Accelerator Physics and Neutrino Beams

Tutorial I – Effects of Errors

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`http://keil.home.cern.ch/keil/MuMu/Doc/NuSchool05/tutorial1.pdf`

Programme of First Tutorial – Effects of Errors

- Field errors
- Correction of the closed orbit
- Gradient errors

Effect of Single Field Error

- Field errors caused by excitation errors of dipoles and alignment errors of quadrupoles
- Simple example of field error in single element, causing kick $\phi = \delta B\ell/B\rho$
- Closed orbit at location of kick with subscript k given by

$$\begin{pmatrix} u \\ u' \end{pmatrix} = \begin{pmatrix} \cos\mu + \alpha_k \sin\mu & \beta_k \sin\mu \\ -\gamma_k \sin\mu & \cos\mu - \alpha_k \sin\mu \end{pmatrix} \begin{pmatrix} u \\ u' + \phi \end{pmatrix}$$

- Solution

$$\begin{pmatrix} u_k \\ u'_k \end{pmatrix} = \begin{pmatrix} (1/2)\beta_k \phi \cot\mu/2 \\ -(1/2)\phi(1 + \alpha_k \cot\mu/2) \end{pmatrix}$$

- Invariant

$$E = (1/2)\beta\left(\phi \cot\mu/2\right)^2$$

- Integral resonance with $u, u', E \rightarrow \infty$ for $\mu \rightarrow 2n\pi$ or $Q \rightarrow n$, where n is an integer

Effect of Many Field Errors

- Use normalised phase space (v, v') and Green function of driven harmonic oscillator

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\varphi^2} + Q^2 v = Q^2 \beta^{3/2} \Delta B(\varphi)/B\rho$$

- Find periodic solution with $f(\varphi) = \beta^{3/2} \Delta B(\varphi)/B\rho$

$$v(\varphi) = \frac{Q}{2 \sin \pi Q} \int_{\varphi}^{\varphi+2\pi} f(\psi) \cos Q(\pi + \varphi - \psi)$$

- Get insight into resonant behaviour of closed orbit by Fourier expansion

$$f(\varphi) = \sum_k f_k \exp(ik\varphi) \qquad f_k = \frac{1}{2\pi} \int_0^{2\pi} f(\varphi) \exp(-ik\varphi)$$

- Periodic solution

$$v(\varphi) = \sum_k \frac{Q^2 f_k \exp(ik\varphi)}{Q^2 - k^2}$$

- Closed orbit most sensitive to harmonics $k \approx Q$

Correction of Closed Orbit

- Closed orbits get corrected in practically all machines by
  - Measuring orbit positions with beam position monitors, typically three to four in betatron wavelength
  - Calculating suitable set of corrections
  - Exciting orbit correction dipoles or windings on existing magnets to counteract effects of alignment and excitation errors
- Essentially two styles of correction
  - Harmonic correction: Fourier analyse orbit offsets, compensate harmonics near to Q
  - Most efficient correctors: Find corrector which reduces orbit distortion most, then find a second, third, $\ldots$ corrector
- Achieve first closed orbit inside aperture by threading, i.e. using few upstream orbit readings and correctors to advance injected beam through pieces of machine

Effects of Single Gradient Error I

- Treat gradient error as short element with strength δ and get new matrix for one turn, where subscripts 0 mark unperturbed values

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \begin{pmatrix} \cos\mu_0 + \alpha_0\sin\mu_0 & \beta_0\sin\mu_0 \\ -\gamma_0\sin\mu_0 & \cos\mu_0 - \alpha_0\sin\mu_0 \end{pmatrix}$$

- Trace of M yields tune shift ΔQ ans stopband width δQ

$$(1/2)Tr(M) = \cos\mu = \cos\mu_0 + (\beta_0\delta/2)\sin\mu_0$$

- Phase shift $\Delta\mu$ and tune shift ΔQ for $\delta \ll 1$ and $\sin\mu_0 \neq 0$

$$\Delta\mu = -\beta_0\delta/2 \qquad \Delta Q = -\beta_0\delta/4\pi$$

- Observing ΔQ for given δ yields β_0
- Half-integral stopbands occur with $|\cos\mu| > 1$ around $\mu_0 = n\pi$ or $Q = n/2$ with width $\delta Q = \beta_0\delta/2\pi$

Effects of Single Gradient Error II

- Element m_{12} of perturbed matrix M yields change $\Delta\beta$ at gradient perturbation for $\delta \ll 1$

$$\Delta\beta = -(\beta\delta/2)\cot\mu_0$$

- Change $\Delta\beta_2$ at phase μ_2 from δ at β_1 and phase μ_1 is for $\delta \ll 1$

$$\Delta\beta_2 = \frac{\delta\beta_1\beta_2\cos[2\pi Q - 2(\mu_2 - \mu_1)]}{\sin 2\pi Q}$$

- Denominator vanishes at $Q = n/2$ where n is an integer
- Generalisation for gradient perturbation $k(s)$ around circumference for ΔQ and δQ

$$\Delta Q = -\frac{1}{4\pi}\int_0^C \beta(s)k(s)\mathrm{d}s \qquad \delta Q = 2|\Delta Q|$$

- Generalisation for gradient perturbation $k(s)$ around circumference for $\Delta\beta$

$$\delta\beta(s) = \frac{\beta(s)}{2\sin 2\pi Q}\int_s^{s+C} k(\sigma)\beta(\sigma)\cos[2\pi Q + \mu(\sigma) - \mu(s)]\mathrm{d}\sigma$$