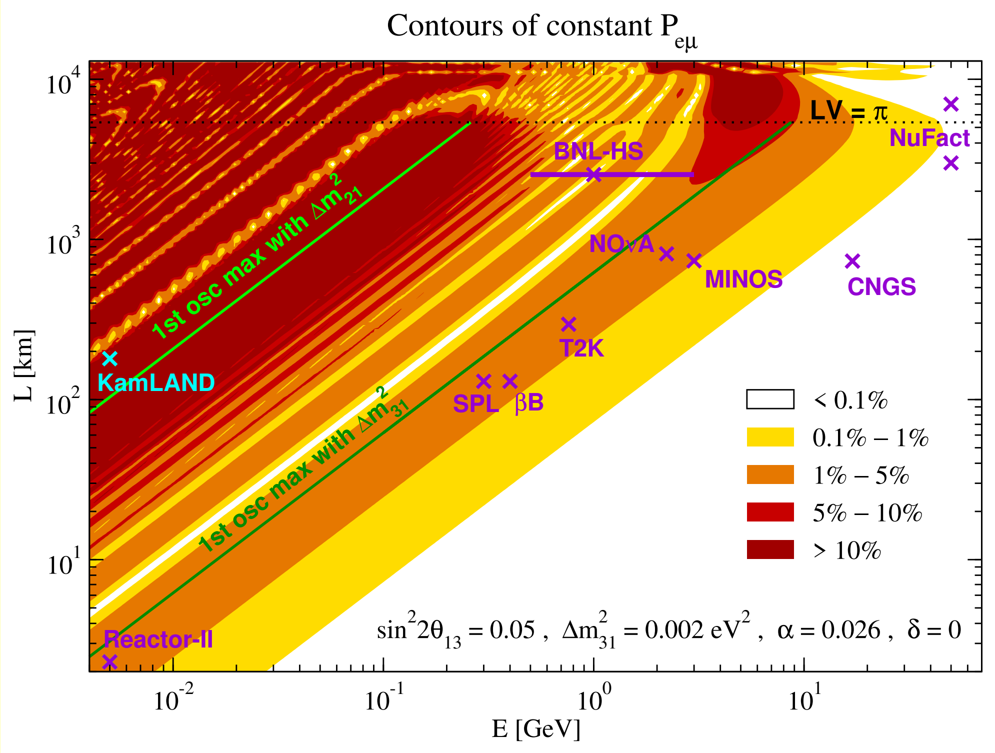
$A = \cos 2\theta$: resonance, $E_{res} = \cos 2\theta \frac{\Delta m^2}{2V}$

LBL probabilities – const. matter



contours of $P_{e\mu}$ for $\sin^2 2\theta_{13} = 0.02$, $\Delta m^2_{31} = 0.002$eV^2, $\alpha = 0.03$, $\theta_{12} = 33°$, $\theta_{23} = 45°$, $\delta_{\rm CP} = 0$, and constant matter dentsity of $\rho = 3$ g/cm^3

LBL probabilities – earth matter profile



Improving on the 'atmospheric' parameters

θ_{23} and $|\Delta m^2_{31}|$

Atmospheric parameters $|\Delta m^2_{13}|$ *and* θ_{23}

determined in ν_μ-disappearance experiments:

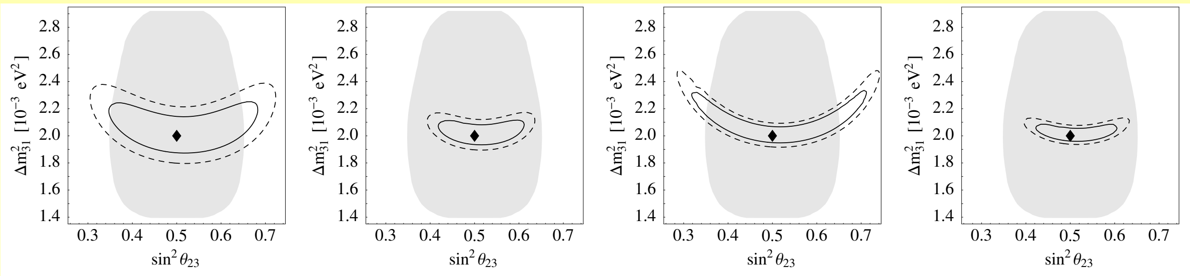
$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \mathcal{O}(\alpha, s_{13}\alpha, s_{13}^2)$$

- large number of events $\rightarrow$ small stat. errors
- $\sin^2 2\theta_{23}$ large $\rightarrow$ large effect

Atmospheric parameters $|\Delta m^2_{13}|$ *and* θ_{23}

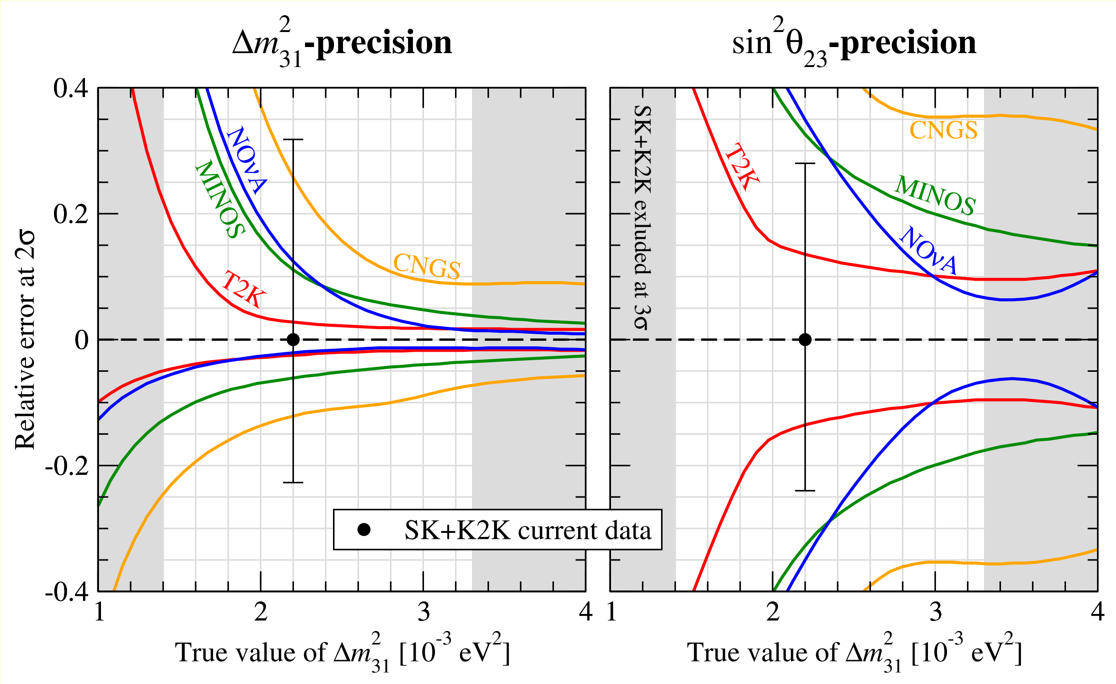
ν_μ-disappearance in LBL accelerator experiments

MINOS+CNGS **T2K** **NOνA*** combined



* **NOνA** with TASD slightly better

Atmospheric parameters $|\Delta m^2_{13}|$ *and* θ_{23}



Atmospheric parameters $|\Delta m^2_{13}|$ *and* θ_{23}

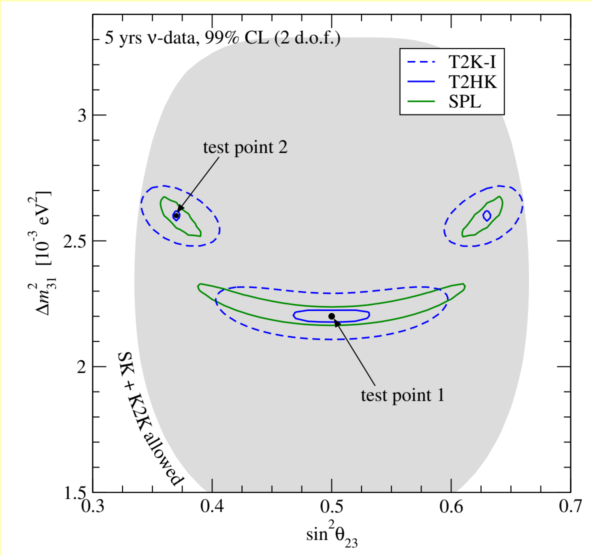
going beyond 10 yrs:

subsequent generation of LBL experiments like

T2HK, **SPL**, **NuFact**

will provide a

sub-% determination of $|\Delta m^2|$ and $\sin^2 2\theta_{23}$!



Atmospheric parameters $|\Delta m^2_{13}|$ *and* θ_{23}

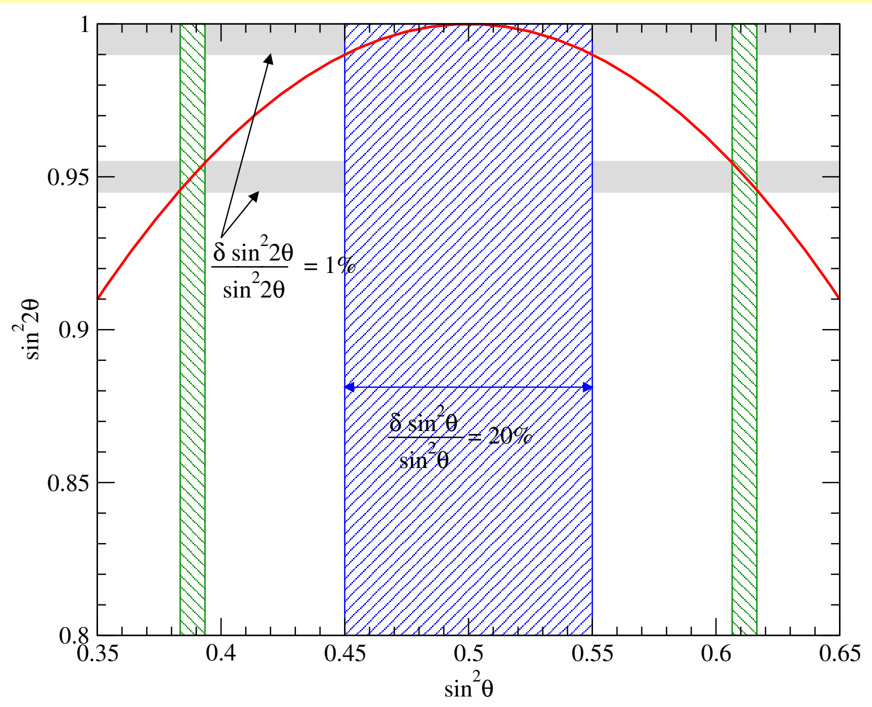
$$P_{\mu\mu} \approx 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

$|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$ are measured very precisely by energy spectral information

BUT:

- no information on sign(Δm^2_{31})
- cannot distinguish between first and second octant, two possibilities: θ_{23} and $\pi - \theta_{23}$
- if $\sin^2 2\theta_{23} \approx 1$ accuracy on $\sin^2 \theta_{23}$ becomes poor

Atmospheric parameters $|\Delta m^2_{13}|$ *and* θ_{23}



Determination of θ_{13}

Why is it so interesting?

- naively one would expect $\theta_{12} \sim \theta_{23} \sim \theta_{13}$
  $\rightarrow \theta_{13}$ around the corner
- $\theta_{13} \ll 1$ hint for some symmetry
- relatively large θ_{13} opens the possibilty to observe generic 3-flavour effects (CP-violation)

Measuring θ_{13}

- $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance reactor experiments with near and far detectors
  D-Chooz, KASKA, Daya Bay, Angra, Braidwood, RENO
- LBL $\nu_\mu \rightarrow \nu_e$ (or $\nu_e \rightarrow \nu_\mu$) appearance experiments
  MINOS, **T2K**, **NOνA**, **T2HK**, **BNL**, **SPL**, **βB**, **NuFact**

Measuring θ_{13} by $\nu_\mu \to \nu_e$ at beams

The measurement of θ_{13} with the $\nu_\mu \to \nu_e$ appearance channel suffers from correlations and degeneracies:

G.L. Fogli, E. Lisi, Phys. Rev. D54 (1996) 3667
J. Burguet-Castell et al., Nucl. Phys. B608 (2001) 301
H. Minakata, H. Nunokawa, JHEP 10 (2001) 001
V.Barger, D.Marfatia, K.Whisnant, Phys. Rev. D65 (2002) 073023; D66 (2002) 053007
P.Huber, M.Lindner, W.Winter, Nucl. Phys. B645 (2002) 3; Nucl. Phys. B654 (2003) 3
and many more

Not $\sin^2 2\theta_{13}$, but only a specific parameter combination is measured very accurately

The LBL appearance oscillation probability

$$\begin{aligned} P_{\mu e} \simeq\ & \sin^2 2\theta_{13}\, \sin^2\theta_{23}\, \frac{\sin^2(1-A)\Delta}{(1-A)^2} \\ + & \sin 2\theta_{13}\, \hat{\alpha}\, \sin 2\theta_{23}\, \frac{\sin(1-A)\Delta}{1-A}\frac{\sin A\Delta}{A}\cos(\Delta + \delta_{\rm CP}) \\ + & \hat{\alpha}^2\, \cos^2\theta_{23}\, \frac{\sin^2 A\Delta}{A^2} \end{aligned}$$

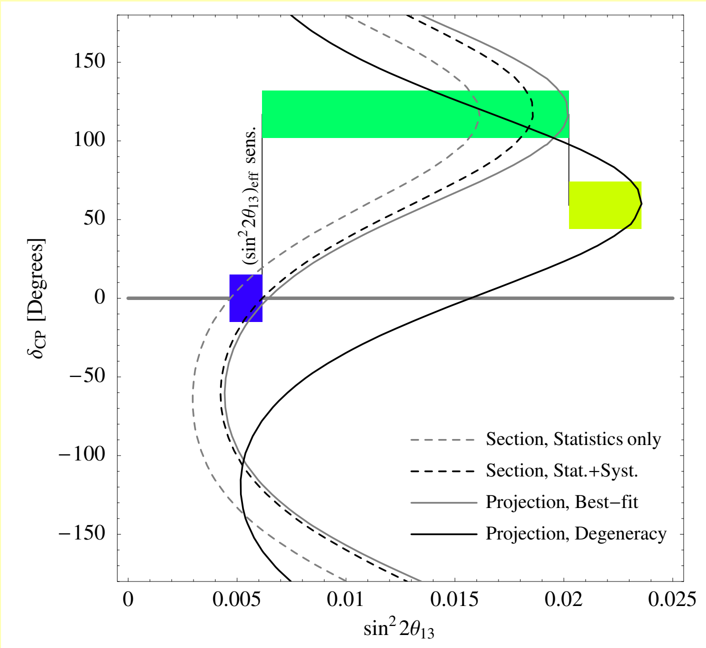
with

$$\Delta \equiv \frac{\Delta m^2_{31} L}{4E_\nu}\,, \quad \hat{\alpha} \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\sin 2\theta_{12}\,, \quad A \equiv \frac{2E_\nu V}{\Delta m^2_{31}}$$

anti-ν: $\delta_{\rm CP} \to -\delta_{\rm CP}$, $A \to -A$, $P_{e\mu}$: $\delta_{\rm CP} \to -\delta_{\rm CP}$
other hierarchy: $\Delta \to -\Delta$, $A \to -A$, $\hat{\alpha} \to -\hat{\alpha}$

Measuring θ_{13} *by* $\nu_\mu \to \nu_e$ *at beams*

T2K: $\delta - \theta_{13}$ correlation:



Measuring $\sin^2 2\theta_{13}$ *at reactors*

“Clean” measurement of $\sin^2 2\theta_{13}$:

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \, \sin^2 \frac{\Delta m^2_{31} L}{4E_\nu} + \left(\frac{\Delta m^2_{21} L}{4E_\nu} \right)^2 \cos^4 \theta_{13} \sin^2 2\theta_{12}$$

last term negligible for $\frac{\Delta m^2_{31} L}{4E_\nu} \sim \pi/2$ and $\sin^2 2\theta_{13} \gtrsim 10^{-3}$

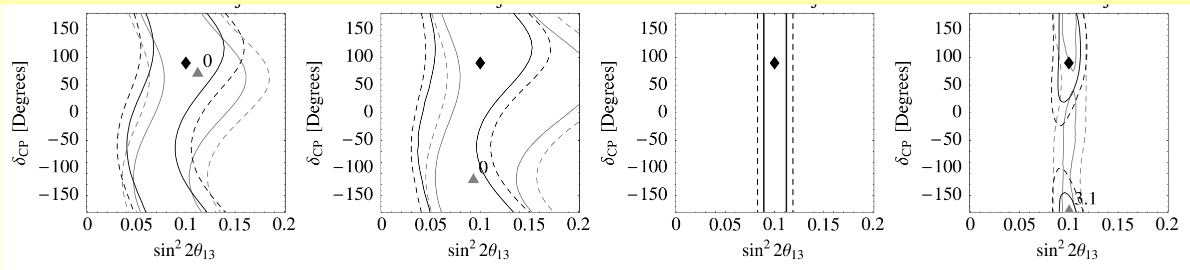
determination of θ_{13} is free of correlations and degeneracies

P. Huber, M. Lindner, T. Schwetz and W. Winter, Nucl. Phys. B **665** (2003) 487 [hep-ph/0303232]
H. Minakata, H. Sugiyama, O. Yasuda, K. Inoue and F. Suekane, Phys. Rev. D **68** (2003) 033017

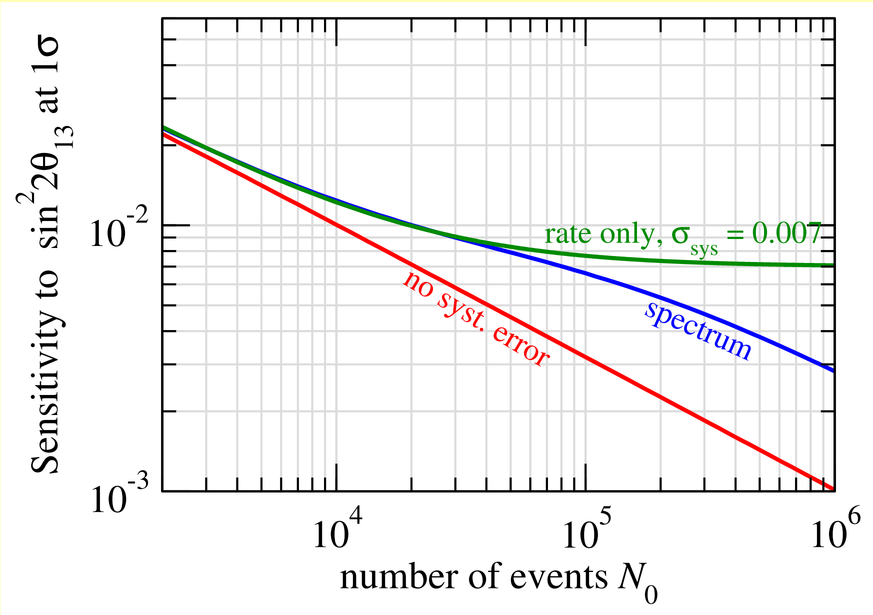
Measuring $\sin^2 2\theta_{13}$ *at beams or reactors*

assume $\sin^2 2\theta_{13} = 0.1$

T2K **NOνA** **Reactor-II** combined

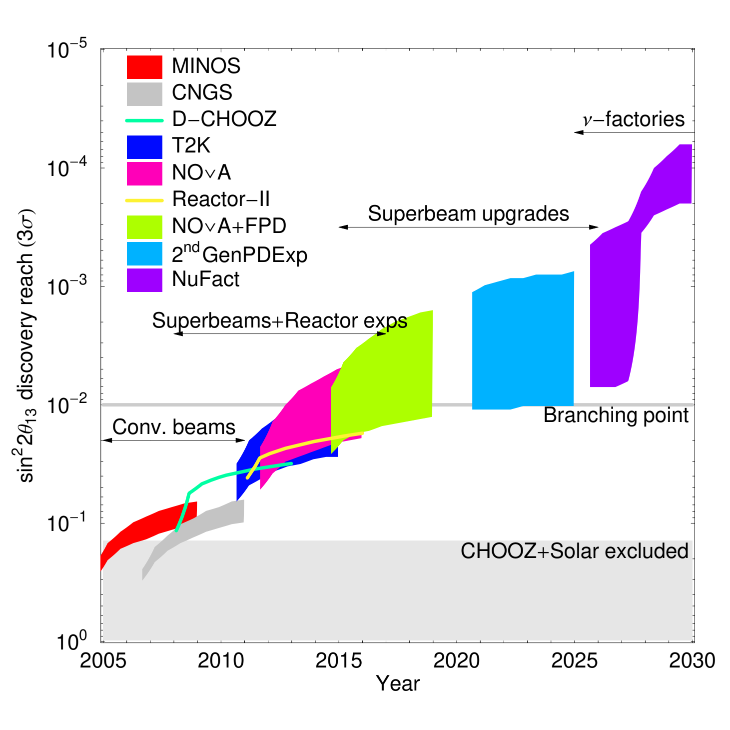


Measuring $\sin^2 2\theta_{13}$ *at reactors*



rate only limit at $n\sigma$: $\sin^2 2\theta_{13} < n\sqrt{\frac{1}{N_0} + \sigma^2_{\rm sys}}$

The $\sin^2 2\theta_{13}$*-sensitivity evolution*



plot by W. Winter from Albrow et al., hep-ex/0509019

$\Delta m^2_{31} = +2.5 \times 10^{-3}$ eV^2
$\sin^2 2\theta_{23} = 1$

FPD = Fermilab Proton Driver
LBL exps.: neutrinos only
$2^{\rm nd}$GenPDExp = T2HK
NuFact anti-ν after 2.5 yr

Determination of θ_{13}, $\delta_{\rm CP}$, sgn(Δm^2_{31})

or the problem of

Degeneracies

Degeneracies

Suppose that the true osc. params. in nature are

$$\hat{\boldsymbol{\theta}} = (\Delta\hat{m}^2_{21}, \Delta\hat{m}^2_{31}, \hat{\theta}_{12}, \hat{\theta}_{23}, \hat{\theta}_{13}, \hat{\delta}_{\mathrm{CP}})$$

Then an experiment will observe a number of events $\hat{N}$ corresponding to the probability $\hat{P}_{\mu e} = P_{\mu e}(\hat{\boldsymbol{\theta}})$

For fixed $\hat{N}$ there are other values of $\theta_{13} \neq \hat{\theta}_{13}$ and $\delta_{\mathrm{CP}} \neq \hat{\delta}_{\mathrm{CP}}$, which lead to the same osc. probability:

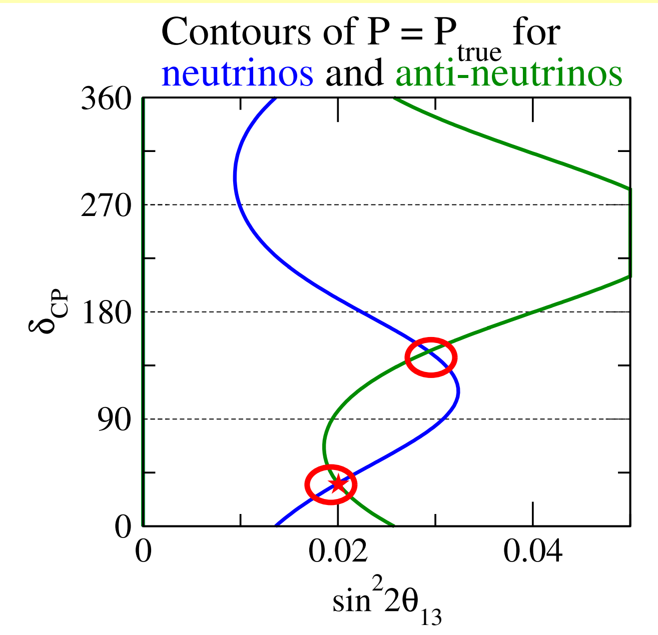
$$\hat{P}_{\mu e} = P_{\mu e}(\theta_{13}, \delta_{\mathrm{CP}})$$

and similar for anti-neutrinos:

$$\hat{\bar{P}}_{\mu e} = \bar{P}_{\mu e}(\theta_{13}, \delta_{\mathrm{CP}})$$

Degeneracies

numerical example:



$\Delta\hat{m}^2_{21} = 7.9 \times 10^{-5}$eV^2
$\Delta\hat{m}^2_{31} = 2.5 \times 10^{-3}$eV^2
$\sin^2\hat{\theta}_{12} = 0.3$
$\sin^2\hat{\theta}_{23} = 0.4$
$\sin^2 2\hat{\theta}_{13} = 0.02$
$\hat{\delta}_{\mathrm{CP}} = 36°$

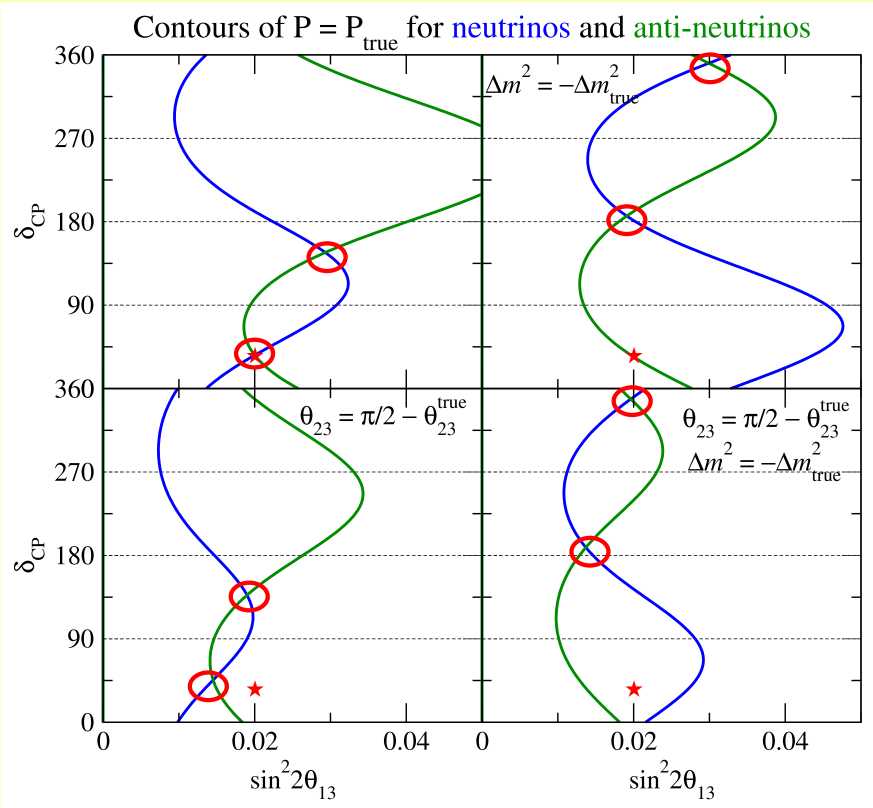
$E_\nu = 2.2$ GeV
$L = 812$ km
(**NOνA**)

Degeneracies

intrinsic or $\delta_{\rm CP} - \theta_{13}$ deg.: $\hat{P} = P(\Delta\hat{m}^2_{31}, \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP})$ $\hat{\bar{P}} = \bar{P}(\Delta\hat{m}^2_{31}, \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP})$	hierarchy or ${\rm sgn}(\Delta m^2_{31})$ deg.: $\hat{P} = P(-\Delta\hat{m}^2_{31}, \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP})$ $\hat{\bar{P}} = \bar{P}(-\Delta\hat{m}^2_{31}, \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP})$
octant or $(\theta_{23}, \pi/2 - \theta_{23})$ deg.: $\hat{P} = P(\Delta\hat{m}^2_{31}, \frac{\pi}{2} - \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP})$ $\hat{\bar{P}} = \bar{P}(\Delta\hat{m}^2_{31}, \frac{\pi}{2} - \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP})$	mixed degeneracy: $\hat{P} = P(-\Delta\hat{m}^2_{31}, \frac{\pi}{2} - \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP}$ $\hat{\bar{P}} = \bar{P}(-\Delta\hat{m}^2_{31}, \frac{\pi}{2} - \hat{\theta}_{23}, \theta_{13}, \delta_{\rm CP}$

$\longrightarrow$ **eight-fold degeneracy**

The eight-fold degeneracy



The eight-fold degeneracy

- ambiguities in the determination of θ_{13} and $\delta_{\rm CP}$
- can involve an ambiguity between CP conserving and CP violating values of $\delta_{\rm CP}$
- the sign(Δm^2_{31}) is not determined (neutrino mass hierarchy)
- the octant of θ_{23} is not determined

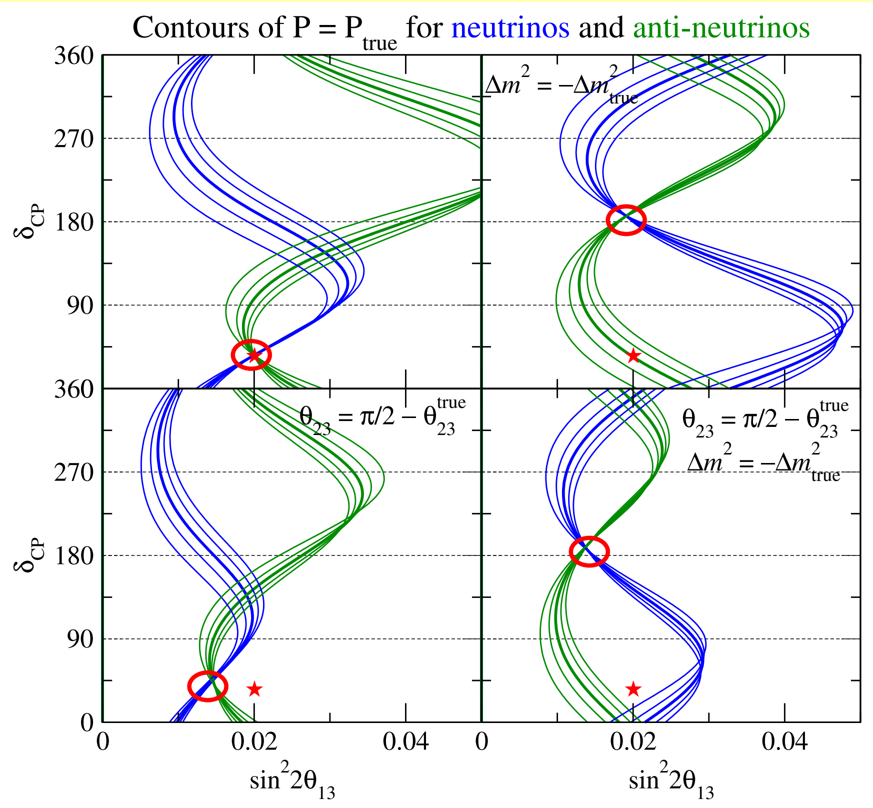
Resolving the degeneracies

several possibilities to resolve the degeneracies are known:

- combining information from detectors at different baselines
- using additional oscillation chanels ($\nu_e \to \nu_\tau$)
- spectral information (broadband beam)
- adding information on θ_{13} from a reactor experiment
- adding information from (Mt scale) atmospheric neutrino experiments
- ...

Spectral information and the intrinsic degeneracy

Spectral information

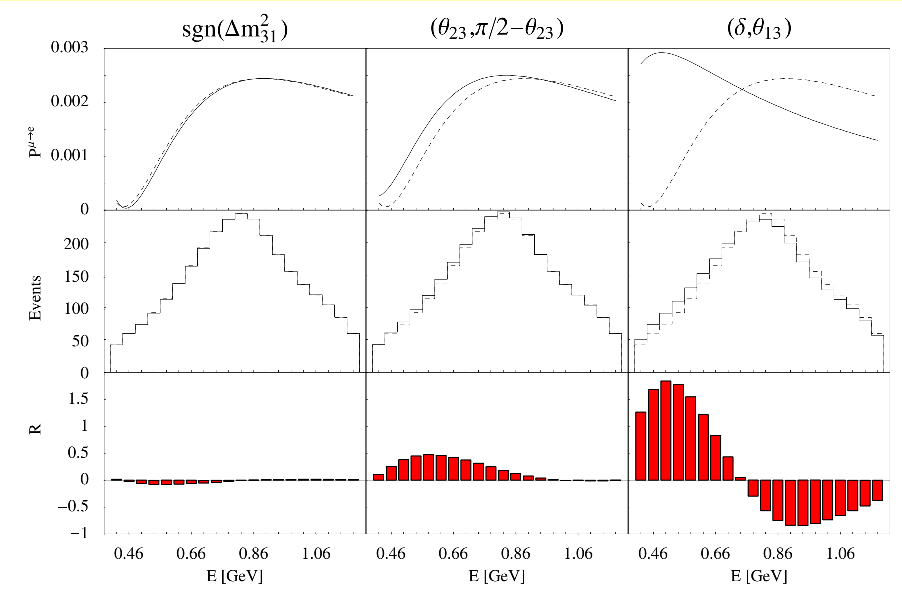


$E_\nu = 2.2$ GeV $\pm 10\%, \pm 20\%$

$L = 812$ km

Degeneracies and **T2K**

Example from actual simulation of experiment: **T2K**



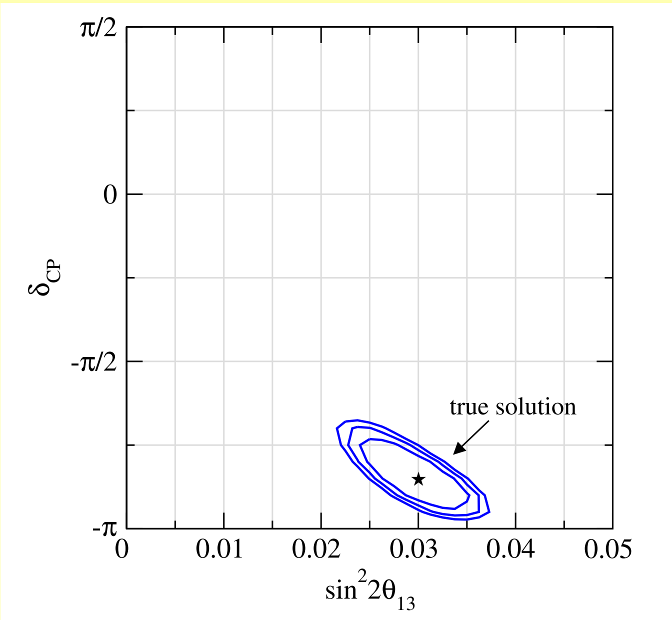
$\sin^2 2\theta_{13} = 0.$
$\delta_{CP} = \pi/4$
$\sin^2 \theta_{23} = 0.3$

$R = \frac{N_i^{tr} - N_i^{deg}}{\sqrt{(N_i^{tr} + N_i^{deg})/}}$

P.Huber, M.Lindner, W.Winter, Nucl. Phys. B645 (2002) 3

Degeneracies and **T2K**

Example from actual simulation of experiment: **T2K**

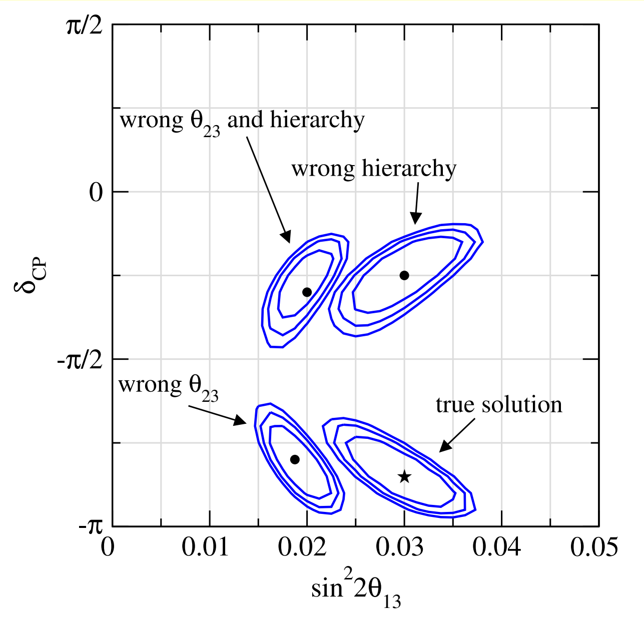


True values:
$\sin^2 2\theta_{13} = 0.03$
$\delta_{CP} = -0.85\pi$
$\sin^2 \theta_{23} = 0.4$
$\Delta m^2_{31} = 2.2 \times 10^{-3}$eV^2

spectral information resolves intrinsic deg.

Degeneracies and **T2K**

Example from actual simulation of experiment: **T2K**



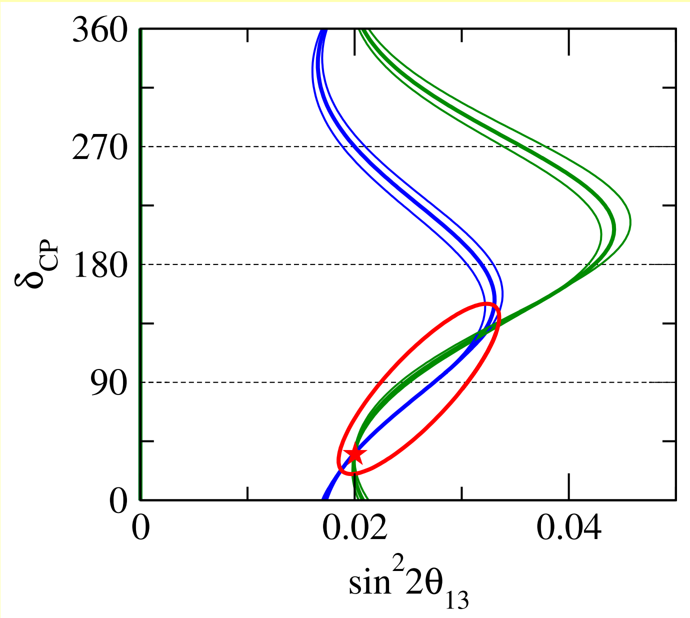
True values:
$\sin^2 2\theta_{13} = 0.03$
$\delta_{\rm CP} = -0.85\pi$
$\sin^2\theta_{23} = 0.4$
$\Delta m^2_{31} = 2.2 \times 10^{-3}\,$eV^2

spectral information resolves intrinsic deg.

octant, hierarchy, and mixed degeneracies remain

Spectral information

WARNING: does not work in all cases!



$E_\nu = 20$ GeV
$\pm 20\%$

$L = 3000$ km

(**NuFact**)

On the hierarchy and octant degeneracies

Hierarchy and octant degeneracies

consider experiments with small matter effects

$$A \equiv \frac{2EV}{\Delta m^2_{31}} \ll 1 \quad \rightarrow \quad E \ll E_{res}$$

at the oscillation maximum $\Delta \simeq \pi/2$ this implies

$$VL \ll \pi \quad \rightarrow \quad L \ll 5400\,\mathrm{km}$$

octant degeneracy affects mainly θ_{13}:

$$\sin^2 2\theta_{13}^{\mathrm{fake}} \approx \tan^2 \hat{\theta}_{23} \sin^2 2\hat{\theta}_{13}$$

hierarchy degeneracy affects mainly δ_{CP}:

$$\delta_{\mathrm{CP}}^{\mathrm{fake}} \approx \pi - \hat{\delta}_{\mathrm{CP}}$$

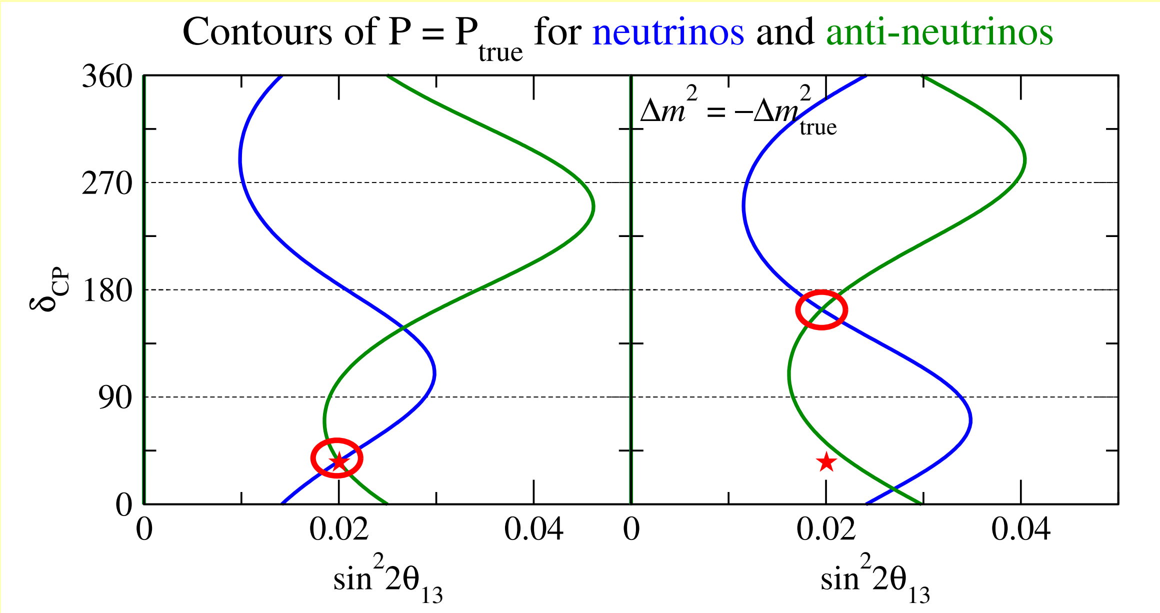
Resolving the hierarchy degeneracy

the key to resolve the hierarchy degeneracy is the matter effect

$\longrightarrow$ need large baselines

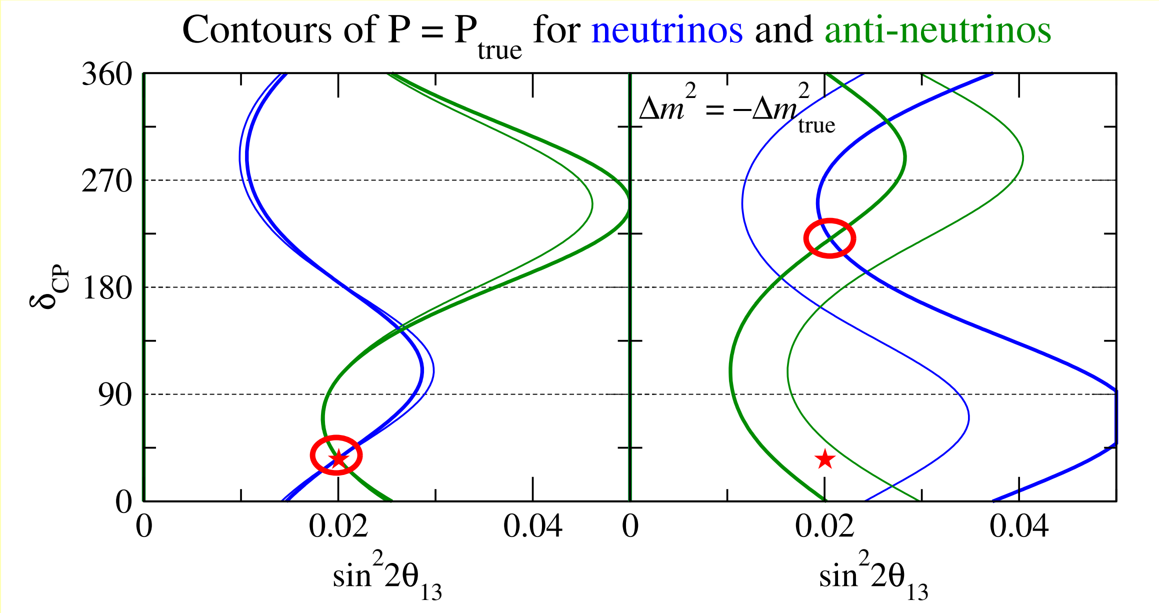
$\Rightarrow$

Resolving the hierarchy degeneracy



$L/E \approx 390\,\mathrm{km/GeV}$, $L = 295\,\mathrm{km}$ (**T2K**)

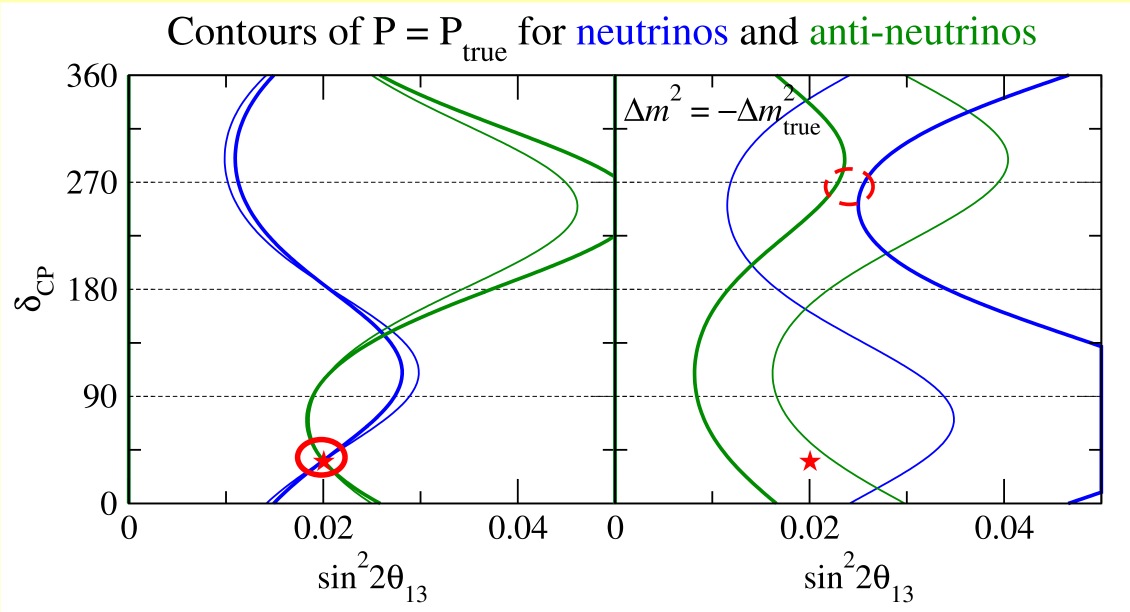
Resolving the hierarchy degeneracy



$L/E \approx 390\,\mathrm{km/GeV}$, $L = 1180\,\mathrm{km}$ (increase L with L/E const)

"Super-NOνA" Mena-Requejo, Palomares-Ruiz, Pascoli, hep-ph/0504015

Resolving the hierarchy degeneracy



$L/E \approx 390\,\mathrm{km/GeV}$, $L = 1620\,\mathrm{km}$

for very long baselines the sign-degeneracy disappears

Resolving the hierarchy degeneracy

Another possibilities is to use a second detector at different L/E:

- different L but same E
  e.g., **T2K** with additional detector in Korea at the same off-axis angle, i.e., same E
- similar L but different E (off-axis angle)
  discussed e.g. as a possible **NOνA** upgrade

The silver channel at **NuFact**

golden channel: $\nu_e \to \nu_\mu$, silver channel: $\nu_e \to \nu_\tau$

Donini, Meloni, Migliozzi, hep-ph/0206034; Autiero et al., hep-ph/0305185

$P_{e\mu} \to P_{e\tau}: \quad s^2_{23} \leftrightarrow c^2_{23}, \; \sin 2\theta_{23} \to -\sin 2\theta_{23} \qquad \Rightarrow$

Akhmedov et al., JHEP **0404**, 078 (2004) [hep-ph/0402175]

$$P_{e\mu} + P_{e\tau} \simeq \sin^2 2\theta_{13} \, \frac{\sin^2 (1-A)\Delta}{(1-A)^2} + \hat{\alpha}^2 \, \frac{\sin^2 A\Delta}{A^2}$$

θ_{23} and $\delta_{\rm CP}$ drop out: deg.-free measurment of θ_{13}

BUT: in practice challenging:
ν_τ detection is very difficult, low efficiencies

$P_{e\mu} + P_{e\tau}$ is not directly observed

The "magic baseline"

consider the condition (see osc. prob $\Rightarrow$)

$$A\Delta \equiv \frac{VL}{2} = \pi$$

$$L_{\rm magic} \approx 7630\,{\rm km} \quad \text{for} \quad \rho \approx 4.3\,{\rm g/cm^3} = {\rm const}$$

numerically:

$$L_{\rm magic} \approx 7250\,{\rm km}$$

independent of osc. parameters and E_ν!

The "magic baseline"

for $A\Delta = \pi$ one has $\Rightarrow$

$$P_{e\mu} \simeq \sin^2 2\theta_{13}\,\sin^2\theta_{23}\,\frac{\sin^2\Delta}{(1-A)^2}$$

- dependence of $\delta_{\rm CP}$ disappears $\rightarrow$ determ. θ_{13}
- very large matter effect ($|A| \approx 2.6$ for $E \approx 20$ GeV) $\rightarrow$ hierarchy determination by $\nu/\bar\nu$ comparison
- no CP violation measurement $\rightarrow$ need second BL (e.g. 3000 km)

P. Huber and W. Winter, Phys. Rev. D **68** (2003) 037301 [hep-ph/0301257]

Resolving the octant degeneracy

resolving the octant degeneracy in LBL experiments is difficult → need complementary information, e.g. from

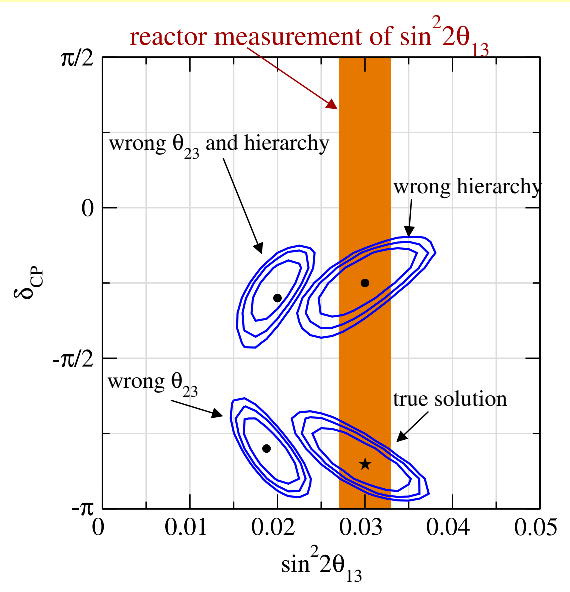
- reactor experiments, or
- atmospheric neutrino experiments.

the best way to resolve the octant degeneracy is

$$\theta_{23}^{\rm true} = 45^\circ\,!$$

Resolving the octant degeneracy

combination of LBL and reactor experiments



works only for relatively large θ_{13}

and a large reactor experiment

hierarchy deg. remains

K. B. McConnel and M. H. Shaevitz, hep-ex/0409028; K. Hiraide et al., hep-ph/0601258

Combining LBL and ATM experiments

In LBL experiments using a Mt scale water Cerencov detector (e.g. **T2HK**, CERN-MEMPHYS β**B**, **SPL**) a large amount of atmospheric neutrino data comes for free!

Subleading effects in atmospheric data can provide complementary information to resolve degeneracies.

Huber, Maltoni, Schwetz, hep-ph/0501037

θ_{13}-effects

$$\frac{N_e}{N_e^0} - 1 \simeq (r\, s_{23}^2 - 1)\, P_{2\nu}(\Delta m_{31}^2, \theta_{13})$$

resonant matter effect in $P_{2\nu}(\Delta m_{31}^2, \theta_{13})$
for multi-GeV events ($r \approx 2.6 - 4.5$)

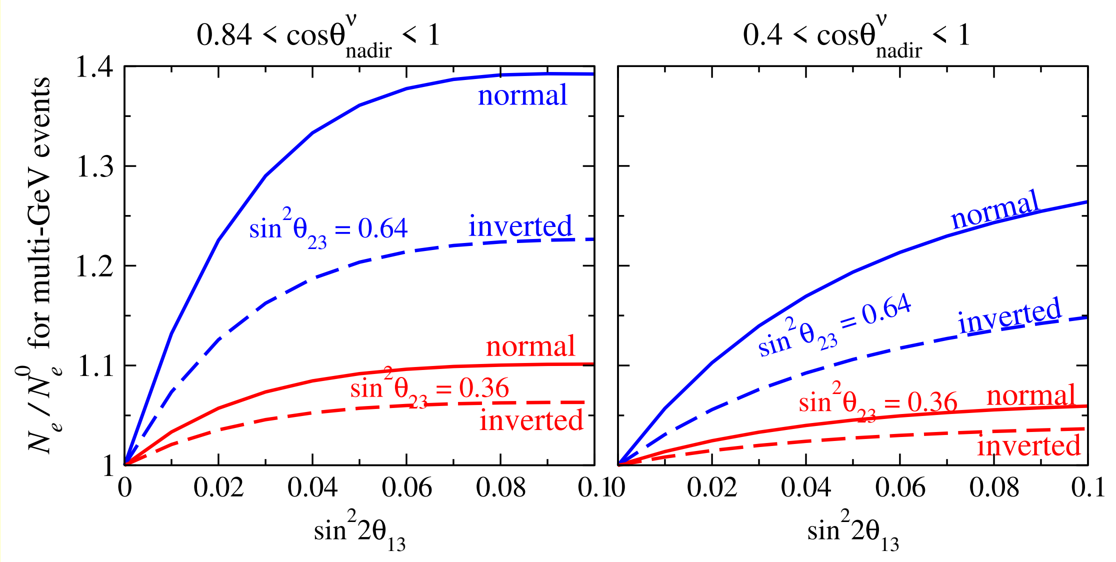
normal hierarchy: enhancement for neutrinos
inverted hierarchy: enhancement for anti-neutrinos

detection cross sections are different for neutrinos and anti-neutrinos

sensitivity to the neutrino mass hierarchy

θ_{13}-effects

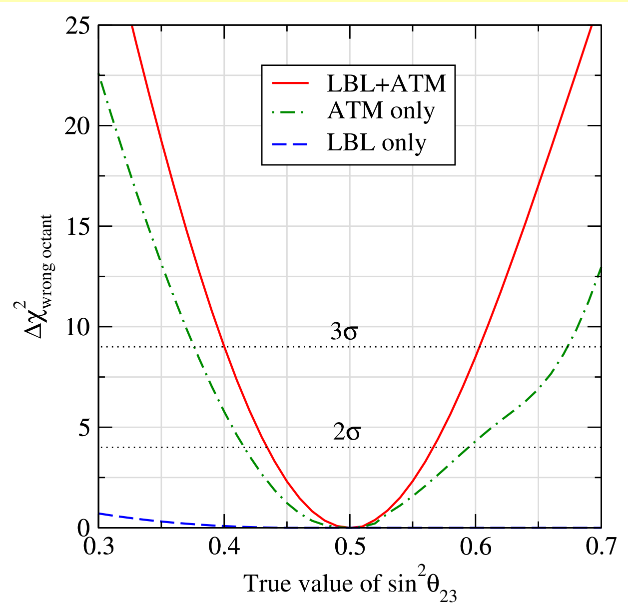
$$\frac{N_e}{N_e^0} - 1 \simeq (r\, s_{23}^2 - 1)\, P_{2\nu}(\Delta m_{31}^2, \theta_{13})$$



Bernabeu, Palomares-Ruiz, Petcov, Nucl. Phys. B669, 255 (2003), hep-ph/0305152

Δm_{21}^2-effects

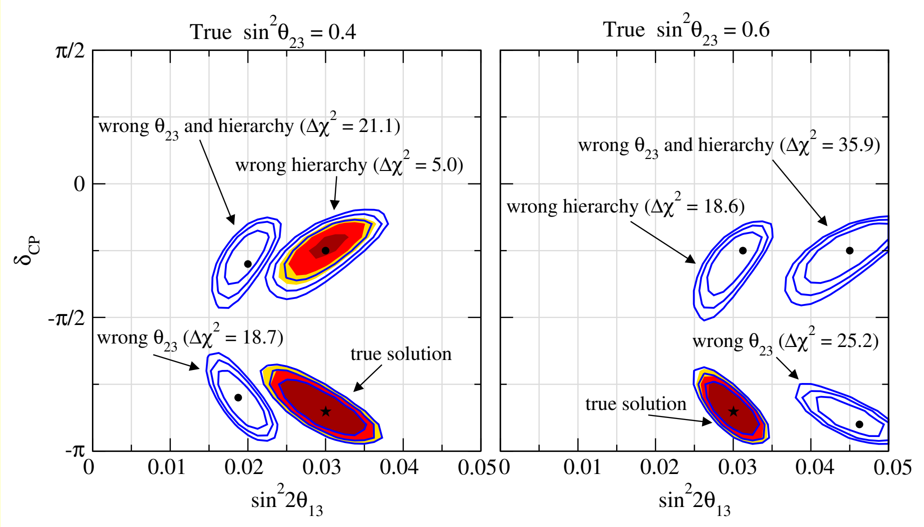
$$\frac{N_e}{N_e^0} - 1 \simeq (r\, c_{23}^2 - 1)\, P_{2\nu}(\Delta m_{21}^2, \theta_{12})$$



relevant for sub-GeV events

sensitivity to the octant of θ_{23} even for $\theta_{13} = 0$

Resolving the degeneracies wit ATM data



Example **T2HK**:

solid curves: LBL only, colored regions: LBL+ATM

true values: $\sin^2 2\theta_{13} = 0.03$, $\delta_{\rm CP} = -0.85\pi$, $\Delta m^2_{31} = 2.2 \cdot 10^{-3}$eV^2

Concluding remarks on degeneracies

- It is essential to include degeneracies in the consideration of future LBL experiments.
- Discussions at the probability level can be useful for the understanding of the underlying physics, but to find out if a method really works a full numerical simulation is necessary (statistics, detector effects, energy smearing, systematical errors, . . .).
- In the numerical analysis all osc. parameters should be varried: In general the degenerate solutions occur at different values of the osc. params (incl. Δm^2_{21}, θ_{12}, $|\Delta m^2_{31}|$)