The impact of solar and atmospheric parameter uncertainties on the measurement of θ_{13} and δ

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based on a collaboration with A. Donini and S. Rigolin

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CONTENTS

1 Introduction

parameter correlations and degeneracies $\left\{ \begin{array}{c} in\ the\ \nu_e \rightarrow \nu_\mu(\nu_\tau)\ appearance \\ in\ the\ \nu_e \rightarrow \nu_e,\ \nu_\mu \rightarrow \nu_\mu\ disappearance \end{array} \right.$

2 The appearance and disappearance channels at the low-gamma β-Beam: the measure of θ_{13} and δ

- impact of solar parameters on the measurement of θ_{13} and δ
- impact of atmospheric parameters on the measurement of θ_{13} and δ
- comparing the effects for present and estimated uncertainties

3 The appearance and disappearance channels at the SPL Super-Beam

- The measure of the atmospheric parameters
- The combination of APP+DIS channels to measure of θ_{13} and δ

Introduction

Present State of Neutrino Oscillation Parameters

- solar angle and mass difference are "known"
- atmospheric sector: $|\Delta m^2_{23}|$ and departure of $\sin^2 2\theta_{23}$ from maximal mixing are "known"

unknown quantities

continuous parameters

the angle θ_{13} and the CP-violating phase δ

discrete parameters

$s_{atm} = sign[\Delta m^2_{23}]$ and
$s_{oct} = sign[\tan(2\theta_{23})]$

our "ignorance" of s_{atm} and s_{oct}

$\oplus$

strong correlation between θ_{13} and δ in appearance

$\Longrightarrow$

PARAMETER DEGENERACY in the measure of θ_{13} and δ

Degeneracies

...in appearance: widely discussed in literature

Fogli96, Burguet-Castell01, Minakata01...

$$\underbrace{N^{\pm}_{\alpha\beta}(\bar\theta_{13}, \bar\delta; \bar s_{atm}, \bar s_{oct})}_{true\ parameters} = \underbrace{N^{\pm}_{\alpha\beta}(\theta_{13}, \delta; s_{atm} = \pm \bar s_{atm}; s_{oct} = \pm \bar s_{oct})}_{guessed\ parameters}$$

$N^{\pm}_{\beta}$=number of charged leptons $l^{\pm}_{\beta}$ observed in a given detector
α, β=neutrino flavours

- four different models (each of them with a definite s_{atm}, s_{oct} choice) must be used to fit the data on the lhs

- all in all, they form an eightfold degeneracy: the *intrinsic*, the *sign*, the *octant* and the *mixed* clones

$$\Downarrow$$

large and dangerous nuisance that increases the resulting uncertainty on θ_{13} and δ

...in disappearance

a similar appoach can be applied to study the presence of degeneracies in disapperance
$$\begin{cases} \text{in the } \nu_e \rightarrow \nu_e \text{ at the } \beta-\text{Beam} \\ \text{in the } \nu_\mu \rightarrow \nu_\mu \text{ at the SPL Super}-\text{Beam} \end{cases}$$

The facilities

The β-Beam

- ν_e ($\bar{\nu}_e$) from ^{18}Ne (^6He) ions accelerated at $\gamma = 100$ ($\gamma = 60$)
- 10 yrs of ν_e and $\bar{\nu}_e$ β-Beam run with a 440 KTon detector
- background and efficiencies

$$\begin{cases} \text{appearance}: & \begin{array}{c} \mathrm{J.\ Bouchez\,et\,al.,\ AIP\ Conf.\ Proc.\ 721, 37(2004)} \\ \mathrm{A.\ Donini\ et\ al., hep-ph/0406132} \end{array} \\ \text{disappearance}: & \begin{array}{c} \mathrm{J.\ Burguet-Castell\ et\ al., Nucl.\ Phys.\ B695, 217(2004)} \\ \mathrm{A.\ Donini\ et\ al., hep-ph/0411402} \end{array} \end{cases}$$

The Super-Beam

- ν_μ ($\bar{\nu}_\mu$) from π^+ (π^-) (60 m decay tunnel assumed)
- 2 yrs of ν_μ and 8 yrs of $\bar{\nu}_\mu$ Super-Beam run with a 440 KTon detector
- background and efficiencies

$$\begin{cases} \text{appearance}: & \begin{array}{c} \mathrm{J.\ J.\ Gomez-Cadenas\ et\ al., hep-ph/0105297} \\ \mathrm{A.\ Donini\ et\ al., hep-ph/0406132} \end{array} \\ \text{disappearance}: & \mathrm{A.\ Donini\ et\ al., hep-ph/0411402} \end{cases}$$

plan of the talk

- the β-Beam setup
  - correlation and degeneracies in ν_e disappearance (appendix)
  - full 3-parameter analysis in the measurement of θ_{13} and δ
    - impact of the parameter uncertainties:
      - the solar sector
      - the atmospheric sector
      - (present and expected uncertainties)
      - comparison between other statistical approaches (appendix)

- the Super-Beam setup
  - correlation and degeneracies in ν_μ disappearance
  - the atmospheric sector : $\delta(\Delta m^2_{atm}), \delta(\theta_{23})$
  - combining appearance and disappearance channels to measure θ_{13} and δ

1 Full 3-par analysis of the appearance channel

- we investigate the impact of uncertainties in the atmospheric and solar parameters in the measure of (θ_{13}, δ)
- in order to understand how any single parameter affect the measurement, we perform three-parameters fits in θ_{13}, δ and only one of the following parameters in turn $x = \theta_{12}$, Δm^2_{12}, θ_{23} and Δm^2_{23}

- For each degeneracy
  - we build a three-parameters χ^2

$$\chi^2(\theta_{13}, \delta, x) = \sum_{p=\pm} \left[\frac{N^p(\theta_{13}, \delta, x, s_{atm}, s_{oct}) - N^p(\bar{\theta}_{13}, \bar{\delta}, \bar{x}, \bar{s}_{atm}, \bar{s}_{oct})}{\delta_{N^p}} \right]^2$$

$$(\delta N^p)^2 = \sigma^2_{N^p} + (s\, N^p)^2 + (s\, B^p)^2 \qquad s = sys,\ B^p = background$$

parameter	present Bahcall04 (solar)-Gonzalez-Garcia03 (atm)	expected LOI of T2K (hep-ex/0106019)	c.v.
$\tan^2\theta_{12}$	**0.30–0.54** $(29^\circ - 36^\circ)$	–	**0.39**
$\Delta m^2_{12}/10^{-5}\ eV^2$	**7.5–9.1**	–	**8.2**
$\tan^2\theta_{23}$	**0.53–2.04** $(36^\circ - 55^\circ)$	**0.57–0.88** $(37^\circ - 43^\circ)$	**0.7***
$\Delta m^2_{23}/10^{-3}\ eV^2$	**1.7–3.5**	**2.3–2.7**	**2.5**
$\tan^2\theta_{23}$	**new analysis** Enrique's talk	**0.62–0.85** $(38^\circ - 43^\circ)$	
$\Delta m^2_{23}/10^{-3}\ eV^2$	**new analysis**	**2.42–2.61 (2.47–2.64)**	

- projection of the 90% CL surface of $\chi^2(\theta_{13}, \delta, r)$ on the (θ_{13}, δ) plane
  - easy to understand
  - easy to compare with the usual bidimensional fit

Full 3-par analysis of the app channel at β-Beam

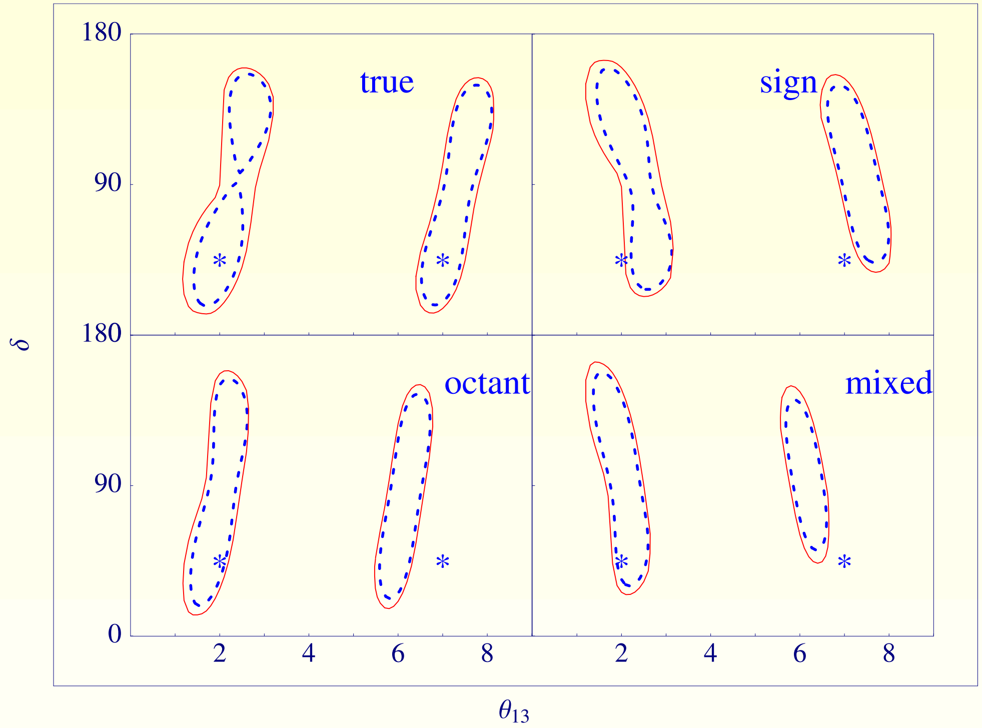
The solar sector

we perform two distinct three-parameters fit in θ_{13}, δ and $x = \theta_{12}$ (for fixed Δm^2_{sol}) or $x = \Delta m^2_{sol}$ (for fixed θ_{12}) $\Longrightarrow$ same results $\Longrightarrow$ only the case $x = \theta_{12}$ shown

- **two(dotted) and three-par fit superimposed**
- **input points:** $(\bar{\theta}_{13}, \bar{\delta}, \bar{\theta}_{12}) = (2/7^\circ, 45^\circ, 32^\circ)$
- **systematic errors at 5% level**

- **simulation performed for other values of** δ **and for the SPL Super-Beam:**

same qualitative results



$\Downarrow$

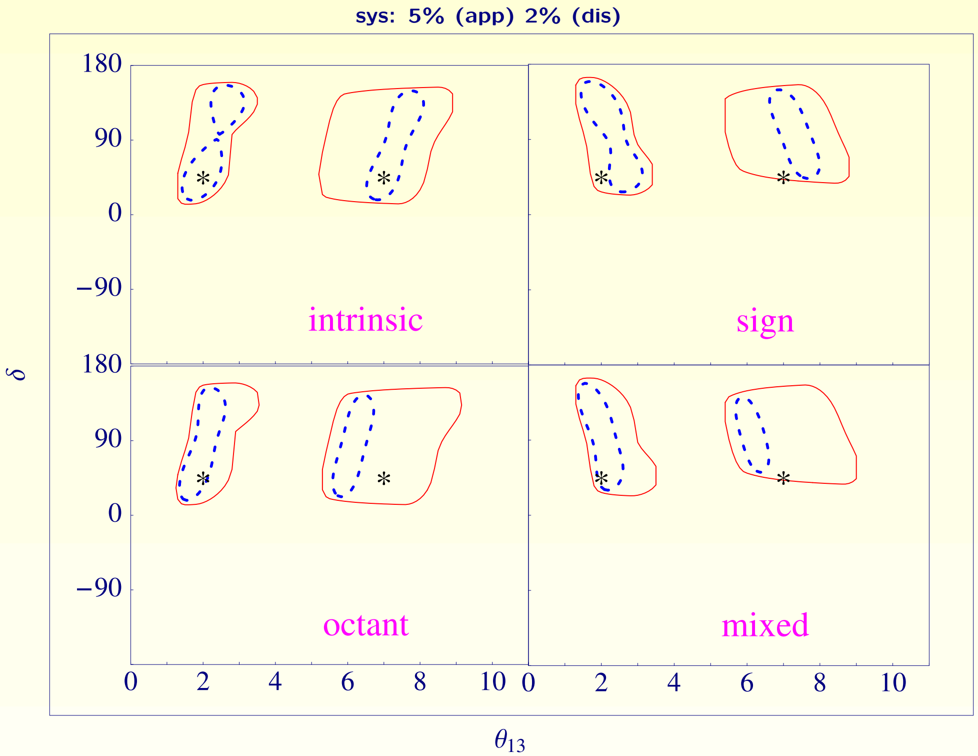
the solar sector does not introduce further uncertainties on the measure of θ_{13} and δ

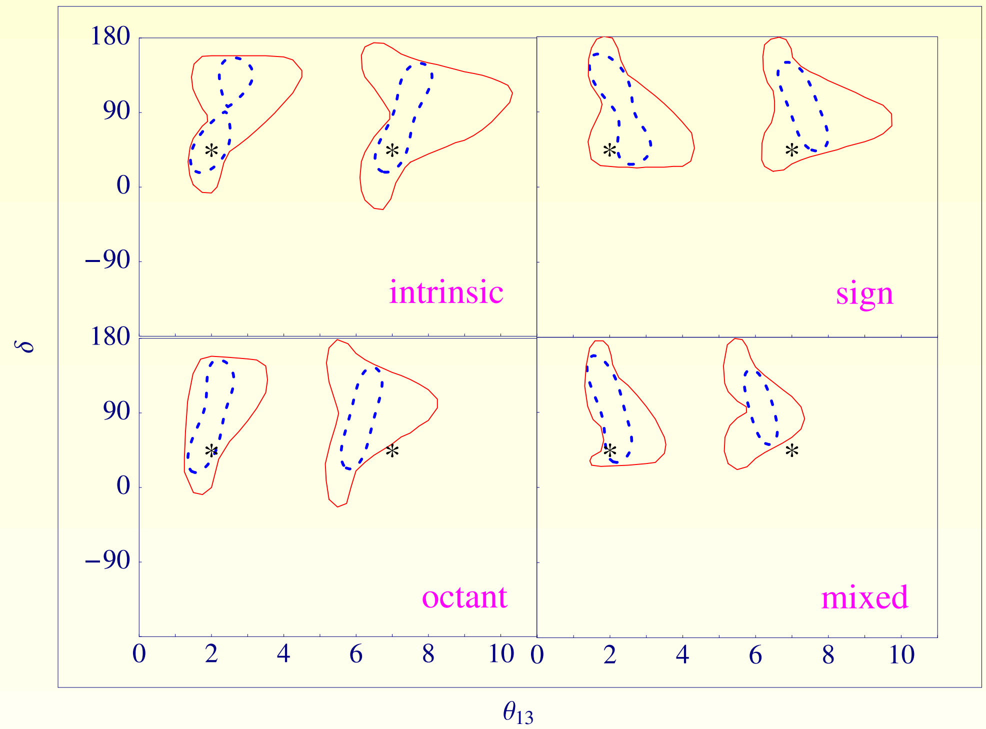
Full 3-par analysis: the β-Beam case

$$\chi^2(\theta_{13}, \delta, x) = \chi^2_{app}(\theta_{13}, \delta, x) + \chi^2_{dis}(\theta_{13}, \delta, x)$$

The atmospheric sector (I)–current errors

$x = \theta_{23}$ $\quad$ $(\bar\theta_{13}, \bar\delta) = (2/7^\circ, 45^\circ)$ $\quad$ $x = \Delta m^2_{23}$





- **two(dotted) and three-par fit superimposed**
- $\delta(\theta_{13}) \sim 3^\circ - 4^\circ$ **(to compesate a change in θ_{23} in the leading term in $P_{e\mu}$: $\sin^2(2\theta_{13}) \sin^2\theta_{23}$)**
- **overall error on δ larger for $x = \Delta m^2_{23}$**

the atmospheric sector strongly affects the measure of θ_{13} and δ

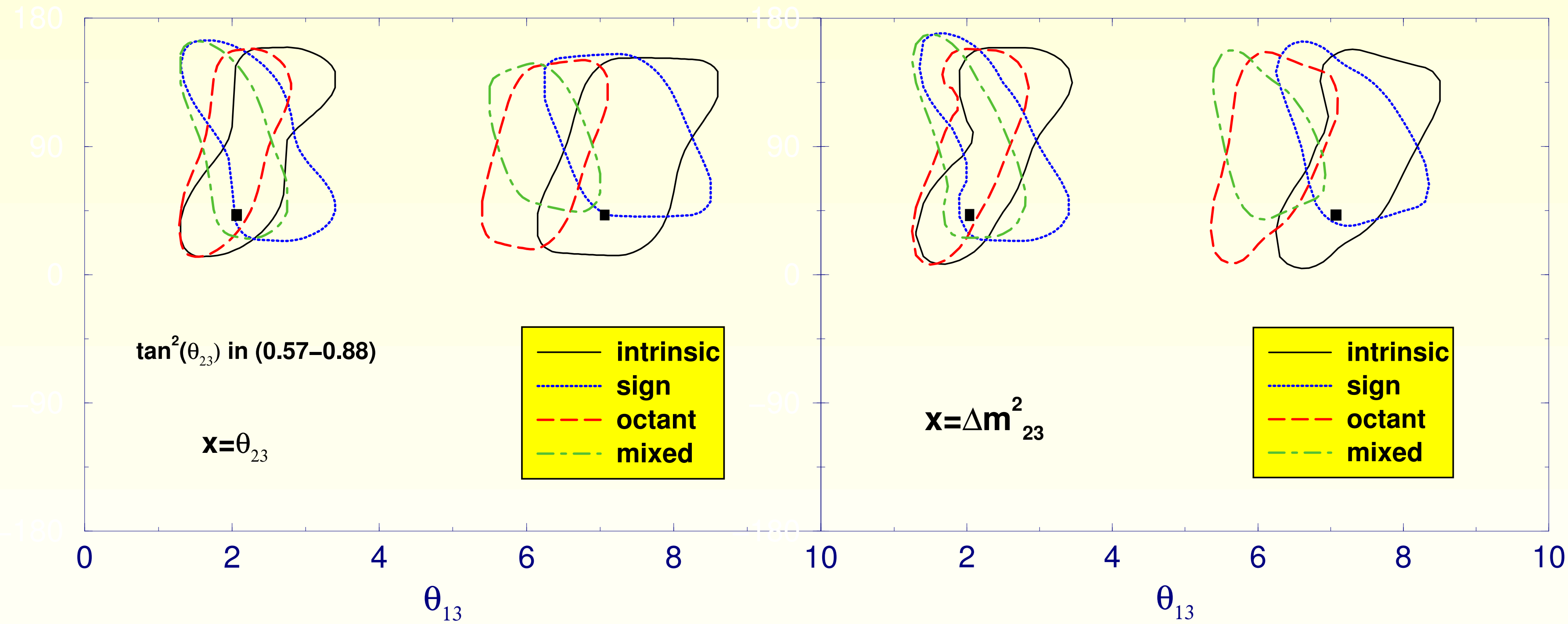
Full 3-par analysis: the β-Beam case

$\chi^2(\theta_{13}, \delta, x) = \chi^2_{app}(\theta_{13}, \delta, x) + \chi^2_{dis}(\theta_{13}, \delta, x)$

The atmospheric sector (II)–expected errors from LOI of T2K

"The overall sensitivity of the first phase is expected to be 1% in precision for $\sin^2 2\theta_{23}$ *and better than* $1 \times 10^{-4}\ \mathrm{eV}^2$ *for* Δm^2_{23}*";* **we estimate the error on** $\bar\theta_{23} = 40^\circ$ **and** $\bar\Delta m^2_{23} = 2.5 \cdot 10^{-3} eV^2$

$(\bar\theta_{13}, \bar\delta) = (2/7^\circ, 45^\circ)$



significant reduction on the θ_{13}-spread is achieved

for larger values of θ_{13} the error is enhanced due to the octant ambiguity

also the δ-spread is reduced considerably with respect to the results obtained with present uncertainties

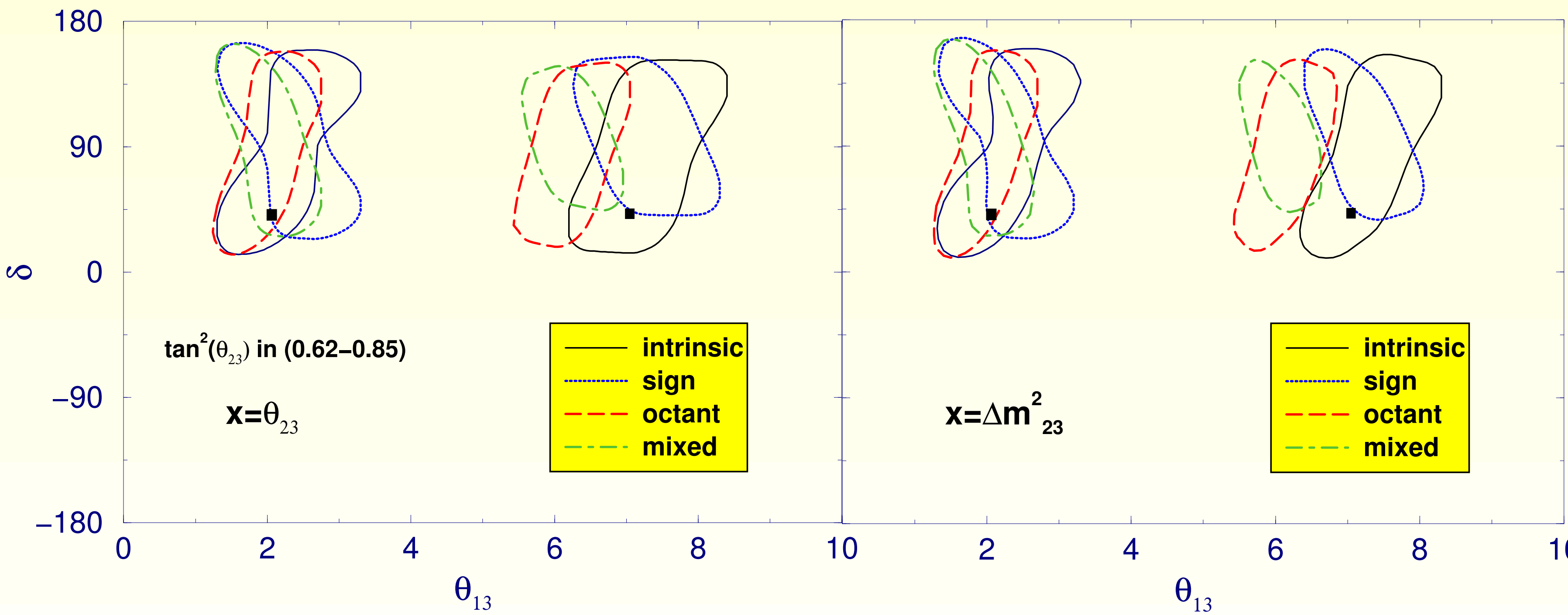
Full 3-par analysis: the β-Beam case

$\chi^2(\theta_{13}, \delta, x) = \chi^2_{app}(\theta_{13}, \delta, x) + \chi^2_{dis}(\theta_{13}, \delta, x)$

The atmospheric sector (III)–new analysis of expected performance of T2K

we consider $\theta_{23} \in [38.2, 42.7]$ and $\Delta m^2_{23} \in [2.42, 2.61] \cdot 10^{-3}\, eV^2$ computed for $\bar{\theta}_{23} = 40^\circ$ and $\bar{\Delta} m^2_{23} = 2.5 \cdot 10^{-3}\, eV^2$

$(\bar{\theta}_{13}, \bar{\delta}) = (2/7^\circ, 45^\circ)$



If θ_{23} and Δm^2_{atm} can be really measured at the T2K-phase I experiment with the expected precision and for any value of $\bar{\theta}_{23}$, the results of two-parameters studies can be considered reliable

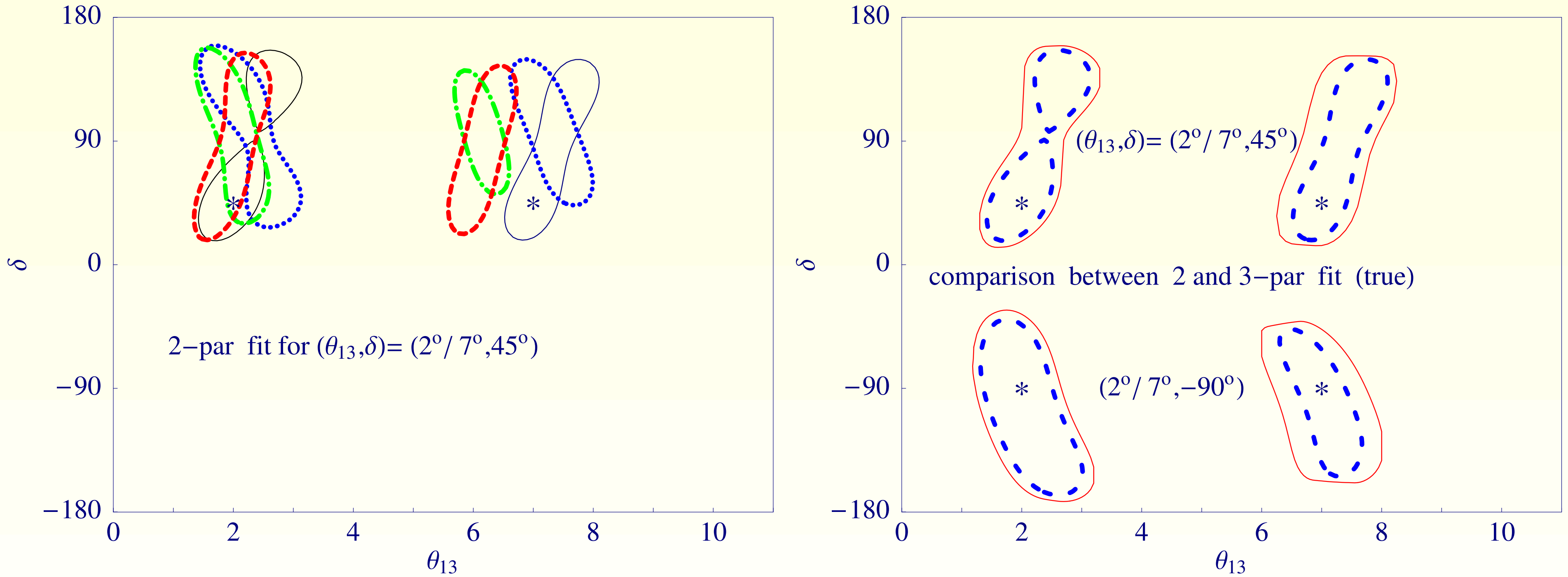
Full 3-par analysis: the β-Beam case

$$\chi^2(\theta_{13}, \delta, x) = \chi^2_{app}(\theta_{13}, \delta, x) + \chi^2_{dis}(\theta_{13}, \delta, x)$$

The atmospheric sector (III)–new analysis of expected performance of T2K

we consider $\theta_{23} \in [38.2, 42.7]$ **and** $\Delta m^2_{23} \in [2.42, 2.61] \cdot 10^{-3}\, eV^2$ **computed for** $\bar{\theta}_{23} = 40^\circ$ **and** $\bar{\Delta} m^2_{23} = 2.5 \cdot 10^{-3}\, eV^2$

$$(\bar{\theta}_{13}, \bar{\delta}) = (2/7^\circ, 45^\circ)$$



If θ_{23} and Δm^2_{atm} can be really measured at the T2K-phase I experiment with the expected precision and for any value of $\bar{\theta}_{23}$, the results of two-parameters studies can be considered reliable

Conclusions on the low-gamma β-Beam analysis

with present uncertainties, the measurement of the two unknowns in the PMNS mixing matrix is severely spoiled

errors on θ_{13}

present uncertainties

$\delta(\theta_{13})$ as large as 4° are found
(a little bit smaller for $\bar{\delta} = 90^\circ$)

expected uncertainties

$\delta(\theta_{13})$ as large as $2.5 - 3^\circ$ are found
(a little bit smaller for $\bar{\delta} = 90^\circ$)

errors on δ

present uncertainties

half of the parameter space is spanned
(a little bit smaller for $\bar{\delta} = 90^\circ$)

expected uncertainties

less than half of the parameter space is spanned

A significant reduction in the uncertainties on the atmospheric parameters (especially on θ_{23}) is mandatory if we plan to use such a facility to look for θ_{13} (and δ)

2 The ν_μ disappearance channel at the SPL SB

useful to measure the atmospheric parameters

$$
\begin{aligned}
P(\nu_\mu \to \nu_\mu) &= 1 - (\sin^2 2\theta_{23} - s^2_{23}\sin^2 2\theta_{13}\cos 2\theta_{23})\,\sin^2\left(\frac{\Delta_{atm} L}{2}\right) \\
&- \left(\frac{\Delta_{sol} L}{2}\right)[s^2_{12}\sin^2 2\theta_{23} + \tilde{J} s^2_{23}\cos\delta]\,\sin(\Delta_{atm} L) \\
&- \left(\frac{\Delta_{sol} L}{2}\right)^2 [c^4_{23}\sin^2 2\theta_{12} + s^2_{12}\sin^2 2\theta_{23}\cos(\Delta_{atm} L)]
\end{aligned}
$$

The main uncertainties in the measure of θ_{23} and Δm^2_{23} comes from:

<u>intrinsic clone</u>

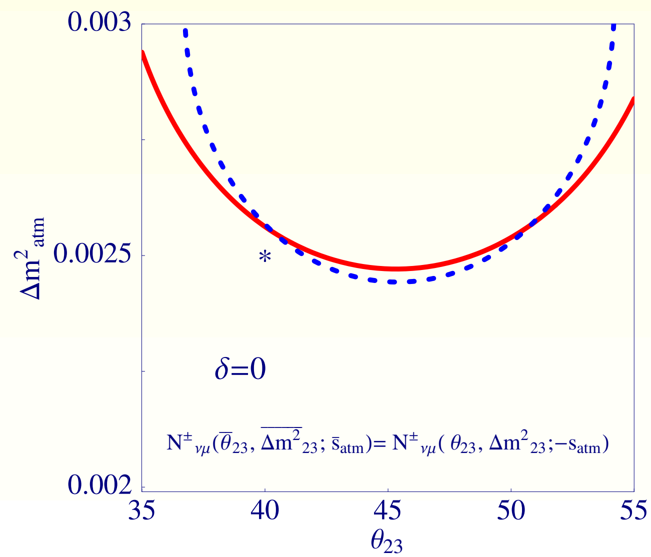
it trivially comes from the main term
$\sin^2 2\theta_{23}$: $\theta_{23} \to \pi/2 - \theta_{23}$

<u>**other sources**</u>

- **error on** $\theta_{13} (\leq 10^\circ)$
- δ **is completely unknown**

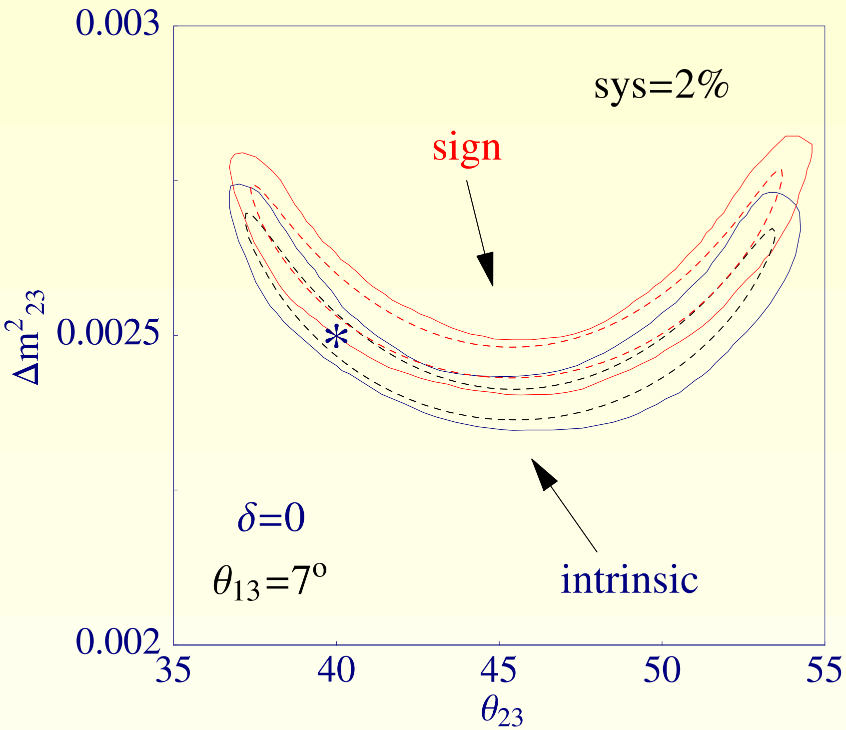
we found a negligible contribution from $\Delta\delta$ and a relatively large from $\Delta\theta_{13}$

<u>sign clone</u>



The ν_μ disappearance channel at the SPL-SB

full 3-par analysis of disappearance channel: $\chi^2(\theta_{23}, \Delta m^2_{atm}, \theta_{13})$



solid line: projection of $\chi^2(\theta_{23}, \Delta m^2_{atm}, \theta_{13})$
dotted line: 2-par $\chi^2(\theta_{23}, \Delta m^2_{atm})$ **at fixed** θ_{13}

a second allowed region in the parameter space appears at $\Delta m^2_{atm} > \bar{\Delta} m^2_{atm}$ when the wrong s_{atm} is considered

$\Downarrow$

it enlarges the errors on the atmospheric parameters

to estimate the errors on the atmospheric parameters we superimposed 3-par fits performed with:

$$\left\{ \begin{array}{c} \theta_{23} \text{ non} - \text{maximal} \\ \theta_{13} \in [0, 10^\circ] \\ \delta \in [0, 2\pi] \end{array} \right.$$

$\theta_{23} \in [37^\circ, 54^\circ]$ and
$\Delta m^2_{atm} \in [2.3, 2.9] 10^{-3}\, eV^2$

$\Downarrow$

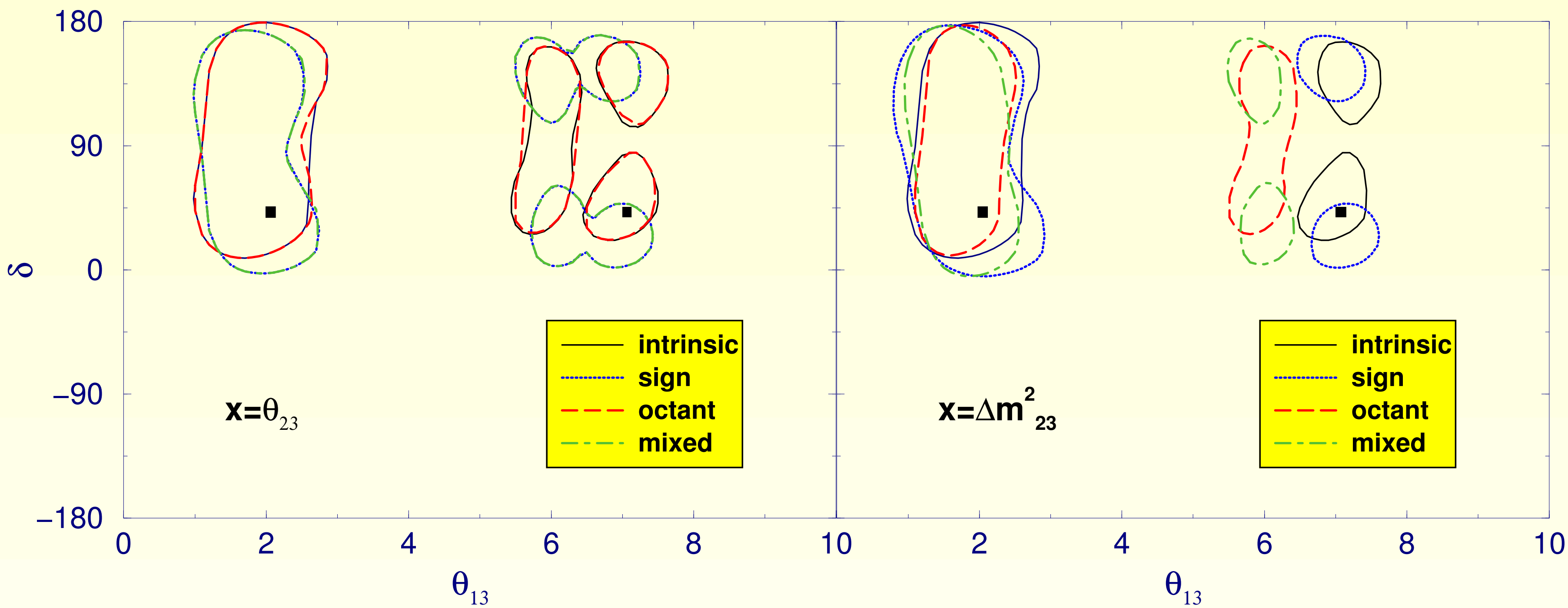
no significant reduction on θ_{23}
strong improvement on Δm^2_{23}

if sys=5% : $\theta_{23} \in [35^\circ, 55^\circ]$ **and** $\Delta m^2_{atm} \in [2.3, 3] 10^{-3}\, eV^2$

3 Combining APP+DIS channels at the SPL SB

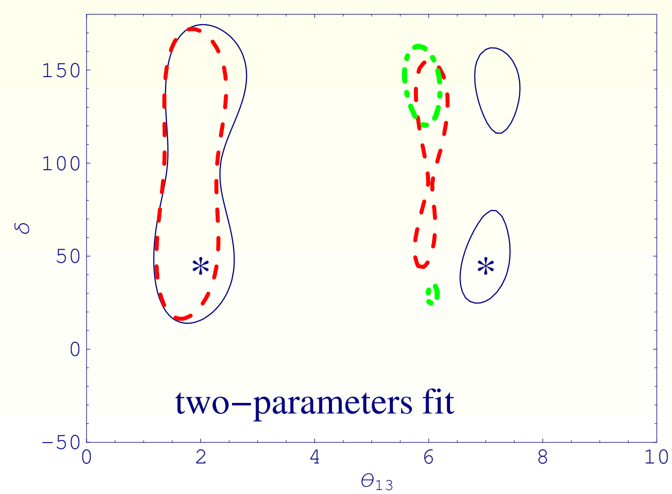
the previous intervals on θ_{23} and Δm^2_{atm} are used to study the combination

$$(\bar\theta_{13}, \bar\delta) = (2/7^\circ, 45^\circ)$$



for $x = \Delta m^2_{23}$, 2- and 3-par contours much more similar than for $x = \theta_{23}$

the degeneracy structure is not reduced, worst for $x = \theta_{23}$: remnant of sign clones still present



Even reducing the errors on the atmospheric parameters, θ_{13} will not be measured better than $\sim 2^\circ$

Conclusions on the SPL SB analysis

atmospheric parameters measurement from disappearance channel is mainly affected by *sign* ambiguity

↓

- errors on Δm^2_{23} roughly doubled but similar to T2K and Enrique's analysis
- errors on θ_{23} comparable to the current uncertainties

combining appearance and disappearance channels to measure θ_{13} and δ

↓

- degeneracies are still present but reduced in size
- physics reach: it performs slightly better than β-Beam in measuring θ_{13}
- half of the parameter space for δ is spanned, θ_{13} measured with error of $\mathcal{O}(2^\circ)$
- octant and mixed ambiguities cause a shift towards smaller θ_{13} (for any value of $\bar{\delta}$)

A significant reduction in the uncertainties on the atmospheric parameters is mandatory if we plan to use such a facility to look for δ

4 The ν_e disappearance channel

$$P_{ee}^{\pm} = 1 - \left(\frac{\Delta_{atm}}{B_{\mp}}\right)^2 \sin^2\left(\frac{B_{\mp}\,L}{2}\right)\sin^2(2\theta_{13}) - \left(\frac{\Delta_{sol}}{A}\right)^2 \sin^2\left(\frac{A\,L}{2}\right)\sin^2(2\theta_{12})$$

It well describes the behaviour of the full transition probability in the energy range covered by the considered β-Beam setup

It does not depend on θ_{23} and δ

Useful to measure θ_{13}

Two remaining sources of ambiguities that can affect the measure of θ_{13}:

- s_{atm} (for large values of θ_{13}, i.e. in the "atmospheric" region)
- $(\theta_{13} - \theta_{12})$ correlation (for small values of θ_{13}, i.e. in the "solar" region)

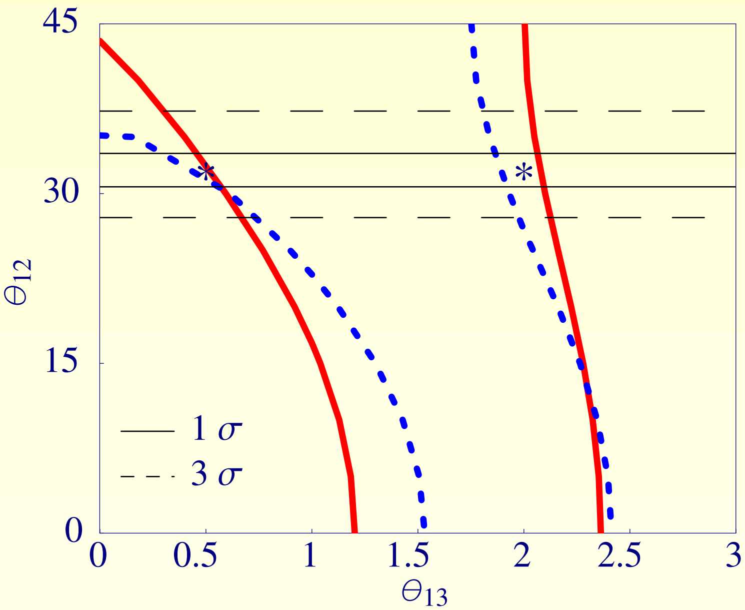
$$\begin{cases} N_{\nu_e}^{\pm}(\bar\theta_{13}, \bar\theta_{12}; \bar s_{atm}) = N_{\nu_e}^{\pm}(\theta_{13}, \theta_{12}; s_{atm} = \bar s_{atm}) & \text{intrinsic} \\ N_{\nu_e}^{\pm}(\bar\theta_{13}, \bar\theta_{12}; \bar s_{atm}) = N_{\nu_e}^{\pm}(\theta_{13}, \theta_{12}; s_{atm} = -\bar s_{atm}) & \text{sign} \end{cases}$$

The intrinsic ambiguity is not a problem since the clone point is at: $\theta_{13} = \bar\theta_{13}, \theta_{12} \sim \pi/2 - \bar\theta_{12}$

In principle the sign clone could be dangerous for values of $\bar\theta_{13} < 3^\circ$

The ν_e disappearance channel

the sign ambiguity from the $\theta_{13} - \theta_{12}$ correlation



two different input points: $(\bar{\theta}_{13}, \bar{\theta}_{12}) = (0.5/2^\circ, 32^\circ)$

-red (blue) lines for neutrinos (antineutrinos)

-also plotted the 1 and 3 σ errors on θ_{12}

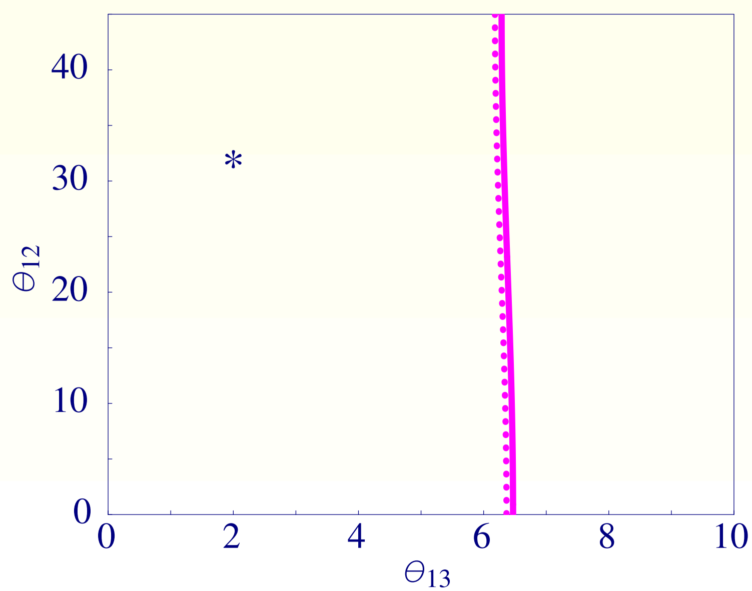
the sign clone could in principle induce a relatively large spread on the measure of θ_{13}

BUT the statistics is too low to separate the true from the clone region (example for $\bar{\theta}_{13} = 2^\circ$)

-solid (dotted) line stands for intrinsic (sign) degeneracy

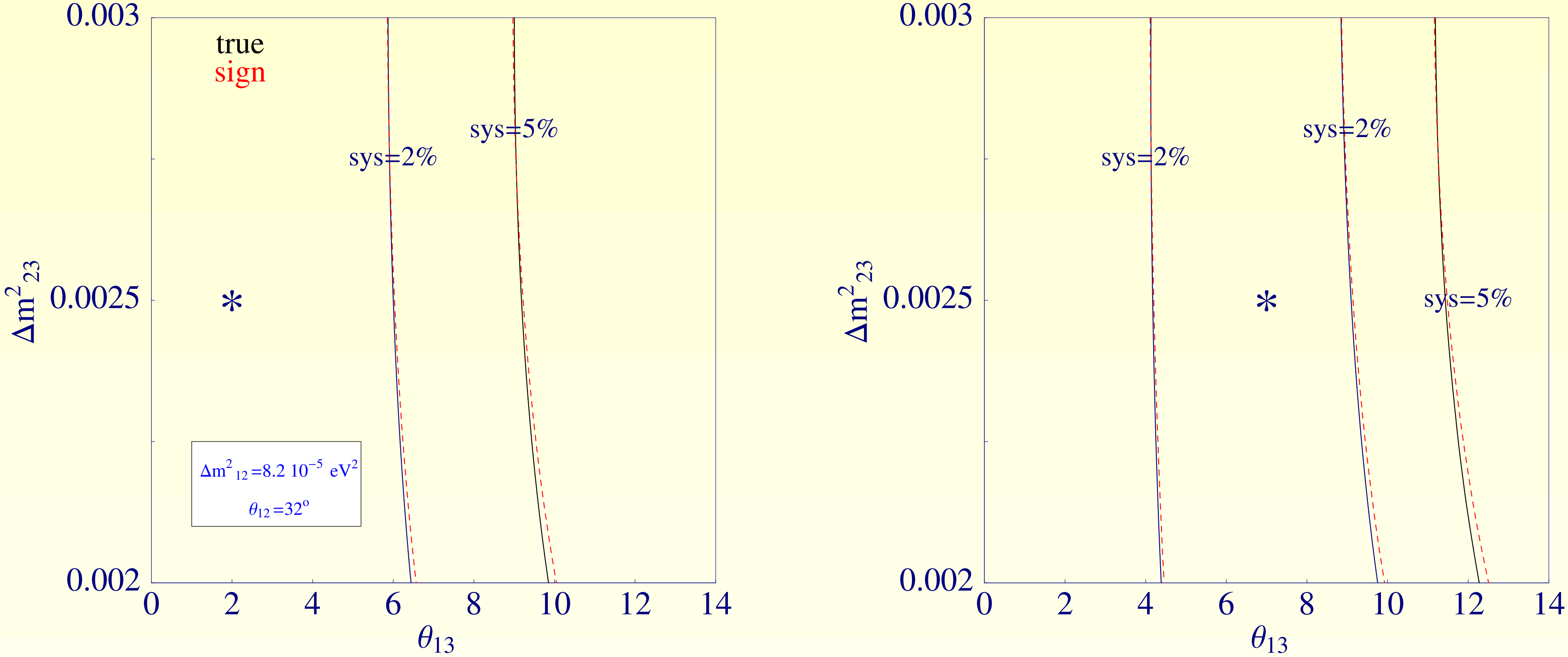
-background and systematics (2 %) included

the effect of the sign clone is screened by low statistics $\Rightarrow$ the clones from $\theta_{13} - \theta_{12}$ correlation are a marginal problem



The ν_e disappearance channel

sensitivity to θ_{13}



2% sys:

- $\bar{\theta}_{13} = 2^\circ$: the dis channel alone could test θ_{13} down 6°
- $\bar{\theta}_{13} = 7^\circ$: θ_{13} measured with a precision of $\pm 2/3^\circ$

5% sys: no improvement of the present bound

we expect very small improvement combining app+dis channels to measure θ_{13}, even with an "optimistic" systematic error

Comparison with the CP-coverage

P. Huber et al., hep-ph/0412199

one builds $\chi^2(\theta_{13}, \delta, N_\alpha)$, N_α external parameters

$\Downarrow$

marginalization over N_α parameters

$\Downarrow$

for each $\bar\theta_{13}, \bar\delta$, a 2-dim $\chi^2(\theta_{13}, \delta, N_\alpha^{min})$

$\Downarrow$

we then minimize the resulting function in θ_{13}

$\Downarrow$

a one-dimensional function of δ: $\chi^2_{min}(\delta, \bar\delta)$

$\Downarrow$

imposing $\chi^2_{min}(\delta, \bar\delta) = CL$, intervals $\Delta_I(\bar\delta)$ can be found

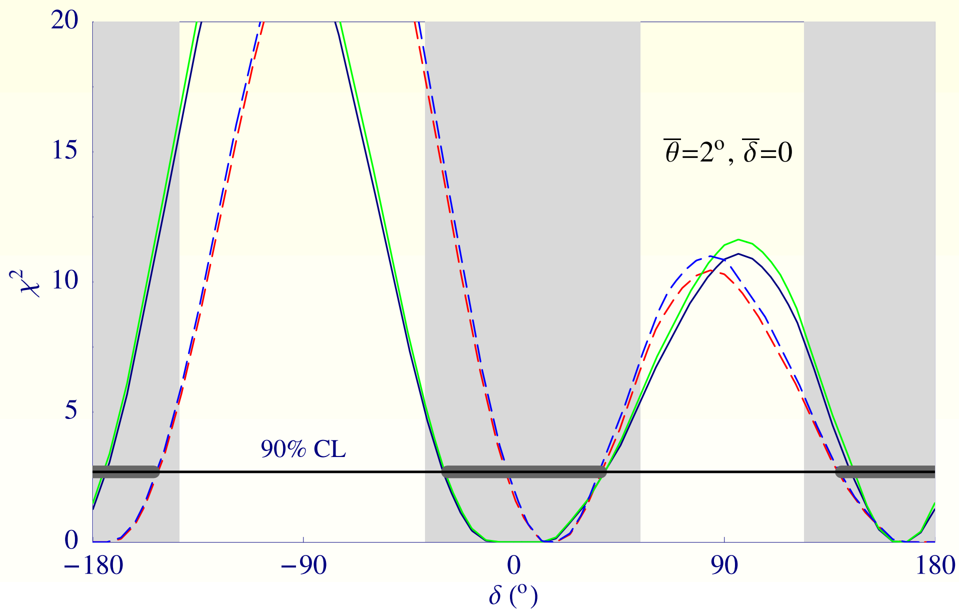
$$\xi(\bar\delta) = \text{Coverage in } \delta = \frac{1}{2\pi} U_{I=1}^{N_{deg}} \Delta_I(\bar\delta)$$

$\Downarrow$

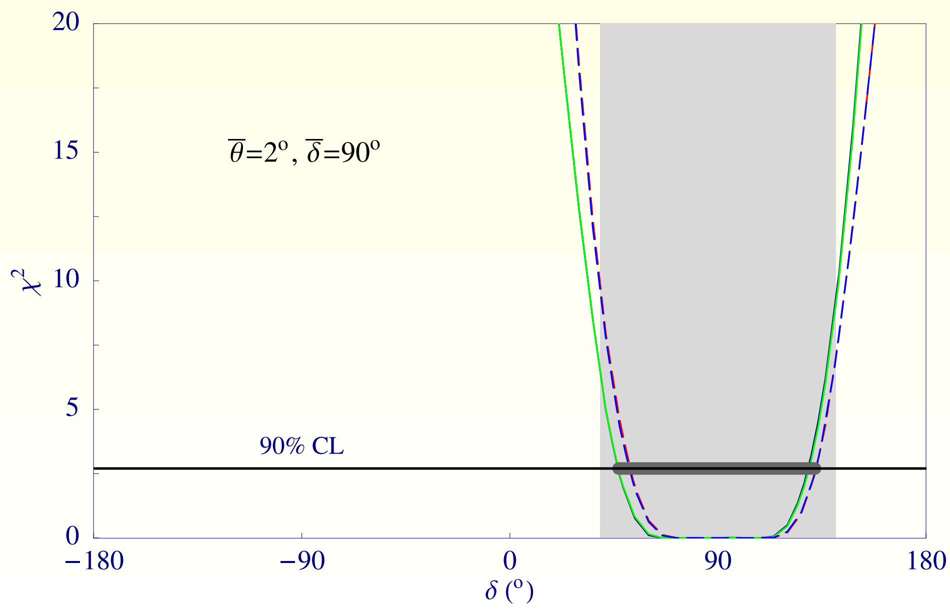
- compact way to compare performacies of different experiments

general underestimation of the error on $\bar\delta$

we performed this procedure starting from a 4-D grid in θ_{13}, δ and the atmospheric parameters and compared with the projection of our 3-D fit (vertical gray bands)

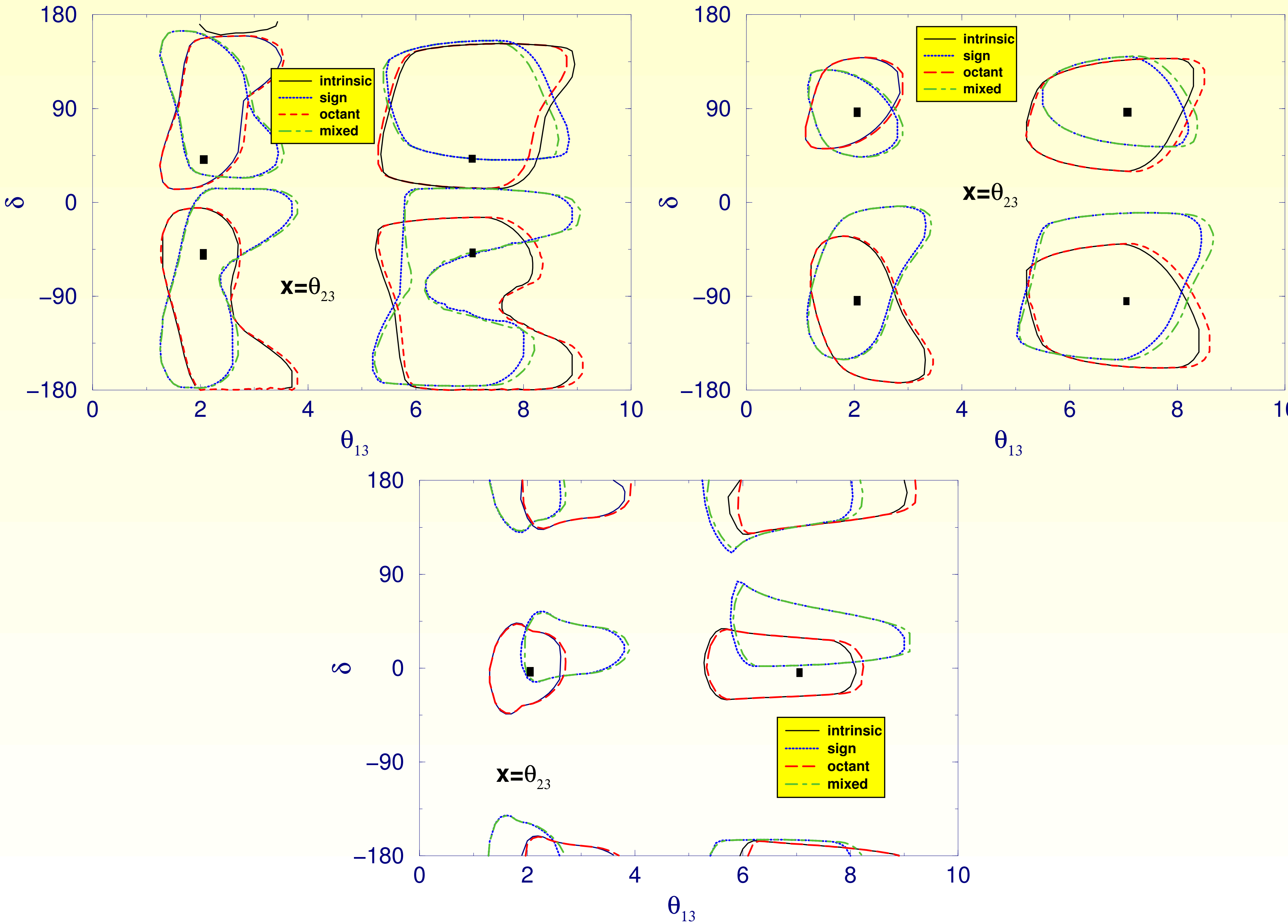


$\xi(0) \sim 0.38$ **(Δ(3-par)=0.58)**

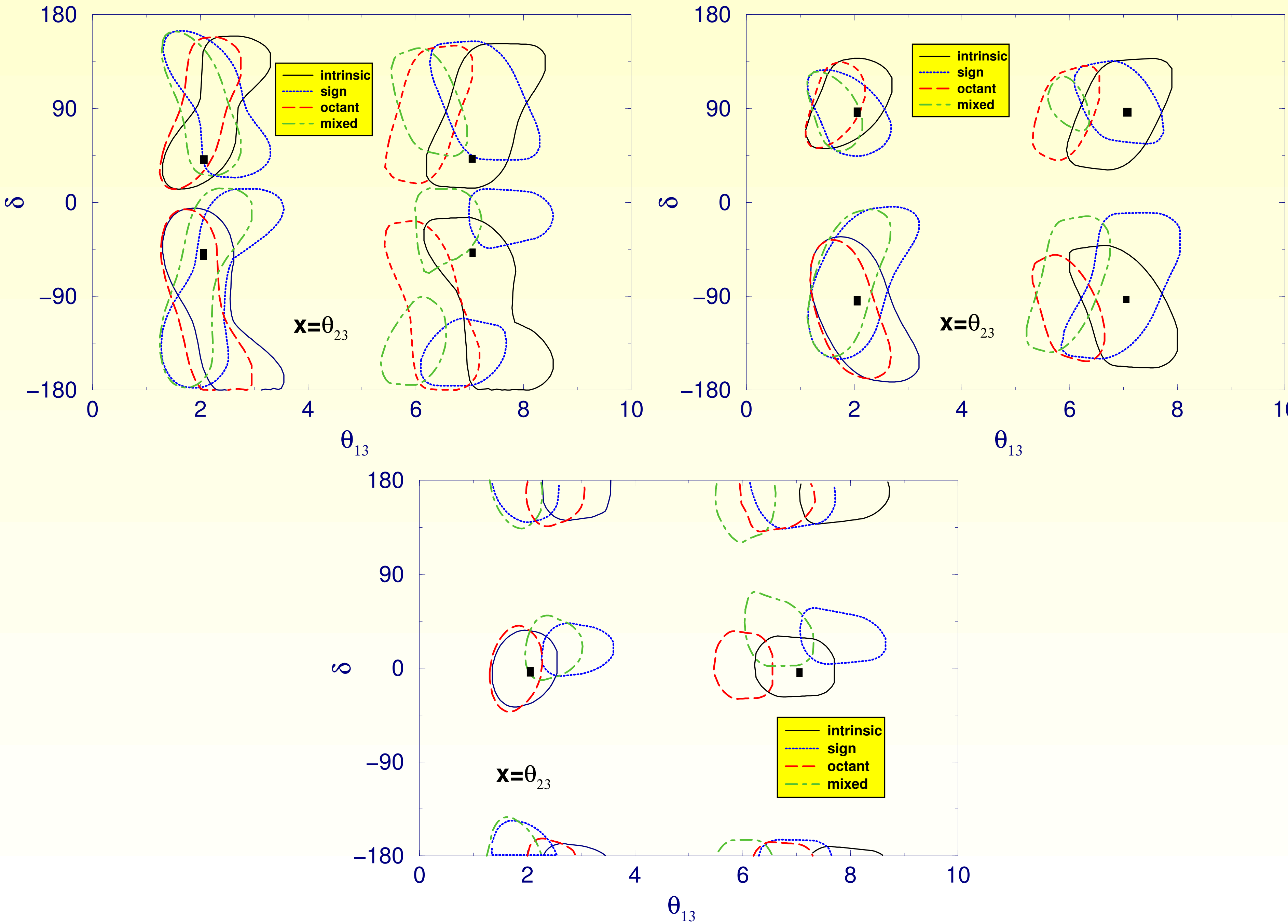


$\xi(90) \sim 0.25$ **(Δ(3-par)=0.31)**

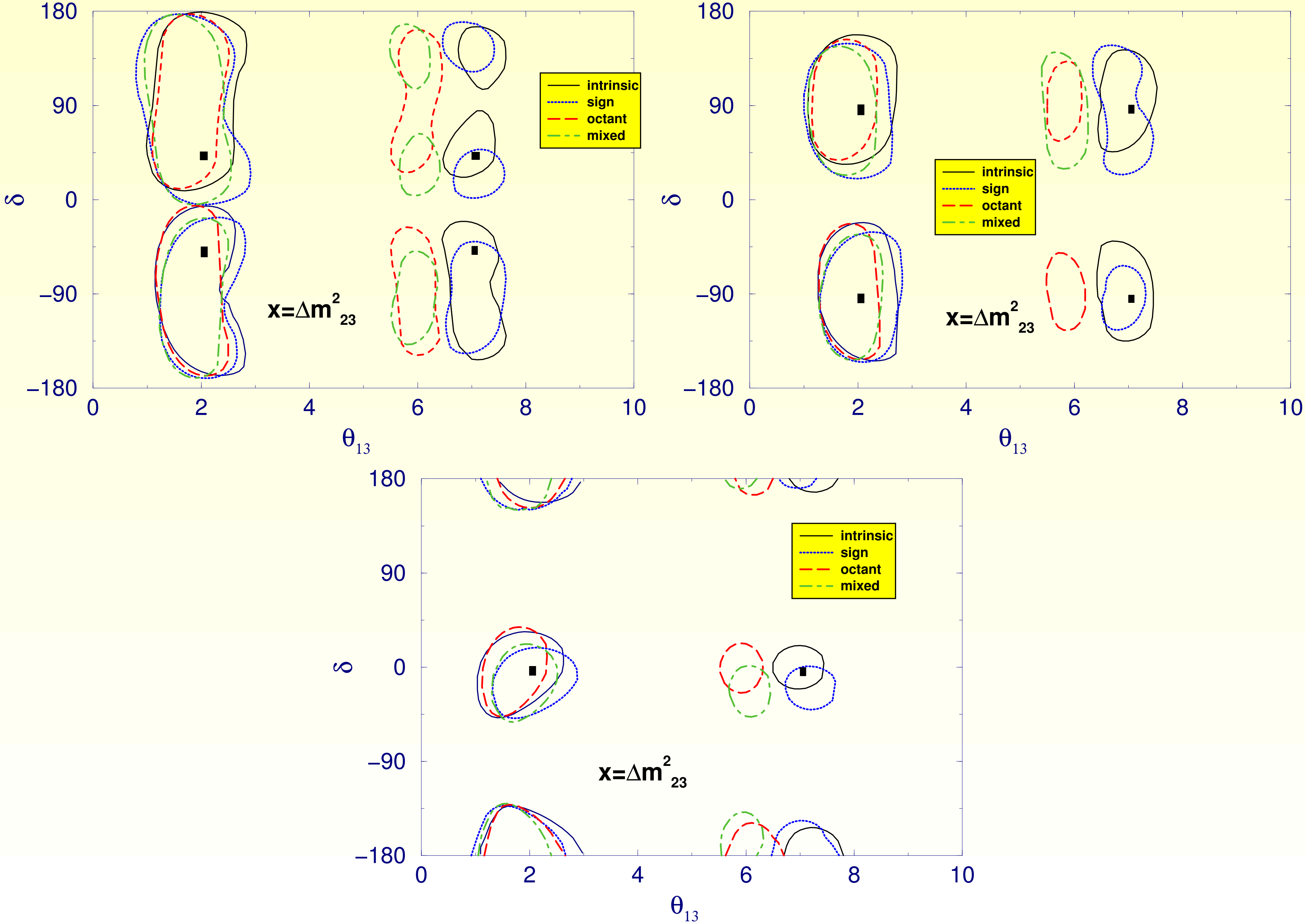
β-Beam fits for $\bar{\delta} = \pm 90°, \pm 45°, 0$ present uncertainties



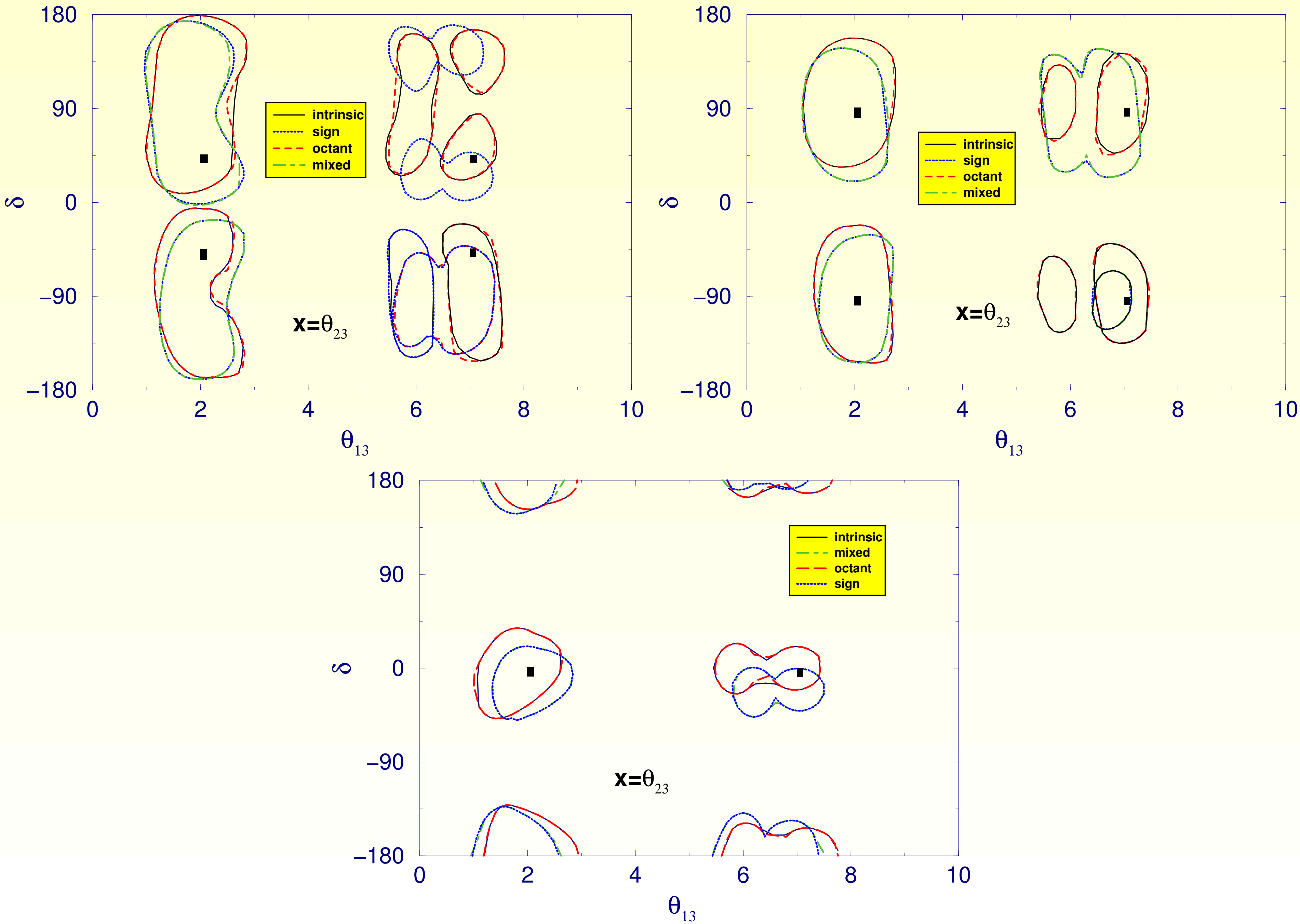
β-Beam fits for $\bar{\delta} = \pm 90^\circ, \pm 45^\circ, 0$ expected uncertainties



SPL Super-Beam fits for $\bar{\delta} = \pm 90°, \pm 45°, 0$



SPL Super-Beam fits for $\bar{\delta} = \pm 90°, \pm 45°, 0$

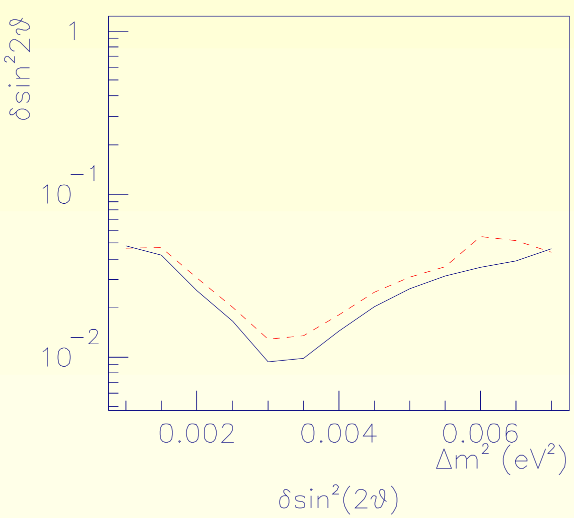


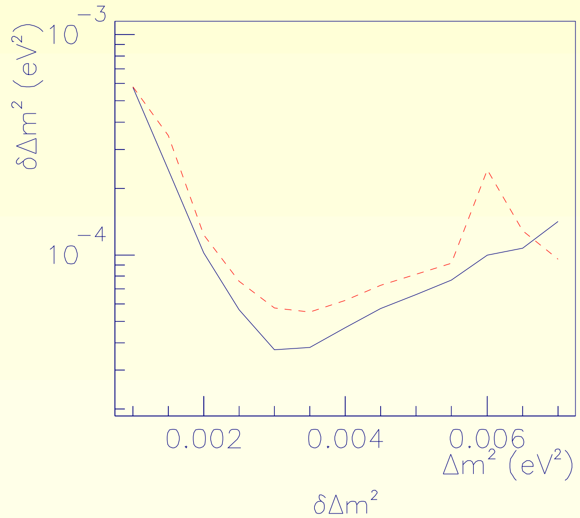
Physics in the first stage of the T2K project

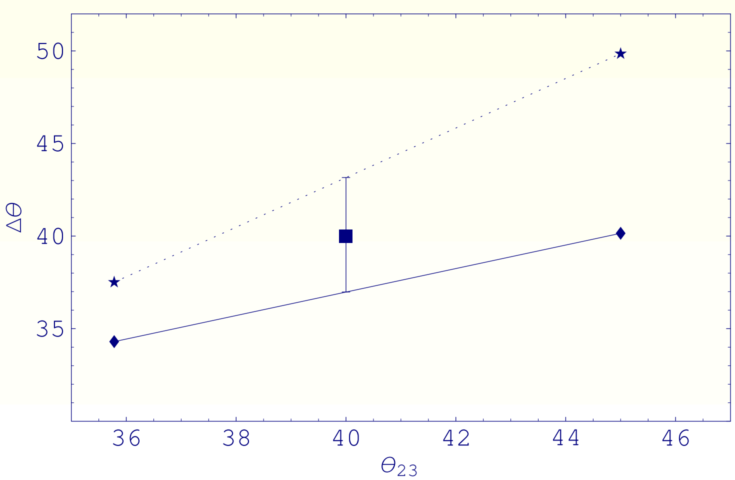
sensitivity of the neutrino oscillation parameters

solid lines: curves for $\sin^2(2\theta_{23}) = 1$

dashed lines: curves for $\sin^2(2\theta_{23}) = 0.9$







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