NuFact Summer Institute Capri 2005

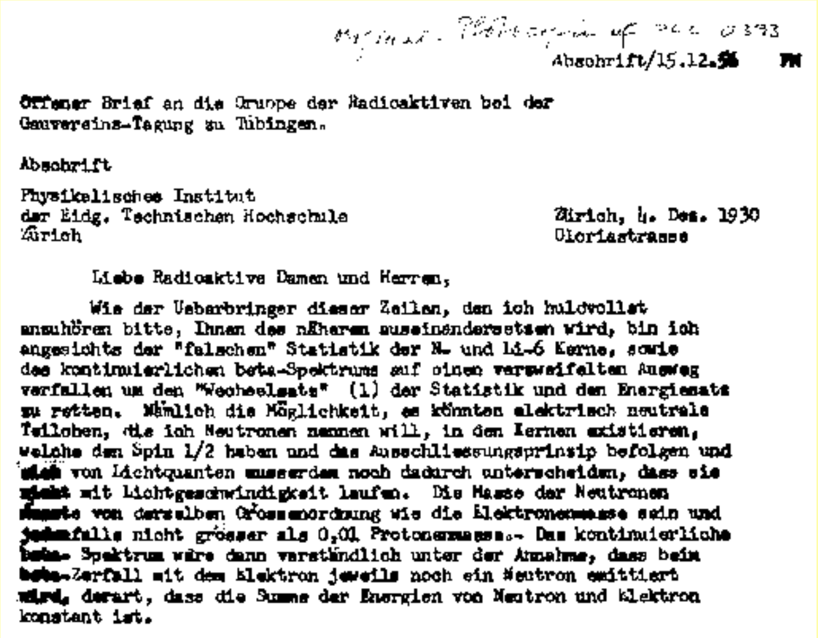
Physics of massive νs

Eligio Lisi, INFN, Bari, Italy

LECTURE I

A brief introduction

ν The oldest **fundamental** particle after the **electron** and the **photon** **(Pauli, 1930)**



Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

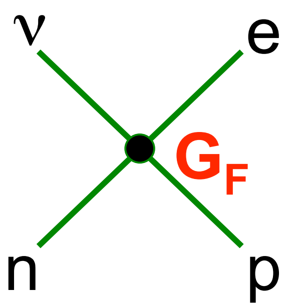
Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

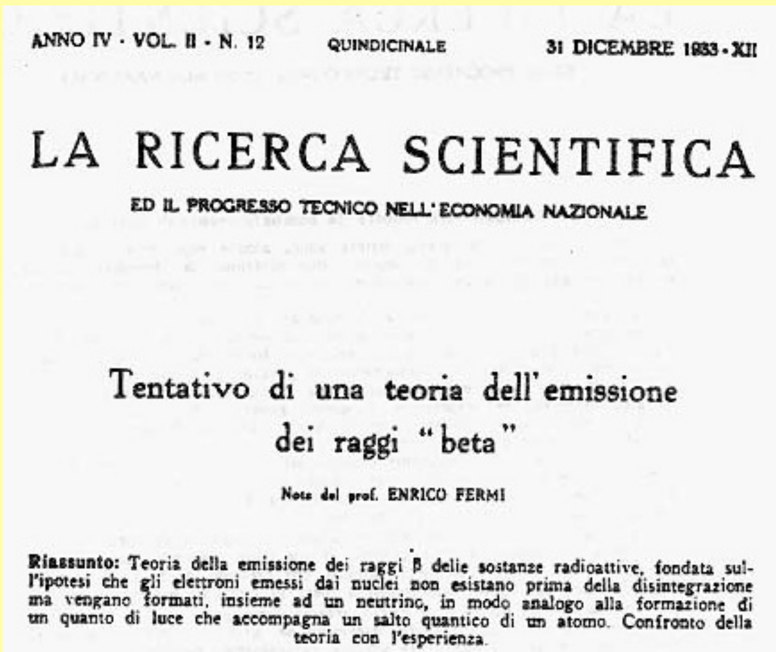
Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

First kinematical properties: spin 1/2, small mass, no charge



Baptised and **quantized** within four-fermion effective interaction **(Fermi, 1933-34)**



ANNO IV · VOL. II · N. 12 QUINDICINALE 31 DICEMBRE 1933 · XII

LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

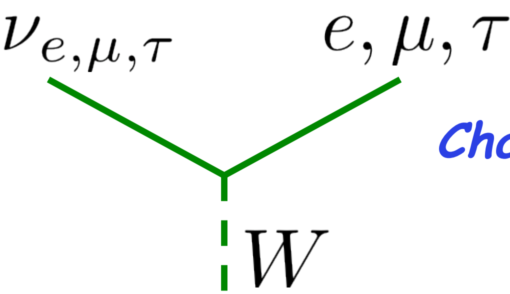
Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

First dynamical properties: Weak interactions, Fermi constant

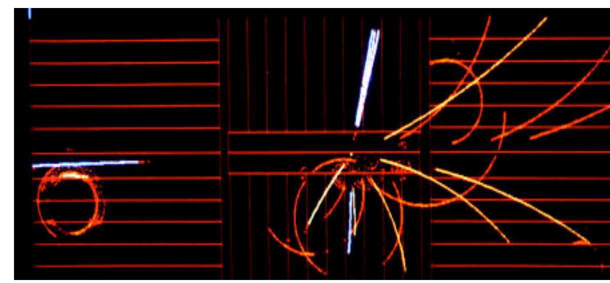
After > 70 years of research we have learned a lot more, e.g., that **neutrinos come in three flavors,**

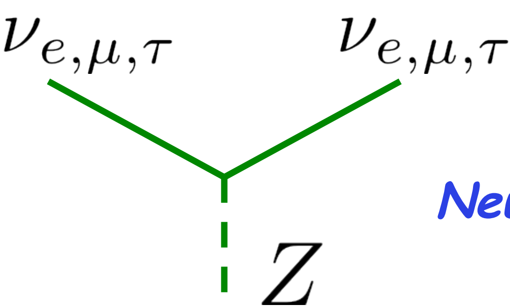
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad \begin{array}{l} \leftarrow \quad q = 0 \\ \leftarrow \quad q = -1 \end{array} \quad (\Delta q = 1)$$

and that the Fermi interaction is mediated by a charged vector boson **W**, with a neutral counterpart: the vector boson **Z**

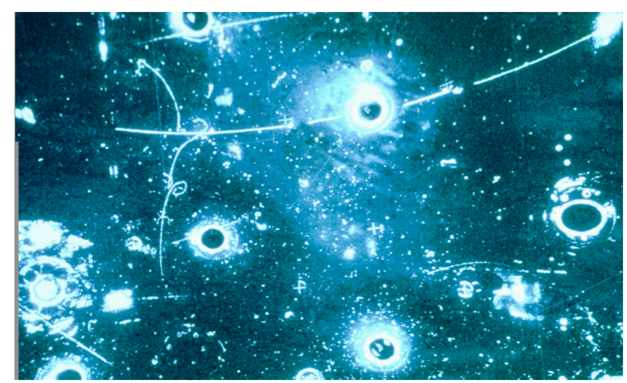


Charged current (Δq=1)





Neutral current (Δq=0)



Despite great progress, only recently we have got (or can reasonably hope to get “soon”) an answer to some fundamental questions asked in the last century:

How small is the neutrino mass ?

(Pauli, Fermi, '30s)

Is the neutrino its own antiparticle?

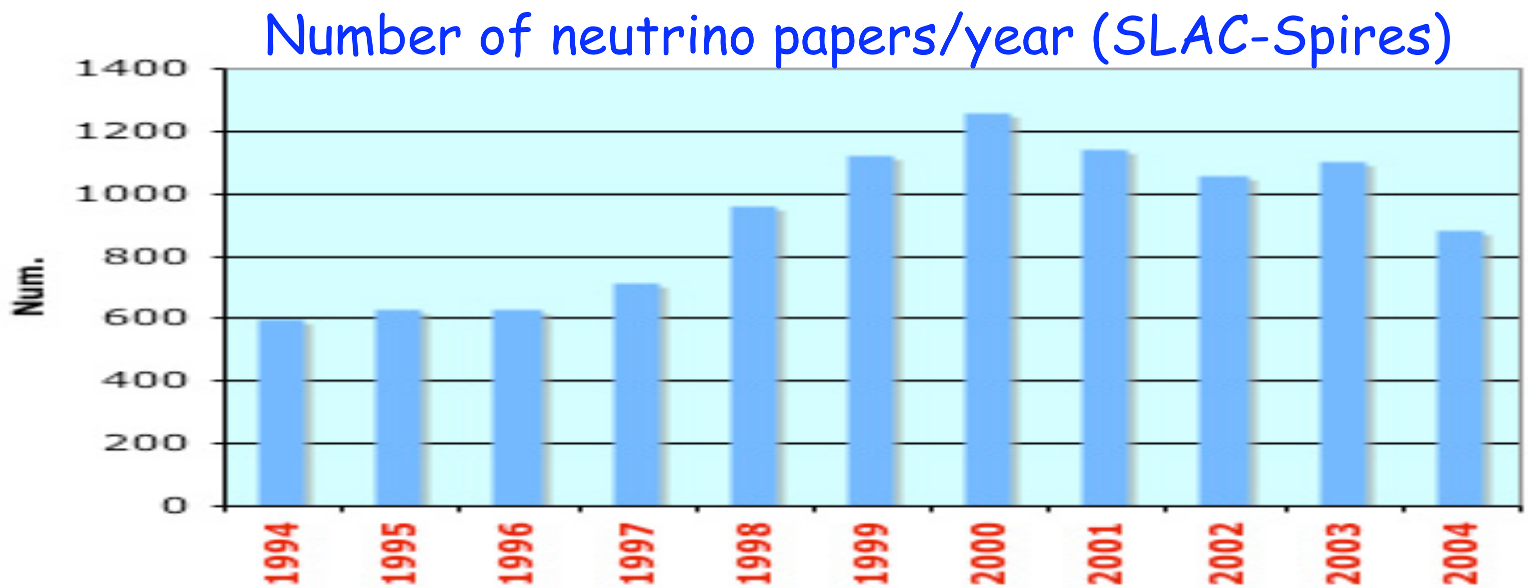
(Majorana, '30s)

Do ν_s of different flavors trasform (“oscillate”) among them?

(Pontecorvo, Maki-Nakagawa-Sakata, '60s)

In particular, one can give an affirmative -and rather detailed- answer to the last question. Explosion of interest (both expt. and theor.)

O(10^4) neutrino papers in the last decade. Boost after 1998 (evidence for atmospheric ν oscillations)



Many excellent neutrino reviews and books exist. Ask me for refs. or browse the "ν unbound" website: www.nu.to.infn.it

Hereafter, I will only touch a few selected topics, and cite literature only occasionally - with apologies to ν colleagues

ν interactions & masses:
elements of theory

Fermion currents in the Standard Model $SU(2)_L \times U(1)_Y$

Building blocks:

$$\begin{pmatrix} U^\alpha \\ D^\alpha \end{pmatrix}_L \qquad \begin{matrix} U_R^\alpha \\ D_R^\alpha \end{matrix}$$

$U =$ "up" fermions

$D =$ "down" fermions

$\alpha = 1, 2, 3 =$ generation index

$$P_{L,R} = \frac{1 \mp \gamma_5}{2}$$

Charges:

$(T_\pm, T_3) = SU(2)_L$ charges

$Y \quad = 2(Q - T_3) = U(1)_Y$ charge

$Q \quad =$ e.m. charge

Gauge bosons (after SSB):

$W_\mu^\pm \; (m = M_W)$

$Z_\mu \; (m = M_Z)$

$A_\mu \; (m = 0)$

Fermion currents:

$$W^{\pm}_{\mu} \begin{cases} J^{+}_{\mu} = \sum_{\alpha} \bar{D}^{\alpha}_{L} \gamma_{\mu} U^{\alpha}_{L} \\ J^{-}_{\mu} = \sum_{\alpha} \bar{U}^{\alpha}_{L} \gamma_{\mu} D^{\alpha}_{L} \end{cases}$$

$$Z_{\mu} \begin{cases} J^{Z}_{\mu} = \sum_{\alpha} \bar{U}^{\alpha}_{L} (T_3 - Q \sin^2\theta_W) \gamma_{\mu} U^{\alpha}_{L} \\ \qquad + \bar{U}^{\alpha}_{R} (-Q \sin^2\theta_W) \gamma_{\mu} U^{\alpha}_{R} + (U \to D) \end{cases}$$

$$A_{\mu} \left\{ J^{EM}_{\mu} = \sum_{\alpha} \bar{U}^{\alpha} Q \gamma_{\mu} U^{\alpha} + (U \to D) \right.$$

Low-energy limit:

$$\mathcal{L}_{CC+NC} = -\frac{4G_F}{\sqrt{2}} \left[J^{+}_{\mu} J^{-}_{\mu} + \rho J^{Z}_{\mu} J^{\mu Z} \right]$$

$\rho = 1$ if SSB induced by Higgs doublet

$\tan\theta_W = g'/g$
$g = SU(2)_L$ coupling
$g' = U1(Y)_L$ "

θ_W = bookkeeping parameter
(can be eliminated in terms of mass spectrum $+ (\alpha, G_F) + \alpha_s$)

Probing fermion currents with neutrinos

Neutrinos have been used to:

1) Assess strength of weak inter. (G_F)
2) Probe V-A structure of $J_\mu^\pm$ (CC)
3) Probe $(T_3 - Q s_W^2)$ charge of J_μ^Z (NC)
4) Probe CC+NC interference

...

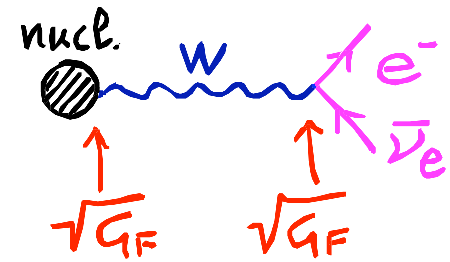
Examples:

1) β-decay, μ decay
2) $\pi \to \mu\bar{\nu}, e\bar{\nu}$ decay
3) $\overset{(-)}{\nu}_\mu e$ scattering
4) $\overset{(-)}{\nu}_e e$ scattering

1) Probing G_F in beta-decay and muon decay

β-decay

rate: $d\Gamma \propto G_F^2 \times (\text{phase sp.})$

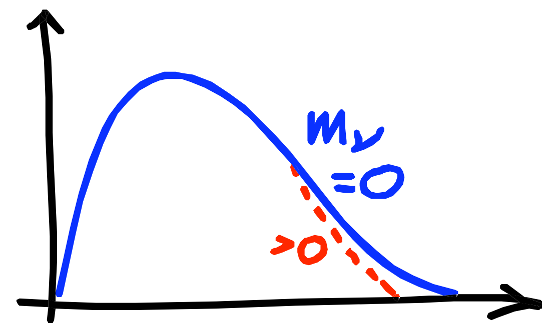


$\left| \cdots \right|^2 \propto G_F^2$

energy spectrum:

$$\frac{d\Gamma}{dE_e} \propto G_F^2\, p_e E_e (Q - E_e)^2 \qquad (m_\nu \equiv 0)$$

$$G_F^2\, p_e E_e (Q - E_e)\sqrt{(Q-E_e)^2 + m_\nu^2} \qquad (>0)$$



μ-decay

$$\Gamma_\mu = \frac{1}{\tau_\mu} \propto G_F^2 m_\mu^5$$

"defines" G_F

2) Probing V-A structure in pion decay

Dirac eq. for free particle (Weyl repres.)

$$\begin{bmatrix} -\frac{m}{E} & 1+h \\ 1-h & -\frac{m}{E} \end{bmatrix} \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

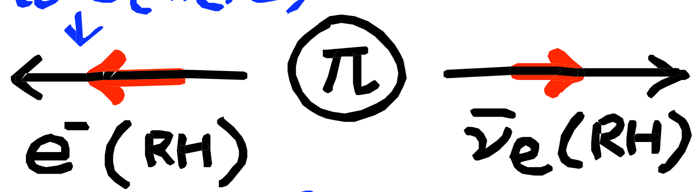
$h = \frac{\vec{p}\cdot\vec{\sigma}}{E}$ ← helicity

$\phi_{R,L} = \frac{1 \pm \gamma_5}{2}$ ← chirality

For $m/E \to 0$: helicity $\simeq$ chirality

$$h\,\phi_{R,L} \simeq \pm\phi_{R,L} + O(m/E)$$

"wrong" chirality up to $O(m_e/E)$



$\pi \to e^- \bar{\nu}_e$ forbidden by V-A for $m_e \to 0$

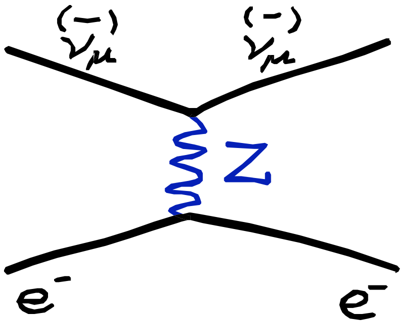
$$\frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} \simeq$$

$$\simeq \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 \ll 1$$

$\ll 1$ chirally suppressed

> 1 phase space

3) Probing (T_3-$Q\sin^2\theta_W$) NC structure with neutrinos



$\overset{(-)}{\nu}_\mu$ scattering on electrons

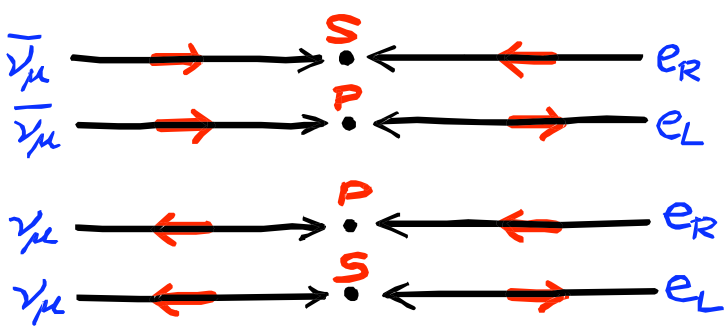
NC electron charges:

$\epsilon_L = (T_3 - Qs_W^2)e_L = -\frac{1}{2} + s_W^2$

$\epsilon_R = (T_3 - Qs_W^2)e_R = 0 + s_W^2$

At high energy, helicity ~ chirality and total (ν,e) spin J=0 (S-wave) or J=1 (p-wave) in C.M. system

$d\sigma/dy \qquad (y = \frac{E_e}{E_\nu})$



$\propto \epsilon_R^2$

$\epsilon_L^2(1-y)^2$

$\epsilon_R^2(1-y)^2$

ϵ_L^2

At low energy, helicity ≠ chirality and a further LR correction appear

$\propto \epsilon_L \epsilon_R \frac{m_e}{E_\nu} \cdot y$

Differential cross sections:

$$\frac{d\sigma}{dy}(\bar{\nu}_\mu e^-) \simeq \frac{2G_F^2 m_e E_\nu}{\pi}\left(\epsilon_R^2 + \epsilon_L^2(1-y)^2\right)$$

$$\frac{d\sigma}{dy}(\nu_\mu e^-) \simeq \frac{2G_F^2 m_e E_\nu}{\pi}\left(\epsilon_L^2 + \epsilon_R^2(1-y)^2\right)$$

Total cross sections:

$\int(1-y)^2 dy = 1/3 \leftarrow$ only 1/3 of $\vec{J}=1$ states allowed by J conservat.

$$\sigma(\bar{\nu}_\mu e^-) \propto \left(\epsilon_R^2 + \tfrac{1}{3}\epsilon_L^2\right)$$

$$\sigma(\nu_\mu e^-) \propto \left(\epsilon_L^2 + \tfrac{1}{3}\epsilon_R^2\right)$$

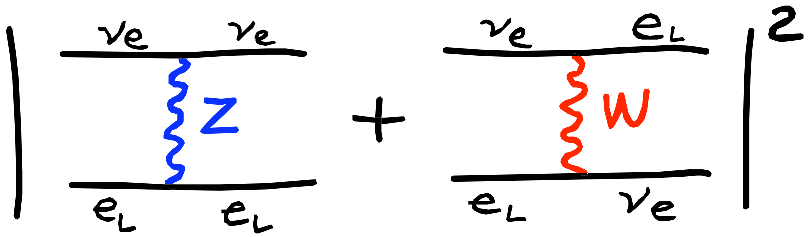
"History":

$$R = \frac{\sigma(\nu)}{\sigma(\bar{\nu})} = \frac{3\epsilon_L^2 + \epsilon_R^2}{3\epsilon_R^2 + \epsilon_L^2} = 3\,\frac{1 - 4s_W^2 + \frac{16}{3}s_W^4}{1 - 4s_W^2 + 16 s_W^4}$$

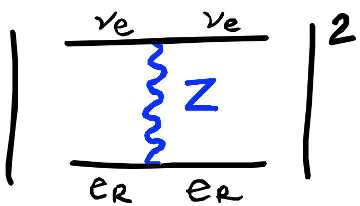
allowed first estimates of s_W^2 and of tree-level M_W and M_Z from:

$$s_W^2 = \pi\alpha/\sqrt{2}\,G_F M_W^2 \quad ; \quad s_W^2 = 1 - M_W^2/M_Z^2$$

4) Probing W-Z interference with neutrinos



$$\propto (\epsilon_L + 1)^2$$



$$\propto \epsilon_R^2 (1-y)^2$$

$$\frac{d\sigma}{dy}(\nu_e e^-) \simeq \frac{2 G_F^2 m_e E_\nu}{\pi}\left[(\epsilon_L + 1)^2 + \epsilon_R^2 (1-y)^2\right]$$

$$\frac{d\sigma}{dy}(\bar{\nu}_e e^-) \simeq \frac{2 G_F^2 m_e E_\nu}{\pi}\left[(\epsilon_R + 1)^2 + \epsilon_L^2 (1-y)^2\right]$$

W-Z INTERFERENCE

Implications →

- $\sigma(\nu_\mu) < \sigma(\nu_e)$

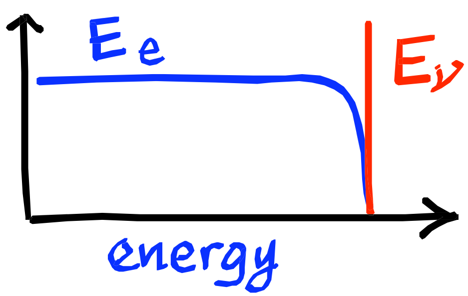
$$\frac{\sigma(\nu_\mu e)}{\sigma(\nu_e e)} \simeq \frac{\epsilon_L^2 + \epsilon_R^2/3}{(\epsilon_L+1)^2 + \epsilon_R^2/3} \simeq \frac{1}{7}$$

- $\nu_e e^- \to \nu_e e^-$: flat spectrum

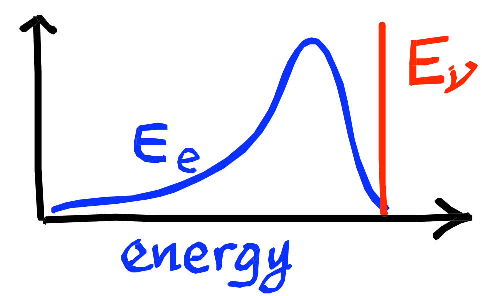
$$\frac{d\sigma}{dy}(\nu_e e^-) \propto 1 + \underbrace{\frac{\epsilon_R^2}{(\epsilon_L+1)^2}}_{\text{small}}(1-y)^2 \sim \text{const}$$

...to be compared with

$\nu_e e^- \to \nu_e e^-$



$\nu_e d \to p p e^-$



Important for solar ν experiments

Fermion masses in the Standard Model

$$\Phi_{\text{Higgs}} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\text{SSB}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

Yukawa lagrangian:

$$-\mathcal{L}_Y = \sum_{\alpha\beta} f_D^{\alpha\beta}\, \overline{(U^\alpha, D^\alpha)}_L \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} D_R^\beta$$

$$+ \sum_{\alpha\beta} f_U^{\alpha\beta}\, \overline{(U^\alpha, D^\alpha)}_L \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} U_R^\beta$$

$\leftarrow$ Get mass from $\tilde{\Phi} = i\sigma_2 \phi^*$

$$= \sum_{\alpha\beta} \bar{D}_L^\alpha M_D^{\alpha\beta} D_R^\beta + \bar{U}_L^\alpha M_U^{\alpha\beta} U_R^\beta$$

Generic 3×3 complex matrices

$\rightarrow$ Diagonalization

Theorem: Generic M $(N \times N)$ is diagonalizable through biunitary transformation: $S^{+}MT = M_d$

where $M_d = \mathrm{diag}(m_1, m_2, \ldots, m_N)$

and $SS^{+} = 1 = TT^{+}$

Proof: MM^{+} is hermitian

$$\to S^{+}(MM^{+})S = M_d^2 = \mathrm{diag}(m_1^2, \ldots, m_N^2)$$

with $m_i^2 = (M_d^2)_{ii} = [(S^{+}M)(S^{+}M)^{+}]_{ii}$

$$= \sum_j (S^{+}M)_{ij} (S^{+}M)^{*}_{ij}$$

$$= \sum_j |S^{+}M|^2_{ij} > 0$$

$\to MM^{+}$ has real, positive eigenvalues m_i^2

Define $M_d = \sqrt{M_d^2} = \mathrm{diag}(m_1, m_2, \cdots, m_N)$

Then: $H = S M_d S^{+}$ ← hermitian

$V = H^{-1} M$ ← unitary

$T = V^{+} S$ ← unitary

$$M_d = S^{+} H S = S^{+} M V^{+} S = S^{+} M T$$

Invariance: the currents

$$J_\mu^- = \sum_\alpha \bar{U}_L^\alpha \gamma_\mu D_L^\alpha$$

$$J_\mu^Z = \sum_\alpha \bar{U}_L^\alpha (T_3 - Q s_W^2) \gamma_\mu U_L^\alpha + \bar{U}_R^\alpha (-Q s_W^2) \gamma_\mu U_R^\alpha + (U \to D)$$

$$J_\mu^{EM} = \sum_\alpha \bar{U}^\alpha Q \gamma_\mu U^\alpha + (U \to D)$$

are invariant under the transformations

(i) $U_R^\alpha \longrightarrow T^{\alpha\beta} U_R^\beta$

(ii) $U_L^\alpha \longrightarrow S^{\alpha\beta} U_L^\beta$

(iii) $D_L^\alpha \longrightarrow S^{\alpha\beta} D_L^\beta$

(ii), (iii): same S

(iv) $D_R^\alpha \longrightarrow W^{\alpha\beta} D_R^\beta$

$SS^\dagger = 1$

$TT^\dagger = 1$

$WW^\dagger = 1$

This fact implies that either M_U or M_D can be diagonalized without affecting currents

Usual "trick" for quarks:

Use properties (i), (ii), (iii) to identify T and S with the matrices diagonalizing M_U: $M_U = S^+ M_U^{diag} T$

Then use (iv) to identify W with one of the matrices diagonalizing M_D:
$M_D = V^+ M_D^{diag} W$ → only V physical:

$$D_L^\alpha \to V^{\alpha\beta} D_L^\beta$$

The V-transformation affects $J_\mu^\pm$ (but not J_μ^{EM}, J_μ^Z):

$$J_\mu^- \to \sum_{\alpha\beta} \overline{U}_L^\alpha \gamma_\mu \underset{\substack{\uparrow \\ \text{CKM matrix}}}{V^{\alpha\beta}} D_L^\beta$$

What about leptons?

If $m_\nu = 0$ (no ν_R) then only 1 matrix needed for diagonalization $\rightarrow$ no observable CKM lepton matrix

If we introduce ν_R^α in the same way as for quarks then ...

can get $m_\nu > 0$ but ...

- no hint for smallness of m_ν
- mass terms for ν can be more general than for quarks since ν is neutral

Massless and massive (neutral) fermions

In Dirac representation:

$$\gamma^0_D = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \qquad \vec{\gamma}_D = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix} \qquad \gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

"PARTICLE" solution $\quad \psi_P \sim \begin{bmatrix} \xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\xi \end{bmatrix} e^{-ip_\mu x^\mu} \qquad \xi\xi^+ = 1$

"ANTIPARTICLE" solution $\quad \psi_A \sim \begin{bmatrix} \frac{\sigma\cdot p}{E+m}\chi \\ \chi \end{bmatrix} e^{ip_\mu x^\mu} \qquad \chi\chi^+ = 1$

ξ, χ = Pauli spinors

Nonrelativistic (particle) limit : $\psi_P \sim \begin{bmatrix} \xi \\ 0 \end{bmatrix} \qquad \overline{\psi}_P \sim \begin{bmatrix} \xi^+ \\ 0 \end{bmatrix}^T$

$$S = \overline{\psi}\psi \simeq |\xi|^2$$
$$P = \overline{\psi}\gamma^5\psi \simeq 0$$
$$\left.\begin{aligned} V &= \overline{\psi}\gamma^\mu\psi \simeq (|\xi|^2, \vec{0}) \\ A &= \overline{\psi}\gamma^\mu\gamma^5\psi \simeq (0, \xi^+\vec{\sigma}\xi) \end{aligned}\right\} \text{useful later}$$

Dirac representation useful to define the particle--antiparticle conjugation operator $\mathcal{C}$

$$\psi^c = \mathcal{C}(\psi)$$
$$\psi_{P,A} = \mathcal{C}(\psi_{A,P})$$

$$\mathcal{C}(\psi) = i\gamma^2\psi^* = i\gamma^2\gamma^0\overline{\psi}^T = C\,\overline{\psi}^T = \psi^c$$

$C = i\gamma^2\gamma^0$

1) Prove that $\mathcal{C}(\psi_P) = \psi_A$; hint : use $\sigma_2\vec{\sigma}^* = -\vec{\sigma}\sigma_2$ and set $\chi = -i\sigma_2\xi^*$

2) Prove that if ψ is e.m. charged, $[i\gamma^\mu(\partial_\mu - iqA_\mu) - m]\psi = 0$, then $[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi^c = 0$

Convention :

When operations such as $P_{L,R}$, $\overline{(-)}$, and $\mathcal{C}$, are involved :

$P_{L,R}$ acts before C which acts before $\overline{(\cdot)}$

$$\psi^c_{L,R} = (P_{L,R}\psi)^c = (\psi_{L,R})^c = P_{R,L}(\psi^c)$$

$$\bar{\psi}_{L,R} = \overline{(P_{L,R}\psi)} = \overline{(\psi_{L,R})} = \bar{\psi}\, P_{R,L}$$

$$\bar{\psi}^c = \overline{(\psi^c)}$$

$$\bar{\psi}^c_{L,R} = \overline{(P_{L,R}\psi)^c} = \overline{(\psi_{L,R})^c} = \overline{P_{R,L}(\psi^c)} = \bar{\psi}^c\, P_{L,R}$$

Weyl representation and Lorentz group

Let's change basis (from "Dirac" to "Weyl"):

$$\psi \to T\psi$$

$$\gamma^\mu \to T\gamma^\mu T^{-1}$$

$$T = \frac{1}{\sqrt{2}}\left(\gamma^0_D + \gamma^5_D\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}$$

$$\gamma^0_W = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \quad \vec{\gamma}_W = \begin{bmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \quad \gamma^5_W = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

Then:

$$\psi_R = \frac{1+\gamma_5}{2}\psi = \begin{bmatrix} \phi_R \\ 0 \end{bmatrix}$$

$$\psi_L = \frac{1-\gamma_5}{2}\psi = \begin{bmatrix} 0 \\ \phi_L \end{bmatrix}$$

"fundamental" objects under Lorentz group

If x^μ transforms as :

$$x'^\mu = e^{i(\underset{\text{ROTATION}}{\vec{\omega}}\cdot\vec{J} + \underset{\text{BOOST}}{\vec{u}}\cdot\vec{K})}\, x^\mu$$

then $\phi_{R,L}$ transform as :

$$\phi'_R = e^{i(\vec{\omega} - i\vec{u})\frac{\vec{\sigma}}{2}}\,\phi_R$$

$$\phi'_L = e^{i(\vec{\omega} + i\vec{u})\frac{\vec{\sigma}}{2}}\,\phi_L$$

coupled by Dirac equation; decoupled only if $m=0$ (Weyl spinors)

Theorem : Given ϕ_R (RH), $i\sigma_2\phi_R^*$ is LH ; given ϕ_L (LH), $-i\sigma_2\phi_L^*$ is RH
(Hint: use $\sigma_2\vec{\sigma}^* = -\vec{\sigma}\sigma_2$ and infinitesimal transform.)

→ can build a Dirac spinor ψ from two RH spinors u & v : $\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix} = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$

(or from two LH ones)

$\overset{(-)}{\psi}{}^{(c)}_{(L,R)}$ components in Weyl basis:

$$\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix} \qquad \psi_L = \begin{bmatrix} 0 \\ i\sigma_2 v^* \end{bmatrix} \qquad \psi_R = \begin{bmatrix} u \\ 0 \end{bmatrix}$$

$$\bar{\psi} = [-iv^T\sigma_2, u^+] \qquad \bar{\psi}_L = [-iv^T\sigma_2, 0] \qquad \bar{\psi}_R = [0, u^+]$$

$$\psi^c = \begin{bmatrix} v \\ i\sigma_2 u^* \end{bmatrix} \qquad \psi^c_L = \begin{bmatrix} v \\ 0 \end{bmatrix} \qquad \psi^c_R = \begin{bmatrix} 0 \\ i\sigma_2 u^* \end{bmatrix}$$

$$\bar{\psi}^c = [-iu^T\sigma_2, v^+] \qquad \bar{\psi}^c_L = [0, v^+] \qquad \bar{\psi}^c_R = [-iu^T\sigma_2, 0]$$

...with $\mathcal{C}$ swapping $u \leftrightarrow v$

In general, no relation between u and v

Given $\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix}$ (u, v R.H.):

$u \neq v \rightarrow$ Dirac ν

$u = v \rightarrow$ Majorana ν

For Majorana neutrinos, $u = v$ implies that $\psi = \psi^c$ (see previous slide)

→ Majorana ν are their own antiparticles

→ They must be completely neutral (no e.m. charge, no generalized charge)

More generally, for Majorana ν's :

$$\psi_M = \psi_M^c \cdot e^{i\varphi_M}$$

← "Majorana creation phase" can be different from +1 (examples later)

Summary of ν representations:

$m=0$ **Weyl**

$$\psi = \begin{bmatrix} \nu_R \\ 0 \end{bmatrix} = \psi_R$$

or: $$\psi = \begin{bmatrix} 0 \\ \nu_L \end{bmatrix} = \psi_L$$

simplest massless case, 2 dof

$m \neq 0$ **Major.**

$$\psi = \begin{bmatrix} \nu_R \\ i\sigma_2 \nu_R^* \end{bmatrix} = \psi_R + \psi_R^c = \psi^c$$

or: $$\psi = \begin{bmatrix} -i\sigma_2 \nu_L^* \\ \nu_L \end{bmatrix} = \psi_L + \psi_L^c = \psi^c$$

simplest massive case, 2 dof

$m \neq 0$ **Dirac**

$$\psi = \begin{bmatrix} \nu_R \\ \nu_L \end{bmatrix} = \psi_R + \psi_L \neq \psi^c$$

general massive case, 4 dof

Paradox and resolution:

Define ν_e as the neutral fermion produced in β^+ decay of some nucleus

Define $\bar{\nu}_e$ as the neutral fermion produced in β^- decay of some nucleus

Q. How can it be $\nu_e = \bar{\nu}_e$ if:

$$\nu_e + n \longrightarrow p + e^-$$
$$\bar{\nu}_e + n \not\longrightarrow p + e^-$$
$$\bar{\nu}_e + p \longrightarrow n + e^+$$
$$\nu_e + p \not\longrightarrow n + e^+$$

?

A(1). Indeed, $\nu_e \neq \bar{\nu}_e$ (Dirac case) $\rightarrow$ Lepton number is conserved: $\Delta L_e = 0$

A(2). It is $\nu = \bar{\nu}$ (Majorana) and we are naming:

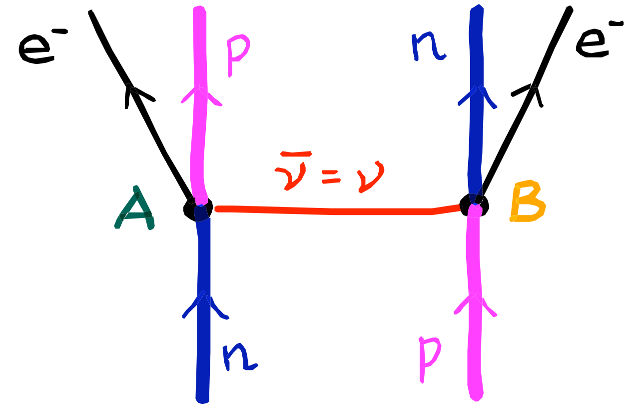
$$\begin{cases} \text{“}\nu_e\text{”} = P_L \nu \\ \text{“}\bar{\nu}_e\text{”} = P_R \nu \end{cases}$$

The initial “ν_e” is produced LH in β^+ decay and remains so up to $O(m/E)$. The reaction $\nu_e p \to n e^+$ is thus chirally suppressed by V-A

However, $O(m/E)$ does not mean “never”. Such reaction can take place at small energies: $\Delta L_e = 2$ at $O(m/E)$

Majorana neutrinos and neutrinoless 2β decay

$0\nu 2\beta$ decay : a low-energy and extremely rare ([weak]^2!) reaction. A nucleus changes charge by two units and emits a couple of electrons :



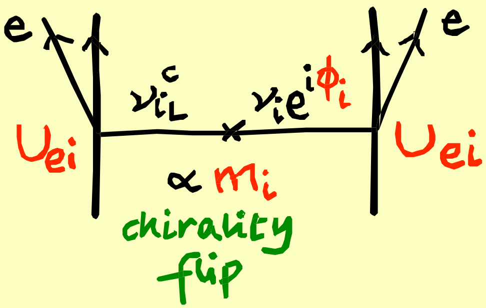
Intuitive picture :

- A $\bar{\nu}_e$ (RH) is emitted in A
- If it is massive, at $O(m/E)$ it develops a LH component ← not possible for Weyl ν
- If $\nu = \bar{\nu}$, such component is a LH neutrino ← not possible for Dirac ν
- The ν_L is absorbed in B and an electron is emitted
- Init. state: no electrons; final stat: 2 electrons → $\Delta L_e = 2$ ← $0\nu 2\beta$ and $\Delta L_e = 2$ only possible for Majorana ν

Relevant parameter in $0\nu 2\beta$

In general, ν_e = superposition of Majorana fields ν_i with masses m_i, coefficients U_{ei}, and creation phases $\exp(i\phi_i)$

$$\Bigg| \sum_i$$



$$\Bigg|^2 \propto \left|\sum_i U_{ei}^2 m_i e^{i\phi_i}\right|^2 = \left|\sum_i |U_{ei}|^2 m_i e^{i\phi_i'}\right|$$

$$= \langle m_{ee} \rangle^2 \text{ or}$$

$$= m_{\beta\beta}^2$$

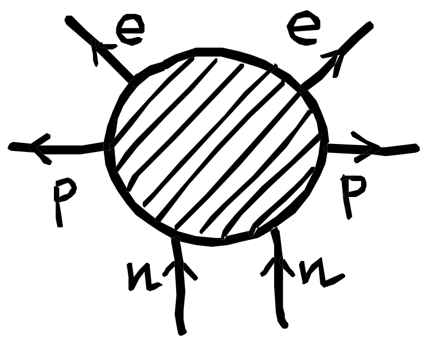
$m_{\beta\beta}$ = "effective Majorana mass"

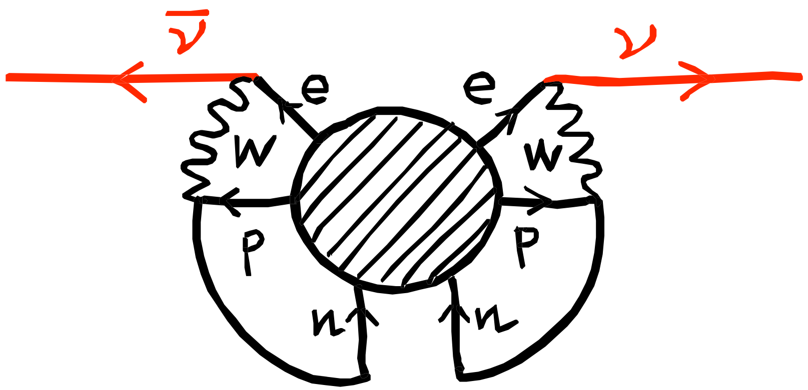
Global phases ϕ_i' (mixing + Majorana) are physical → can get constructive or destructive interf. in (i,j) channels → $m_{\beta\beta}$ may be small due to "cancellations"

Deep link between $0\nu2\beta$ decay and Majorana ν

Independently of the mechanism for $0\nu2\beta$ decay...

... get a Majorana neutrino mass term if $0\nu2\beta$ occurs





Neutrino mass terms for ONE FAMILY

Can generate $m\bar{\psi}\psi$ in 3 possible ways:

1) $\psi = \psi_L + \psi_R$ (Dirac) $\rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$

2) $\psi = \psi_L + \psi_L^c$ (Major.) $\rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_L^c + \bar{\psi}_L^c\psi_L$

3) $\psi = \psi_R + \psi_R^c$ (Major.) $\rightarrow \bar{\psi}\psi = \bar{\psi}_R\psi_R^c + \bar{\psi}_R^c\psi_R$

How? E.g. Higgs
- doublet Φ ← standard model Higgs
- triplet $\vec{\Phi}$ ← beyond standard model
- singlet φ ← " " "

$$\mathcal{L} \ni h(\bar{\nu}_L \bar{e}_L)\Phi\,\nu_R + h'(\bar{\nu}_L \bar{e}_L)\vec{\Phi}\cdot\frac{\vec{\sigma}}{2}\begin{pmatrix}\nu_L^c\\ e_L\end{pmatrix} + h''(\bar{\nu}_R^c\nu_R + \bar{\nu}_R\nu_R^c)\varphi$$

$$\xrightarrow{\text{SSB}}$$

$$m_D(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) \quad \text{Dirac}$$
$$+\,m_L(\bar{\nu}_L\nu_L^c + \bar{\nu}_L^c\nu_L) \quad \text{Major.}$$
$$+\,m_R(\bar{\nu}_R\nu_R^c + \bar{\nu}_R^c\nu_R) \quad \text{Major.}$$

doublet × doublet × singlet
doublet × triplet × doublet
singlet × singlet × singlet

Majorana mass terms not invariant under any global $U(1)$: $\psi \rightarrow e^{i\phi}\psi$
→ no additive (lepton) number conserved

Mass lagrangian in matrix form

(Majorana basis)

$$-\mathcal{L}_m = \left(\bar{\nu}_L + \bar{\nu}_L^c,\ \bar{\nu}_R + \bar{\nu}_R^c\right) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L + \nu_L^c \\ \nu_R + \nu_R^c \end{pmatrix}$$

→ Intuitively clear that, in general, diagonalization will give Majorana ν as eigenstates (not Dirac ν)

Diagonalization exercise:

$$M = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \qquad \begin{aligned} T &= \mathrm{Tr}\, M = m_L + m_R \\ D &= \det M = m_L m_R - m_D^2 \end{aligned}$$

Eigenvalues: $m_\pm = \frac{1}{2}\left(T \pm \sqrt{T^2 - 4D}\right)$

$$\sin 2\theta = \frac{2 m_D}{\sqrt{T^2 - 4D}} \qquad \cos 2\theta = \frac{m_L - m_R}{\sqrt{T^2 - 4D}}$$

$$\begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{bmatrix}$$

$$[v_1, v_2] \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \underbrace{[v_1', v_2']}_{\text{eigenvec.}} \begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

θ is <u>not</u> a "Cabibbo" angle (1 family only!)

"Dirac" case : $M = \begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix}$

(recover Dirac ν)

- **Eigenvectors:**

$$\phi_1 = \frac{1}{\sqrt{2}}\left[(\nu_L + \nu_L^c) + (\nu_R + \nu_R^c)\right] \quad \text{mass } m$$

$$\phi_2 = \frac{1}{\sqrt{2}}\left[-(\nu_L + \nu_L^c) + (\nu_R + \nu_R^c)\right] \quad \text{mass } -m$$

- "Negative mass" not a problem (Majorana phase $= -1$). Define:

$$\tilde{\phi}_2 = \gamma_5 \phi_2 = \frac{1}{\sqrt{2}}\left[(\nu_L - \nu_L^c) + (\nu_R - \nu_R^c)\right]$$

which obeys Dirac eq. with $+m$

note: $\tilde{\phi}_2^c = -\tilde{\phi}_2$

- ϕ_1 and $\tilde{\phi}_2$ have both mass m. Observable (active) component is:

$$\nu_L = P_L \nu = P_L(\nu_L + \nu_R) = P_L \frac{1}{\sqrt{2}}(\phi_1 + \tilde{\phi}_2)$$

$$= \frac{1}{\sqrt{2}}(\phi_1 + \phi_2)_L$$

$\rightarrow$ get a Dirac spinor $\nu \, (\neq \nu^c)$ with mass $m = m(\phi_1) = m(\phi_2)$

"See-saw" case $\mathcal{M} = \begin{bmatrix} 0 & m \\ m & M \end{bmatrix}$

- $\exists\, \nu_R$ in fermion multiplets of many SM extensions; e.g., $\underline{16}$ of SO(10):
  → get a Majorana mass term
  $M(\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c)$
  presumably large mass scale characterizing SM extension

$$\begin{pmatrix} u_L & u_L & u_L & \nu_L \\ d_L & d_L & d_L & e_L \\ u_R & u_R & u_R & \nu_R \\ d_R & d_R & d_R & e_R \end{pmatrix}$$

- Eigenvectors at $O(m/M)$:

$$\phi_1 = (\nu_R + \nu_R^c) + \frac{m}{M}(\nu_L + \nu_L^c) \quad \leftarrow \text{heavy mass } M$$

$$\phi_2 = (\nu_L + \nu_L^c) + \frac{m}{M}(\nu_R + \nu_R^c) \quad \leftarrow \text{negative mass } -m^2/M$$

$$\tilde{\phi}_2 = \gamma_5 \phi_2 = (\nu_L - \nu_L^c) + \frac{m}{M}(\nu_R - \nu_R^c) \quad \leftarrow \text{light mass } +m^2/M$$

- The light state is active $(\nu_L \in \tilde{\phi}_2)$
- The light state mass can be very small (see-saw)

$$m(\phi_2) = \frac{m^2}{M}$$

← Dirac scale (SSB)

← Beyond SM "heavy" scale

Neutrino masses for MORE FAMILIES

1) In the general case (Dirac + Majorana) start from:

- 3 LH gauge doublets $\nu_{\alpha L}$ $\alpha = e\mu\tau$
- n_s RH gauge singlets ν_{sR} $s = 1, 2, \ldots, n_s$

2) Build column of LH fields

$$\nu_L = \begin{pmatrix} \nu_{\alpha L} \\ \nu_{sR}^c \end{pmatrix}$$ dim = $3 + n_s$

3) Write mass term: ... and diagonalize M

$$\mathcal{L}_M = -\frac{1}{2} \overline{\nu_L^c} M \nu_L$$

row of RH fields

column of LH fields

$$M = \begin{bmatrix} M_L & M_D \\ M_D' & M_R \end{bmatrix}$$

$M_L = 3 \times 3$ ← Majorana
$M_D^{(\prime)} = 3 \times n_s$ ← Dirac
$M_R = n_s \times n_s$ ← Majorana

C antisimmetry + anticommutation rules → $\begin{cases} M_L = M_L^T \\ M_R = M_R^T \\ M_D' = M_D^T \end{cases}$

→ $M = M^T$ (symmetric matrix)

Diagonalization in general (Dirac+Majorana) case

→ at least 3 important differences w.r.t. pure Dirac (quark-like) case

1) Eigenvectors ν_k (mass eigenstates) are generally Majorana ⟶ Expect $0\nu2\beta$ decay

2) The LH column $\begin{pmatrix}\nu_{\alpha L}\\ \nu^c_{sR}\end{pmatrix}$ is a linear combination of ν_{kL} ⟶

$$\begin{bmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\\ \nu^c_{1R}\\ \nu^c_{2R}\\ \vdots\\ \nu^c_{n_s R}\end{bmatrix} = U\cdot\begin{bmatrix}\nu_{1L}\\ \nu_{2L}\\ \nu_{3L}\\ \vdots\\ \vdots\\ \vdots\\ \nu_{(3+n_s)L}\end{bmatrix}$$

Conversely, massive states are superpositions of active ν_α and sterile ν_s (active/sterile ν mixing)

3) Since M is symmetric, only one matrix needed for diagonaliz. (not biunitary) → less freedom to reabsorb phases

E.g. for 3 ν generations:

Dirac case ("quark-like") $\quad U \ni \delta_{CP}$

Major. case $\quad U \ni \delta_{CP}, \phi', \phi''$

RECAP

- Neutrino currents well understood and tested

- Neutrino nature (Weyl? Majorana? Dirac?) difficult to explore in practice, due to chirality of interactions and smallness of ν mass.
  However: $m_\nu \neq 0 \rightarrow$ not Weyl; $\exists\ 0\nu 2\beta \rightarrow$ not Dirac

- Neutrino mass terms can be more general than in the quark sector, and point towards new physics in general
  - non standard Higgs sector
  - heavy RH scale
  - active-sterile mixing
  - ...