Accelerator Physics and Neutrino Beams

II. Linear Machine Lattices

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`http://keil.home.cern.ch/keil/MuMu/Doc/NuSchool05/talk2.pdf`

Programme of Second Lecture – Linear Machine Lattices

- Thin-lens approximation
- Parameters of a thin-lens FODO lattice
- Insertions
  - Principle
  - Strings of insertions
  - Dispersion suppressors

Linear Machine Lattices

- Linear machine lattice is arrangement of linear elements, such as drift spaces, bending magnets, quadrupoles, which is possibly repeated periodically around circumference C
- Introduce thin-lens approximation to get simple algebraic expressions for orbit functions α, β, μ and D
- Calculate of orbit functions for real machines with finite-length elements with computer programs

Thin-Lens Approximation

- Often argument of trigonometric and hyperbolic functions $L\sqrt{K} \ll 1$
- Achieve considerable simplification by $L \to 0$, keeping $\delta = LK$ constant, and assuming $\varphi \ll 1$
- Algebraic matrix elements

$$\begin{pmatrix} 1 & 0 & 0 \\ \mp\delta & 1 & \varphi \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ \pm\delta & 1 \end{pmatrix}$$

- Upper sign applies to radially focusing and vertically defocusing element
- Parameter δ is reciprocal of focal length
- Neglect edge focusing
- Use $|K|$?

Focusing in Thin-Element FODO Lattice I

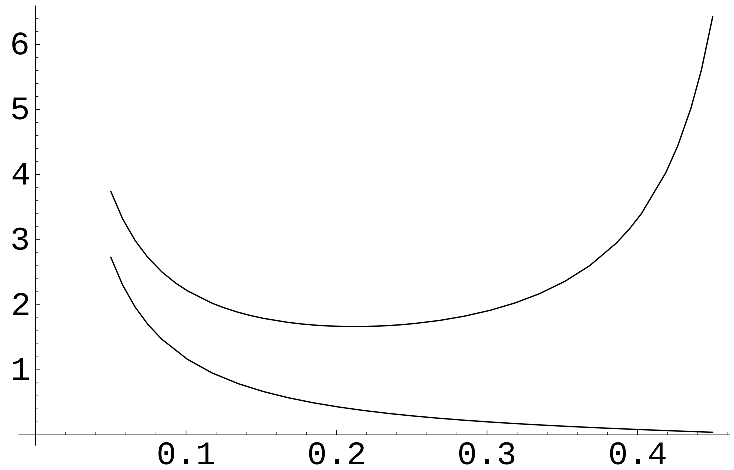
- FODO lattices were used in synchrotrons like SPS and Tevatron, and storage rings like HERA, KEK-B, LEP, LHC, PEP, PETRA, and in neutrino factory
- For cells with length L and equal and opposite δ, starting at entrance of F quadrupole

$$
\begin{aligned}
M_F &= \begin{pmatrix} 1 & L/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\delta & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 - L\delta/2 - L^2\delta^2/4 & L + L^2\delta/4 \\ -L\delta^2/4 & 1 + L\delta/2 \end{pmatrix}
\end{aligned}
$$

- Comparing coefficients with standard cell matrix and introducing phase advance μ as independent variable yields

$$
\begin{aligned}
L\delta &= 4\sin\mu/2 \\
\beta_{F/D} &= L(1 \pm \sin\mu/2)/\sin\mu \\
\alpha_{F/D} &= \mp(1 \pm \sin\mu/2)/\cos\mu/2
\end{aligned}
$$

Focusing in Thin-Element FODO Lattice II



- With transformation between two points in lattice given by

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- Phase advance $\Delta\mu$ between point 1 and point 2

$$\tan \Delta\mu = \frac{m_{12}}{\beta_1 m_{11} - \alpha_1 m_{12}}$$

- Min $\check{\beta}/L$ and max $\hat{\beta}/L$ vs. $\mu/2\pi$
- Stability limit at $\mu/2\pi = 0.5$
- Min $\hat{\beta}/L$ at $\mu/2\pi = 0.212$
- Orbit functions at point 2 obtained from those at point 1 by Steffen matrix

$$\begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} m_{11}m_{22} + m_{12}m_{21} & -m_{11}m_{21} & -m_{12}m_{22} \\ -2m_{11}m_{12} & m_{11}^2 & m_{12}^2 \\ -2m_{21}m_{22} & m_{21}^2 & m_{22}^2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix}$$

Dispersion in Thin-Element FODO Lattice I

- Use 3×3 matrices, include bending angles, and use symmetry to compute only for half period from centre of F quadrupole to centre of D quadrupole with bending angle $\varphi \ll 1$ for full period

$$
\begin{aligned}
M &= \begin{pmatrix} 1 & 0 & 0 \\ \delta/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L/4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \varphi/2 \\ 0 & 0 & 1 \end{pmatrix} \times \\
&\quad \times \begin{pmatrix} 1 & L/4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\delta/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 - L\delta/4 & L/2 & L\varphi/8 \\ -L\delta^2/8 & 1 + L\delta/4 & (1 + L\delta/8)\varphi/2 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

Dispersion in Thin-Element FODO Lattice II

- Since the slopes of the dispersion D'_F and D'_D must vanish at the centre of the quadrupoles, because of symmetry, D_F and D_D must satisfy

$$\begin{pmatrix} D_D \\ 0 \\ 1 \end{pmatrix} = M \begin{pmatrix} D_F \\ 0 \\ 1 \end{pmatrix}$$

- Solution is

$$D_{F/D} = \frac{L\varphi}{4} \frac{1 \pm (1/2)\sin\mu/2}{\sin^2\mu/2}$$

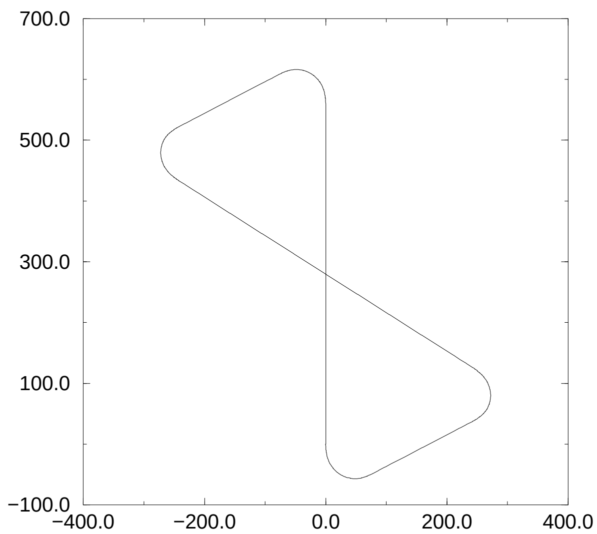
- Average dispersion $\bar{D}$ in FODO lattice with sector dipoles filling all space between thin quadrupoles and finite μ, but neglecting edge focusing

$$\bar{D} = \frac{L\varphi}{4}\left(\frac{1}{\sin^2\mu/2} - \frac{1}{12}\right)$$

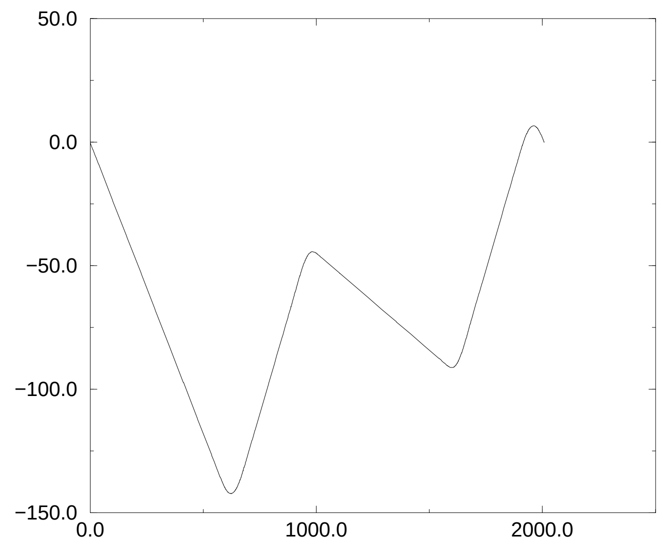
is smaller than the mean value of D_F and D_D

Bowtie μ Storage Ring μSR

- Real-world examples of cells and insertions taken from bowtie μ storage ring design
- Finite elements, computer programs, graphics, ...

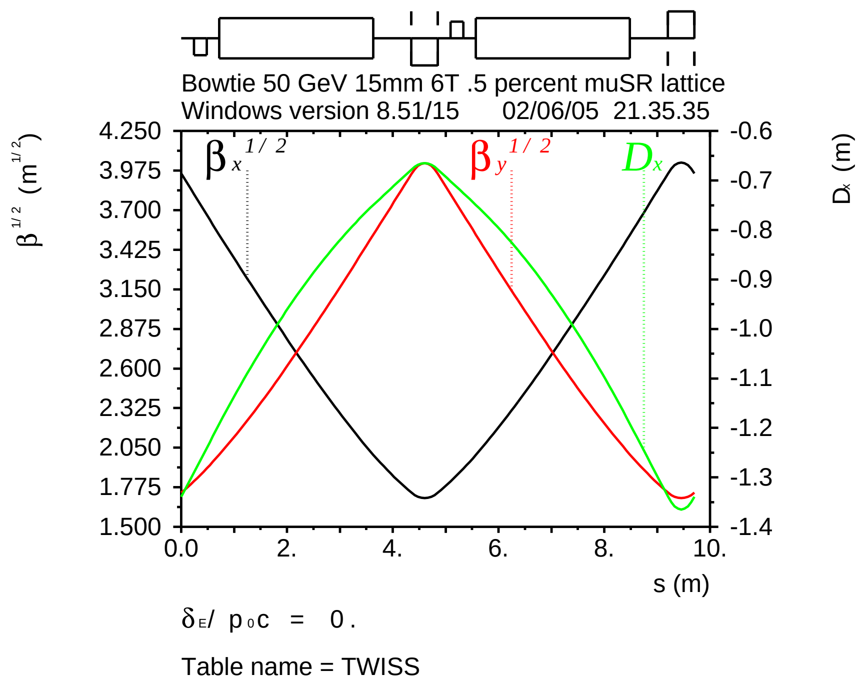


Projection of μSR on plane tangent to Earth



Vertical position along circumference of μSR

Orbit Functions in Cell of Bowtie μ Storage Ring



Schematic layout and orbit functions

- Bowtie has arcs with opposite curvature
- $D < 0$ in half the arcs
- Large centred rectangles are dipoles
- Large rectangles above and below the line are focusing and defocusing quadrupoles
- Small rectangles above and below the line are focusing and defocusing sextupoles
- Collimators are also shown

Insertions

- Insertions overcome shortcomings of simple periodic lattices
  1. No long drift spaces for RF cavities, and injection/ejection equipment
  2. No long straight lattices pointing at remote ν detector(s) with small μ beam divergence
  3. No interaction regions in $\mu^+\mu^-$ colliding-beam storage rings with particularly small transverse beam radii, in order to achieve the desired luminosity
- Discuss a few types of insertion
  1. Dispersion suppressors
  2. Straight lattice in μ decay ring of ν factory
  3. Low-β insertion in e^+e^-storage ring
- Joining insertions is called matching, and done with computer programs
- Insertions invented at about the time when computers started to be used in lattice design

Strings of Insertions

- Typical matching problem consists of joining an arc with cells with orbit functions α_x, α_y, β_x, β_y, and dispersion D and D' to another piece of lattice with different orbit function and different dispersion
- In large machines, i.e. LEP, LHC, μ storage ring in ν factory, it is advantageous to divide matching into steps, matching first the dispersion, doing little if anything to the orbit functions, and then matching the orbit functions, knowing that this step will not have an effect on the dispersion
- $\ldots$
- Number of variables and constraints smaller in each step, making fitting easier
- Unfortunately this is not always possible in small machines with limited space

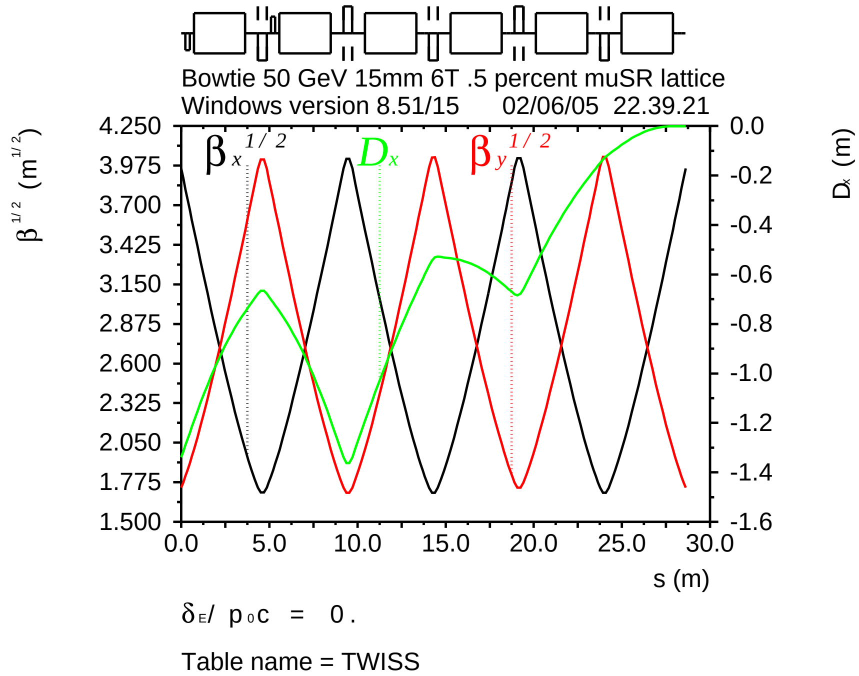
Dispersion Suppressors

- Dispersion suppressors are inserted between arc with dispersion and straight lattice without dispersion
- Two free parameters needed to satisfy conditions on D and D'
- Particularly simple arrangement consists of two FODO cells similar to those in arc, but with modified bending angles, but unchanged focusing

$$\varphi_1 = \varphi \left(1 - \frac{1}{4\sin^2\mu/2}\right) \qquad \varphi_2 = \frac{\varphi}{4\sin^2\mu/2}$$

- φ_1 in cell next to arc with $D \neq 0$
- Works only at design phase advance μ
- $\varphi_1 = \varphi_2$ for $\mu = \pi/2$
- No reverse bending $\varphi_1 \geq 0$ for $\pi/3 \leq \mu \leq 2\pi/3$
- Note $\varphi_1 + \varphi_2 = \varphi$, i.e. dispersion suppressor has half the bending of arc cells

Dispersion Suppressor of Bowtie μ Storage Ring

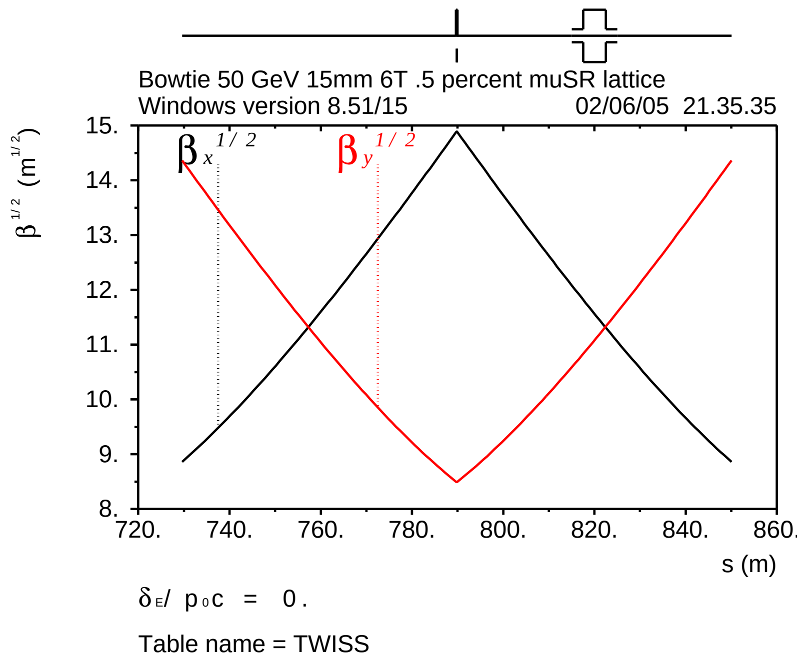


Schematic layout and orbit functions

- One arc cell at left and two dispersion suppressor cells at right
- $D < 0$ in the arcs
- D and D' vanish at right edge of graph
- Also works starting at a D quadrupole

Muon Decay Straight Section

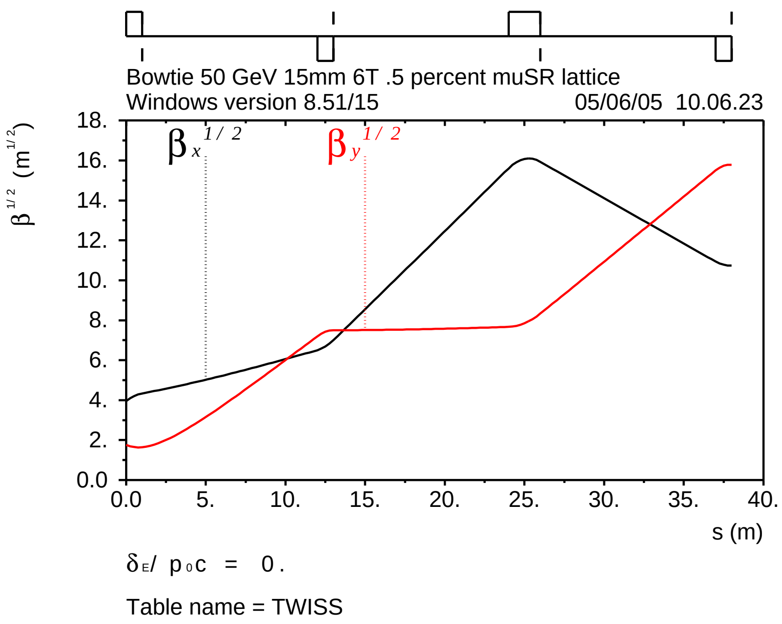
- FODO lattice without bending magnets
- Design dominated by upper limit on normalised divergence $\beta\gamma\sigma' = \beta\gamma\sqrt{\gamma_\perp\varepsilon} = 0.1$ due to 1 % limit on ν energy error
- $\gamma_\perp$ constant in drift space between quadrupoles, has minima in quadrupoles
- Use permanent-magnet quadrupoles?
- RF cavity to keep μ's bunched and polarised



Schematic layout and orbit functions

Matching Insertion between Arc and Decay Straight Section

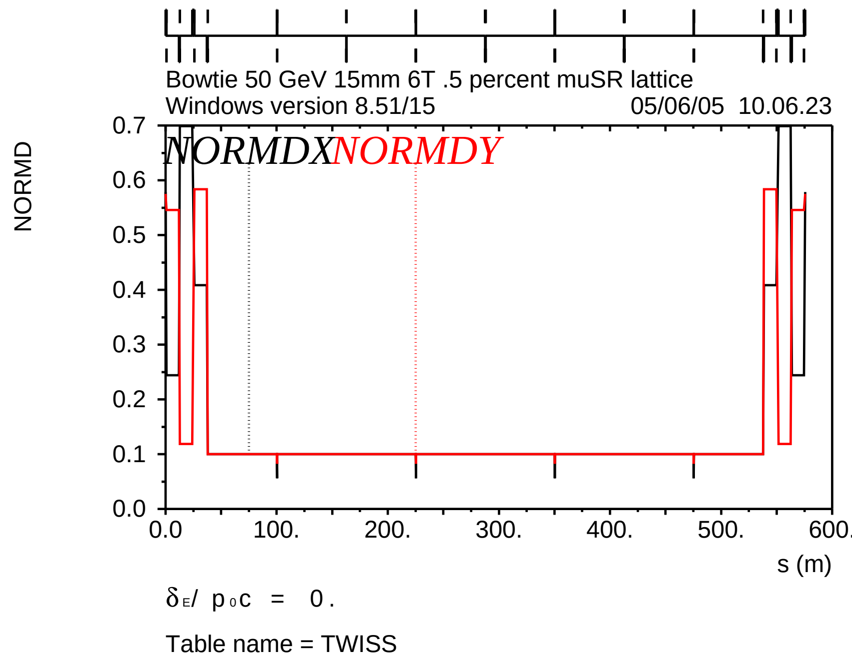
- $D = 0$ and $D' = 0$ at exit of dispersion suppressor and in decay straight section $\Rightarrow$ no dispersion matching needed
- Match 2 α's and 2 $\sqrt{\beta}$'s from $2 \ldots 4$ m to $9 \ldots 15$ m
- 4 constraints and 4 variables
- Pick length of straight sections to achieve alternating quadrupole polarity, avoiding large oscillations of $\sqrt{\beta}$



Schematic layout and orbit functions

- Third quadrupole twice as long, because first neutrino factory study at Fermilab found that long quadrupole was needed to avoid reduction of dynamic aperture

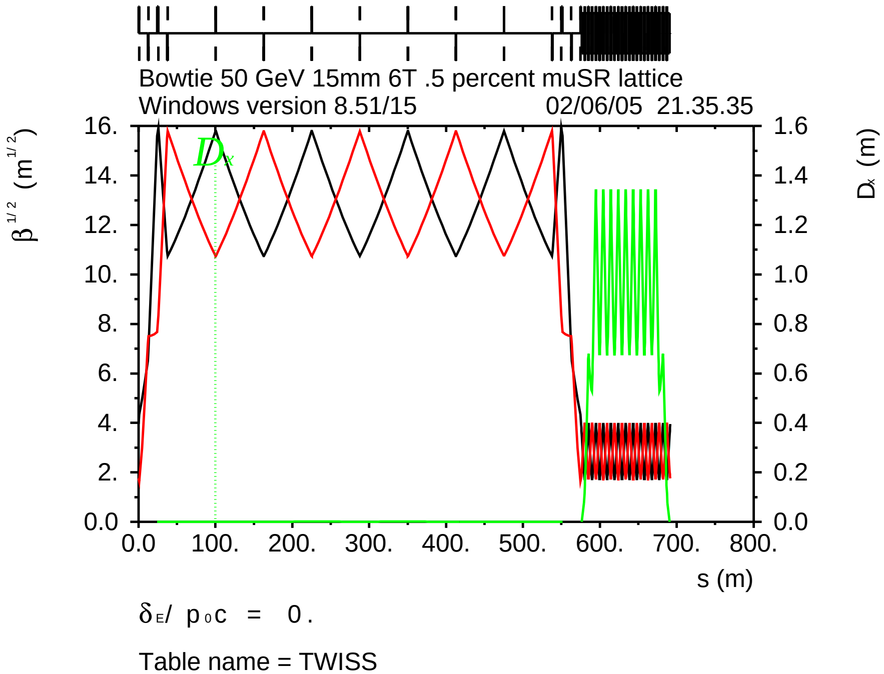
Normalised Divergence in Matching Insertion and Decay Section



Normalised divergence

- Meet design criterion $\beta\gamma\sigma' = 0.1$
- Why are there minima in the quadrupoles?
- Miss design criterion in matching insertions at either end
- Small dipole at ends of decay section ensures that ν's from matching insertion don't reach detector
- Include bending magnet in dispersion matching at ends of arcs

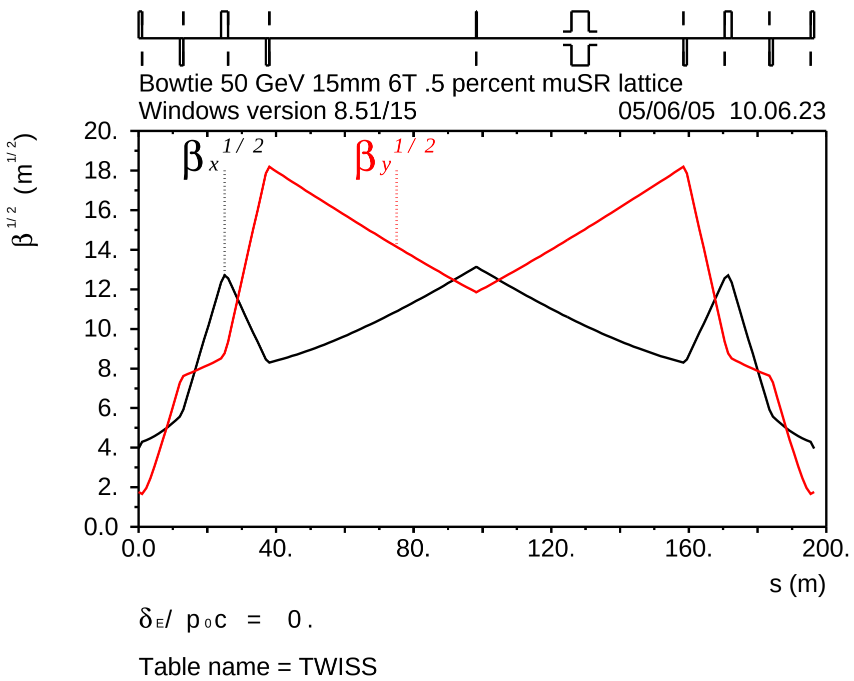
Muon Decay Straight Section and Arc



Schematic layout and orbit functions

- Decay straight section with periodic β-functions and vanishing dispersion D, surrounded by two matching insertions
- Arc with periodic β-functions and dispersion D, surrounded by two dispersion suppressors

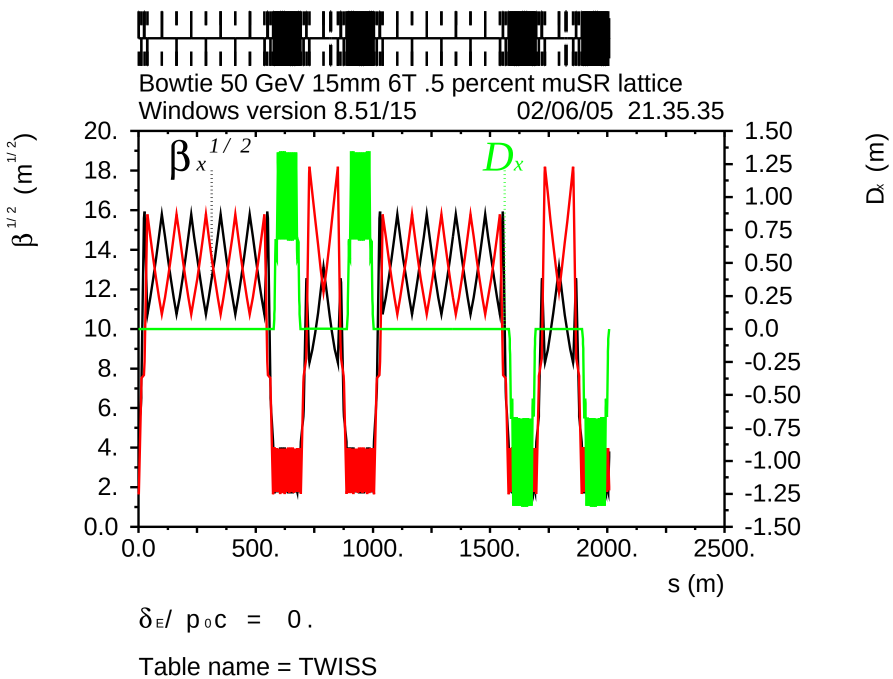
Non-Physics Straight Section



Schematic layout and orbit functions

- Joins two arcs symmetrically
- Matching insertions at both ends
- Use symmetry and impose only 2 constraints $\alpha_x = \alpha_y = 0$ at centre
- 2 variables: 2 quadrupoles in matching insertion, 1 quadrupole at centre

Entire Muon Decay Storage Ring



Schematic layout and orbit functions

- Two long decay straight sections surrounded by matching insertions
- Two pairs of arcs with opposite curvature and dispersion
- Arcs surrounded by dispersion suppressors at both ends
- Two short straight sections joining arcs in a pair

Conclusions II

- Thin-lens approximation yields algebraic expressions for many parameters, easily understood and manipulated
- Discussed dispersion suppressors
- Presented bowtie muon storage ring
- Discussed insertions in bowtie muon storage ring
- Assembled complete muon storage ring