NuFact Summer Institute Capri 2005

Physics of massive νs

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LECTURE II

Neutrino oscillations

- THEORY -

ν oscillations: general consequence of mixing of flavor states ν_α with massive states ν_β

$$\begin{array}{l} \text{3 active states} \\ \\ \text{sterile states} \end{array} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \\ \vdots \end{pmatrix} = U_{\alpha i} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{pmatrix} \qquad (UU^\dagger = 1)$$

Smallness of ν mass splittings
→ macroscopic oscillation lengths

Smallness of neutrino masses
(w.r.t. to observable energies)

→ Can ignore exceedingly small chirality flips during propagation

→ Can use "Dirac-like" terminology
"ν" $= \nu_L$, "$\bar{\nu}$" $= \nu_R$, even for $\nu_{Major.}$

→ Can often treat ν's as "wavefunctions"
(and use QM-like notation)

Explore propagation Hamiltonians of increasing complexity
(especially in experimentally manageable flavor basis)

$$i\frac{\partial}{\partial t}\nu_\alpha = H_{\alpha\beta}\,\nu_\beta$$

3 massless ν in vacuum

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \qquad m(\nu_\alpha)=0$$

for a beam of momentum p :

$$H = \begin{bmatrix}E_e & & \\ & E_\mu & \\ & & E_\tau\end{bmatrix} = \begin{bmatrix}p & & \\ & p & \\ & & p\end{bmatrix} = p\cdot\mathbb{1}$$

$$|\nu_\alpha\rangle_t = e^{-ipt}|\nu_\alpha\rangle_0$$ ← flavor is conserved

Overall phase $\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \to e^{i\phi}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$ unobservable in squared amplitudes $|\langle\nu_\beta|\nu_\alpha\rangle|^2$

→ H defined mod. $\lambda\mathbb{1}$ in general

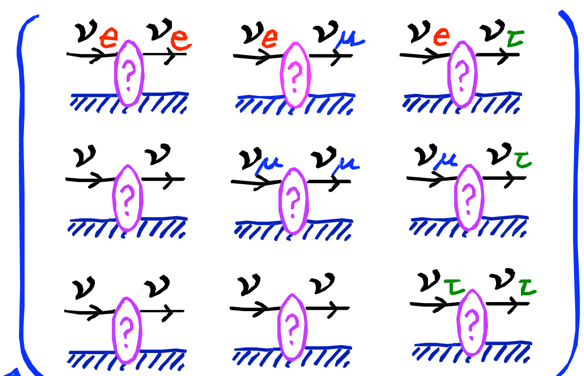
3 massless ν in matter

$$i\frac{\partial}{\partial t}\nu_\alpha = H_{\alpha\beta}\,\nu_\beta \qquad m(\nu_\alpha)=0 \qquad H = \underbrace{p\mathbb{1}}_{\text{kinematics}} + \underbrace{V}_{\text{dynamics}}$$

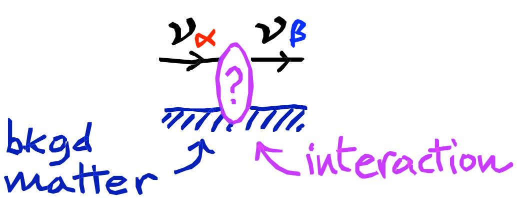
V = interaction energy in matter (Wolfenstein)

$$p\mathbb{1} = \begin{bmatrix} \xrightarrow{\nu_e} & 0 & 0 \\ 0 & \xrightarrow{\nu_\mu} & 0 \\ 0 & 0 & \xrightarrow{\nu_\tau} \end{bmatrix}$$

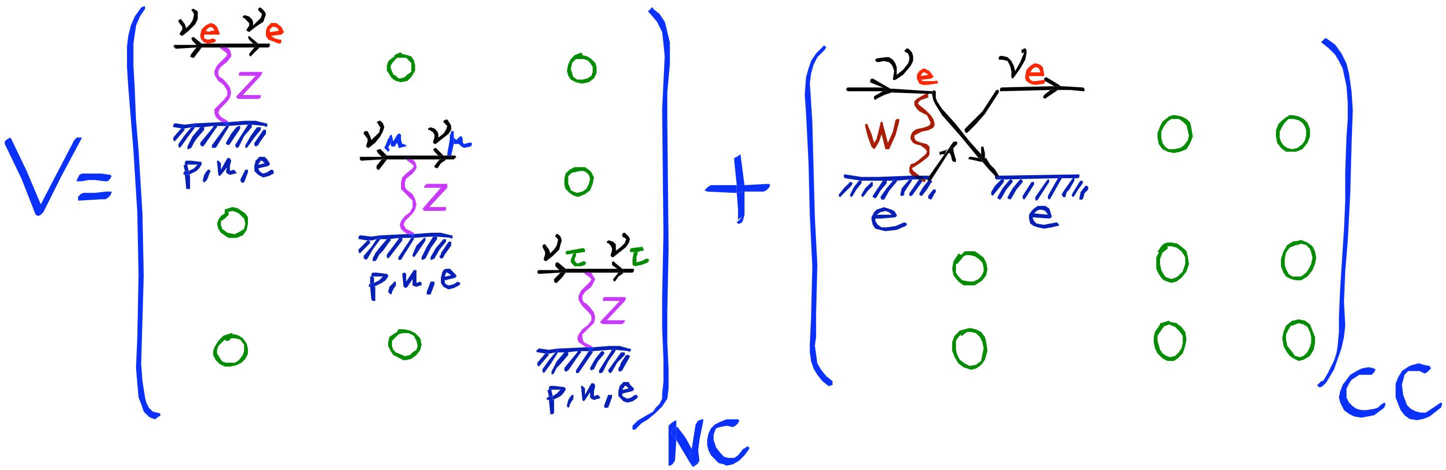
ν free streaming

$V =$ 

dynamical contribution to forward scattering

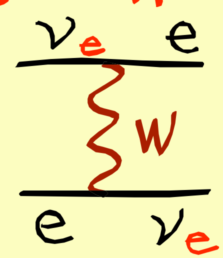


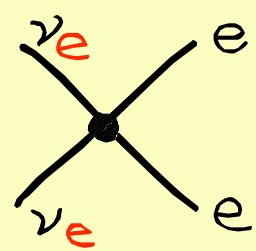
Interaction "blob" well defined in standard EW model $\longrightarrow$



$V = V_{NC} + V_{CC}$ with $V_{NC} \propto \mathbb{1}$
(up to small higher-order corrections)

→ Relevant term is the interaction energy difference V_{cc}

$V_{cc}^{ee} =$ 

$\simeq$ 

Evaluation of V_{CC}^{ee}

$$H_{CC} = \frac{G_F}{\sqrt{2}} \underbrace{\bar{e}\gamma^\mu(1-\gamma_5)\nu_e}_{J^{CC}} \otimes \underbrace{\bar{\nu}_e\gamma_\mu(1-\gamma_5)e}_{J^{CC}} \overset{\text{Fierz}}{=\!=} \frac{G_F}{\sqrt{2}} \underbrace{\bar{e}\gamma^\mu(1-\gamma_5)e}_{J_e} \underbrace{\bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e}_{J_\nu}$$

From the ν viewpoint, the e^- is ~nonrelativistic and ~unpolarized

→ Dirac representation, $e \simeq \begin{bmatrix}\xi \\ 0\end{bmatrix}$

$$\bar{e}\gamma^\mu(1-\gamma_5)e \simeq (\underbrace{\xi^+\xi}_{\substack{\text{density} \\ N_e}}, \underbrace{\xi^+\vec{\sigma}\xi}_{\substack{\text{polarization} \\ \sim 0}}) \simeq N_e \delta_{\mu 0}$$

$$H_{CC} = \frac{G_F}{\sqrt{2}} N_e \bar{\nu}_e \gamma_0 (1-\gamma_5)\nu_e = \underbrace{\sqrt{2}G_F N_e}_{\text{coupling}} \underbrace{\bar{\nu}_{eL}\gamma_0\nu_{eL}}_{\text{"static" term}}$$

$$\boxed{V_{CC}^{ee} = \sqrt{2}\, G_F N_e}$$

Exercise : prove that

$$\frac{A}{eV^2} = 1.526 \times 10^{-7} \left(\frac{N_e}{mol/cm^3}\right)\left(\frac{E}{MeV}\right)$$

where $A = 2EV = 2\sqrt{2}\, G_F N_e E$

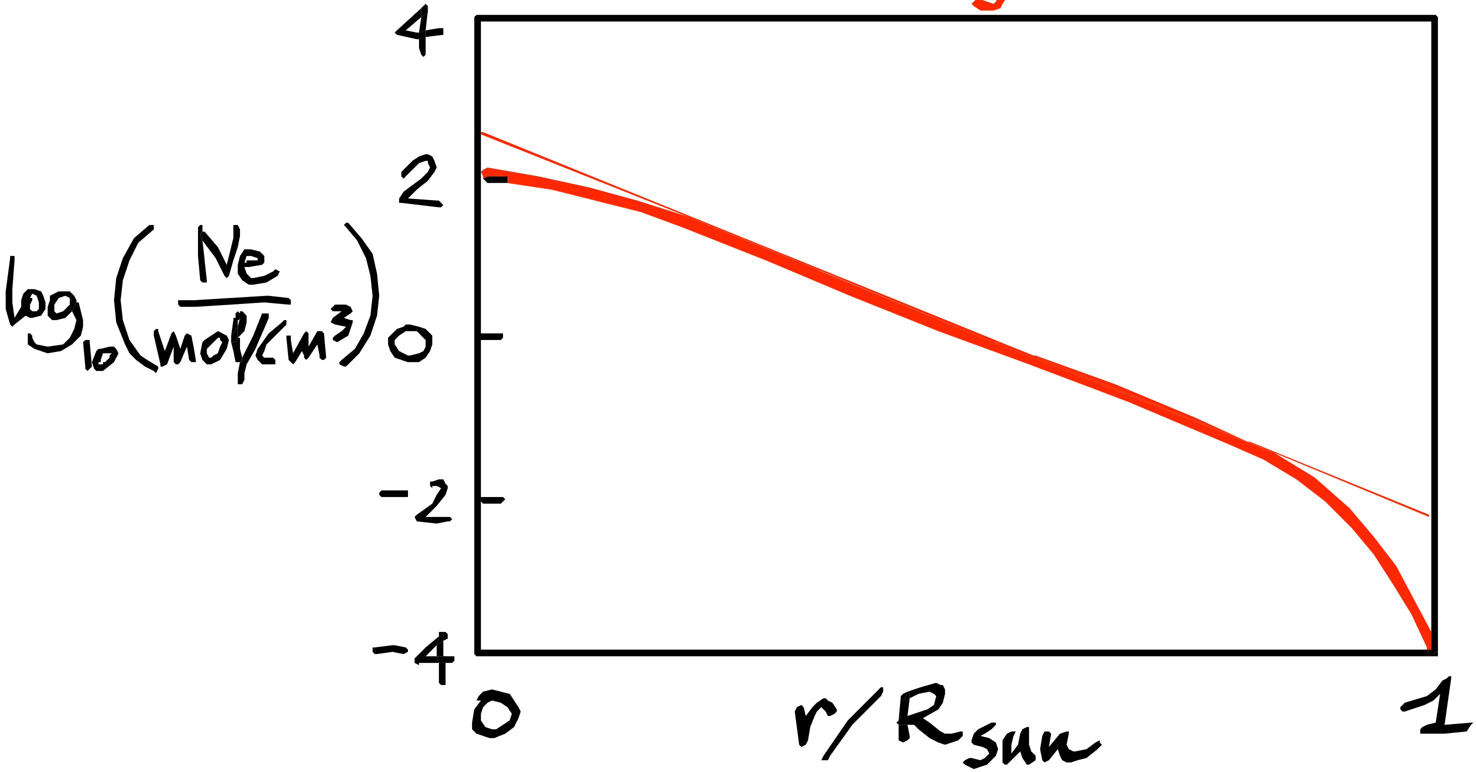
hint : remember that

$G_F = 1.16637 \times 10^{-5}\ GeV^{-2}$

and use (see tutorials)

$1\ \frac{mol}{cm^3} = 4.627 \times 10^{-9}\ MeV^3$

electron density in the Sun



Exp. approximation:

$N_e \simeq N_e(0)\, e^{-x/r_0}$

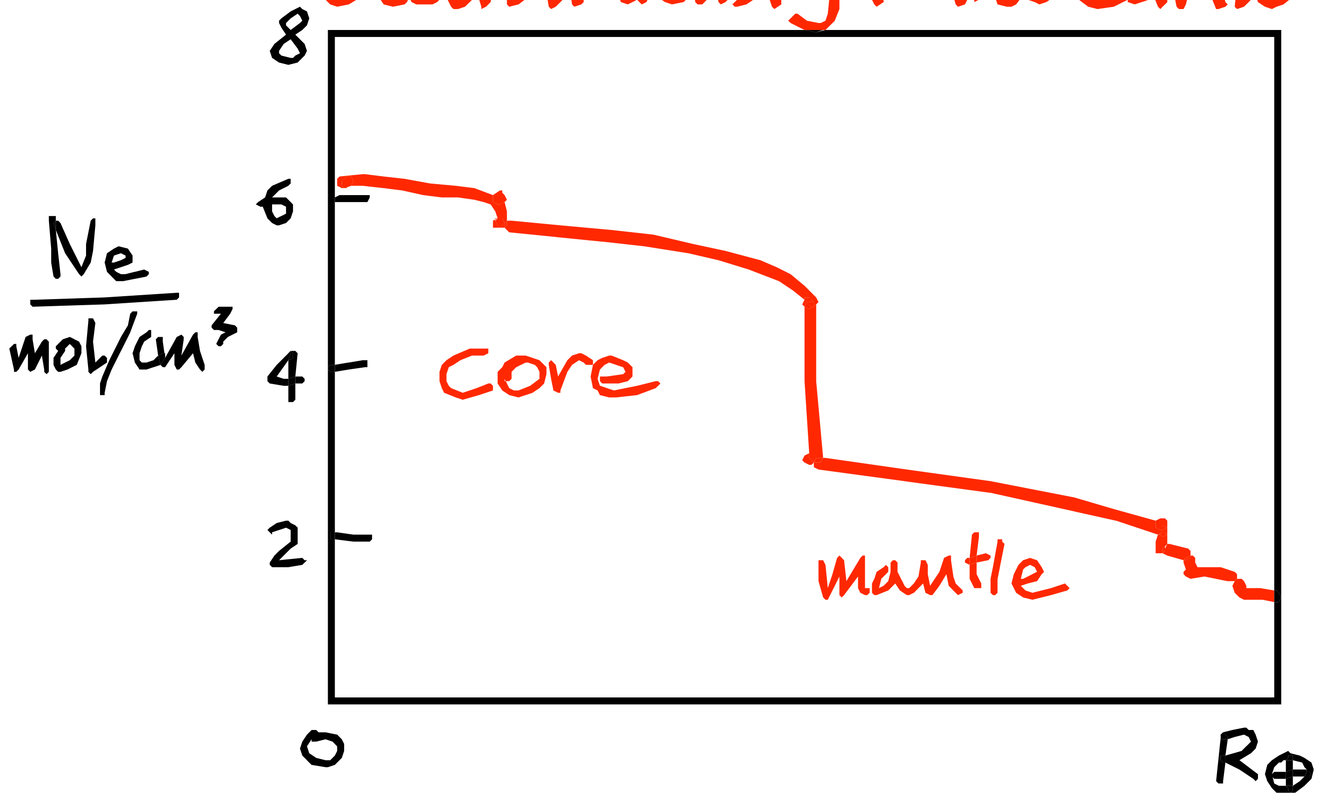
$N_e(0) \simeq 245$ mol/cm^3

$r_0 \simeq R_\odot/10.54$

But in true SSM:

$N_e(0) \simeq 100$ mol/cm^3

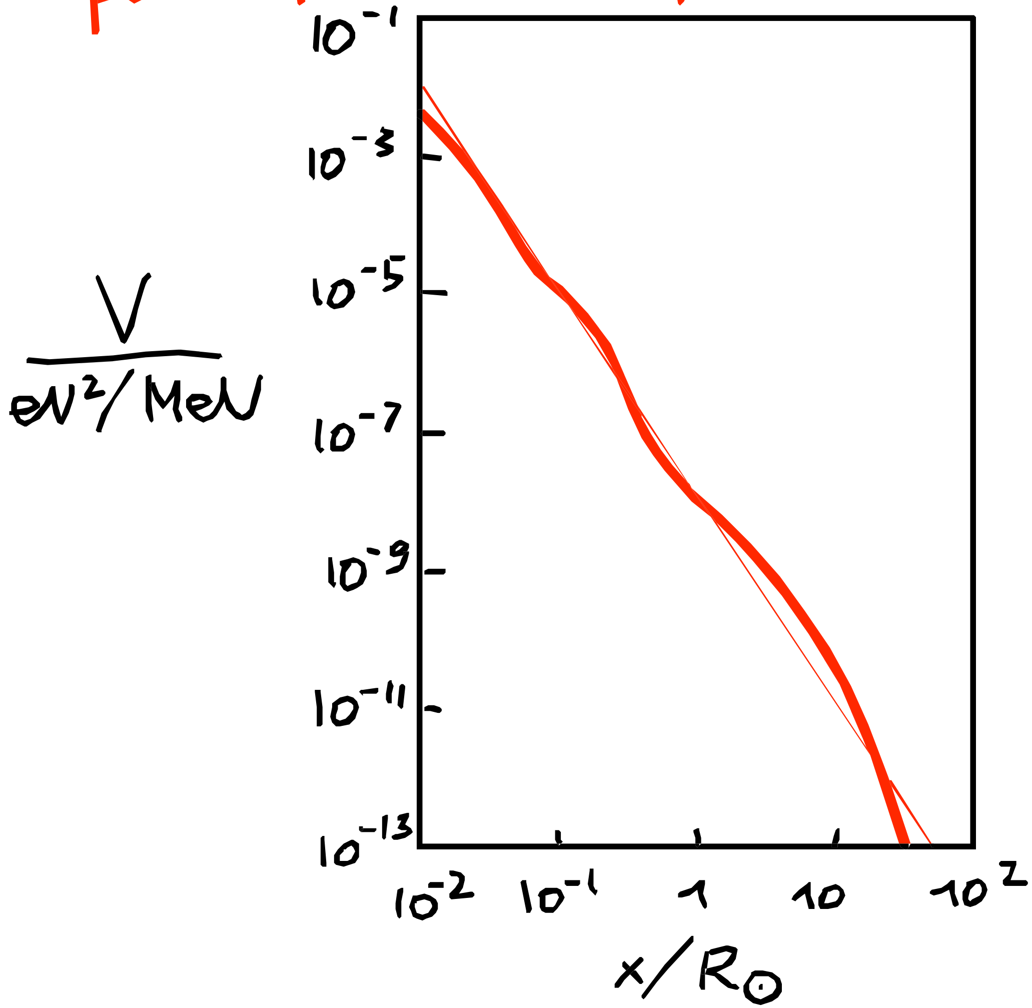
Electron density in the Earth



mantle : $N_e \sim 2 \div 3$ mol/cm^3

core : $N_e \sim 5 \div 6$ mol/cm^3

ν potential in a Supernova



$V \propto x^{-3}$

More on standard EW interaction energies

ν type	bkgd matter	Interaction energy V
ν_e	e	$\frac{1}{\sqrt{2}} G_F (4s_w^2+1)(N_e - N_{\bar{e}})$
$\nu_{\mu,\tau}$	e	$\frac{1}{\sqrt{2}} G_F (4s_w^2-1)(N_e - N_{\bar{e}})$
$\nu_{e,\mu,\tau}$	n	$\frac{1}{\sqrt{2}} G_F (N_{\bar{n}} - N_n)$
$\nu_{e,\mu,\tau}$	p	$\frac{1}{\sqrt{2}} G_F (1-4s_w^2)$
ν_s	e, p, n	0

for $\nu \to \bar{\nu}$:
$V \to -V$

In ordinary matter: $N_e = N_p$, $N_{\bar{e}} = N_{\bar{p}} = N_{\bar{n}} = 0$

$$V_e - V_{\mu,\tau} = \sqrt{2} G_F N_e$$ } as before

$$V_\mu - V_\tau = 0$$ } vacuum-like

$$V_s - V_{\mu,\tau} = \sqrt{2} G_F \frac{N_n}{2}$$

$$V_s - V_e = \sqrt{2} G_F \left(N_e - \frac{1}{2} N_n\right)$$

} relevant for sterile ν phenomenology

Back to 3 massless ν in matter

Standard EW inter.
+ ordinary matter $\rightarrow$ $H = \begin{pmatrix} P+V_{CC} & & \\ & P & \\ & & P \end{pmatrix}$

$\rightarrow$ no off-diagonal elements in flavor basis
$\rightarrow$ flavor is conserved

However, flavor changing neutral currents may arise in theories beyond the standard model:

$$V_{FCNC} = \begin{pmatrix} 0 & \nu_e \, \nu_\mu \, (\text{bkgd}) & \nu_e \, \nu_\tau \, (\text{bkgd}) \\ & 0 & \nu_\mu \, \nu_\tau \, (\text{bkgd}) \\ & & 0 \end{pmatrix} \propto \epsilon_{\alpha\beta} G_F N_f$$

(e.g. SUSY with R-parity breaking, violations or equivalence principle ...)

In such cases, flavor transitions could take place even for massles ν

3 massive ν in vacuum, no mixing

Assume $m(\nu_\alpha) = \delta_{\alpha i}\, m_i$ $(U \equiv 1)$; then, for ultrarelativistic ν:

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$

$$H = \begin{bmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{bmatrix} \simeq \begin{bmatrix} p & & \\ & p & \\ & & p \end{bmatrix} + \frac{1}{2E}\begin{bmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{bmatrix}$$

$$= p\,\mathbb{1} + \frac{\mathcal{M}^2}{2E}$$

$$\mathcal{M}^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$$

→ H diagonal in flavor (= mass) basis

→ no flavor transitions

3 massive ν in vacuum, with mixing

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$UU^\dagger = \mathbb{1}$

$m(\nu_i) = m_i$

Hamiltonian diagonal in mass basis:

$$\mathcal{H}_{mass} = \frac{\mathcal{M}^2}{2E} + p\mathbb{1}$$

$\mathcal{M}^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$

... but not diagonal in flavor basis

$$\mathcal{H}_{flav.} = \underbrace{U\frac{\mathcal{M}^2}{2E}U^\dagger}_{\text{off-diag}} + \underbrace{p\mathbb{1}}_{\text{diag}}$$

If no ~~CP~~, U real; usual parametrization: ($\theta_{ij} \in [0, \pi/2]$)

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}\\ 0 & 1 & 0\\ -s_{13} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

If ~~CP~~, and mass terms are Dirac, one phase (quark-like):

$$\begin{pmatrix}c_{13} & 0 & s_{13}\\ 0 & 1 & 0\\ -s_{13} & 0 & c_{13}\end{pmatrix} \rightarrow \begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}$$

However, if $\cancel{CP}$, and mass terms are Majorana (or Dirac-Majorana):

$$U \to U U_M \quad , \quad U_M = \begin{pmatrix} 1 & & \\ & e^{i\phi'} & \\ & & e^{i\phi''} \end{pmatrix}$$

Majorana phases

... but: no effect on oscillations

$$U U_M \frac{\mathcal{M}^2}{2E} (U U_M)^+ = U \frac{U_M \mathcal{M}^2 U_M^+}{2E} U^+ = U \frac{\mathcal{M}^2}{2E} U^+$$

→ Oscillations do not distinguish Dirac vs Majorana neutrinos

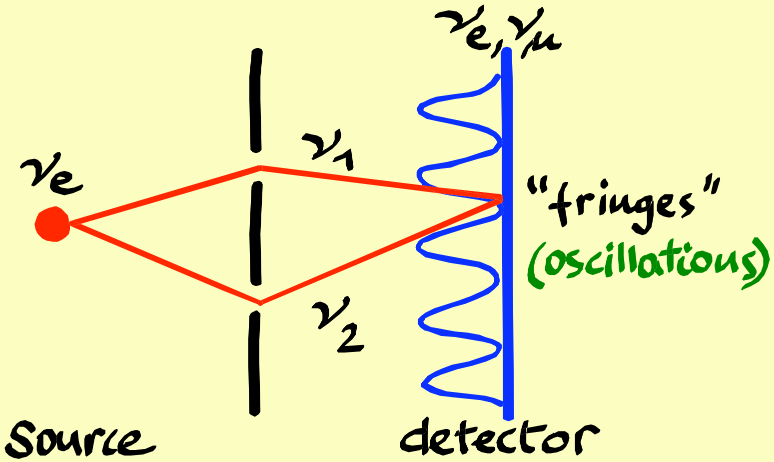
2ν oscillations in vacuum

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \;;\quad \Delta m^2 = m_2^2 - m_1^2$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

see tutorials

Analogy with 2-slit expt.:



Length scales:

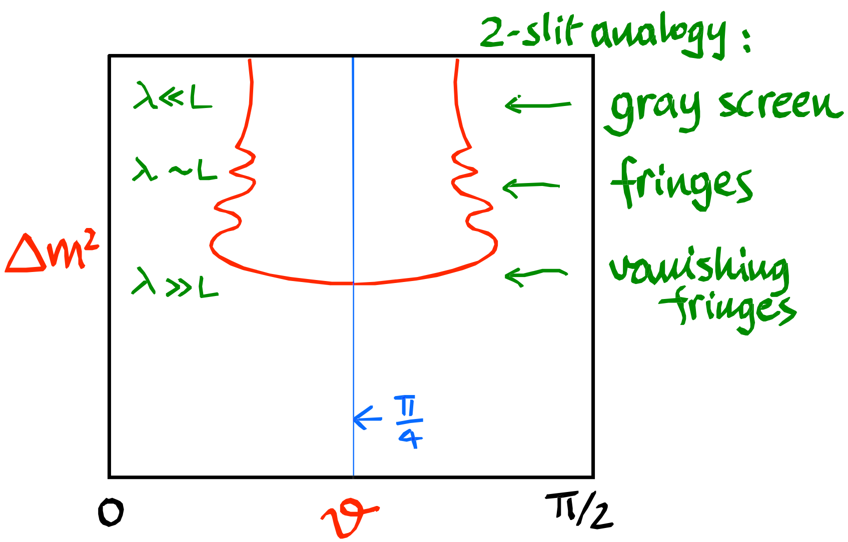
L = baseline

$\lambda = \dfrac{4\pi E}{\Delta m^2}$ = osc. length

Fringes may not be visible for
$\lambda \ll L$ ("fast oscillations) or
large expt. smearing ($\Delta E/E$ etc.)

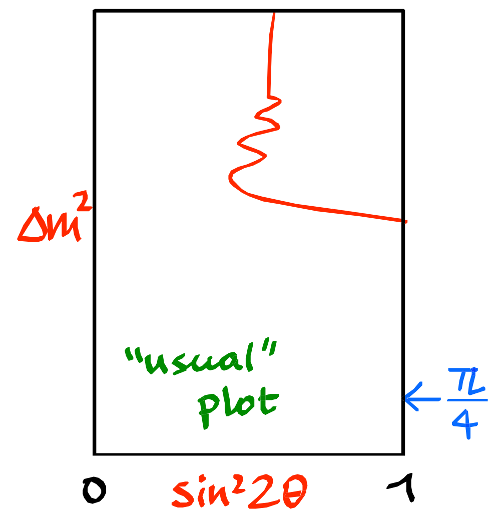
$$\rightarrow \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle \sim \frac{1}{2}$$

Typical iso-$\langle P_{e\mu} \rangle$ contours



Octant symmetry: $\theta \to \frac{\pi}{2} - \theta$ in $P_{e\mu}$

If 2nd octant folded onto the 1st one:



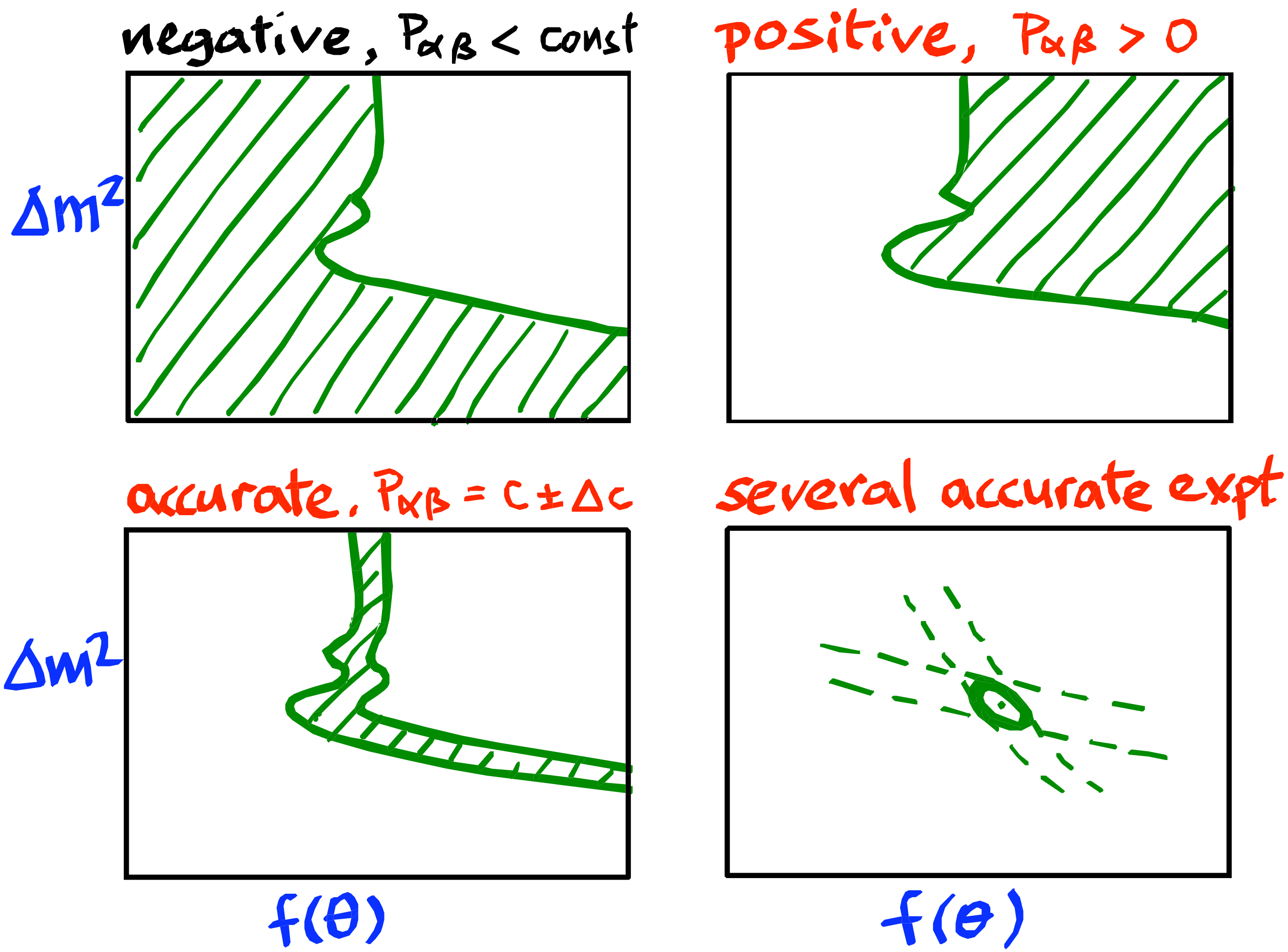
Basically obsolete

$(\Delta m^2, \sin^2 2\theta)$ plot still used for pure 2ν $\nu_\mu \to \nu_\tau$ oscillations (they are vacuum-like even in matter)

In general, better to use $\log \tan^2\theta$ or $\sin^2\theta$ (preserve octant-symmetry)

Typical experimental results

allowed



2ν oscill. in constant-density matter

$$P_{e\mu} = \sin^2 2\tilde{\theta}\, \sin^2\left(\frac{\Delta\tilde{m}^2 L}{4E}\right)$$ (tutorial)

$$\sin 2\tilde{\theta} = \frac{\sin 2\theta}{\sqrt{\left(\cos 2\theta - \frac{A}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$$

$$\frac{\Delta\tilde{m}^2}{\Delta m^2} = \frac{\sin 2\theta}{\sin 2\tilde{\theta}}$$

"Breit-Wigner" resonance form

Can get a MSW resonant behavior for $c_{2\theta} \sim A/\Delta m^2$

$\rightarrow \Delta m^2 c_{2\theta} = 2\sqrt{2}\, G_F N_e E$

$\rightarrow \sin^2 2\tilde{\theta} \sim 1$ (enhanc.)

$\rightarrow \Delta\tilde{m}^2$ minimized

Can get suppression for $A \gg \Delta m^2 \rightarrow \sin^2 2\tilde{\theta} \sim 0$

Matter can profoundly modify osc. amplitude (enhancement - suppression) and its energy dependence. New length scale $\tilde{\lambda} = \frac{\sqrt{2}\,\pi}{G_F N_e}$

(important effects for $\lambda \sim \tilde{\lambda}$)

Note: MSW = Mikheyev-Smirnov-Wolfenstein

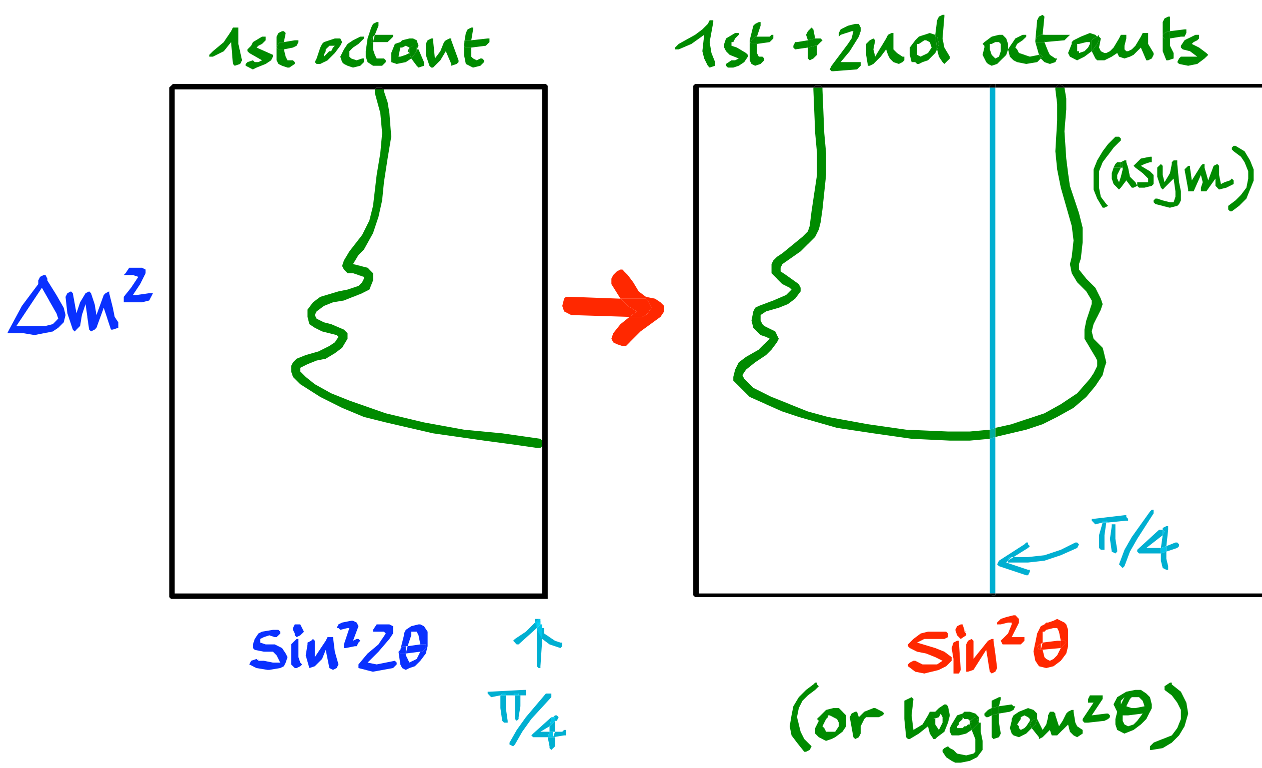
For $\bar{\nu}$: $A \rightarrow -A$ (no MSW resonance)

Matter effects are not octant-symmetric

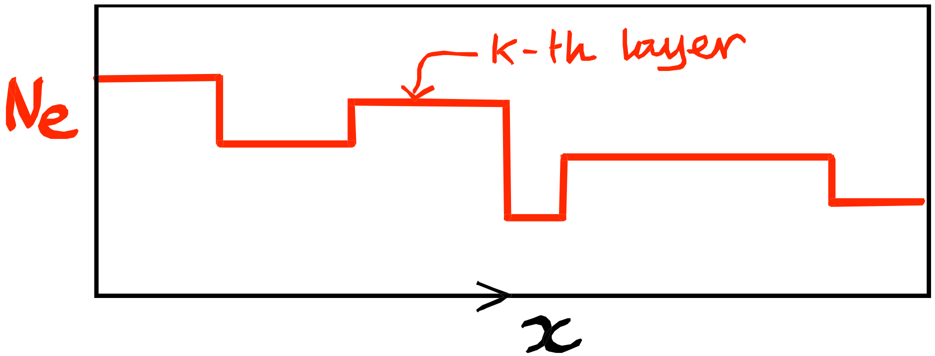
$$Q(\theta) \neq Q(\tfrac{\pi}{2}-\theta)$$

where $Q = \Delta\tilde{m}^2, \tilde{\theta}, P_{e\mu} \cdots$

→ must unfold second octant



2ν oscillations in layered matter



Approx. valid in Earth:
- Mantle + core
- Inhomogeneous crust

Tutorial: evolution operator S in 1 layer

For N layers:
$$S = S_N S_{N-1} \cdot \ldots \cdot S_k \cdot \ldots \cdot S_3 S_2 S_1$$
(time-ordered product)

$$P_{\alpha\beta} = |S_{\alpha\beta}|^2$$
can be somewhat complicated

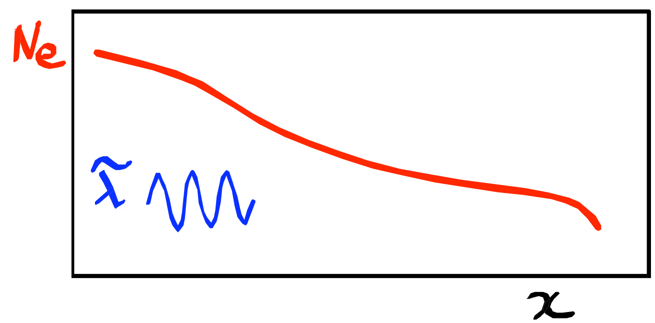
Enhancement conditions for $P_{\alpha\beta}$ contain
(but do not reduce to) MSW-reson. conditions
→ Further conditions arise for constructive interference

2ν oscillations in variable density

Solution requires, in general, numerical evolution

But: Analytical approximations exist in several cases of phenomenological interest

We'll start with the case of adiabatic evolution i.e., of "slowly varying density"



many cycles while N_e changes

We'll consider then nonadiabatic corrections to the adiabatic evolution

Note: adiabatic evolution relevant for the LMA solution to the solar ν deficit.
Nonadiabatic corrections relevant in other contexts (e.g., supernova ν)

Adiabatic evolution

At each point x :

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\tilde\theta(x) & \sin\tilde\theta(x)\\ -\sin\tilde\theta(x) & \cos\tilde\theta(x)\end{pmatrix}\begin{pmatrix}\tilde\nu_1(x)\\ \tilde\nu_2(x)\end{pmatrix}$$

with $P(\tilde\nu_1 \to \tilde\nu_2)$ "no crossing"

Typically, $\tilde\lambda \ll L \to$ phase information lost

$\to$ can propagate "probabilities" (rather than amplitudes)

$$P(\nu_e \to \nu_e) = \underset{\substack{\uparrow\\ \text{final } \nu_e}}{(1,0)} \underset{\substack{\uparrow\\ \text{rotate back}\\ \text{at } x=x_f}}{\begin{pmatrix}\cos^2\tilde\theta_f & \sin^2\tilde\theta_f\\ \sin^2\tilde\theta_f & \cos^2\tilde\theta_f\end{pmatrix}} \underset{\substack{\uparrow\\ \text{no}\\ \text{crossing}}}{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}} \underset{\substack{\uparrow\\ \text{rotate at}\\ x=x_i \text{ to}\\ \tilde\nu_{1,2} \text{ basis}}}{\begin{pmatrix}\cos^2\tilde\theta_i & \sin^2\tilde\theta_i\\ \sin^2\tilde\theta_i & \cos^2\tilde\theta_i\end{pmatrix}} \underset{\substack{\uparrow\\ \text{initial}\\ \nu_e}}{\begin{pmatrix}1\\ 0\end{pmatrix}}$$

from right to left:

$$\boxed{P_{ee} = \frac{1}{2}\left(1 + \cos 2\tilde\theta_i \cos 2\tilde\theta_f\right)}$$

For solar neutrinos: $\tilde\theta_f = \theta$ (vacuum), up to Earth matter effects

$$P_{ee}^{\odot} = \frac{1}{2}\left(1 + \cos 2\underset{\substack{\uparrow\\ \text{production point}}}{\tilde\theta(x)} \cos 2\theta\right)$$

Nonadiabatic corrections

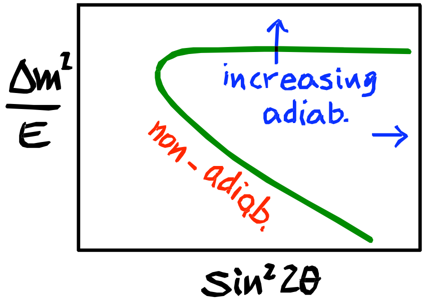
In $(\tilde{\nu}_1, \tilde{\nu}_2)$ basis: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1-P_c & P_c \\ P_c & 1-P_c \end{pmatrix}$

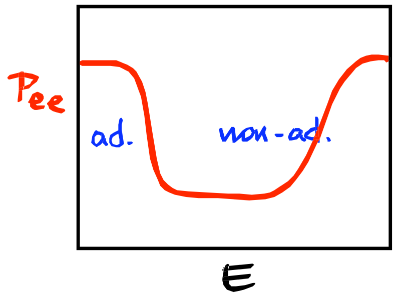
P_c = crossing prob.
$\tilde{\nu}_1 \to \tilde{\nu}_2$ "tunnelling"

$$P_{ee} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right) \cos 2\tilde{\theta}_i \cos 2\tilde{\theta}_f$$

enormous literature on P_c evaluation

Historically relevant in solar ν solutions:

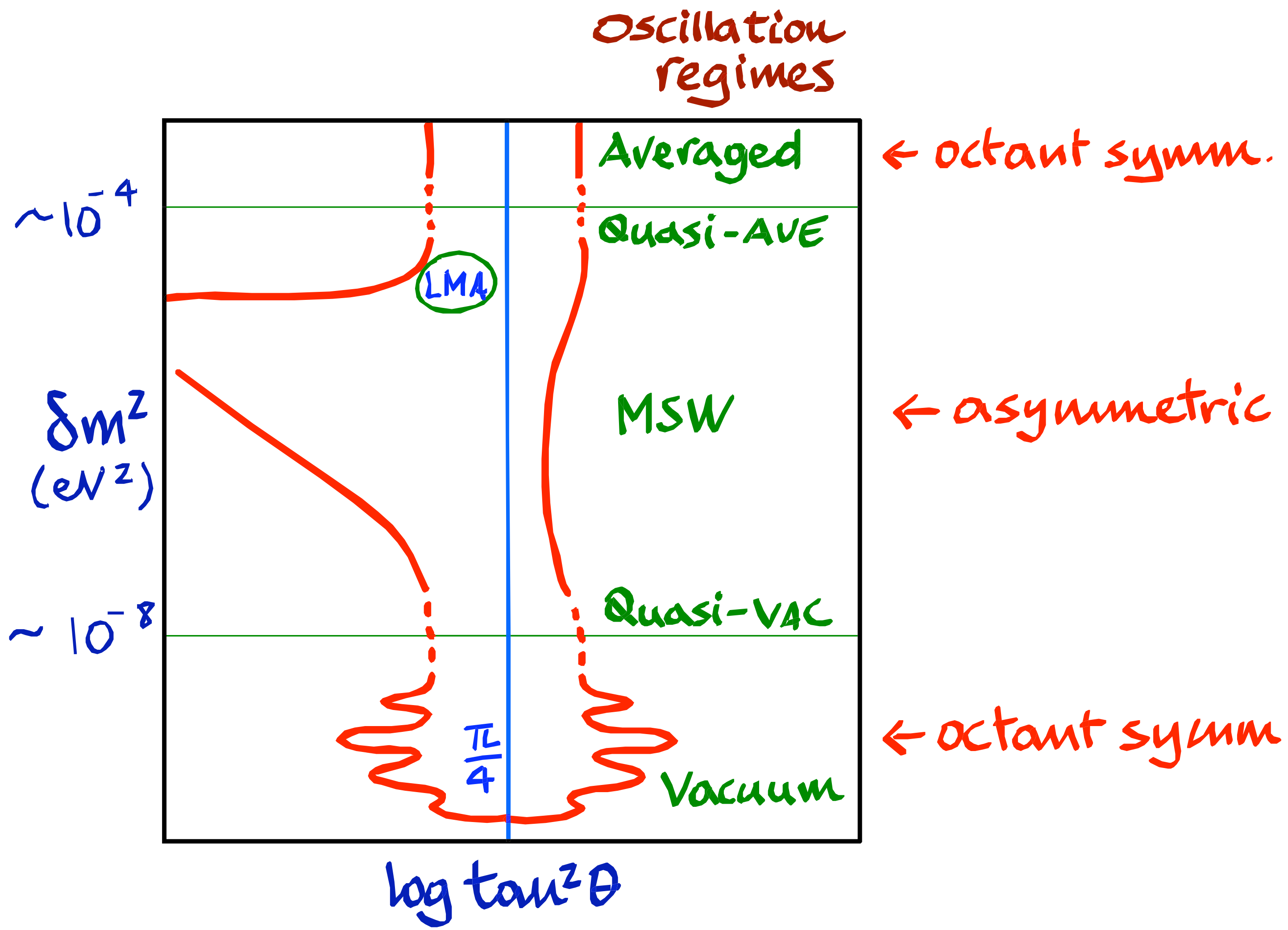




MSW "triangle", zone of small P_{ee}

Strong difference from vacuum case

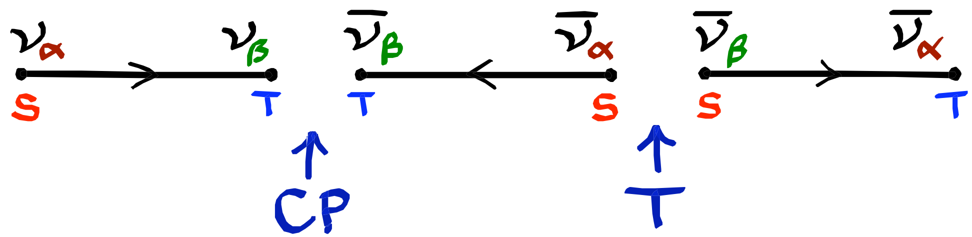
Solar P_{ee} suppression: param. space



3ν : CP violation

Requires $n \geq 3$ neutrinos;

S = source
T = target



CPT : $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$

if CP: $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$

if $\cancel{CP}$: $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$

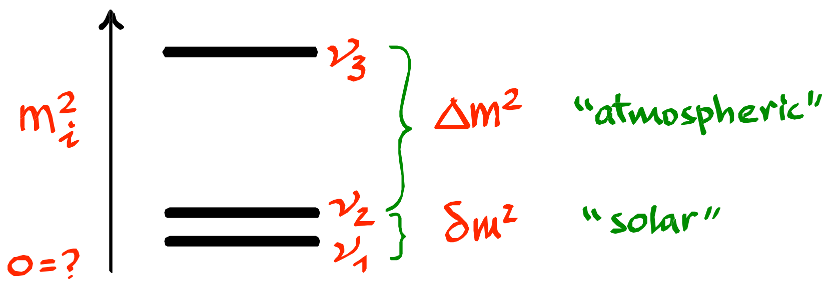
In vacuum: (See tutorials)

$$\Delta P_{\cancel{CP}} \propto \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \cdot \sin\delta \times \sin\left(\frac{\Delta m^2_{12} L}{4E}\right) \cdot \sin\left(\frac{\Delta m^2_{23} L}{4E}\right) \cdot \sin\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\Delta P_{\cancel{CP}} \neq 0 \text{ only if } \begin{cases} \sin\delta \neq 0 \\ \text{all } \theta_{ij} \neq 0 \\ \text{all } \Delta m^2_{ij} \neq 0 \end{cases}$$

3ν : one dominant mass scale

SQUARED MASS SPECTRUM :



(or up-side-down)

$\delta m^2 \ll \Delta m^2$

$$\mathcal{M}^2 \simeq \begin{pmatrix} 0 & & \\ & \delta m^2 & \\ & & \Delta m^2 \end{pmatrix} \quad \text{mod } \mathbb{1}$$

From the viewpoint of atmospheric ν:

$$\mathcal{M}^2 \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & \Delta m^2 \end{pmatrix}$$

$\rightarrow$ ~~CP~~ unobservable

From the viewpoint of solar ν:

$$\mathcal{M}^2 \sim \begin{pmatrix} 0 & & \\ & \delta m^2 & \\ & & \infty \end{pmatrix}$$

$\rightarrow$ ~~CP~~ unobservable

Atmospheric ν, o.d.m.s.

$\mathcal{M}^2 \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & \Delta m^2 \end{pmatrix}$, no CP ($U = U^*$), imply in vacuum

$$P_{\alpha\alpha} = 1 - 4U_{\alpha 3}^2(1 - U_{\alpha 3}^2)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{\alpha\beta} = 4U_{\alpha 3}^2 U_{\beta 3}^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$\rightarrow$ parameter space $(\Delta m^2, U_{e3}^2, U_{\mu 3}^2, U_{\tau 3}^2)$

$= (\Delta m^2, \sin^2\theta_{23}, \sin^2\theta_{13})$

Corrections to above approx. from:
- matter effects
- $\delta m^2 > 0$
- CP violation
- $\pm \Delta m^2$ (hierarchy)

Solar ν, o.d.m.s. approximation

$\mathcal{M}^2 \sim \begin{pmatrix} 0 & & \\ & \delta m^2 & \\ & & \infty \end{pmatrix}$ imply, in vacuum:

$$P_{ee} = 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\left(\frac{\delta m^2 L}{4E}\right) - 4U_{e1}^2 U_{e3}^2 \sin^2(\infty) - 4U_{e2}^2 U_{e3}^2 \sin^2(\infty)$$

$\leftarrow \sin^2(\infty) \sim \frac{1}{2}$

$$= (1 - U_{e3}^2)^2 - 4U_{e1}^2 U_{e2}^2 \sin^2\left(\frac{\delta m^2 L}{4E}\right) + U_{e3}^4$$

$$= \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\delta m^2 L}{4E}\right)\right] + \sin^4\theta_{13}$$

$\rightarrow$ parameter space $(\delta m^2, U_{e1}^2, U_{e2}^2, U_{e3}^2)$

$= (\delta m^2, \sin^2\theta_{12}, \sin^2\theta_{13})$

Structure $P_{3\nu} = c_{13}^4 P_{2\nu} + s_{13}^4$
preserved in matter (with $V \to c_{13}^2 V$)
Parameter space unaltered,
negligible corrections from $\Delta m^2 < \infty$

3ν: more precise definitions and conventions

Previous notation somewhat "sloppy":

- ν = field (QF operator),
  state (QM ket),
  component (number) ?
- U, U^* ?
- $\Delta m^2 = m_3^2 - m_1^2$ or $m_3^2 - m_1^2$
- $\Delta m^2 \gtrless 0$, and δm^2 ?
- CP phases ?

But: Important to be precise, e.g., in prospective NuFact studies

Consistent use of U & U*

Fields: $\nu_{\alpha L} = \sum_{i=1}^{3} U_{\alpha i}\, \nu_{iL}$

(quantized, in the CC Lagrangian)

particle created by $\psi^{+}|0\rangle$ or $\overline{\psi}|0\rangle$

$\downarrow$

States: $|\nu_\alpha\rangle = \sum_i U^{*}_{\alpha i} |\nu_i\rangle$

(one-particle kets)

$\downarrow$

Components: if $|\nu\rangle = \sum_i \nu^i |\nu_i\rangle$
$= \sum_\alpha \nu^\alpha |\nu_\alpha\rangle$

then

$$\nu^\alpha = \sum_i U_{\alpha i}\, \nu^i$$

(components = numbers)

e.g. $|\nu_e\rangle$ components: $\nu^e = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
in flavor basis

$$U = O_{23}\,\Gamma_\delta\, O_{13}\,\Gamma_\delta^{+}\, O_{12}$$

$$O_{ij} \ni \begin{pmatrix} c_{ij} & s_{ij} \\ -s_{ij} & c_{ij} \end{pmatrix} \text{ at } (ij) \; ; \quad \Gamma_\delta = \mathrm{diag}(1,1,e^{i\delta})$$

Note: no ~~CP~~ $\rightarrow U = U^*$

Satisfied for $\delta = 0$ AND $\delta = \pi$: $\Gamma_\delta = (1, 1, \pm 1)$

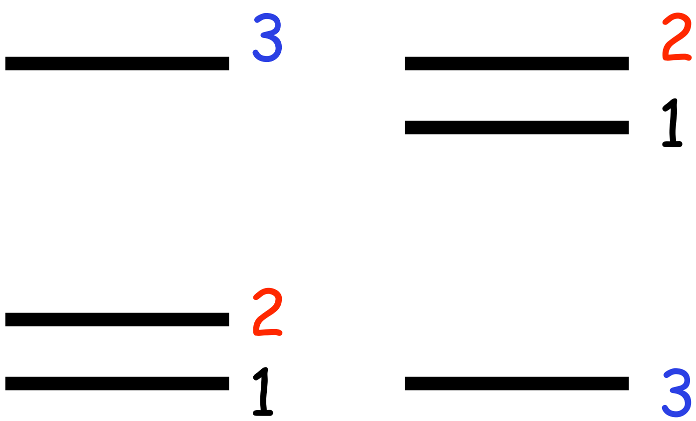
$\delta = 0$: $\Gamma_\delta\, O_{13}\, \Gamma_\delta^{+} = O_{13}$ $\quad \cos\delta = +1$

$\delta = \pi$: $\Gamma_\delta\, O_{13}\, \Gamma_\delta^{+} = O_{13}\big|_{s_{13} \to -s_{13}}$ $\quad \cos\delta = -1$

$\rightarrow$ Two CP-conserving cases

Masses: labels and splittings

Consensus labels: doublet=(ν_1, ν_2), with ν_2 heaviest in both hierarchies



$$\delta m^2 = m_2^2 - m_1^2 > 0$$

Sign of smallest splitting: conventional.
The relative ν_e content of ν_1 and ν_2 is instead physical (given by MSW effect)

$$\text{Note}: \quad |m_3^2 - m_1^2| = \begin{cases} \text{largest splitting (N.H.)} \\ \text{next–to–largest splitting (I.H.)} \end{cases}$$

$\Rightarrow \Delta m^2_{31}$ (or Δm^2_{32}) change physical meaning from NH to IH

We prefer to define the 2nd independent splitting as:

$$\Delta m^2 = \left|\frac{\Delta m^2_{31} + \Delta m^2_{32}}{2}\right| = \left|m^2_3 - \frac{m^2_1 + m^2_2}{2}\right|$$

so that the largest and next-to-largest splittings, in both NH & IH, are given by: $\Delta m^2 \pm \dfrac{\delta m^2}{2}$

and only one physical sign distinguishes NH (+) from IH (-), as it should be:

$$(m^2_1, m^2_2, m^2_3) = \frac{m^2_2 + m^2_1}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

sign($\pm\Delta m^2$) can be determined - in principle - by interference of Δm^2-driven oscillations with some Q-driven oscillations, provided that sign(Q) is known. Two ways (barring exotics):

$Q = A(x)$ $= 2\sqrt{2} G_F N_e(x) E$ **(only in matter & for $s_{13}>0$)**

$Q = \delta m^2$ **(in any case, but hard !)**

Weak sensitivity with current data; challenge for future expts.

Majorana phases

$$U \to U \cdot U_M$$

Useful (not unique) convention:

$$U_M = \mathrm{diag}\left(1, e^{\frac{i}{2}\phi_2}, e^{\frac{i}{2}(\phi_3 + 2\delta)}\right)$$

includes U_M

$$\to m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right|$$

$$= \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right|$$

does not contain δ explicitly

Besides $m_{\beta\beta}$, two relevant observables sensitive to absolute ν masses:

$$m_\beta = \left[\sum_i |U_{ei}^2| m_i^2\right]^{1/2}$$
$$= \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{1/2}$$

(β-decay)

$$\Sigma = m_1 + m_2 + m_3$$

(cosmology)

In the next lecture, we shall see how
the mass and mixing parameters are probed
by oscillation and non-oscillation experiments