Interactions of Neutrinos at High and Low Energies

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“Neutrino Oscillation Experiments” Meta-Outline

- Neutrino Interactions (12 – 13 June), KSM
- Conventional Neutrino Beams (12 – 13 June), D. Harris
- Why New Neutrino Beams (12 June), A. Blondel
- High Energy Neutrino Detectors (14 – 15 June), D. Harris
- Long Baseline Phenomenology (17 – 18 June), A. Donini
- Low Energy Neutrino Detectors (18 – 19 June), T. Kajita
-
- Tutorials follow each lecture

Or at least that <u>was</u> the plan…

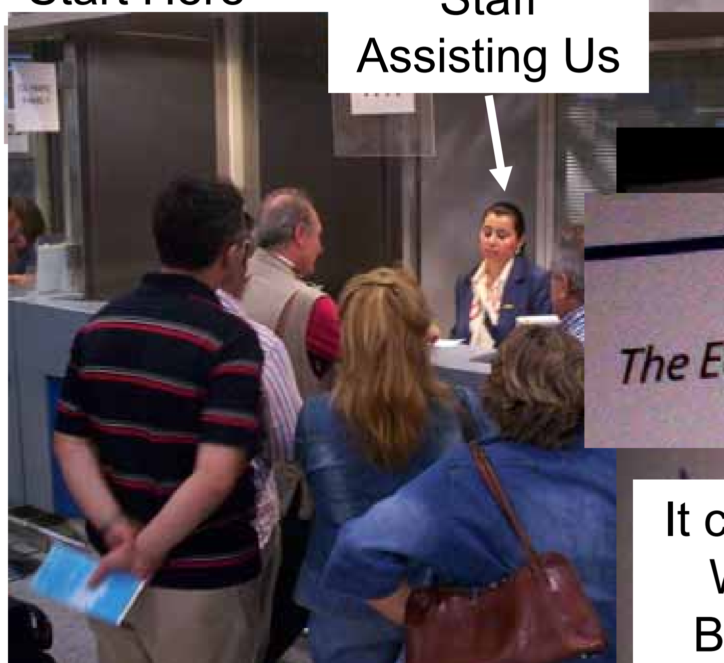
- As you may have gathered, your lecturers coming from WIN05 at Delphi had some difficulty getting to Anacapri…

Of course Olympic Airlines was very helpful…

“Hotel Desk”

New Refugees Start Here

The Many Helpful and Courteous Olympic Air Staff Assisting Us



It could have been worse! Who knows what fate Brussels kept us from?

End of Complaining. Neutrinos, anyone?

These lectures

- Since you haven't **yet** heard about oscillation experiments, I will start with a minimal introduction to important elements
- Then we will delve into cross-sections
  - First from a theoretical point of view, starting from the basics of weak interactions and applying them to point-like scattering
  - As we proceed, the discussion will become increasingly applied.

MINIMAL INTRODUCTION

Ingredients for Oscillations

- After the previous lecture(s), you are all experts in the theory of neutrino oscillations.

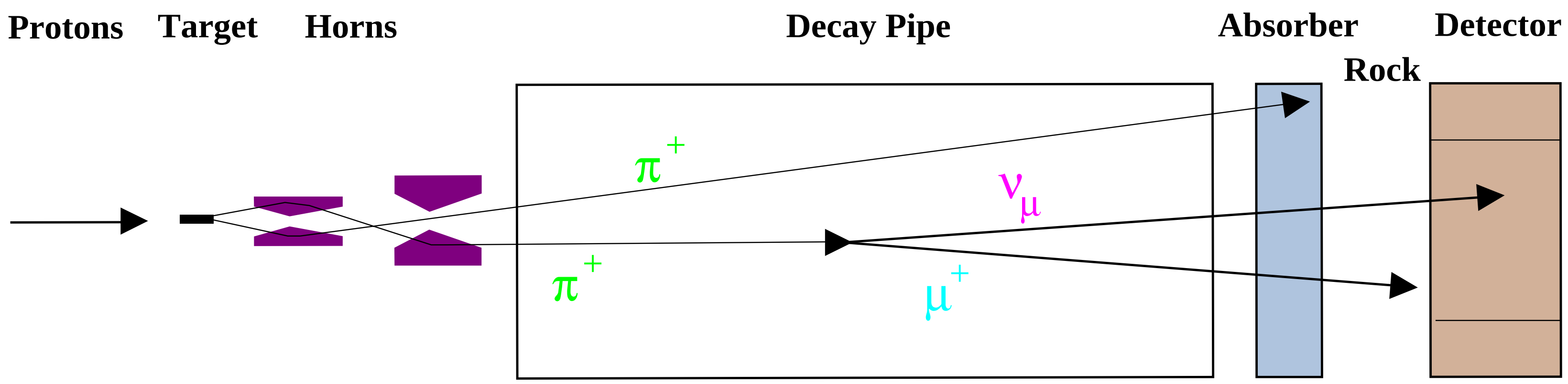
- From a theoretical perspective, how do you do a neutrino oscillation experiment?

Ingredients for Oscillations (cont'd)

- From a theoretical perspective, how do you do a neutrino oscillation experiment?
- "Prepare neutrinos in a flavor eigenstate."
  - Conventional, Muon and Beta Sources
- "Observe flavor eigenstates at far detector…"
  - Disappearance and Appearance Experiments
- "… through the interactions of neutrinos."
  - Charged and Neutral Weak Interactions

NEUTRINO BEAMS

Generic Features of ν Beams



- Produce weakly decaying, relativistic particles
- Focus them towards detector
- Allow them to decay
- Shield detector from the source

Types of Neutrino Beams

- Conventional: $\pi^+, K^+ \rightarrow \mu^+ \nu_\mu$
- Muon Source: $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$
- Reactors and "Beta" Beams: ${}^{A}Z \rightarrow {}^{A}(Z+1) e^- \bar{\nu}_e$

Type	Neutrino Flavors	Flavor Selection	In Use?
Conventional	Muon, neutrino and anti-neutrino	Meson charge	Copiously
Reactors and Beta Beams	Electron neutrino and anti-neutrino	Nucleus. (Anti-nu only at reactors)	A at rest (<5 MeV)
Muon	One from each of: electron, muon, and neutrino and anti-neutrino	Muon charge	μ at rest (~30 MeV)

As you may have gathered, great plans are afoot to create accelerated beams for the latter two types of sources…

Conventional Beams

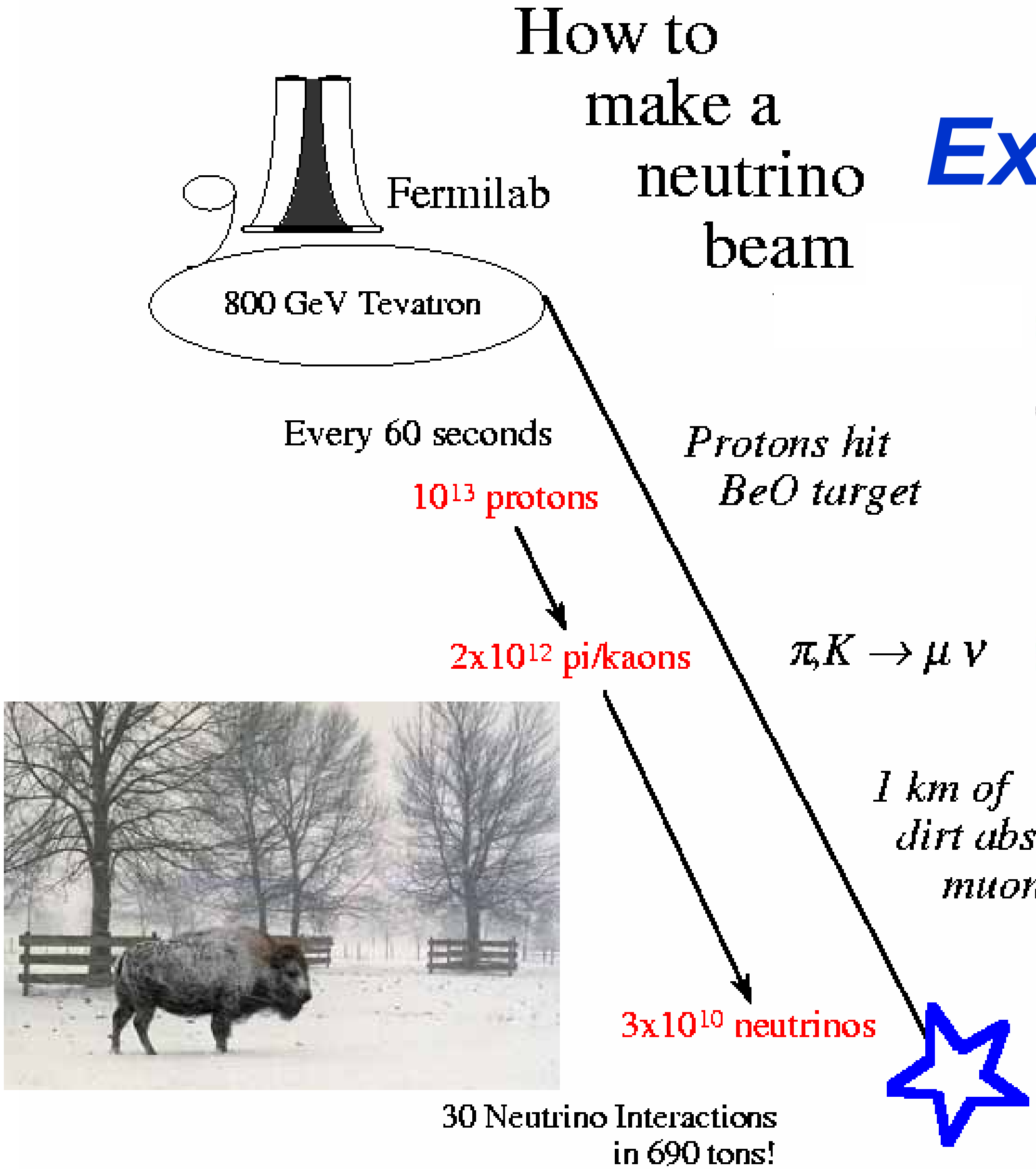
- π and K mesons primarily decay to muon neutrinos or anti-neutrinos
  - meson sign selects which

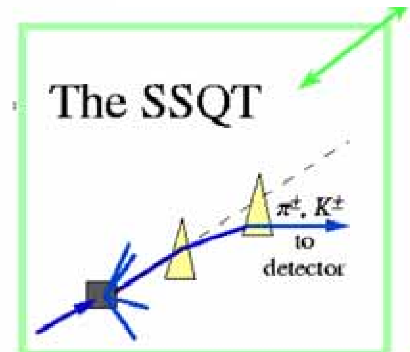
    e.g.,

$$\pi^+ \to \mu^+ \nu_\mu$$
$$\pi^- \to \mu^- \overline{\nu}_\mu$$

- Flavor backgrounds come from
  - Muon decay
  - K_{e3} decay (~7% of $K_{\mu 2}$ decay rate)
  - Charm decay (to electron and D_S to $\tau\nu_\tau$)

Example: NuTeV

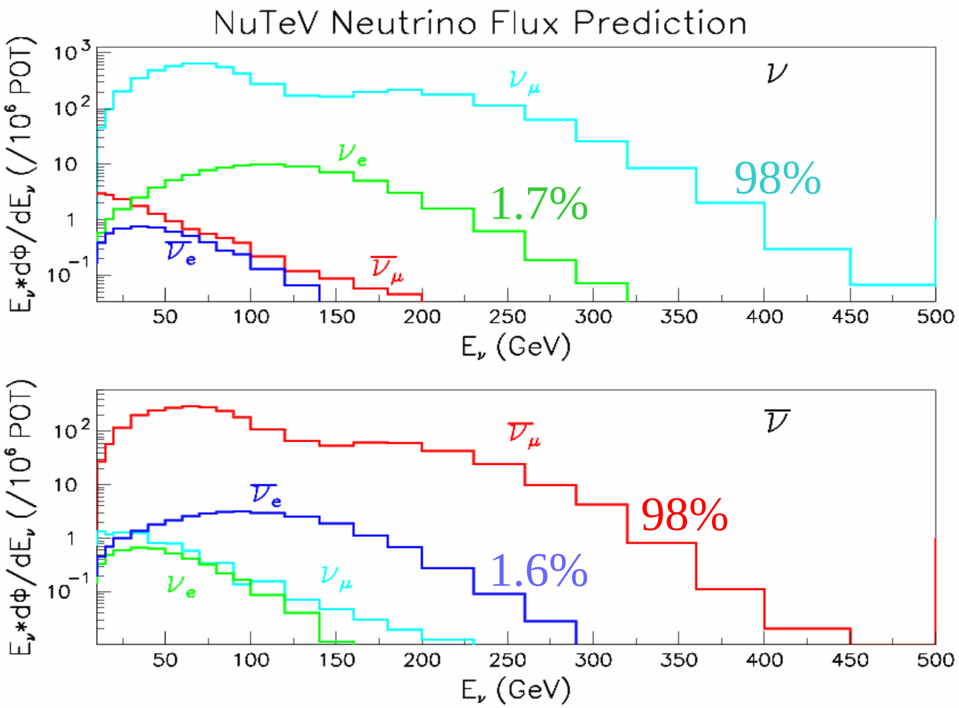




- Beam is very pure ($\bar{\nu}$ in ν mode 3×10^{-4}, ν in $\bar{\nu}$ mode 4×10^{-3})
- Beam has $\sim1.6\%$ electron neutrinos

NuTeV Neutrino Flux

- What processes produce neutrinos in this beam?
  - Energy of secondaries is ~120 – 300 GeV.
  - Decay pipe is 400m vs. $\gamma c \tau_\pi$ ~ 10 km.
  - ν_μ from $\pi^\pm$, $K^\pm$ decays are ~98% of the beam
    - Second hump of spectrum is $K^\pm$. Higher Q of decay.
- Flavor backgrounds (ν_e):
  - ~10^{-2} from $K^\pm$ ($K^\pm_{e3}$ BR)
  - ~10^{-3} from other strange
  - Charm is ~10^{-3}
  - Muon decay is ~10^{-4}
  - ν_τ production is mostly from rare D_s decay. ~10^{-5}



EXPERIMENTAL OBSERVATIONS

What does one actually measure?

- Charged-current interactions of neutrinos

$$\upsilon_l + X \rightarrow l^- + X'$$

  - These almost always tag the “flavor” of the neutrino at the detector by presence of a particular final state lepton

- Neutral current interactions of neutrinos

$$\upsilon_l + X \rightarrow \upsilon_l + X'$$

  - Flavor independent (caveat emptor: “as far as we know for the three neutrinos we know and love”, LEP I)

Disappearance Measurements

- Compare rate at a far detector to prediction or extrapolation from a near detector to measure transition probability, P.
  - Two major sources of uncertainty
    - Predicted rate at far detector
      - Fractional uncertainty, f, directly limits sensitivity to P>f.
    - Statistics at far detector
      - Sensitivity to oscillation probabilities where $1-P<\sqrt{\frac{1}{N}}$
- No observable CP violation because CPT says…

$$P(\nu_l \rightarrow \nu_l) = P(\overline{\nu_l} \rightarrow \overline{\nu_l})$$

- Neutral current disappearance implies sterile neutrinos

Appearance Measurements

- Look for increase in neutrinos of a particular flavor, indicating transitions from another flavor w/ probability P.
- Major sources of uncertainty
  - Background, from beam or misidentifications
    - Fractional background uncertainty, f, limits sensitivity to transitions with probability $P > f\,\frac{N_{\text{background}}}{N_{\text{initial flavor}}}$
  - Appearance statistics affect sensitivity as $\frac{1}{N}$
- Neutrino vs. anti-neutrino rate probes CP violation
- Differences between neutral and charged-current rates signal appearance of neutrinos whose charged current interactions are not observed.

END of MINIMAL INTRODUCTION

partons-ν to the world of

NEUTRINO INTERACTIONS

Outline for Neutrino Interactions

- Weak interactions and neutrinos
  - Elastic and quasi-elastic processes, e.g., νe scattering
  - Deep inelastic scattering, (νq scattering)
  - The difficulties of being in near thresholds…
- Current & future cross-section knowledge
  - What we need to learn and how to learn it

Weak Interactions

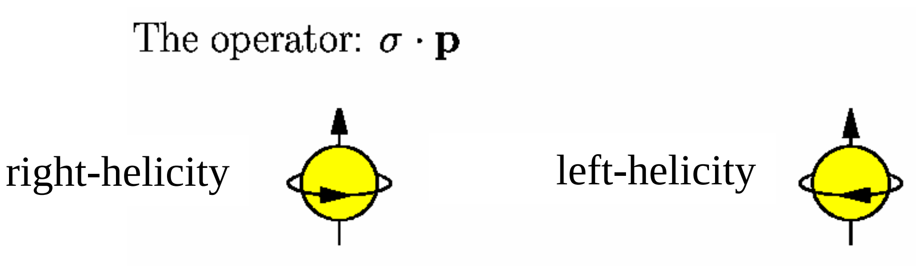
- Current-current interaction (Fermi 1934) $\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$
  - Paper rejected by *Nature* because "it contains speculations too remote from reality to be of interest to the reader"
- Modern version:

$$H_{weak} = \frac{G_F}{\sqrt{2}} \left[\bar{l} \gamma_\mu (1-\gamma_5) \nu \right] \left[\bar{f} \gamma^\mu (V - A\gamma_5) f \right] + h.c.$$

- $P_L = 1/2(1-\gamma_5)$ is a projection operator onto left-handed states for fermions and right-handed states for anti-fermions

Helicity and Chirality

- ***Helicity*** is projection of spin along the particles direction
  - Frame dependent (if massive)



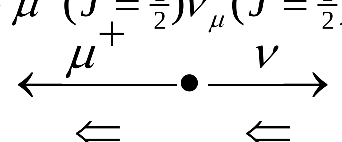
- Neutrinos only interact weakly with a (V-A) interaction
  - All **neutrinos** are **left-handed**
  - All **antineutrinos** are **right-handed**
    - **because of production!**
  - Weak interaction **maximally** violates parity

- However, ***chirality*** ("handedness") is Lorentz-invariant
  - Only same as helicity for massless particles.

- If neutrinos have mass then left-handed neutrino is:
  - Mainly left-helicity
  - But also small right-helicity component $\propto m/E$
- Only left-handed charged-leptons (e^-,μ^-,τ^-) interact weakly but mass brings in right-helicity:

$$R_{theory} = \frac{\Gamma(\pi^\pm \to e^\pm \nu_e)}{\Gamma(\pi^\pm \to \mu^\pm \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi{}^2 - m_e{}^2}{m_\pi{}^2 - m_\mu{}^2}\right)^2 = 1.23 \times 10^{-4}$$

$$\pi^+(J=0) \to \mu^+(J=\tfrac{1}{2})\nu_\mu(J=\tfrac{1}{2})$$



Two Weak Interactions

- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

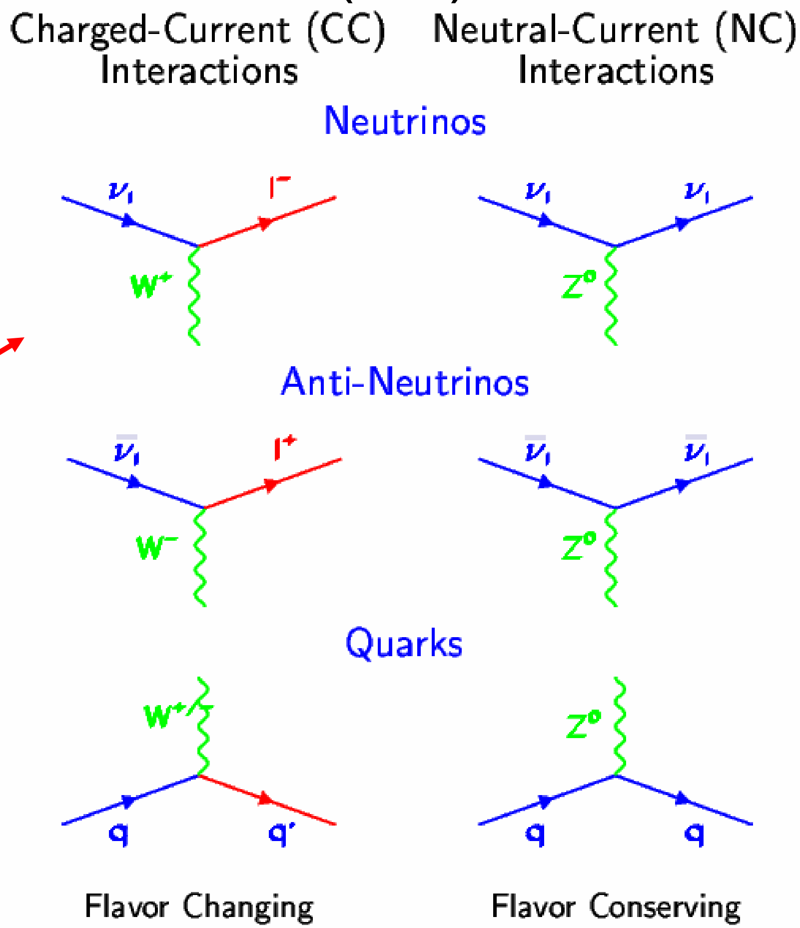
In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

$$l^+ \Rightarrow \overline{\nu}_l$$

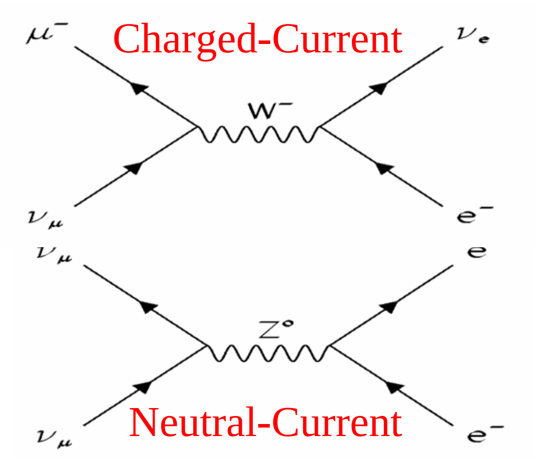


Electroweak Theory

- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    - ⇒ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings, $g' = g \tan\theta_W$

Z Couplings	g_L	g_R
ν_e, ν_μ, ν_τ	1/2	0
e, μ, τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u, c, t	$1/2 - 2/3 \sin^2\theta_W$	$-2/3 \sin^2\theta_W$
d, s, b	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

$$e = g\sin\theta_W,\; G_F = \frac{g^2\sqrt{2}}{8M_W^2},\; \frac{M_W}{M_Z} = \cos\theta_W$$



- Neutrinos are special in SM
  - Right-handed neutrino has ***NO*** interactions!

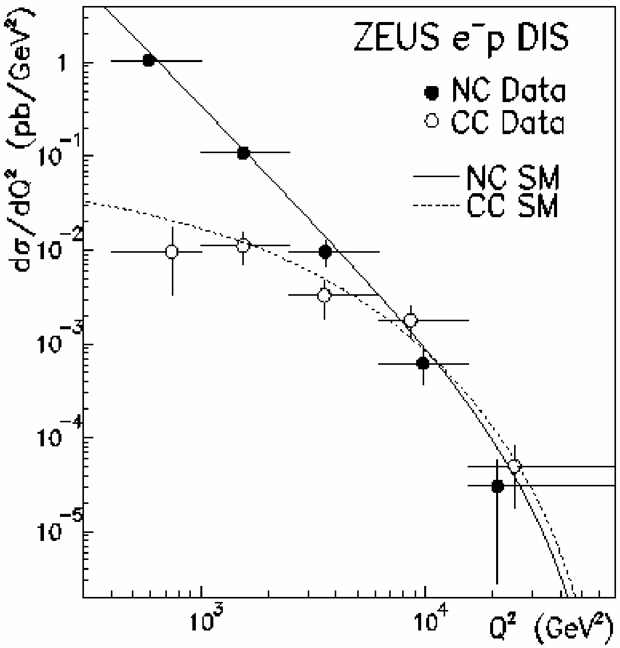
Why "Weak"?

- Weak interactions are weak because of the massive W and Z bosons exchange

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

q is 4-momentum carried by exchange particle
M is mass of exchange particle

At HERA see W and Z propagator effects
- Also weak ~ EM strength



- Explains dimensions of Fermi "constant"

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$$

$$= 1.166 \times 10^{-5} / GeV^2 \quad (g_W \approx 0.7)$$

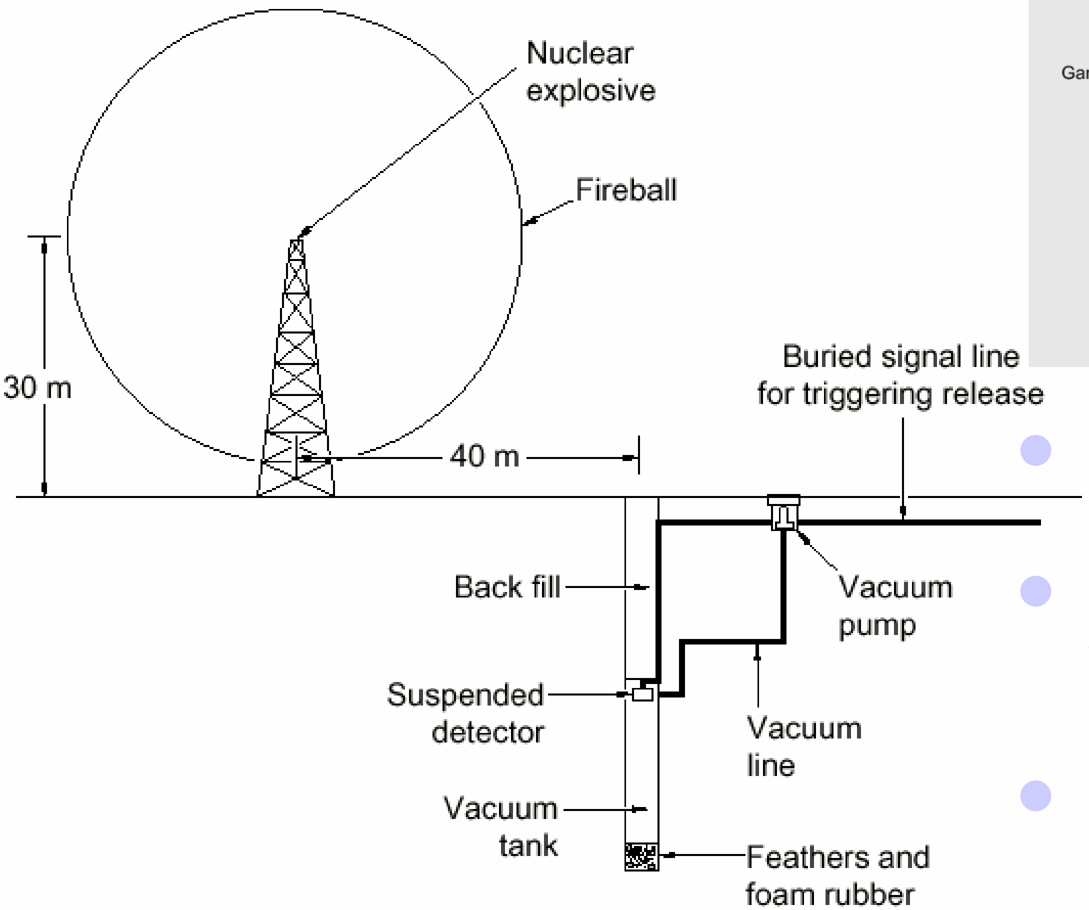
How Weak is Weak?

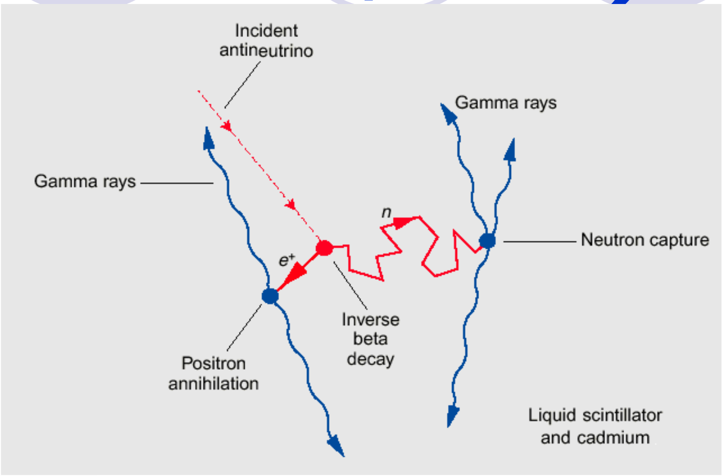
- 100 GeV Neutrinos incident on a target
  - $\sigma(\nu e) \sim 10^{-40}$ and $\sigma(\nu p) \sim 10^{-36}$ cm^2 vs. $\sigma(pp) \sim 10^{-26}$ cm^2
  - ***Mean free path in a steel absorber is 10 light seconds***

"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."

Wolfgang Pauli

Extreme Measures to Overcome Weakness (Reines and Cowan, 1946)





- Ultimately realized at a nuclear reactor (Savannah River)
- 1956: ”We are happy to inform you [Pauli] that we have definitely detected neutrinos…”
- 1995 Nobel Prize for Reines

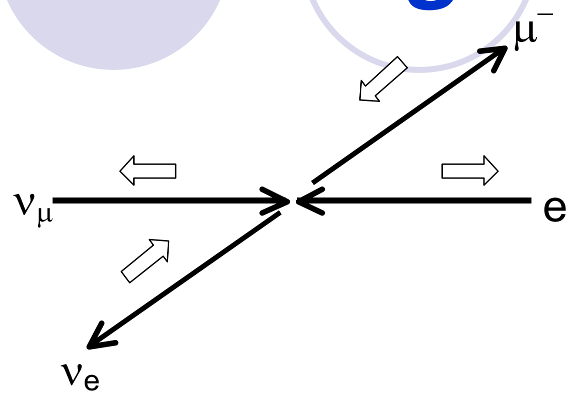
Neutrino-Electron Scattering

- Inverse μ–decay:

  $$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

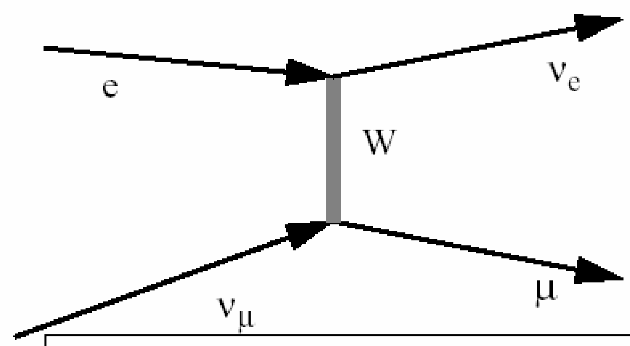
  - Total spin J=0

    (*Assuming massless muon*, helicity=chirality)



$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

$$\approx \frac{Q^2_{\max}}{M_W{}^4}$$



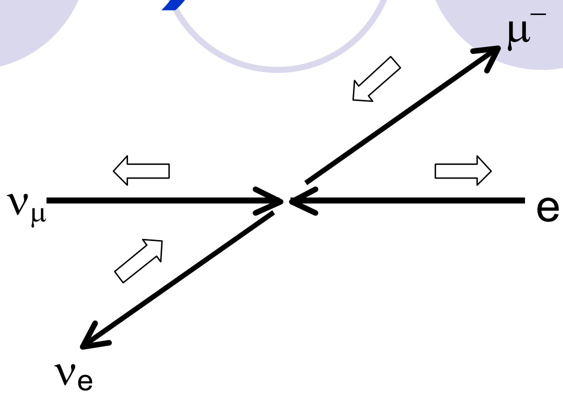
what is Q^2_{max}?

$$Q^2 = -\left(\underline{e} - \underline{\nu}_e\right)^2$$
$$\approx -\left[-2E_\nu^{*2}\left(1-\cos\theta^*\right)\right] \quad \text{(CM frame)}$$
$$< \left(2E_\nu^*\right)^2 = s$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$
$$= 17.2\times10^{-42} cm^2 / GeV \cdot E_\nu(GeV)$$

Neutrino-Electron (cont'd)

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$
$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu (GeV)$$



- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2 m_e E_\nu \quad (\text{e}^- \text{ rest frame})$$

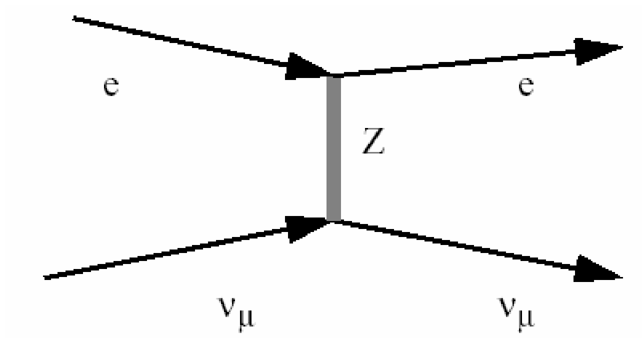
- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant

Neutrino-Electron (cont'd)

- Elastic scattering:

  $$\nu_\mu + e^- \to \nu_\mu + e^-$$

  - Coupling to left *or* right-handed electron
  - Total spin, J=0,1



- Electron-Z^0 coupling
  - (LH, V-A): $-1/2 + \sin^2\theta_W$

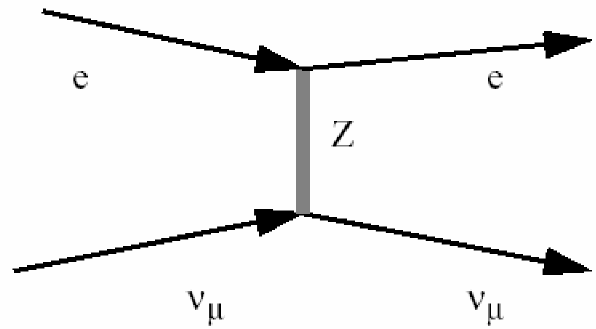
  $$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

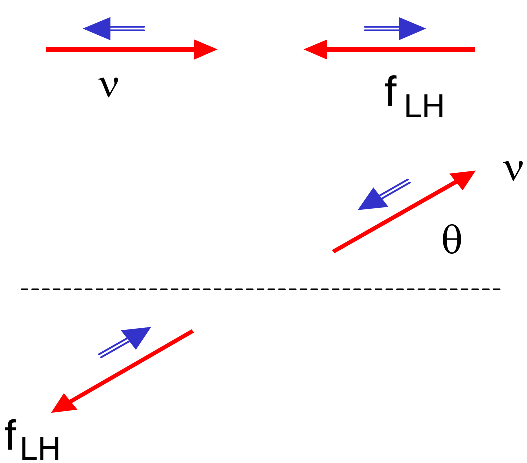
  - (RH, V+A): $\sin^2\theta_W$

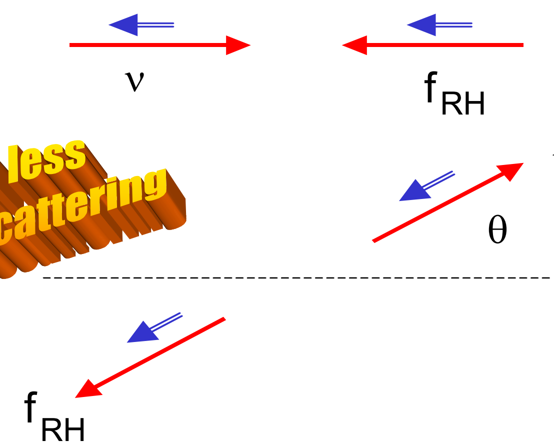
  $$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

Neutrino-Electron (cont'd)

- What are relative contributions of left *and* right-handed scattering from electron?







Now with less backwards scattering

$$\frac{d\sigma}{d\cos\theta} = \text{const}$$

$$\frac{d\sigma}{d\cos\theta} = \text{const} \times \left(\frac{1+\cos\theta}{2}\right)^2$$

Neutrino-Electron (cont'd)

- Electron-Z^0 coupling $\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$ $\quad \sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$

Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1-\cos\theta)$$

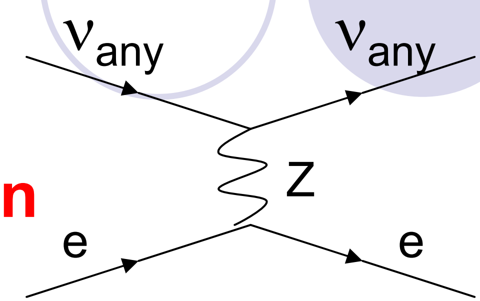
$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times 10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Concept Question #1

- **The reaction**

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$



has a much smaller cross-section than

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

What extra process present in the second makes this so? (Naïve answer)

Show that this increases the rate (precise answer)

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy}$$

$$= \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right]$$

$$= \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{LH} \right|^2$$

For electron…	LH coupling	RH coupling
Weak NC	$-1/2+\sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	-1/2	0

)

Concept Question #1

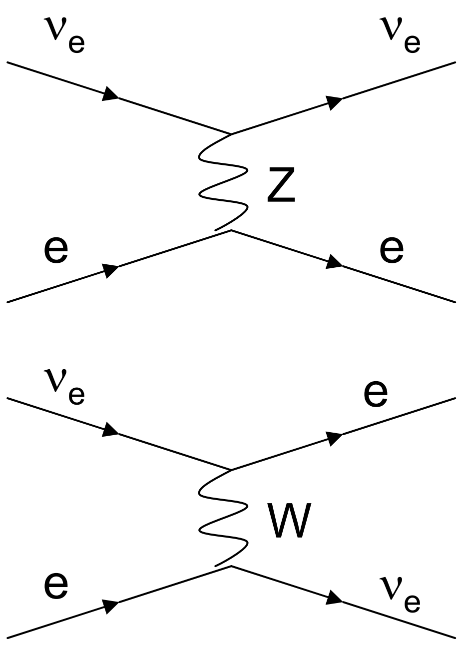
- **The reaction**

$$\nu_\mu + e^- \to \nu_\mu + e^-$$

has a much smaller cross-section than

$$\nu_e + e^- \to \nu_e + e^-$$

Why is this?



Naïve answer: Because there is both a CC and NC reaction!

More precisely: We have to show the interference between the two is constructive.

The total RH coupling is unchanged because there is no RH weak CC coupling

There are two LH couplings: NC coupling is $-1/2+\sin^2\theta_W \approx -1/4$ and the CC coupling is -1/2. We add the associated amplitudes… and get $-1+\sin^2\theta_W \approx -3/4$

Lepton Mass Effects

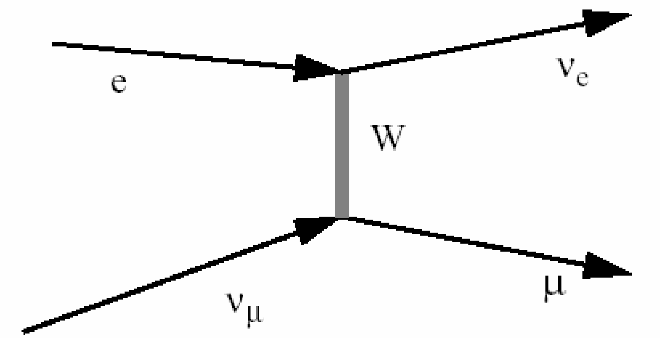
- Let's return to

  Inverse μ–decay:

  $$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

  - What changes in the presence of final state mass?
    - pure CC so always left-handed
    - BUT there must be finite Q^2 to create muon in final state!

$$Q^2_{\min} = m^2_\mu$$



$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

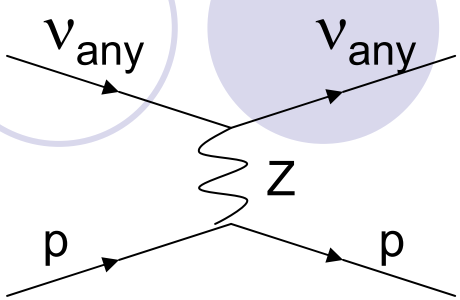
$$\approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

$$\sigma_{TOT} = \frac{G_F^2 (s - m^2_\mu)}{\pi}$$

$$= \left[\sigma_{TOT}{}^{\text{(massless)}}\right]\left(1 - \frac{m^2_\mu}{s}\right)$$

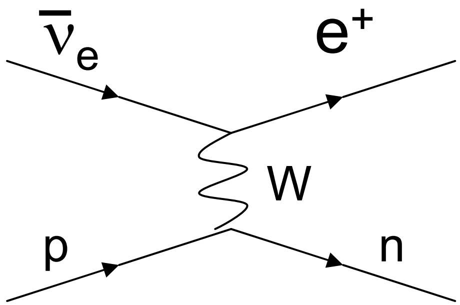
  - see a suppression scaling with **(mass/CM energy)²**
    - can be generalized…

What about other targets?



- Imagine now a proton target
  - Neutrino-proton elastic scattering: $\nu_e + p \rightarrow \nu_e + p$
  - "Inverse beta-decay":

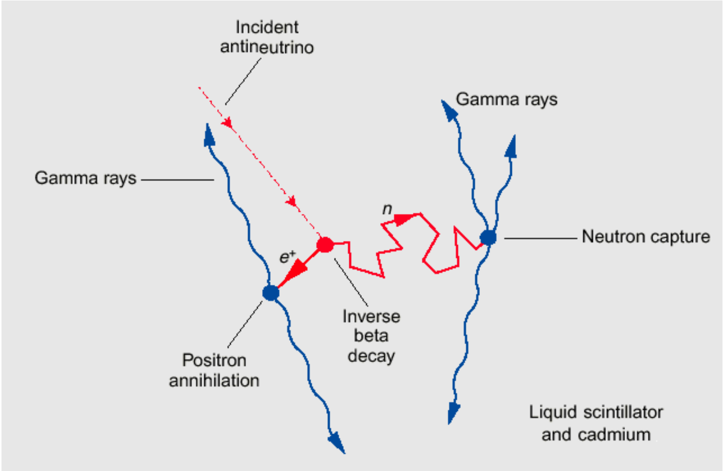
$$\bar{\nu}_e + p \rightarrow e^+ + n$$



  - and its close cousin:

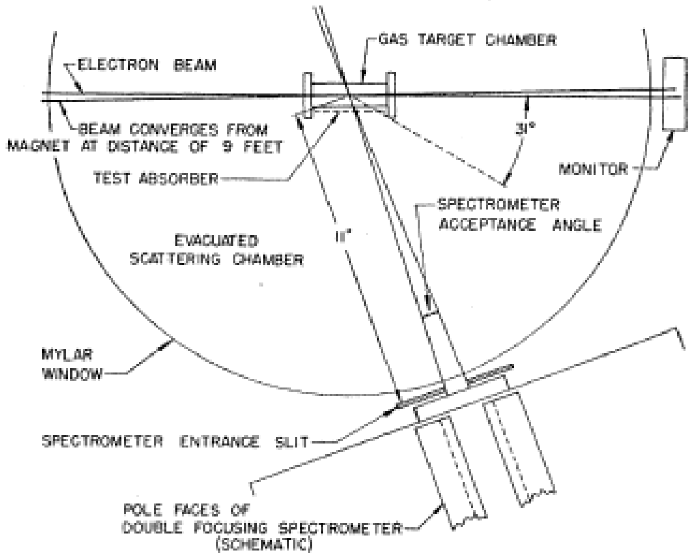
$$\nu_e + n \rightarrow e^- + p$$

  - Inverse beta-decay (IBD) was the Reines and Cowan discovery signal

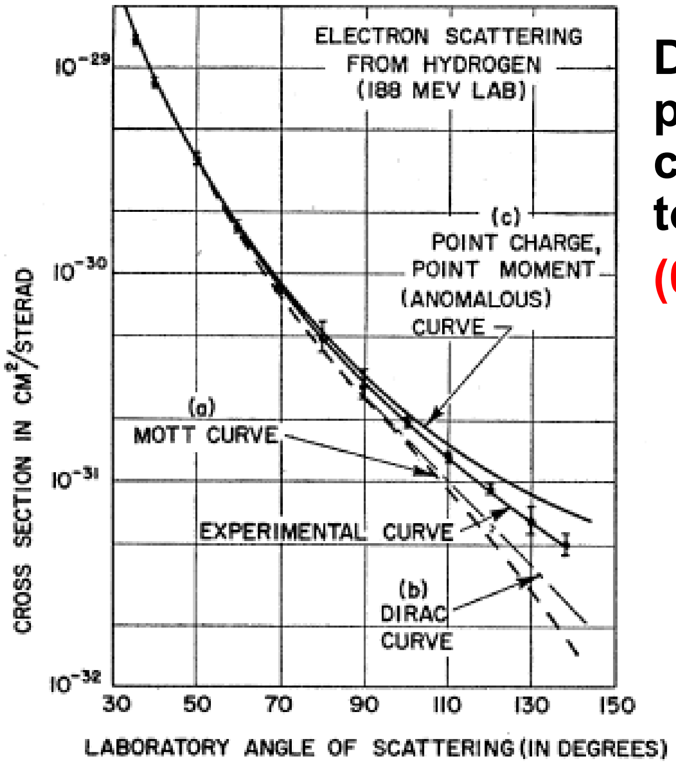


Proton Structure

- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \neq 1$
  - "form factors" related to finite size



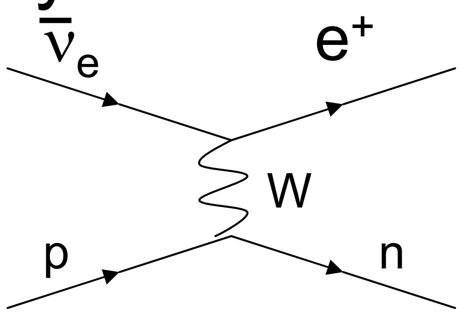
McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford



Determined proton RMS charge radius to be (0.7±0.2) x10^{-13} cm

Final State Mass Effects

- In IBD, $\overline{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*



- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n\left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $E_\nu^{\min} = \frac{(M_n + m_e)^2 - M_p^2}{2M_p} \approx 1.806 \text{ MeV}$

Final State Mass Effects (cont'd)

- Define δE as E_ν-E_ν^{min}, then

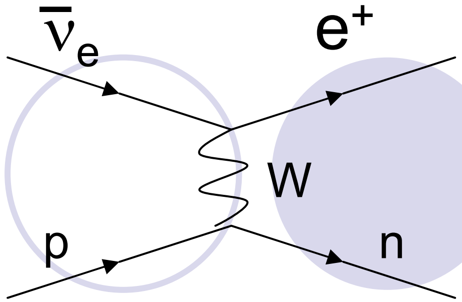
$$s_{\text{initial}} = M_p^2 + 2M_p\left(\delta E + E_\nu^{\min}\right)$$
$$= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2$$
$$= 2\delta E \times M_p + \left(M_n + m_e\right)^2$$

- Remember the suppression generally goes as

$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}^2}{s} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}$$

$$= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p^2}\dfrac{M_p}{\delta E} & \text{high energy} \end{cases}$$

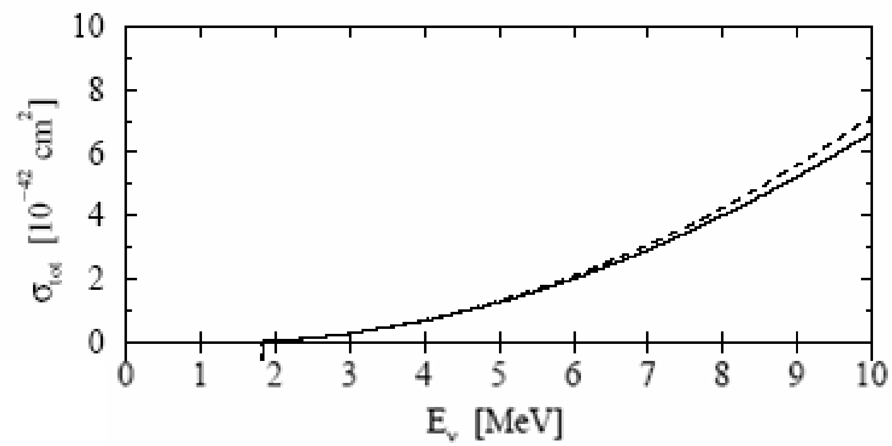
Putting it all together…



$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V{}^2 + 3 g_A{}^2\right)$$

quark mixing! ***final state mass suppression*** ***proton form factors (vector and axial)***

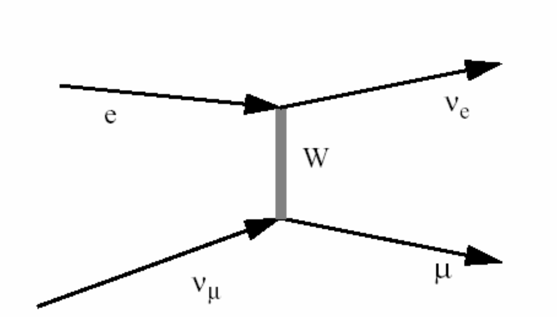
- mass suppression is proportional to δE at low E_ν, so get quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively) g_V, $g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n



Concept Question #2

- **Which is closest to the minimum beam energy in which the reaction**

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



can be observed?

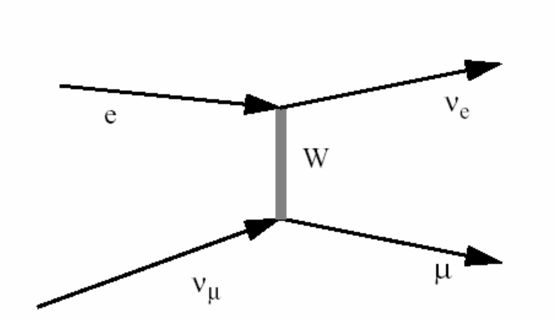
(a) 100 MeV (b) 1 GeV (c) 10 GeV

(It might help you to remember that $Q^2_{\min} = m^2_\mu$
or you might just want to think about the total CM energy required to produce the particles in the final state.)

Concept Question #2

- Which is closest to the minimum beam energy in which the reaction

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



can be observed?

(a) 100 MeV (b) 1 GeV (c) 10 GeV

$$Q^2{}_{\min} = m_\mu^2$$

$$Q^2 < s = (\underline{p}_e + \underline{p}_\nu)^2$$

$$= (m_e + E_\nu, 0, 0, \sqrt{E_\nu{}^2 - m_\nu{}^2})^2 \approx m_e^2 + 2m_e E_\nu$$

$$\therefore E_\nu > \frac{m_\mu^2}{2m_e} \approx 10.9 \text{ GeV}$$

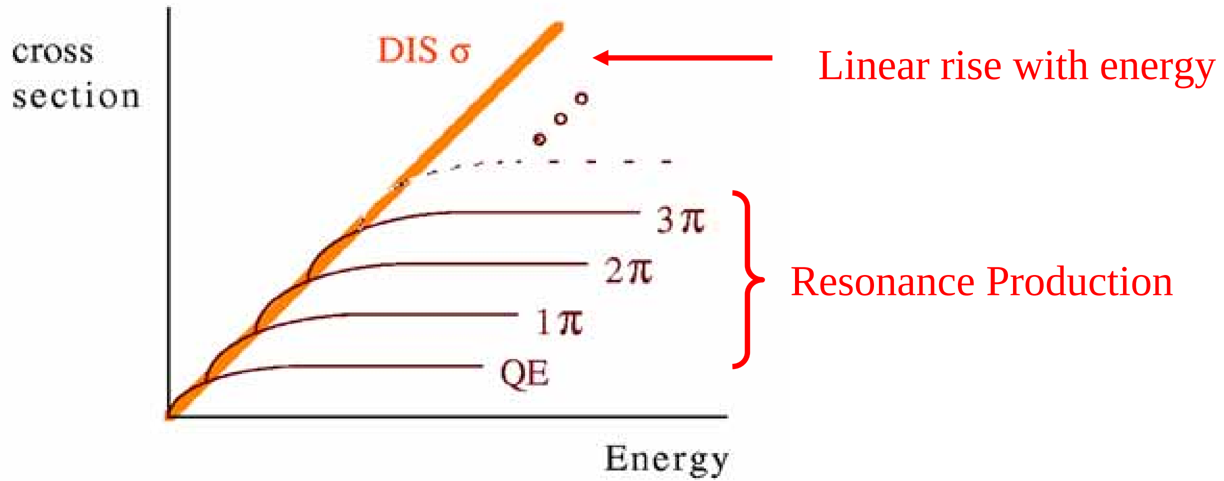
Summary and Outlook

- We know νe^- scattering and IBD cross-sections!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$ is ≈ constant.
    - Integrating gives $\sigma \propto E_\nu$
  - LH coupling enters w/ $d\sigma/dy \propto 1$, RH w/ $d\sigma/dy \propto (1-y)^2$
    - Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum Q^2
    - Integrating gives correction factor in σ of $(1-Q^2_{min}/s)$
  - Structure of target can add form factors
- Deep Inelastic Scattering is also a point-like limit where interaction is ν-quark scattering

Neutrino-Nucleon Deep Inelastic Scattering

Neutrino-Nucleon 'n a Nutshell

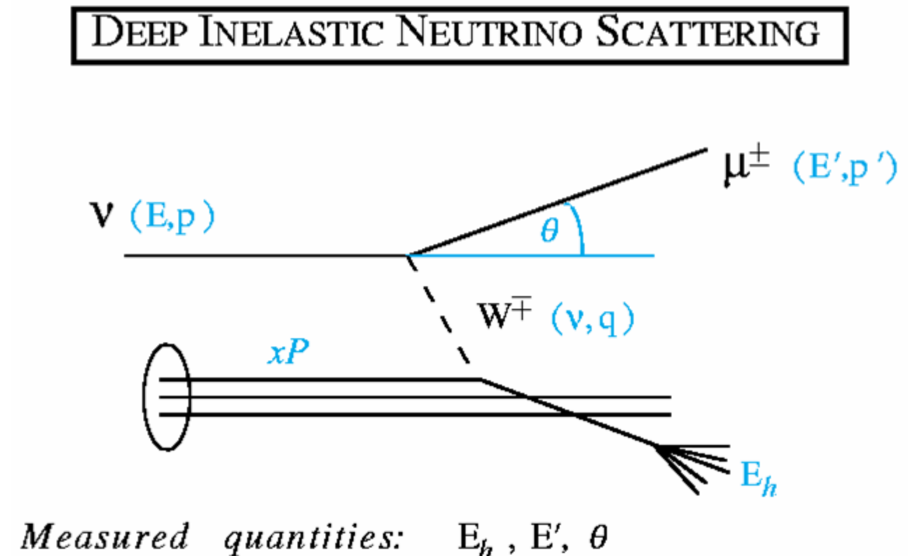
- Charged - Current: $W^{\pm}$ exchange
  - Quasi-elastic Scattering:
    (Target changes but no break up)
    $\nu_\mu + n \rightarrow \mu^- + p$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + n \rightarrow \mu^- + p + \pi^0 \quad (N^* \text{ or } \Delta)$
    $n + \pi^+$
  - Deep-Inelastic Scattering:
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \mu^- + \text{quark'}$
- Neutral - Current: Z^0 exchange
  - Elastic Scattering:
    (Target unchanged)
    $\nu_\mu + N \rightarrow \nu_\mu + N$
  - Nuclear Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + N \rightarrow \nu_\mu + N + \pi \quad (N^* \text{ or } \Delta)$
  - Deep-Inelastic Scattering
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \nu_\mu + \text{quark}$



Scattering Variables

Scattering variables given in terms of invariants

- More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.



$$4\text{-momentum Transfer}^2: \quad Q^2 = -q^2 = -\left(p' - p\right)^2 \approx \left(4EE'\sin^2(\theta/2)\right)_{Lab}$$

$$\text{Energy Transfer}: \quad \nu = (q \cdot P)/M_T = \left(E - E'\right)_{Lab} = \left(E_h - M_T\right)_{Lab}$$

$$\text{Inelasticity}: \quad y = (q \cdot P)/(p \cdot P) = (E_h - M_T)/\left(E_h + E'\right)_{Lab}$$

$$\text{Fractional Momentum of Struck Quark}: \quad x = Q^2/2M_T\nu$$

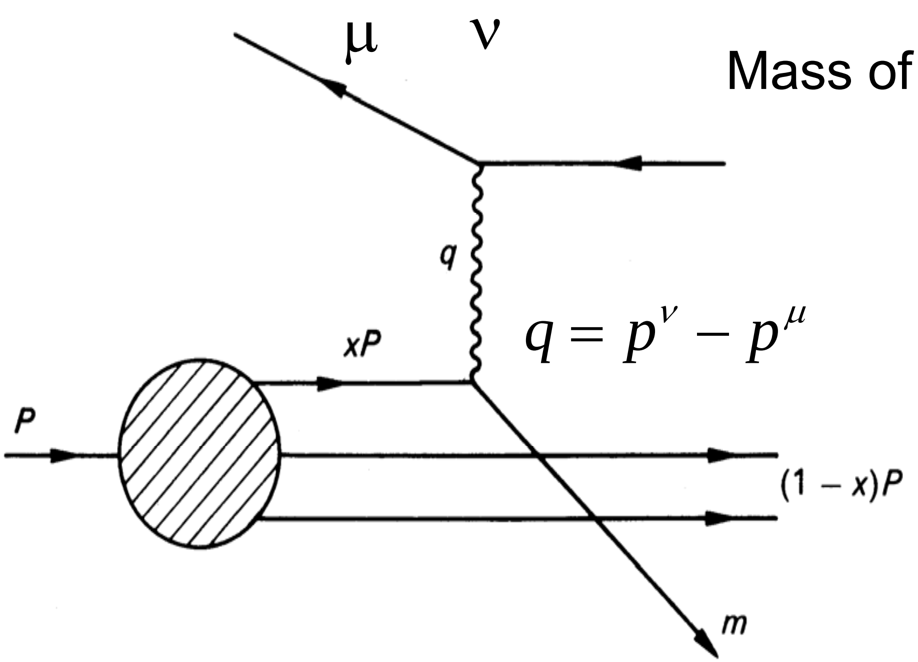
$$\text{Recoil Mass}^2: W^2 = (q+P)^2 = M_T^2 + 2M_T\nu - Q^2$$

$$\text{CM Energy}^2: \quad s = (p+P)^2 = M_T^2 + \frac{Q^2}{xy}$$

Parton Interpretation of DIS

Mass of target quark $m_q^{\;2} = x^2 P^2$

Mass of final state quark $m_{q'}^{\;2} = (xP + q)^2$



$$q = p^{\nu} - p^{\mu}$$

In “infinite momentum frame”, x is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

Neutrino scatters off a parton inside the nucleon

So why is cross-section so large?

- (at least compared to νe^- scattering!)
- Recall that for neutrino beam and target at rest

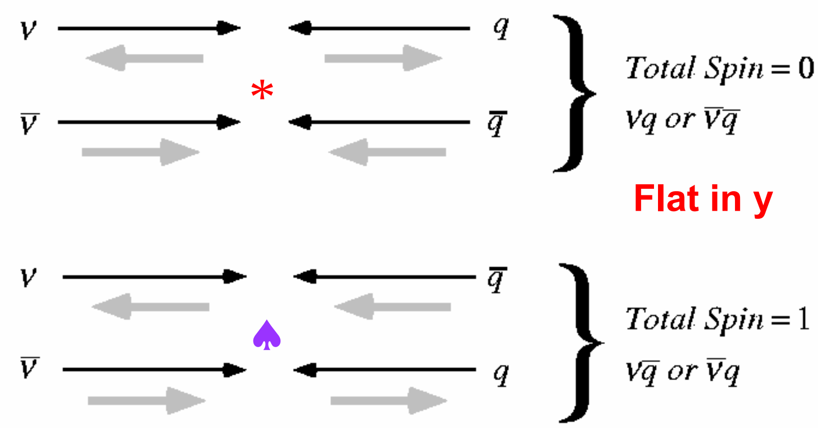
$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q^2_{\max} \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$

$$s = m_e^2 + 2 m_e E_\nu$$

- But we just learned for DIS that effective mass of each target quark is $m_q = x m_{\text{nucleon}}$
- So much larger target mass means larger σ_{TOT}

Chirality, Charge in CC ν-q Scattering

- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



Flat in y

1/4(1+cosθ*)² = (1-y)²

∫(1-y)²dy=1/3

point-like scattering implies linear with energy

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(xd(x) + x\bar{u}(x)(1-y)^2 \right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\bar{d}(x) + xu(x)(1-y)^2 \right)$$

but what is this "q(x)"?

- Neutrino/Anti-neutrino CC each produce particular Δq in scattering

$$\nu d \rightarrow \mu^- u$$

$$\bar{\nu} u \rightarrow \mu^+ d$$

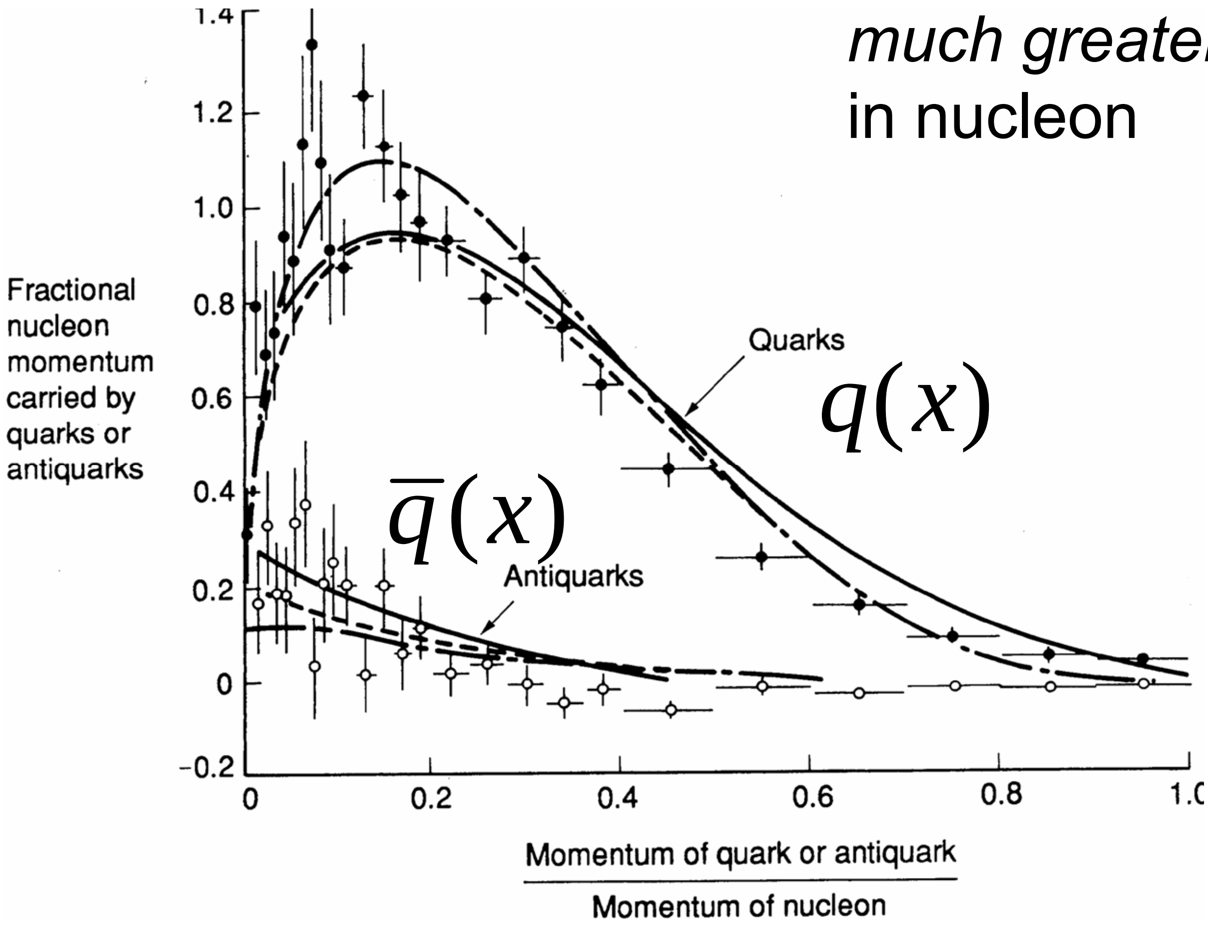
Factorization and Partons

- Factorization Theorem of QCD allows amplitudes for hadronic processes to be written as:

$$A(l+h \rightarrow l+X) = \sum_q \int dx A(l+q(x) \rightarrow l+X) q_h(x)$$

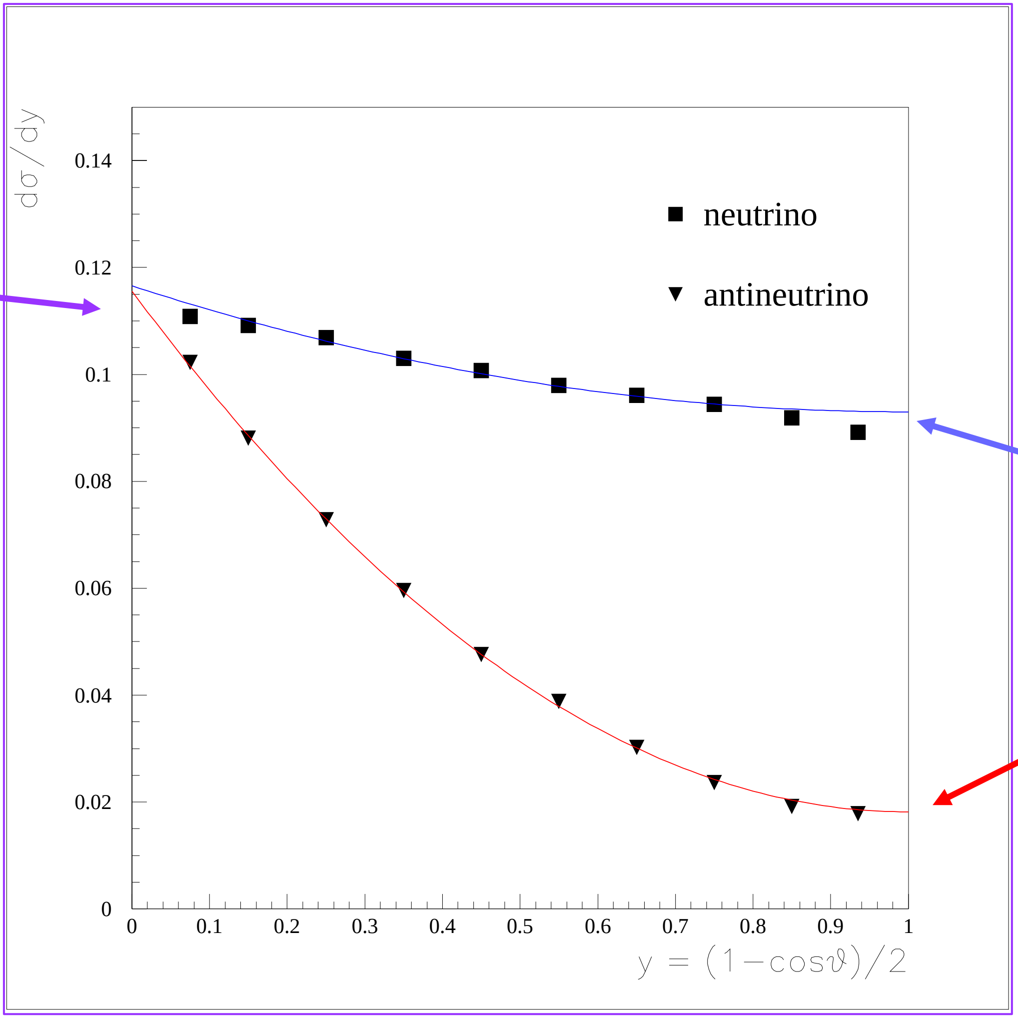
  - Parton distribution functions (PDFs) are universal
  - Processes well described by single parton interactions
  - Parton distribution functions not (yet) calculable from first principles in QCD
- “Scaling”: parton distributions are largely independent of Q^2 scale, and depend on fractional momentum, x.

Momentum of Quarks & Antiquarks



- Momentum carried by quarks *much greater* than anti-quarks in nucleon

y distribution in Neutrino CC DIS



$$\frac{d\sigma(\nu q)}{dxdy} = \frac{d\sigma(\bar{\nu}\bar{q})}{dxdy} \propto 1$$

$$\frac{d\sigma(\nu \bar{q})}{dxdy} = \frac{d\sigma(\bar{\nu} q)}{dxdy} \propto (1-y)^2$$

y=0:

Quarks & anti-quarks

Neutrino and anti-neutrino identical

y=1:

Neutrinos see only quarks.

Anti-neutrinos see only anti-quarks

$$\sigma^{\bar{\nu}} \approx \frac{1}{2}\sigma^{\nu}$$

Concept Question #3

- Given: $\sigma_{CC}^{\bar{\nu}} \approx \frac{1}{2}\sigma_{CC}^{\nu}$ in the DIS regime (CC)

  and $\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu}q)}{dx}$

  for CC scattering from quarks or anti-quarks of a given momentum,

  and that cross-section is proportional to parton momentum, what is the approximate ratio of anti-quark to quark momentum in the nucleon?

(a) $\bar{q}/q \sim 1/3$ (b) $\bar{q}/q \sim 1/5$ (c) $\bar{q}/q \sim 1/8$

Concept Question #3

- **Given:** $\sigma_{CC}^{\bar{\nu}} \approx \frac{1}{2}\sigma_{CC}^{\nu}$ **in the DIS regime (CC)**

and $\sigma(\nu q) = \sigma(\bar{\nu}\bar{q}) = 3\sigma(\nu\bar{q}) = 3\sigma(\bar{\nu} q)$

(a) $\bar{q}/q \sim 1/3$ **(b)** $\bar{q}/q \sim 1/5$ **(c)** $\bar{q}/q \sim 1/8$

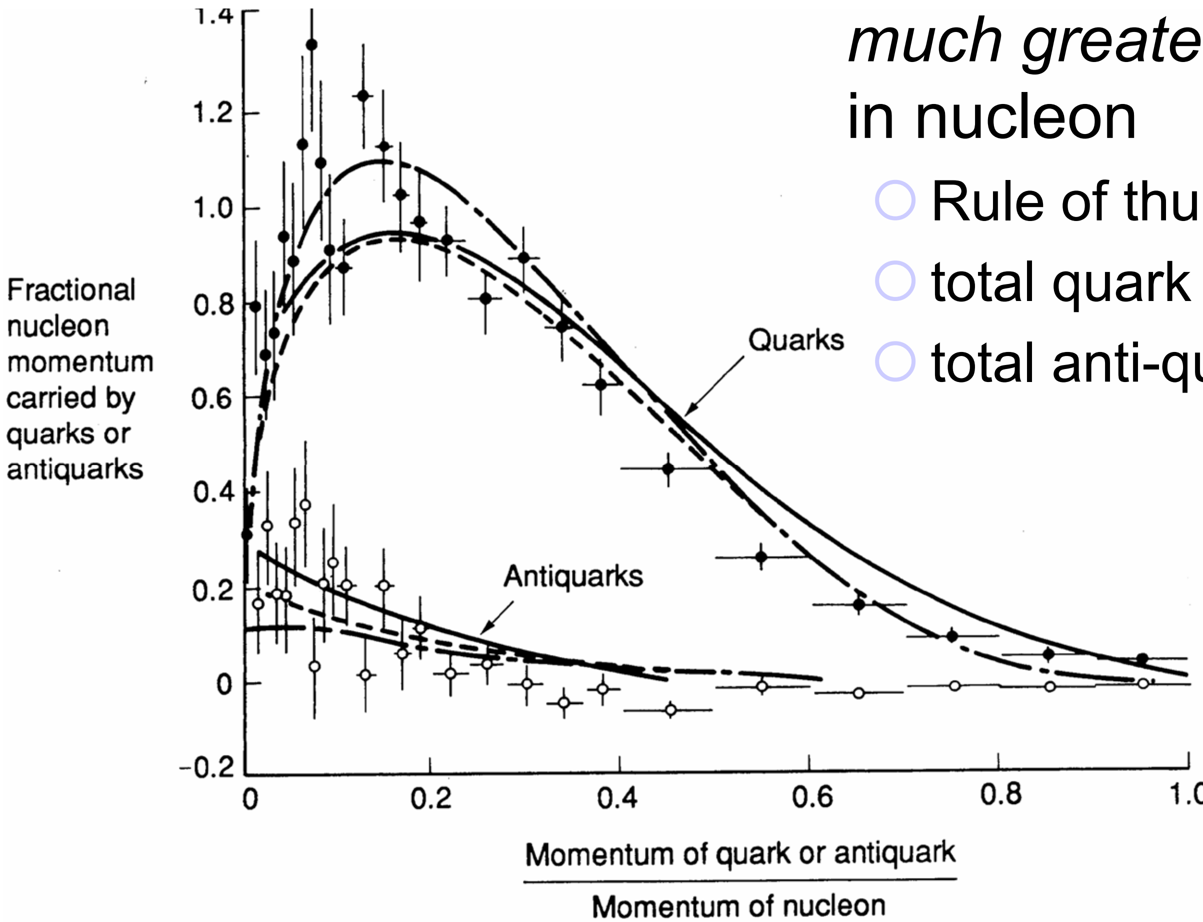
$$\sigma_\nu = \int_{q,\bar{q}} dx \left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx} \right)$$

$$\sigma_{\bar{\nu}} = \int_{q,\bar{q}} dx \left(\frac{d\sigma(\bar{\nu} q)}{dx} + \frac{d\sigma(\bar{\nu}\bar{q})}{dx} \right) = \int_{q,\bar{q}} dx \left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx} \right)$$

$$\therefore \int_{q,\bar{q}} dx \left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx} \right) = 2\int_{q,\bar{q}} dx \left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx} \right)$$

$$\frac{1}{3}\int_{q,\bar{q}} dx \frac{d\sigma(\nu q)}{dx} = 5\int_{q,\bar{q}} dx \frac{d\sigma(\nu\bar{q})}{dx} = \frac{5}{3}\int_{q,\bar{q}} dx \frac{d\sigma(\bar{\nu}\bar{q})}{dx}$$

Momentum of Quarks & Antiquarks



- Momentum carried by quarks *much greater* than anti-quarks in nucleon
  - Rule of thumb: at Q^2 of 10 GeV^2:
  - total quark momentum is 1/3,
  - total anti-quark is 1/15.

Or… Structure Functions (SFs)

- A model-independent picture of these interactions can also be formed in terms of nucleon "structure functions"
  - All Lorentz-invariant terms included
  - Approximate zero lepton mass (small correction)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} \propto \left[y^2 2xF_1(x,Q^2) + \left(2 - 2y - \frac{M_T xy}{E} \right) F_2(x,Q^2) \pm y(2-y) xF_3(x,Q^2) \right]$$

- For massless free spin-1/2 partons, one simplification…
  - Callan-Gross relationship, $2xF_1=F_2$
  - Implies intermediate bosons are completely transverse

Can parameterize transverse cross-section by R_L.
•Callan-Gross violations, M
•NLO pQCD, $g \rightarrow q\bar{q}$

$$R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left(1 + \frac{4M_T^2 x^2}{Q^2}\right)$$

SFs to PDFs

- Can relate SFs to PDFs in naïve quark-parton model by matching y dependence
  - Assuming Callan-Gross, massless targets and partons…
  - F_3: $2y-y^2$, $2xF_1=F_2$: $2-2y+y^2$

$$2xF_1^{\nu p,CC} = x\left[d_p(x) + \overline{u_p}(x) + s_p(x) + \overline{c_p}(x)\right]$$

$$xF_3^{\nu p,CC} = x\left[d_p(x) - \overline{u_p}(x) + s_p(x) - \overline{c_p}(x)\right]$$

- In analogy with neutrino-electron scattering, CC only involves left-handed quarks
- However, NC involves both chiralities (V-A and V+A)
  - Also couplings from EW Unification
  - And no selection by quark charge

$$2xF_1^{\nu p,NC} = x\left[(u_L^2+u_R^2)\left(u_p(x)+\overline{u_p}(x)+c_p(x)+\overline{c_p}(x)\right)+(d_L^2+d_R^2)\left(d_p(x)+\overline{d_p}(x)+s_p(x)+\overline{s_p}(x)\right)\right]$$

$$xF_3^{\nu p,NC} = x\left[(u_L^2-u_R^2)\left(u_p(x)-\overline{u_p}(x)+c_p(x)-\overline{c_p}(x)\right)+(d_L^2-d_R^2)\left(d_p(x)-\overline{d_p}(x)+s_p(x)-\overline{s_p}(x)\right)\right]$$

Isoscalar Targets

- Heavy nuclei are roughly neutron-proton isoscalar
- Isospin symmetry implies $u_p = d_n, d_p = u_n$
- Structure Functions have a particularly simple interpretation in quark-parton model for this case…

$$\frac{d^2\sigma^{\nu(\bar{\nu})N}}{dxdy} = \frac{G_F^2 s}{2\pi}\left\{ \left(1+(1-y)^2\right) F_2(x) \pm \left(1-(1-y)^2\right) xF_3^{\nu(\bar{\nu})}(x) \right\}$$

$$F_2^{\nu(\bar{\nu})N,CC}(x) = x(u(x)+d(x)+\bar{u}(x)+\bar{d}(x)+s(x)+\bar{s}(x)+c(x)+\bar{c}(x) = \boxed{xq(x)+x\bar{q}(x)}$$

$$xF_3^{\nu(\bar{\nu})N,CC}(x) = \boxed{xu_{Val}(x)+xd_{Val}(x)} \pm 2x(s(x)-c(x))$$

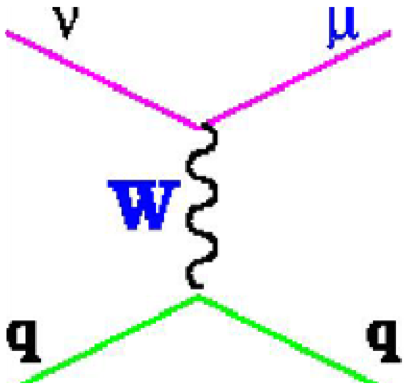
$$\text{where } u_{Val}(x) = u(x) - \bar{u}(x)$$

Neutrino-Nucleon Deep Inelastic Scattering

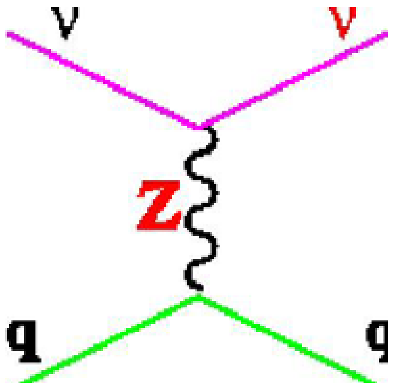
BONUS Example!

Example: NuTeV NC/CC Ratio

- NuTeV experiment measures ratios of neutral to charged current cross-sections on an isoscalar target to extract NC couplings



W-q coupling is I_3



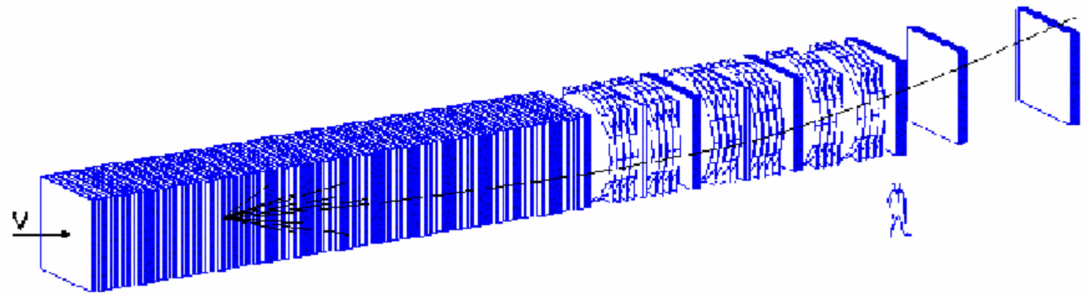
Z-q coupling is $I_3 - Q\sin^2\theta_W$

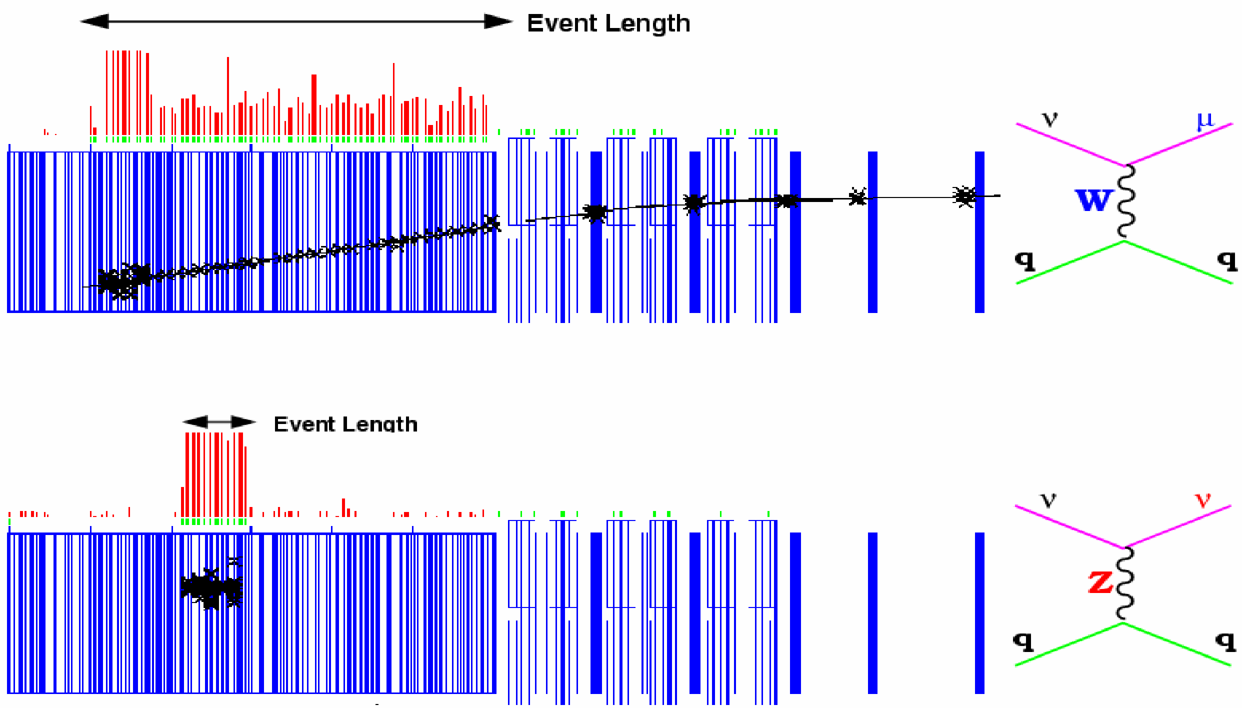
Llewellyn Smith Formulae

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \left(\left(u_L^2 + d_L^2\right) + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\bar{\nu})}} \left(u_R^2 + d_R^2\right) \right)$$

- Holds for isoscalar targets of u and d quarks only
  - Heavy quarks, differences between u and d distributions are corrections
- Isospin symmetry causes PDFs to drop out, even outside of naïve quark-parton model

NuTeV at Work…





NuTeV Fit to R^{ν} and $R^{\nu bar}$

- NuTeV result:

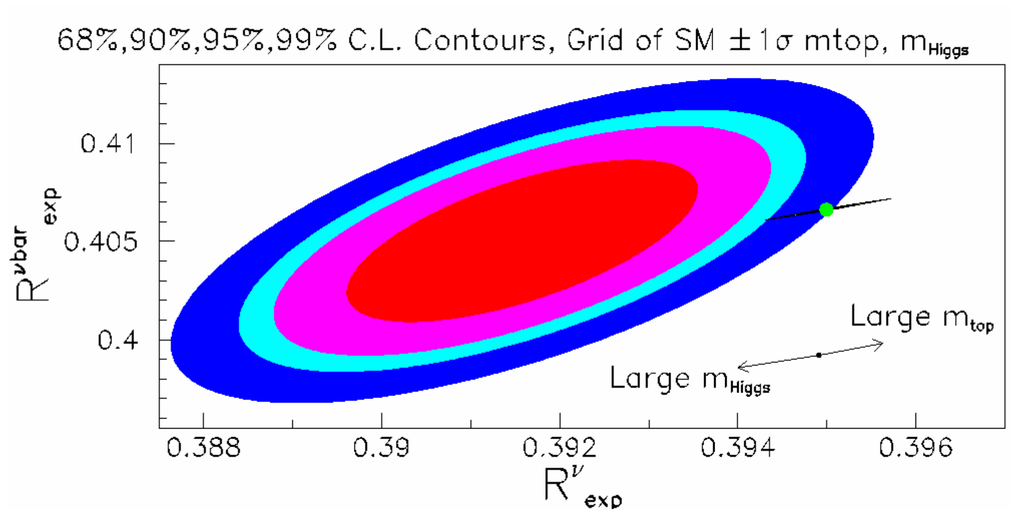
$$\sin^2\theta_W^{(on-shell)} = 0.2277 \pm \pm 0.0013(stat.) \pm 0.0009(syst.)$$
$$= 0.2277 \pm 0.0016$$

(Previous neutrino measurements gave 0.2277 ± 0.0036)

- Standard model fit (LEPEWWG): 0.2227 ± 0.00037

A 3σ discrepancy ....

$$R^{\nu}_{\text{exp}} = 0.3916 \pm 0.0013$$
$$(SM: 0.3950) \quad \Leftarrow 3\sigma\ difference$$
$$R^{\bar{\nu}}_{\text{exp}} = 0.4050 \pm 0.0027$$
$$(SM: 0.4066) \quad \Leftarrow Good\ agreement$$

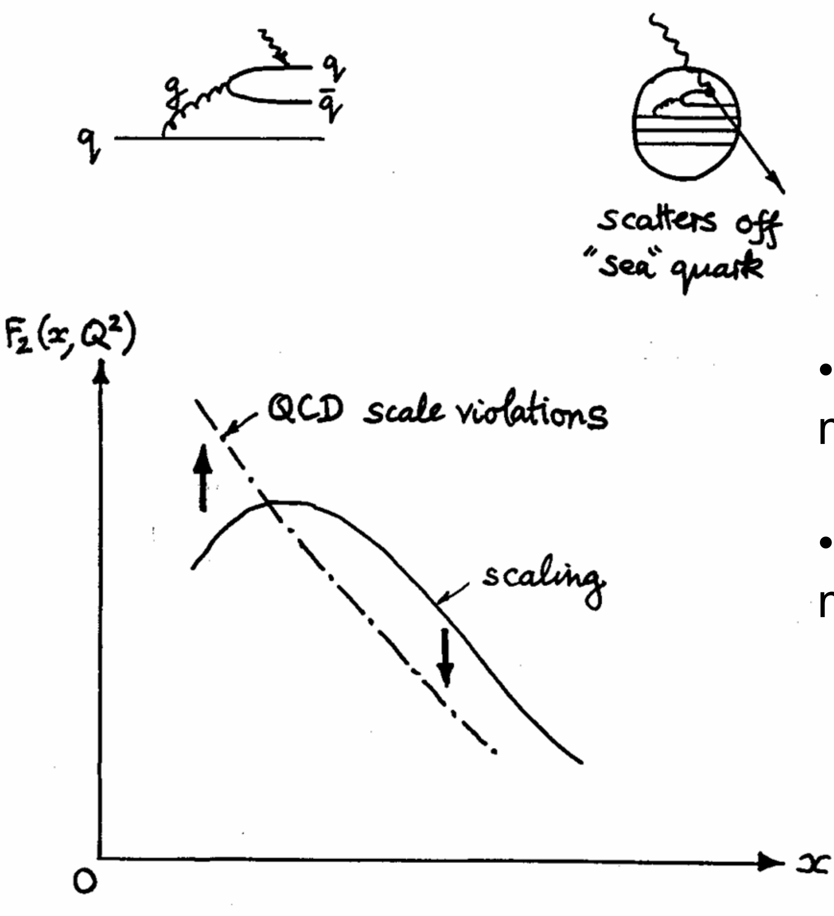


Neutrino-Nucleon Deep Inelastic Scattering

BONUS topics!

Strong Interactions among Partons

Q^2 Scaling fails due to these interactions



$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)q(y,Q^2) + P_{qg}\left(\frac{x}{y}\right)g(y,Q^2)\right]$$

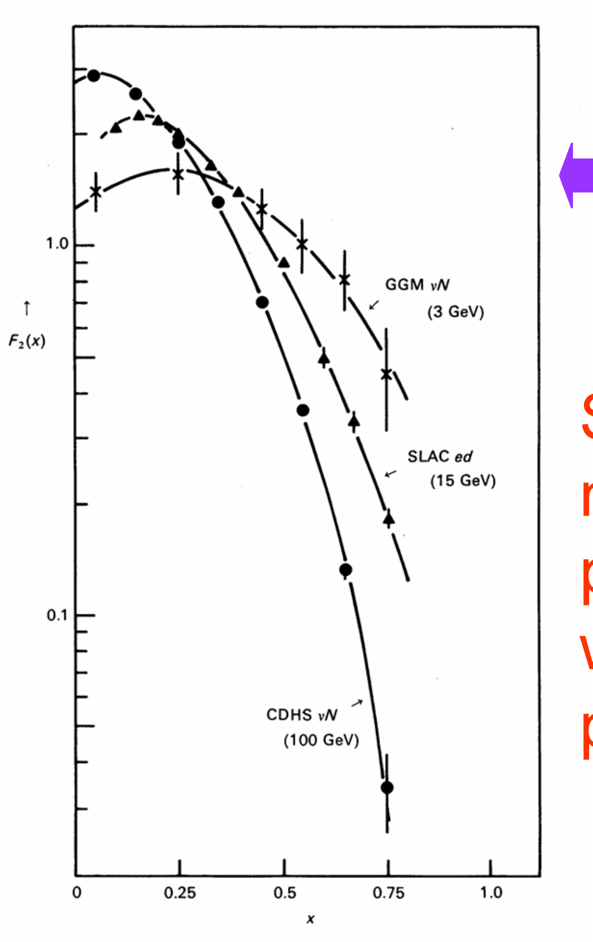
- Pqq(x/y) = probability of finding a quark with momentum x within a quark with momentum y

- Pqq(x/y) = probability of finding a q with momentum x within a gluon with momentum y

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{(1-z)} + 2\delta(1-z)$$

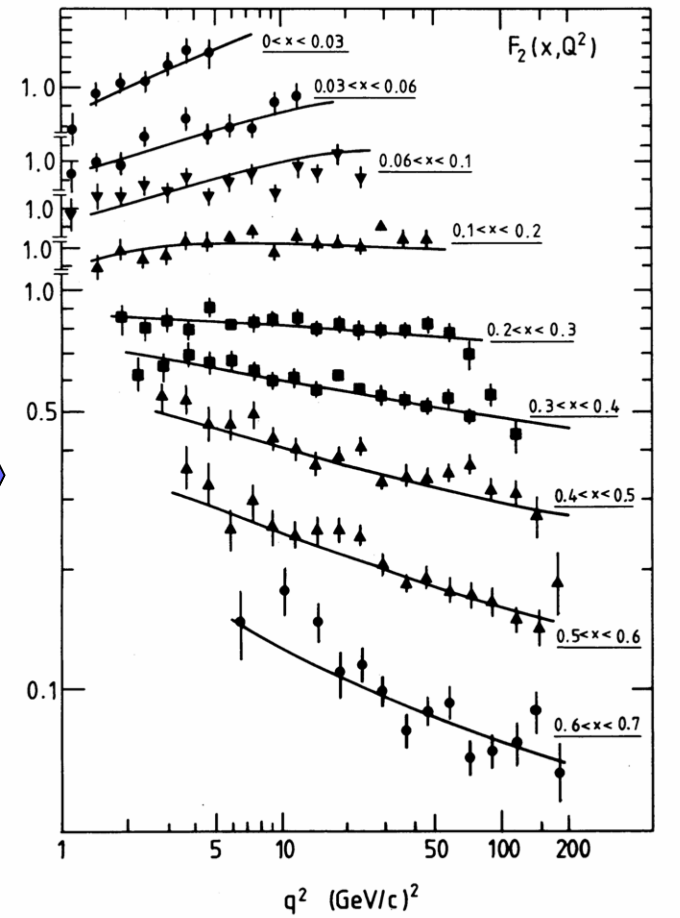
$$P_{gq}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

Scaling from QCD



Observed quark distributions vary with Q^2

Scaling well modeled by perturbative QCD with a single free parameter (α_s)



Lepton Mass Effects in DIS Region

- Recall that final state mass effects enter as corrections:

$$1-\frac{m_\mu^2}{s_{\text{point-like}}} \rightarrow 1-\frac{m_\mu^2}{x s_{\text{nucleon}}}$$

  - *relevant center-of-mass energy is that of the "point-like" neutrino-parton system*
  - *this is high energy approx.*
- For ν_τ charged-current, there is a threshold of

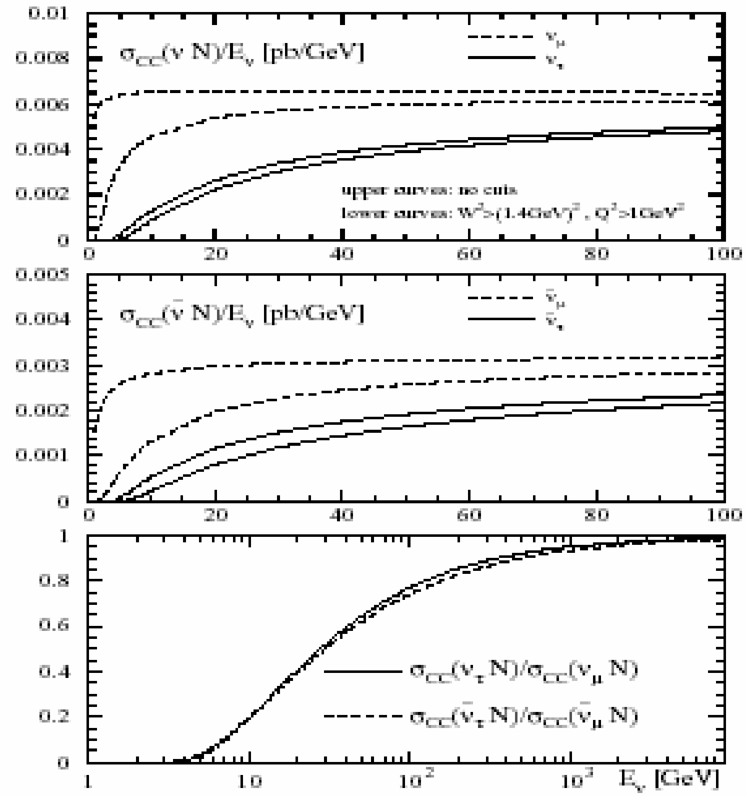
$$s_{\text{min}} = (m_{\text{nucleon}} + m_\tau)^2$$

where

$$s_{initial} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$

$$\therefore E_\nu > \frac{m_\tau^2 + 2m_\tau m_{\text{nucleon}}}{2m_{\text{nucleon}}} \approx 3.5\text{ GeV}$$

"m_{nucleon}" is M_T elsewhere, but don't want to confuse with m_τ...

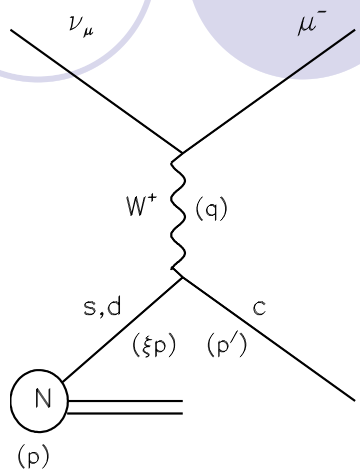


(Kretzer and Reno)

- This is threshold for partons with *entire* nucleon momentum
  - effects big at higher E_ν also

Heavy Quark Production

- Scattering from heavy quarks is more complicated.
  - Charm is heavier than proton; hints that its mass is not a negligible effect…



$$(q+\zeta p)^2 = p'^2 = m_c{}^2$$

$$q^2 + 2\zeta p \bullet q + \zeta^2 M^2 = m_c{}^2$$

Not your father's fractional momentum

$$\text{Therefore } \zeta \cong \frac{-q^2 + m_c{}^2}{2 p \bullet q}$$

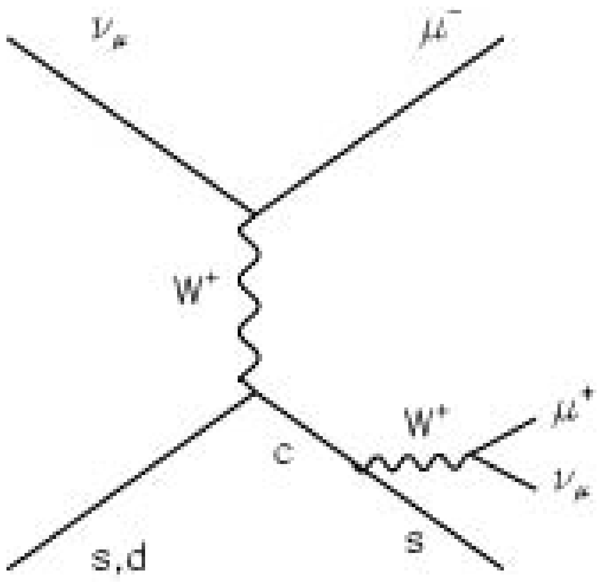
$$\zeta \cong \frac{Q^2 + m_c{}^2}{2M\upsilon} = \frac{Q^2 + m_c{}^2}{Q^2 / x}$$

$$\zeta \cong x\left(1 + \frac{m_c{}^2}{Q^2}\right)$$

"slow rescaling" leads to kinematic suppression of charm production

Neutrino Induced Dilepton Events

- Neutrino induced charm production has been extensively studied
  - Emulsion/Bubble Chambers (low statistics, 10s of events)
  - "Dimuon events" (high statistics, 1000s of events)



$$\nu_\mu + \begin{pmatrix} d \\ s \end{pmatrix} \to \mu^- + c + X$$

$$c \to \mu^+ + \nu_\mu + X'$$

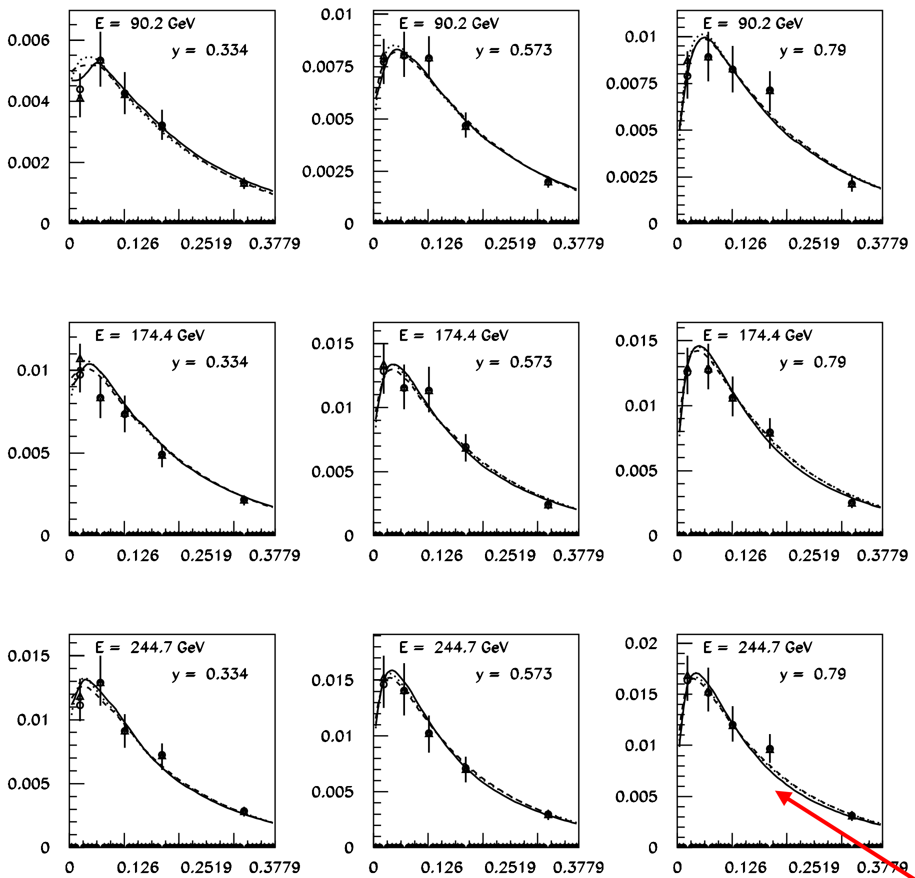
$$\bar{\nu}_\mu + \begin{pmatrix} \bar{d} \\ \bar{s} \end{pmatrix} \to \mu^+ + \bar{c} + X$$

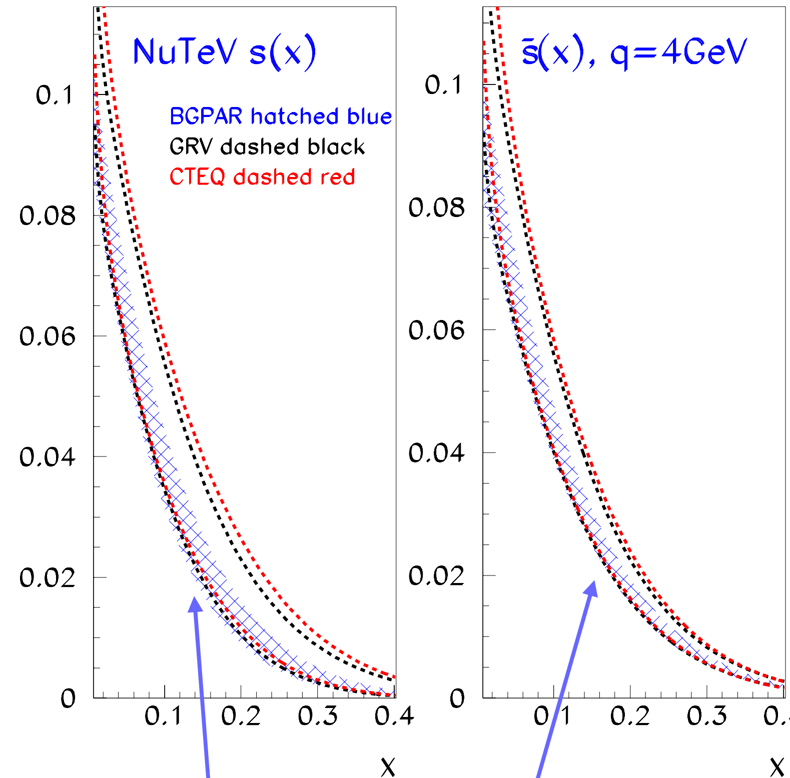
$$\bar{c} \to \mu^- + \bar{\nu}_\mu + X'$$

- Rate depends on:
  - d, s quark distributions
  - |Vcd|
  - Kinematic suppression and fragmentation
- Effects can be separated and measured

NuTeV Dimuon Sample

- Extract production suppression and separate measurement of strange and anti-strange quark distributions



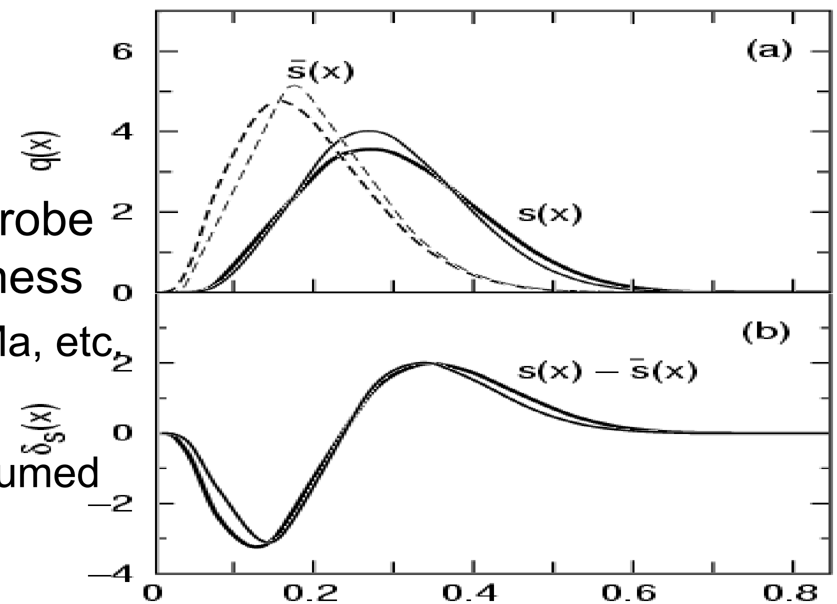


strange and anti-strange seas

Differential cross-sections

QCD at Work: Strange Asymmetry?

- An entertaining aside…
  - The strange sea can be generated perturbatively from g s+sbar.
  - BUT, perturbative generation of differences between s and sbar are suppressed, so s & sbar difference probe non-perturbative ("intrinsic") strangeness
    - Models: Signal&Thomas, Brodsky&Ma, etc.
  - NuTeV has tested this
    - NB: NOT independent of what is assumed about non-strange sea, so caution in applying this is warranted



(Brodsky & Ma, s-sbar)

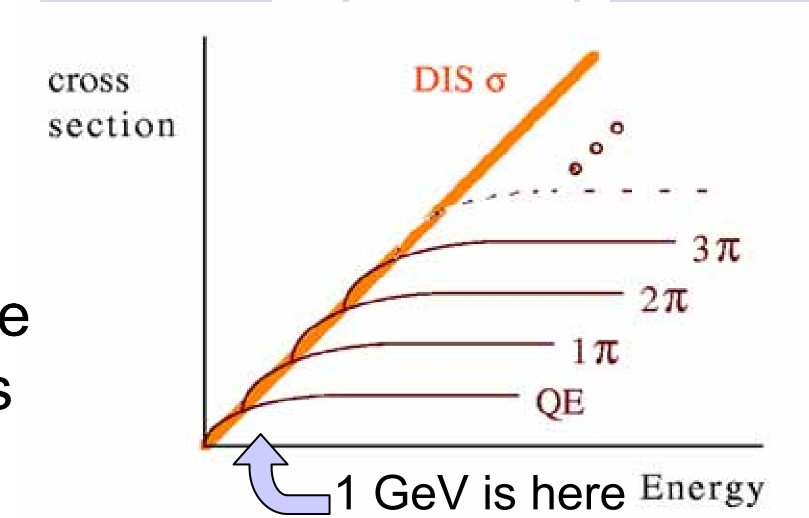
- NuTeV measures:

$$\int dx\left[x\left(s-\bar{s}\right)\right] = -0.0027 \pm 0.0013$$

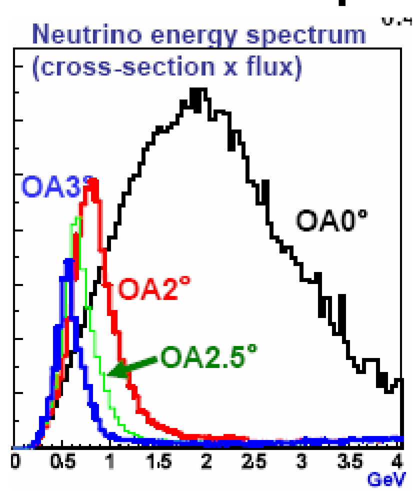
$$\text{c.f.,} \int dx\left[x\left(s+\bar{s}\right)\right] \approx 0.02$$

GeV Cross-Sections

What's special about it? Why do we care?



- Remember this picture?
  - 1-few GeV is exactly where these additional processes are turning on
  - It's not DIS yet! Final states & threshold effects matter
- Why is it important? Example: T2K

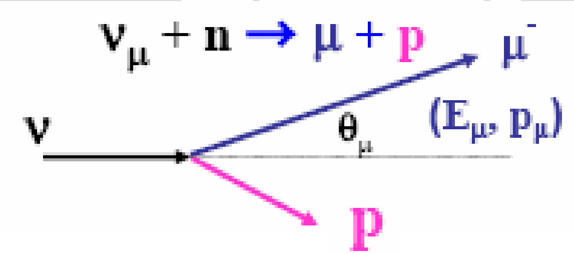


Goals:

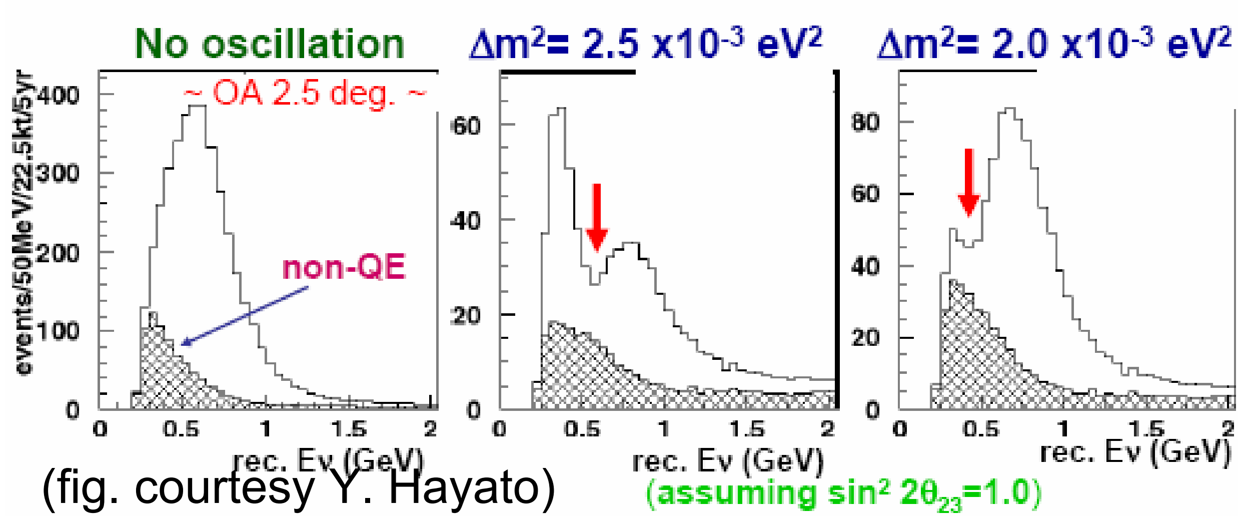
1. $\nu_\mu \rightarrow \nu_e$
2. ν_μ disappearance

E_ν is 0.4-2.0 GeV

How do cross-sections effect oscillation analysis?



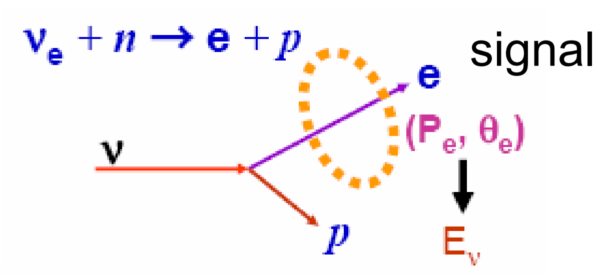
- ν_μ disappearance
  - at Super-K reconstruct these events by muon angle and momentum (proton below Cerenkov threshold in H_2O)
  - other final states with more particles below threshold ("non-QE") will disrupt this reconstruction
- T2K must know these events at few % level to do disappearance analysis to measure Δm^2_{23}, θ_{23}

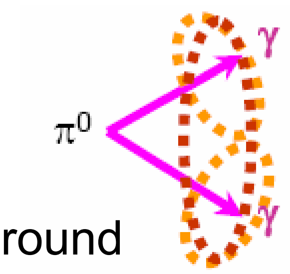


(fig. courtesy Y. Hayato)

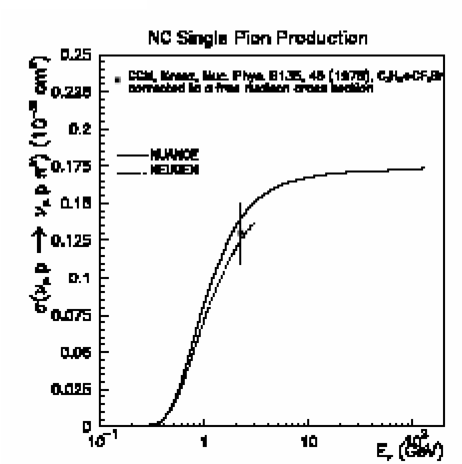
How do cross-sections effect oscillation analysis?

- ν_e appearance
  - different problem: signal rate is very low so even rare backgrounds contribute!





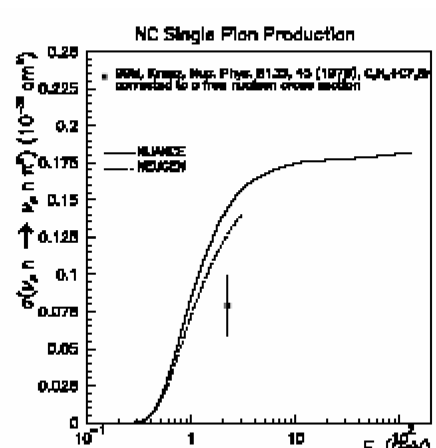
π^0 background from E_ν>peak



$\nu_\mu p \rightarrow \nu_\mu p \pi^0$

the world's data on this background

(compiled by G. Zeller, hep-ex/0312061)

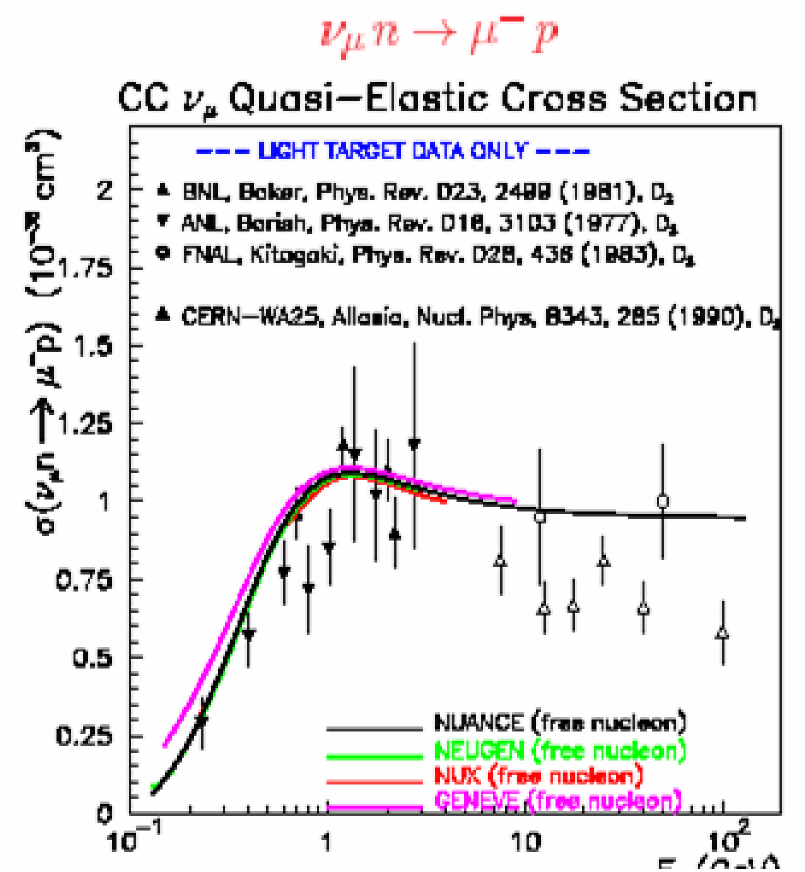


$\nu_\mu n \rightarrow \nu_\mu n \pi^0$

(Quasi-)Elastic Scattering

- Elastic scattering leaves a single nucleon in the final state
  - CC "quasi-elastic" easier to observe

$$\nu n \rightarrow l^- p$$
$$\bar{\nu} p \rightarrow l^+ n$$
$$\overset{(-)}{\nu} N \rightarrow \overset{(-)}{\nu} N$$



- State of data is marginal
  - No free neutrons implies nuclear corrections
  - Low energy statistics poor
- Cross-section is calculable
  - But depends on incalculable form-factors
- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is scale of Q^2 limit

Hmmm…
What was that last cryptic remark?

- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is scale of Q^2 limit

- Inverse μ–decay:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2} \approx \frac{Q^2_{\max}}{M_W{}^4}$$

a maximum Q^2 independent of beam energy $\Rightarrow$ constant σ_{TOT}

Elastic Scattering (cont'd)

$$\nu n \rightarrow l^- p$$
$$\bar{\nu} p \rightarrow l^+ n$$
$$\overset{(-)}{\nu} N \rightarrow \overset{(-)}{\nu} N$$

- How does nucleon structure impact elastic scattering?

C.H. Llewellyn Smith, Phys. Rep. **3C**, 261 (1972)

$$< N'|J_\mu|N > \;=\; \overline{u}(N')\left[\gamma_\mu F_V^1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu \xi F_V^2(q^2)}{2M} + \gamma_5\gamma_\mu F_A(q^2)\right]u(N)$$

$$F_V(q^2) \sim \frac{1}{(1-q^2/M_V^2)^2} \qquad F_A(q^2) = \frac{F_A(0)}{(1-q^2/M_A^2)^2}$$

← "dipole approximation"

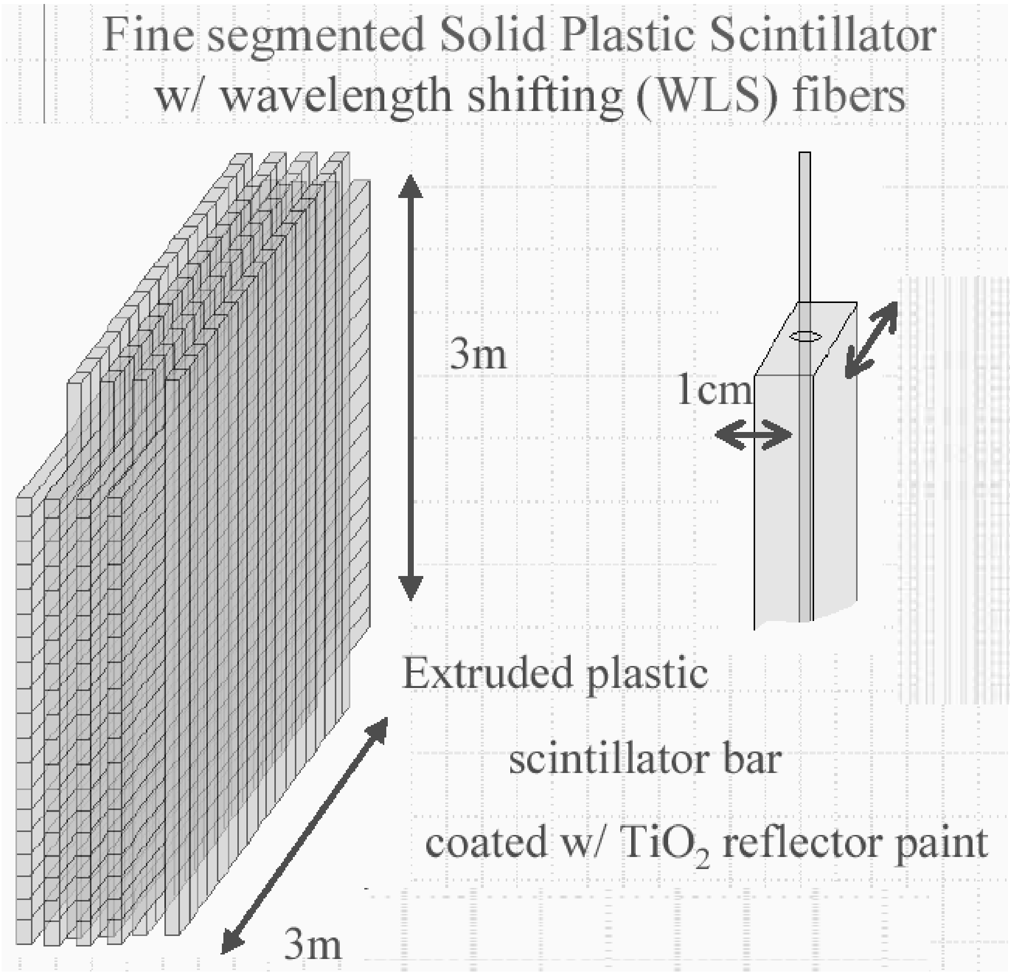
- $\hookrightarrow \mathbf{M_A = 1.032}$ GeV
- $\hookrightarrow \mathbf{M_V = 0.84}$ GeV
- $\hookrightarrow \mathbf{F_A(q^2) = \frac{F_A(0)}{(1-q^2/M_A^2)^2};\ F_A(0) = -1.25}$

} parameters determined from data

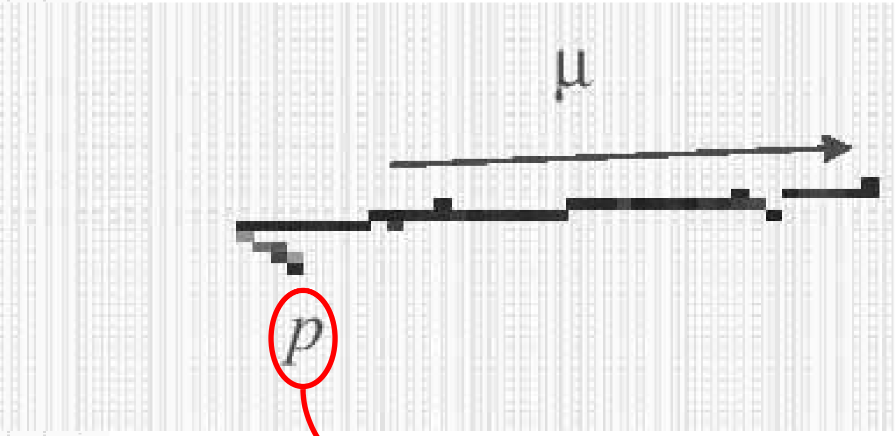
n.b.: we've seen $F_V(0)$ and $F_A(0)$ before in IBD discussion (g_V and g_A)

- "Form factors" modify vanilla V-A prediction of point-like scattering in Fermi theory
  - vector part can be checked in electron elastic scattering

Quasi-Elastic Signature



Simulation of new K2K "SciBar" detector



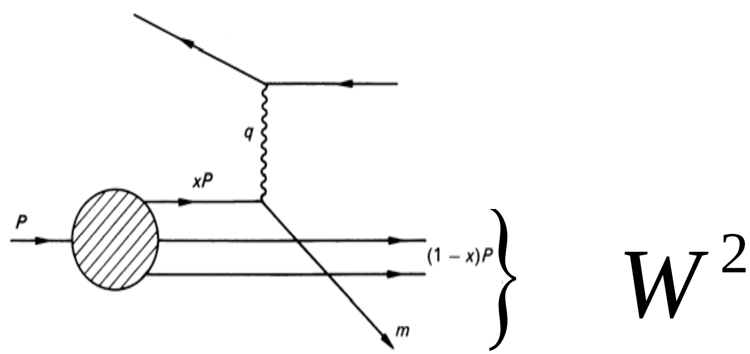
proton is NOT ultra-relativistic!

Low W, the Resonance Region

- Intermediate to elastic and DIS regions is a region of resonance production
  - Recall mass^2 of hadronic final state is given by

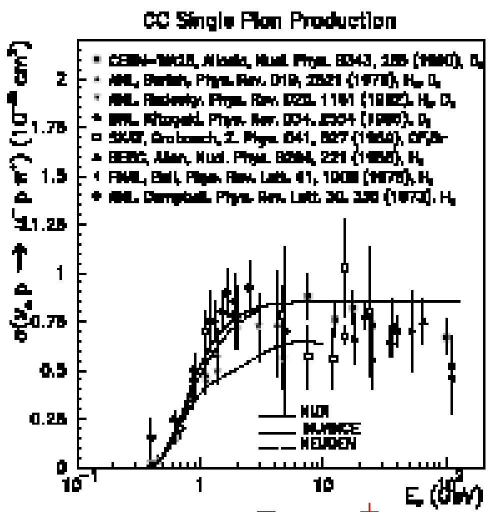
$$W^2 = M_T^2 + 2M_T\nu - Q^2 = M_T^2 + 2M_T\nu(1-x)$$

  - At low energy, nucleon-pion states are dominated by N* and D resonances
- Leads to cross-section dominated by discrete W^2 values
  - Low ν, high x

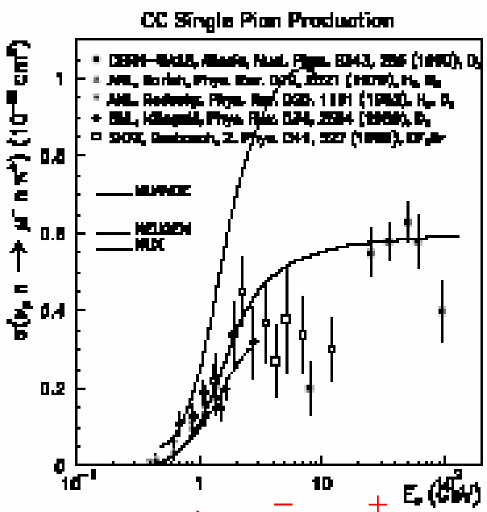


Resonance Region Data

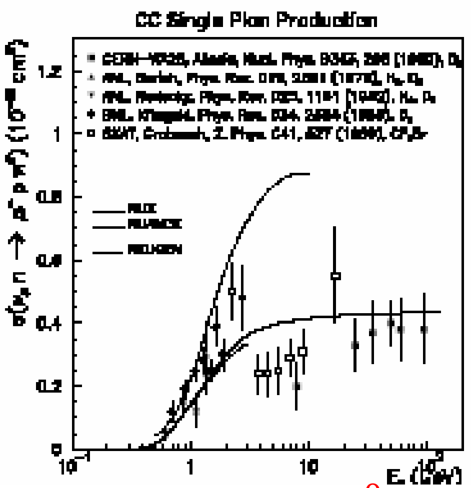
- Data here, again, is impressively imprecise
  - This will be a problem if details of cross-sections are needed where resonance production is dominant. *Need differential distributions*!
    - ~1-2 GeV important for T2K (background), NOvA (signal)



$\nu_\mu p \to \mu^- p \pi^+$



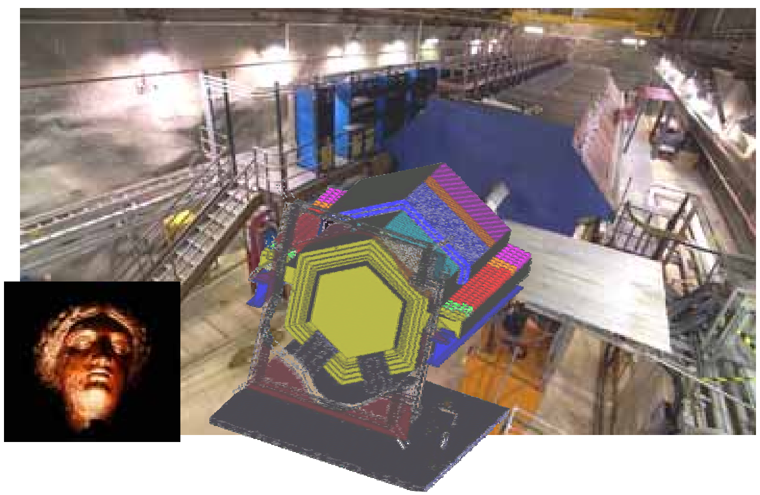
$\nu_\mu n \to \mu^- n \pi^+$

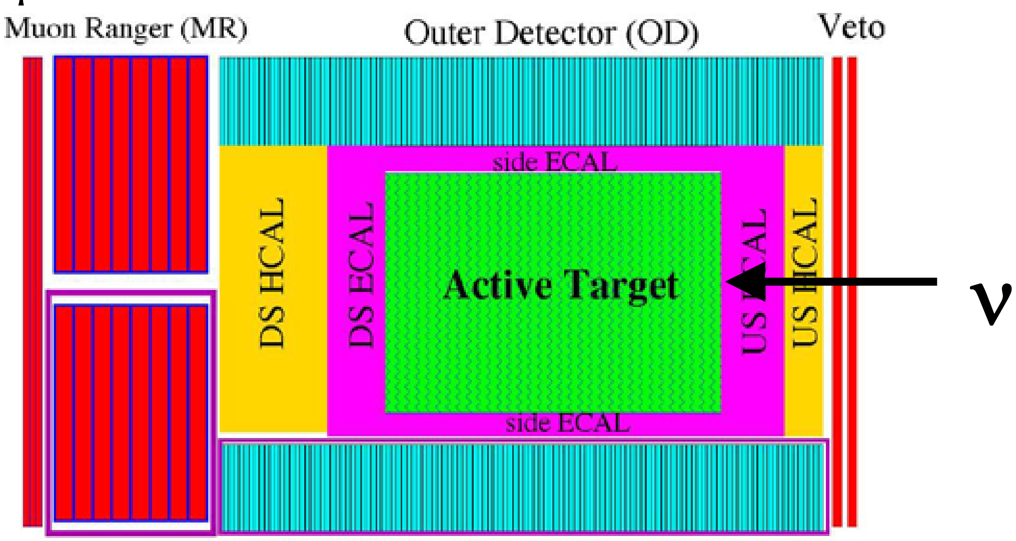


$\nu_\mu n \to \mu^- p \pi^0$

How to measure resonance region cross-sections?

- Need a high granularity detector (like SciBar) but in a higher energy beam and with improved containment of γ, $\pi^{\pm}$, μ

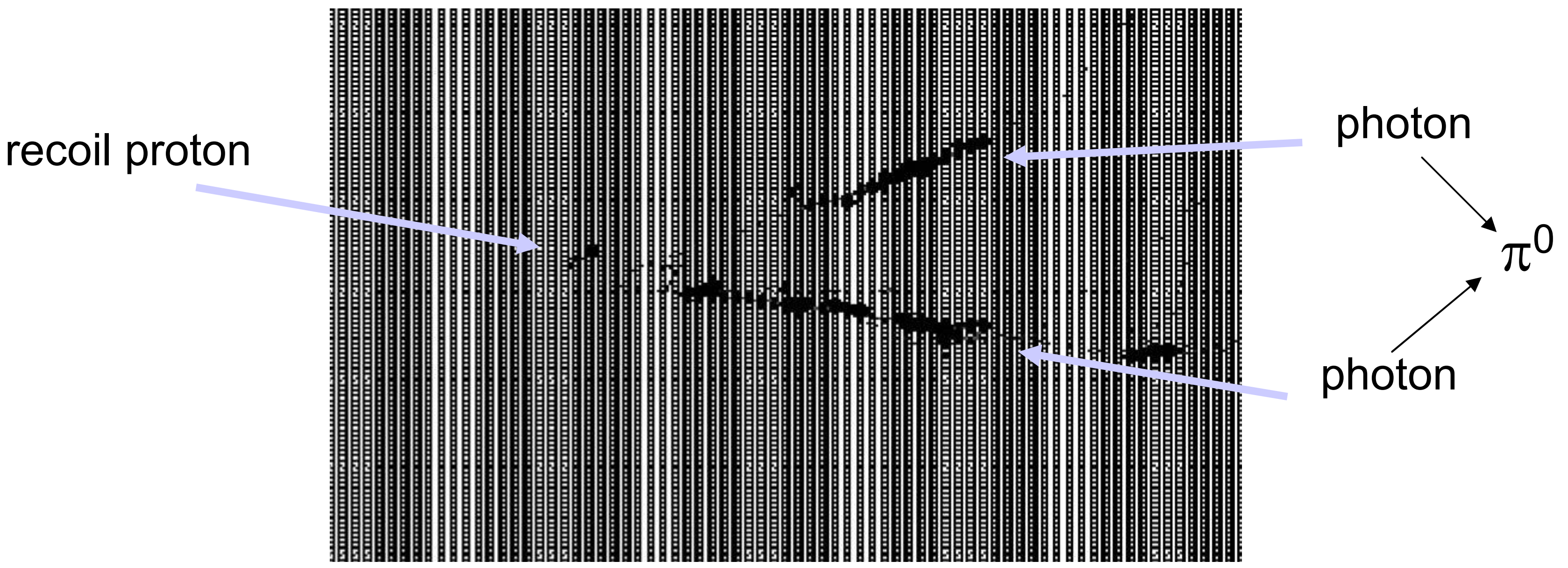




- MINERνA at NuMI
  - “chewy center” (active target)
  - with a crunchy shell of muon, hadron and EM absorbers

What can MINERνA see?

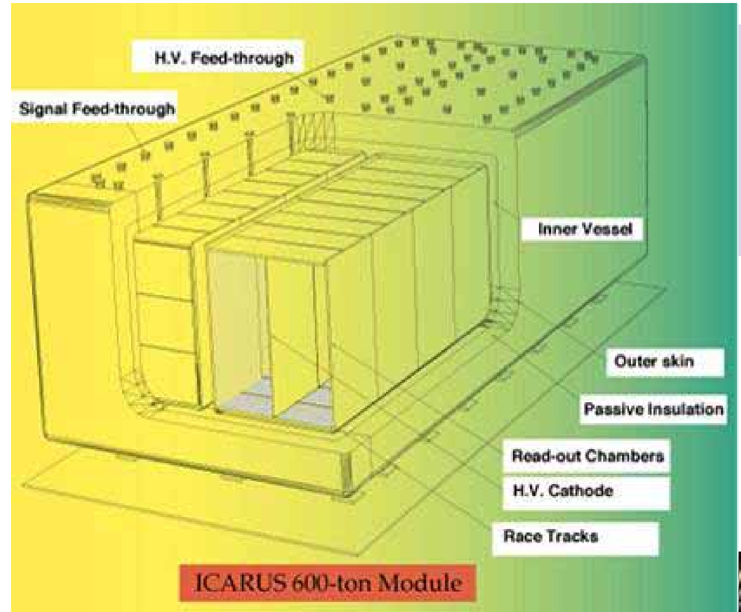
- With high granularity, can reconstruct a broad variety of exclusive final states



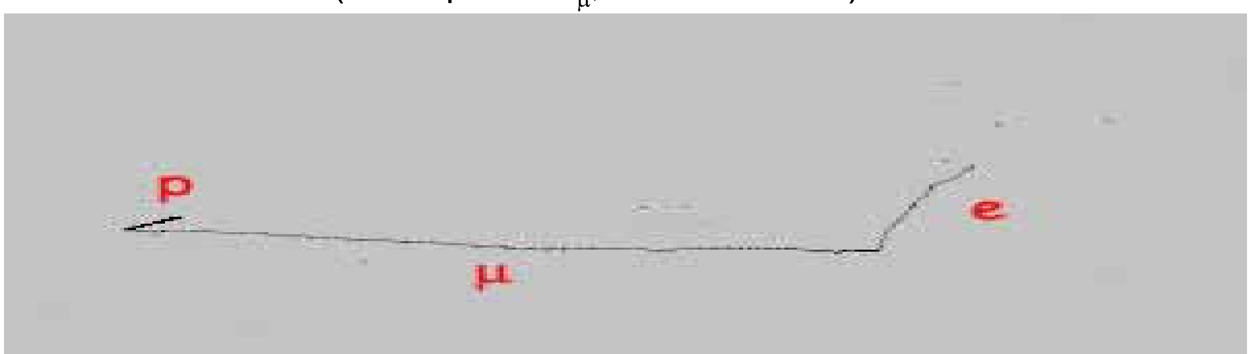
$$\nu_\mu p \rightarrow \nu_\mu p \pi^0,\ \pi^0 \rightarrow \gamma\gamma$$

Even better…

- A Liquid Ar TPC offers near bubble chamber precision…
- Hard to build!



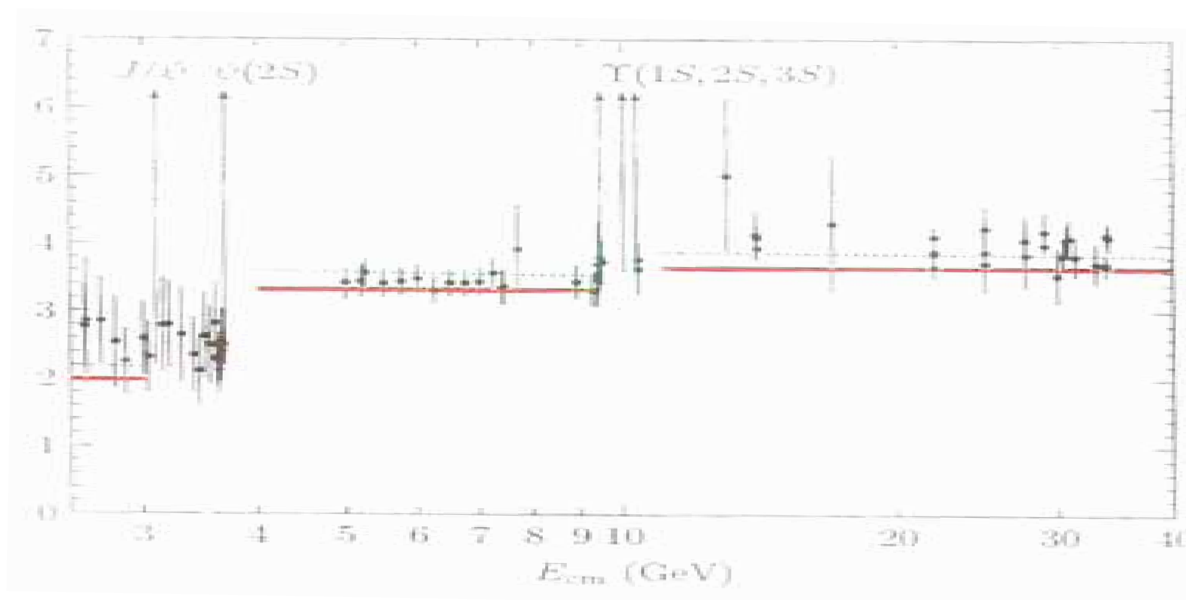
MonteCarlo Event (atmospheric ν_μ, QE interaction) in an ideal LAr detector



Quark-Hadron Duality

- Bloom-Gilman Duality is the relationship between quark and hadron descriptions of reactions. It reflects:
  - link between *confinement* and *asymptotic freedom*
  - transition from *non-perturbative* to *perturbative* QCD

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



parton model calculation:

$$R = N_C \sum_{q\ni's>m_q^2} \left(Q_q^{EM}\right)^2 + O(\alpha_{EM} + \alpha_S)$$

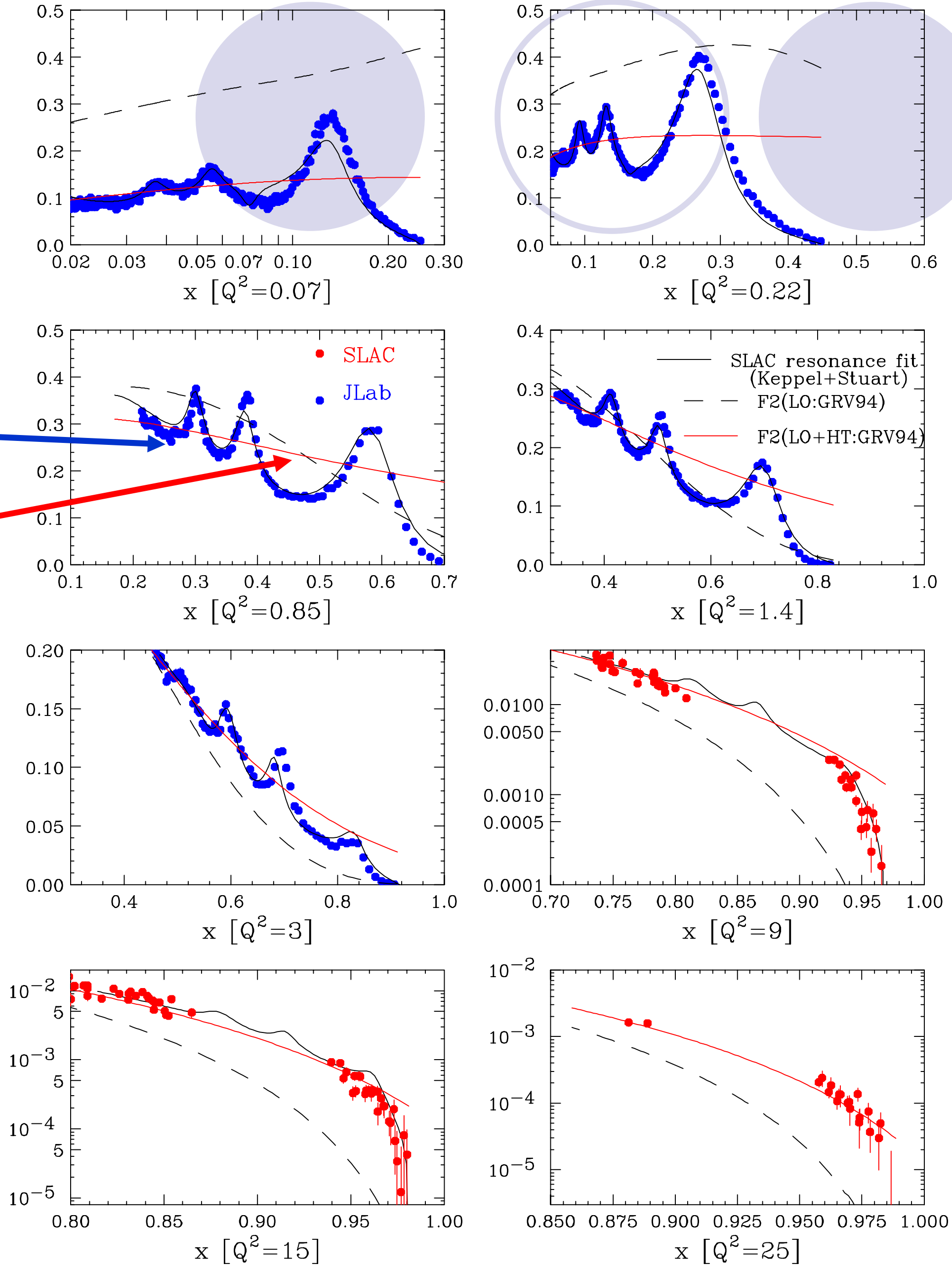
but of course, final state is really sums over discrete hadronic systems

Duality and ν

$$W^2 = M_T^2 + Q^2\left(\frac{1}{x} - 1\right)$$

Low Q^2 data

DIS-Style PDF prediction



- Governs transition between resonance and DIS region
- Sums of discrete resonances approaches DIS cross-section
- *Observe in electron scattering data; apply to ν cross-sections*

Concept Question #4

A difficulty in relating cross-sections of electron scattering (photon exchange) to charged-current neutrino scattering ($W^{\pm}$ exchange) is that some e-scatting reactions have imperfect ν-scattering analogues.

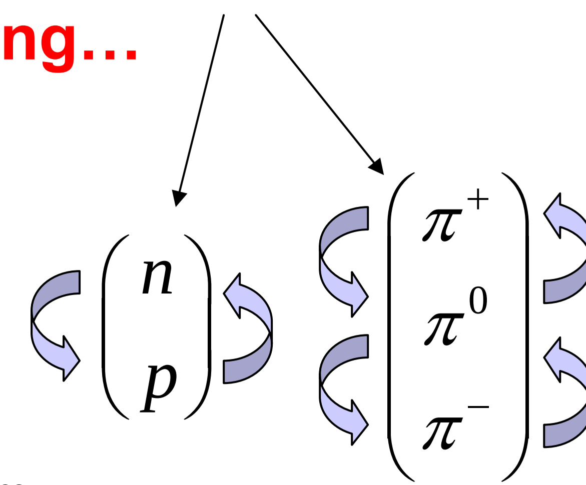
Write all possible ν_{μ} CC reactions involving the same target particle and isospin rotations of the final state for each of the following…

(a) $e^{-}n \rightarrow e^{-}n$

(b) $e^{-}p \rightarrow e^{-}p$

(c) $e^{-}p \rightarrow e^{-}n\pi^{+}$

(d) $e^{-}n \rightarrow e^{-}p\pi^{-}$



Concept Question #4

Write all possible ν reactions involving the same target particle and isospin rotations of the final state for each of the following…

(b) $e^- n \rightarrow e^- n$

$$\nu_\mu n \rightarrow \mu^- p$$

(b) $e^- p \rightarrow e^- p$

there are none!

(c) $e^- p \rightarrow e^- n \pi^+$

$$\nu_\mu p \rightarrow \mu^- p \pi^+$$

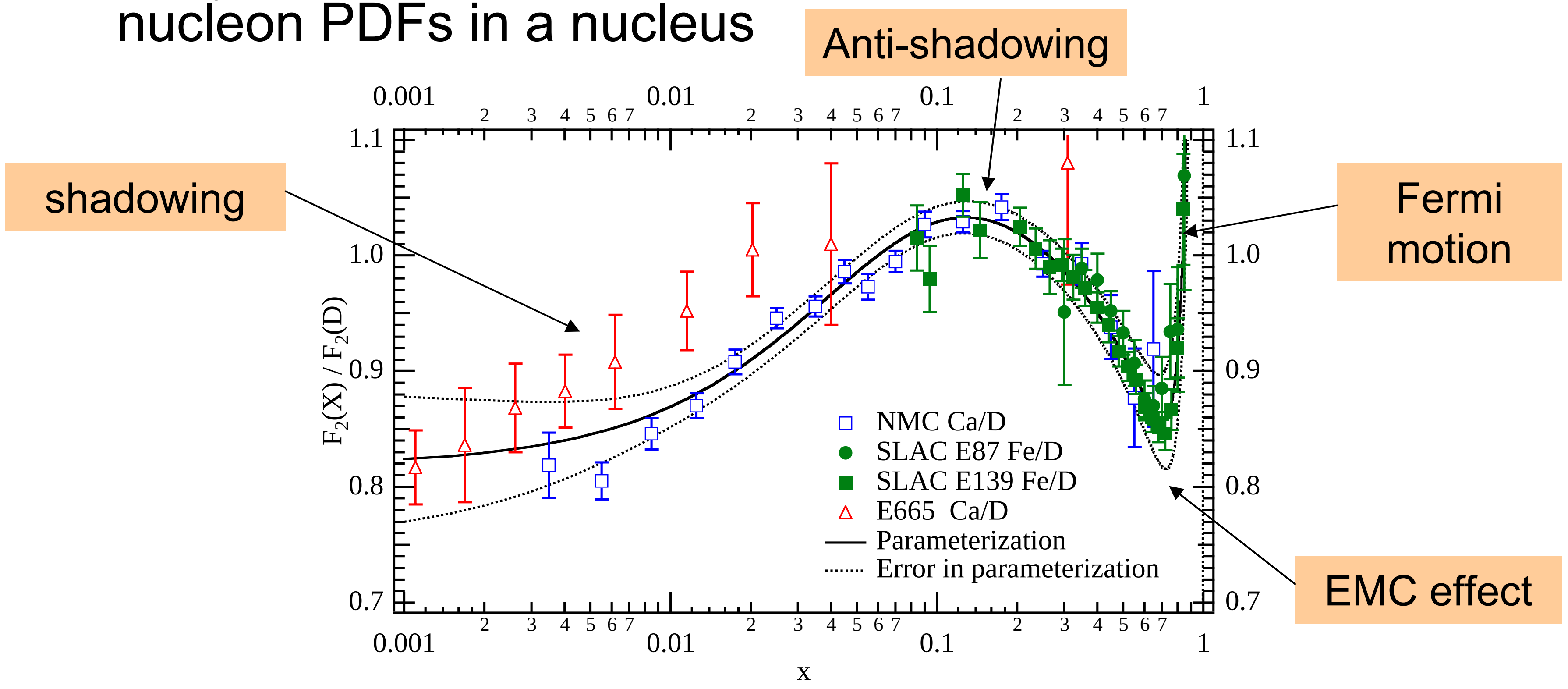
(d) $e^- n \rightarrow e^- p \pi^-$

$$\nu_\mu n \rightarrow \mu^- n \pi^+$$
$$\nu_\mu n \rightarrow \mu^- p \pi^0$$

Cross-Sections on Nucleons in a Nucleus

Nuclear Effects in DIS

- Well measured effects in charged-lepton DIS
  - Maybe the same for neutrino DIS; maybe not… all precise neutrino data is on Ca or Fe targets!
  - Conjecture: these can be absorbed into effective nucleon PDFs in a nucleus

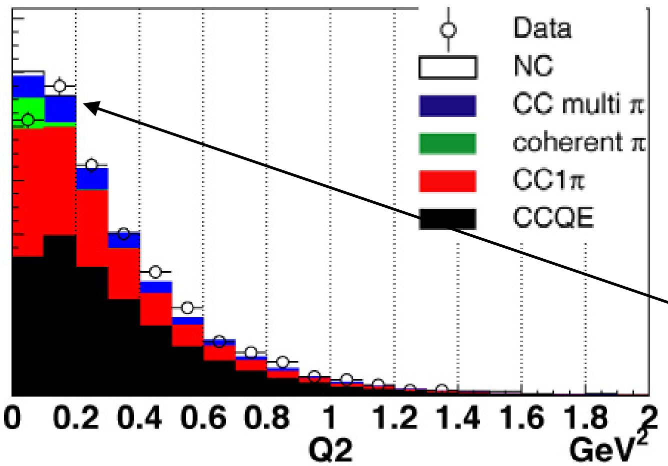


Nuclear Effects in Elastic Scattering

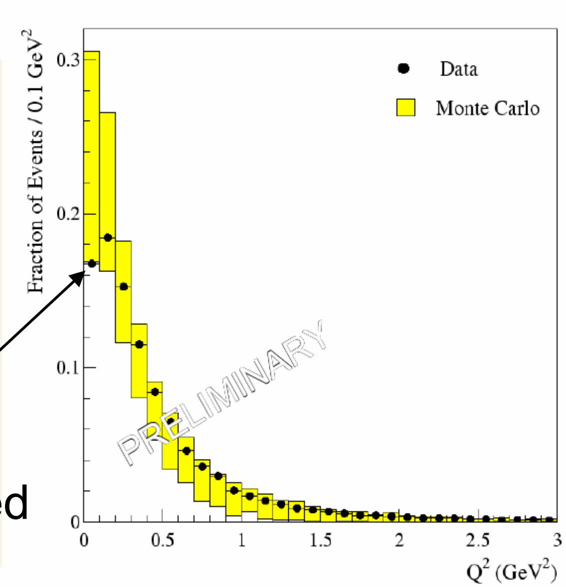
- Two effects
  - In a nucleus, target nucleon has some initial momentum which modifies the observed scattering
    - Often handled in a “Fermi Gas” model of nucleons filling available states up to some initial state Fermi momentum, k_F
  - Outgoing nucleon can interact with the target
    - Usually treated as a simple binding energy
    - Also, Pauli blocking… states are already filled with identical nucleon
    - *However other final states can contribute to “quasi-elastic” scattering through absorption in the nucleus…*
- Theoretical uncertainties are large
  - At least at the 10% level
  - If precise knowledge is needed for target (e.g., water, liquid argon, hydrocarbons), dedicated measurements will be needed
    - Most relevant for low energy experiments

And what does the data look like?

- First glimpses at quasi-elastic rich low Q^2 region on C nuclei…



Larger than expected rollover at low Q^2



Q2 distribution for K2K SciBar detector

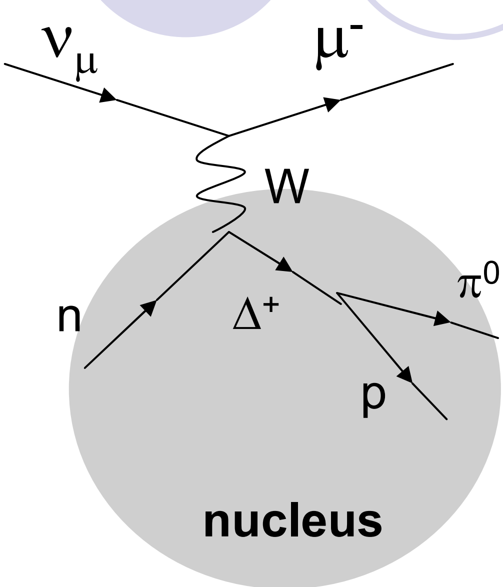
Q2 distribution for MiniBooNE

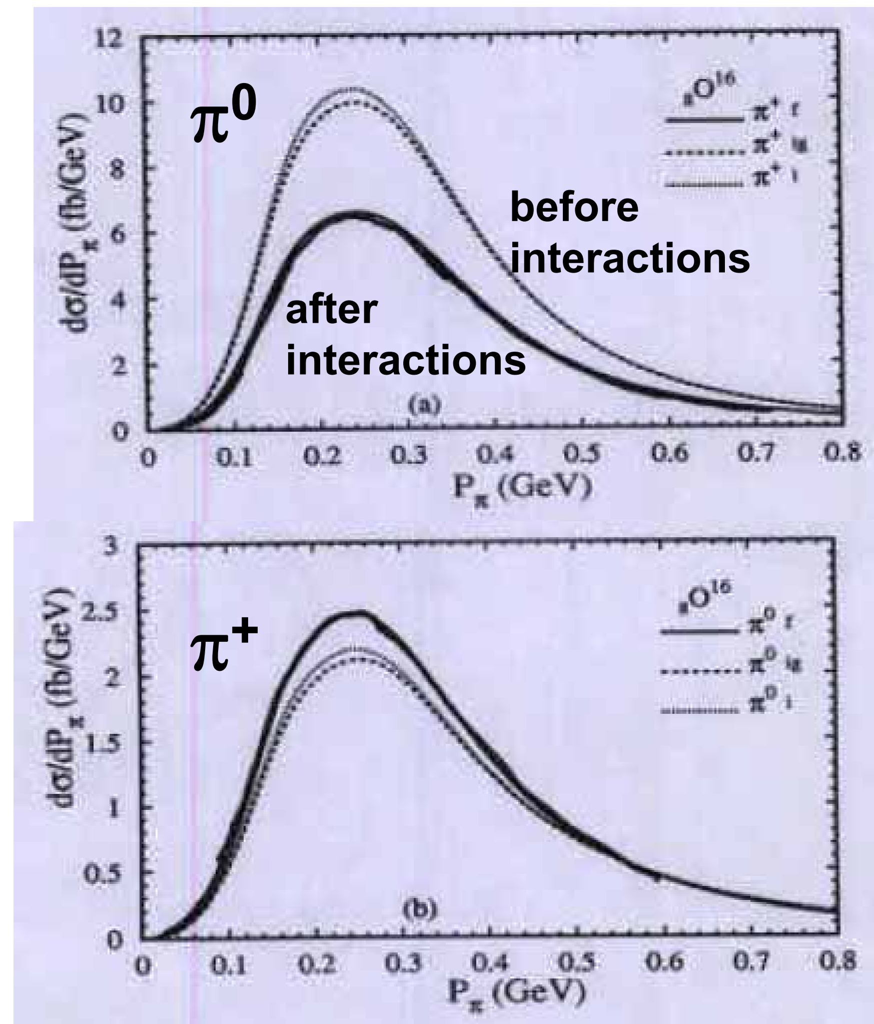
- Data are, not surprisingly, suggesting nuclear effects are not well modeled

Nuclear Effects in Resonance Region

$$\nu_\mu n \rightarrow \mu^- p \pi^0$$

model of
E. Paschos, NUINT04





- How does nucleus affect π^0 production (ν_e background)?
- Rescattering. Absorption.
- Must measure to predict ν_e backgrounds!

Nuclear Effects in IBD

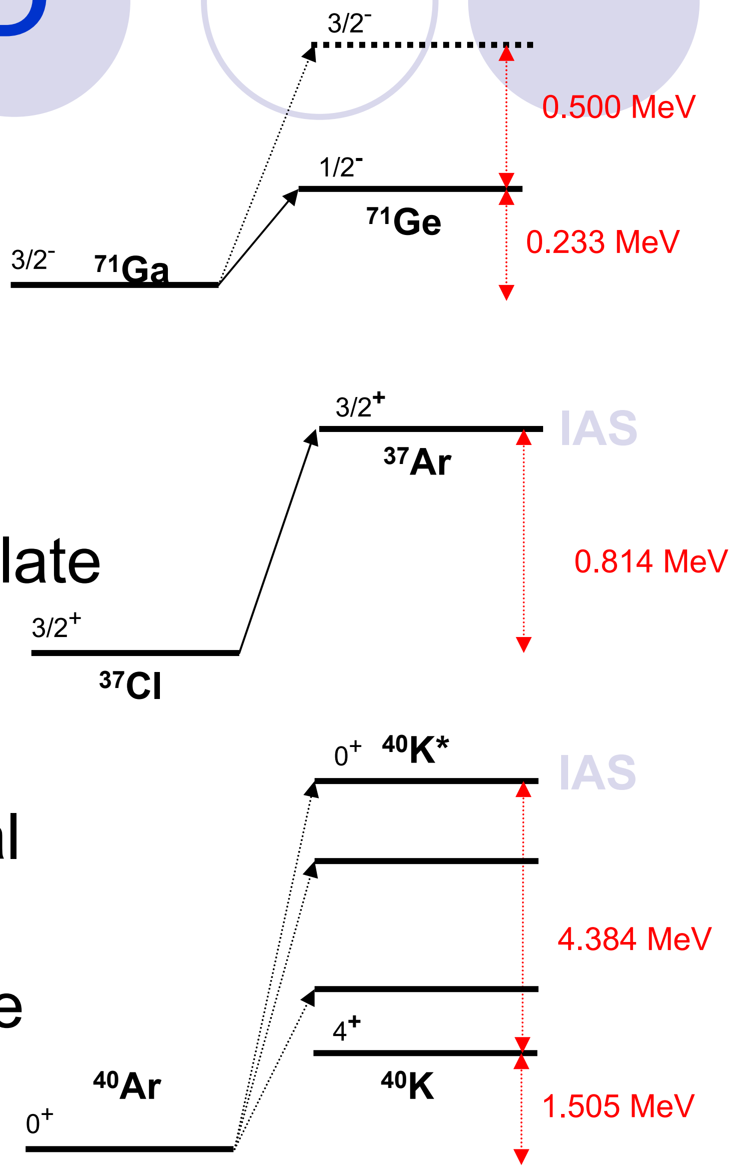
$$\nu_e + A_Z \rightarrow e^- + A_{Z+1}$$

- There is a complicated nuclear physics phenomenology which I don't care to detail here
- Suffice it to say that the form factors are not as simple to calculate

  $\Delta J=0$ (Fermi Trans.),

  $\Delta J=\pm 1$ (Gamow-Teller Trans.)

- Threshold energies are less trivial
  - sometimes multiple states
- Also have corrections due to finite size of nucleus and electron screening



Some Common IBD Nuclei

- here are some nuclei historically important for Solar neutrino experiments

Experiment	*Nuclear Target*	*Reaction*	σ_0 [$10^{-46}cm^2$]	ΔE_{nucl} [MeV] (no det. Thres.)
GALLEX/GNO SAGE	$^{71}Ga_{33}$	$\nu_e + {}^{71}Ga \to e^- + {}^{71}Ge$	8.611 ± 0.4% **(GT)**	0.2327
HOMESTAKE	$^{37}Cl_{17}$	$\nu_e + {}^{37}Cl \to e^- + {}^{37}Ar$	1.725 **(F)**	0.814
SNO	$^{2}H_{1}$	$\nu_e + {}^{2}H \to e^- + p + p$	**(GT)**	1.442
ICARUS	$^{40}Ar_{18}$	$\nu_e + {}^{40}Ar \to e^- + {}^{40}K^*$	148.58 **(F)** ... 44.367 **(GT_2)** ... 41.567 **(GT_6)** ...	1.505 +

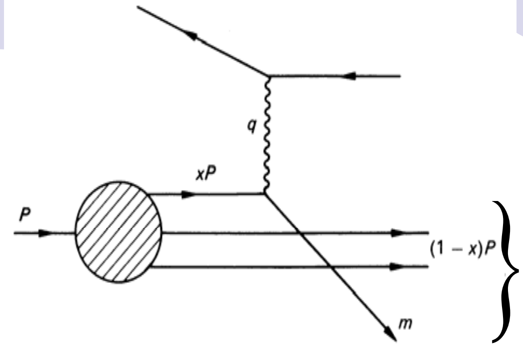
table courtesy F. Cavana

Concept Question #5

- **Two questions with (*hint*) related answers…**

1. Remember that W^2 is…

$$
\begin{aligned}
W^2 &= M_P^2 + 2M_P\nu - Q^2 \\
&= M_P^2 + 2M_P\nu(1-x)
\end{aligned}
$$



W^2

the square of the invariant mass of the hadronic system. ($\nu=E_\nu-E_\mu$; x is the parton fractional momentum)

It can be measured, as you see above with only leptonic quantities (neutrino and muon 4-momentum).

In neutrino scattering on a scintillator target, you observe an event with a recoiling proton and with W reconstructed from the leptonic variables that is $<M_p$. Explain this event.

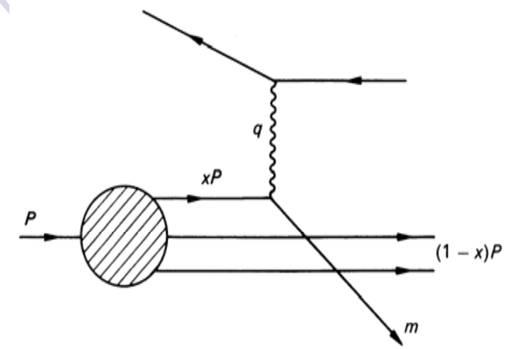
2. In the same scintillator target, you observe the reaction… $\nu_\mu\,{}^{12}\mathrm{C} \to \mu^- p \pi^- + \text{remant nucleus}$

Why is this puzzling? Explain what happened.

Concept Question #5

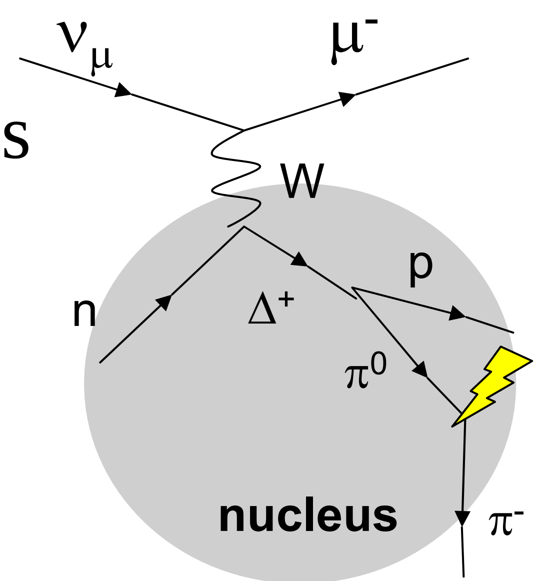
- **Both phenomena occur because of nuclear effects!**

1. $M_P > W^2 = M_P^2 + 2M_P\nu(1-x)$
can only be true if x>1.
That means the fractional momentum by the struck target parton is >1! This can only happen for in a nucleon boosted towards the collision in the CM frame by interactions within the nucleus ("Fermi momentum")



2. $\nu_\mu {}^{12}\mathrm{C} \rightarrow \mu^- p \pi^-$ + remant nucleus
seems to be nonsense. It is forbidden to occur off of a proton or a neutron target by charge conservation!
But remember…

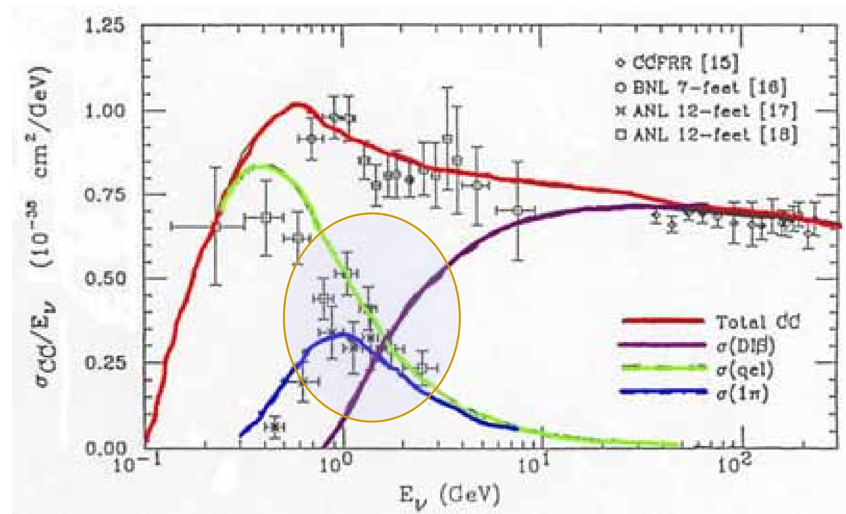
- **reinteraction of pions!**



Connections to Low Energy and Ultra-High Energy Cross-Sections

What is Different at New Energies?

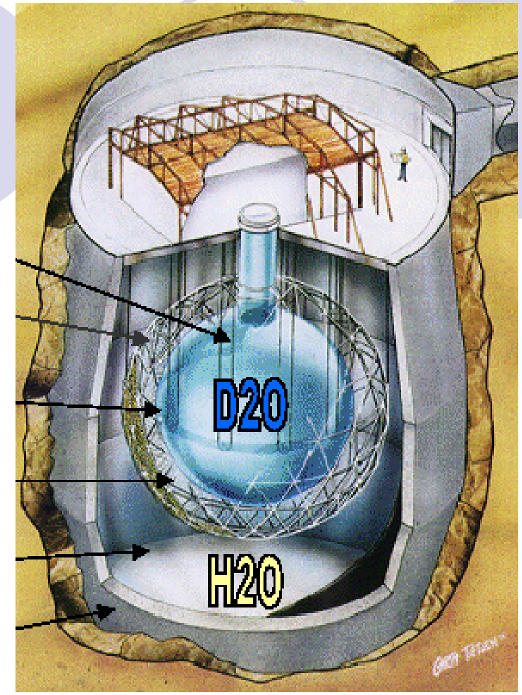
- At 1-few GeV, cross-section makes a transition between DIS-like and resonant/elastic
  - Why? "Binding energy" of target (nucleon) is ~1 GeV, comparable to mean Q^2



- What are other thresholds?
  - Binding energy of nucleus is $>>(M_n - M_p) \approx$ 1 MeV, typically 1/10ths – 10s of MeV
  - Binding energies of atoms are $<\sim Z^2 m_e c^2 \alpha_{EM}/2 \sim$ $10\text{-}10^5$ eV
  - Binding energies of ν, $\ell^{\pm}$, quarks (*into hypothetical constituents that we haven't found yet*) are > 10 TeV

Example: SNO

- Three reactions for observing ν from sun (E_ν ~ few MeV

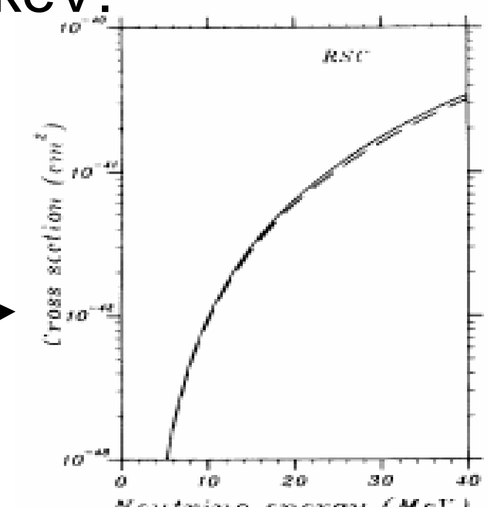


ES $\nu_x + e^- \Rightarrow \nu_x + e^-$

  - ^{2}H, ^{16}O binding energies are 13.6eV, ~1 keV. e^- are "free". $\sigma \propto E_\nu$

CC $\nu_e + d \Rightarrow p + p + e^-$

NC $\nu_x + d \Rightarrow p + n + \nu_x$



(Bahcall, Kuboeara, Nozawa, PRD38 1030)

  - Binding energy of deuteron is 2.2 MeV. Energy threshold for NC of a few MeV.

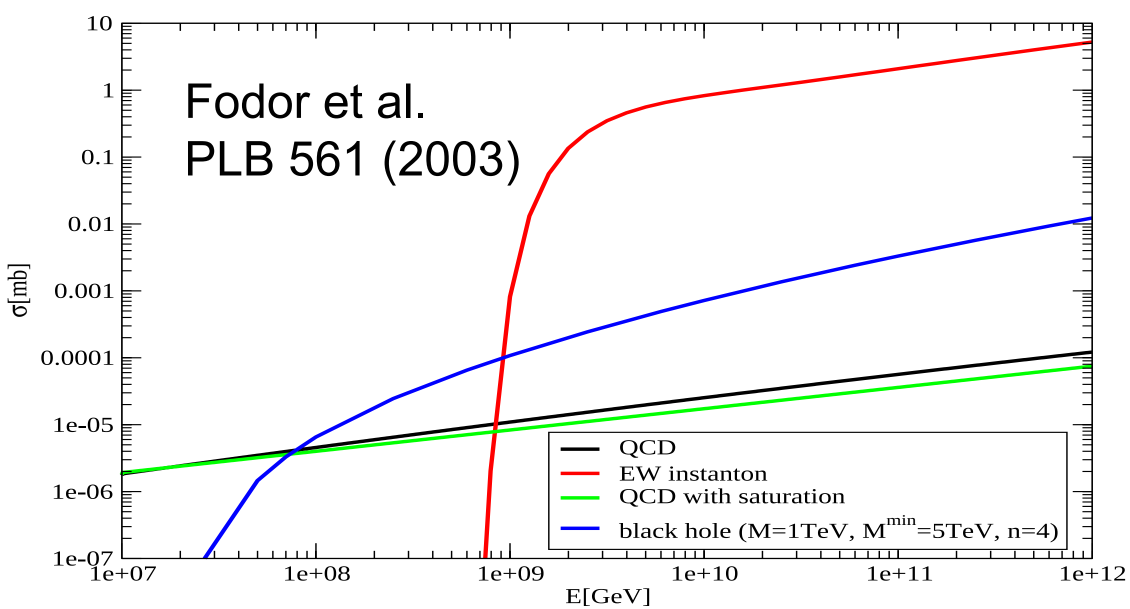
Example: Ultra-High Energies

- At energies relevant for UHE Cosmic Ray studies (e.g., IceCube, ANITA)
  - ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
    - at high Q^2, scaling violations have made most of nucleon momentum carried by sea quarks
    - see a rise in σ/E_ν from growth of sea at low x
    - neutrino & anti-neutrino cross-sections nearly equal
  - *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

Example: Ultra-High Energies

- Unless, of course, non-SM processes are excited! E.g., structure of quark or leptons, black holes from extra dimensions, etc.
  - Then no one knows what to expect



Conclusions

What Should I Remember from This?

- Understanding neutrino interactions *is key* to precision measurements of neutrino oscillations at accelerators
- Weak interactions couple to single chirality of fermions
  - Consequences for scattering on point-like particles
- Neutrino scattering rate proportional to energy
  - Point-like target (electron, quark), below real boson exchange
- Target (proton, nucleus) structure is a significant complication to theoretical prediction of cross-section
  - Particularly problematic near inelastic thresholds
  - can learn things by analogy with DIS (duality) and electron scattering, but improved neutrino cross-section measurements are required by next generation oscillation experiments