Massive neutrinos

We do not know...[if] neutrinos are massive or massless. We do not know if potentially massive neutrinos are Majorana or Dirac, and we do not know if these neutrinos can oscillate among flavours... In short, there is a great deal we do *not* know about neutrinos.

Jeremy Bernstein, 1984

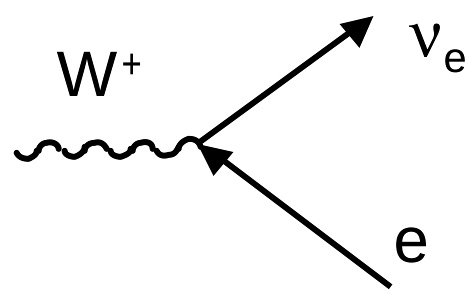
ν in the Standard Model

SM has three weak isodoublets
and
three charged lepton isosinglets

$$\begin{pmatrix} n_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} n_m \\ m \end{pmatrix}_L \quad \begin{pmatrix} n_t \\ t \end{pmatrix}_L$$

$$e_R \quad m_R \quad t_R$$

The active neutrino flavours (ν_e, ν_μ, ν_τ) are defined by the associated leptons in CC processes



The neutrino mass puzzle

In the SM the neutrino mass is put to zero arbitrarily.
It is not like in the photon case where the gauge invariance
$\Rightarrow$ massless photon

Is the neutrino massless or massive?

Charged leptons are Dirac particles, what about neutrinos?

Is the neutrino a Dirac or a Majorana Particle?

Massive neutrinos in the SM(I)

In the SM the electron mass is obtained as

$$m_e = g_e \frac{v}{\sqrt{2}}$$

where g_e is an ad hoc constant and v is the vacuum expectation value for the Higgs

For the neutrino we could use the same approach

$$m_n = g_n \frac{v}{\sqrt{2}}$$

BUT

Massive neutrinos in the SM(II)

Experimental evidence: the neutrino mass is very small!

Being ν the same for all the leptons $\Rightarrow$ $g_\nu \ll g_e$ ($g_e > 10^4 g_\nu$)

The troublesome feature of this approach is that does not explain why the relative couplings are so different

An alternative approach is to consider Majorana mass terms as well as Dirac mass terms

Dirac or Majorana?

- ✓ If ν is Dirac particle $\Rightarrow$ $\nu \neq$ anti-ν (lepton number is conserved)
- ✓ If ν is Majorana particle $\Rightarrow$ ν=anti-ν (lepton number is violated)
- ✓ If the weak interaction is left-handed a massless Majorana neutrino cannot be distinguished from a massless Dirac neutrino!
- ✓ For massive neutrinos it is possible to distinguish Majorana from Dirac (i.e. neutrinoless double β decay)

➢ Interesting physics

The mass lagrangian(I)

The most general lagrangian mass (for a single flavour) can be written as

$$\mathrm{L} = \overbrace{m_D\left[\left(\bar{n}_L + \bar{n}_L^c\right)\left(n_R + n_R^c\right)\right] + m_D\left[\left(\bar{n}_R + \bar{n}_R^c\right)\left(n_L + n_L^c\right)\right]}^{\text{Dirac mass terms}} + \underbrace{m_L\left[\left(\bar{n}_L + \bar{n}_L^c\right)\left(n_L + n_L^c\right)\right] + m_R\left[\left(\bar{n}_R + \bar{n}_R^c\right)\left(n_R + n_R^c\right)\right]}_{\text{Majorana mass terms}}$$

The mass lagrangian(II)

$$\mathrm{L} = \bar{S}\mathrm{M}\, S$$

$$S = \begin{pmatrix} n_L + n_L^c \\ n_R + n_R^c \end{pmatrix}$$

$$\mathrm{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

To obtain the physical masses one has to diagonalise the matrix

Such a simple form allows to introduce the “see-saw” mechanism

The see-saw mechanism(I)

✓ Same Grand Unified Theories (GUT) requires m_L=0

⇒ only one Dirac(m_D) and one Majorana(m_R=M) masses are left

✓ Diagonalising the mass matrix under this assumption

$$m_{light} = \frac{m_D^2}{M} \qquad m_{heavy} = M$$

⇒ we are left with two neutrino masses

$$m_{light} = \frac{\left(10^2\ \mathrm{GeV}\right)^2}{10^{13}\ \mathrm{GeV}} \approx 1\,\mathrm{eV} \qquad m_{heavy} = 10^{13}\ \mathrm{GeV}$$

The see-saw mechanism(II)

- ✓ Advantages
  - It generates a left-handed light neutrino which matches the observations
  - It generates a right-handed neutrino which is not yet observed because it is too massive
- ✓ Disadvantages
  - GUT predict a proton decay rate much higher than experimental limits!
  - The proton decay rate can be adjusted, but the theories become very elaborated

Beyond the see-saw

Many models and theories have been developed in order to generate small neutrino masses in a simple and understandable

Our main concern in the following will be the discussion of

- Processes sensitive to neutrino masses (neutrino oscillations)
- Searches for neutrino oscillations and their results
- Direct and indirect neutrino mass searches and their results

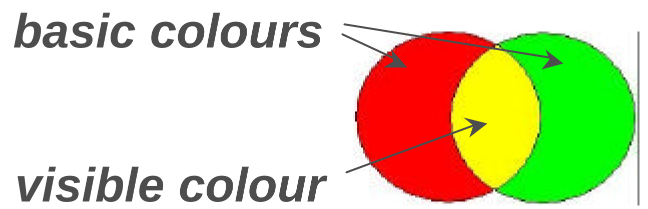
Neutrino oscillations

… neutrinos induce courage in theoreticians
and perseverance in experimenters

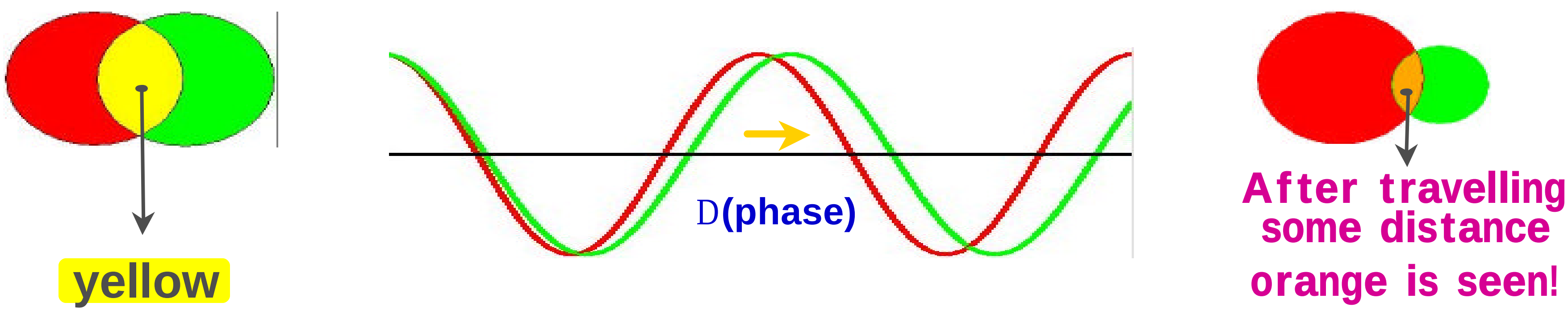
Maurice Goldhaber, 1974

An analogy to ν metamorphosis (“oscillation”)

- <u>Mixing</u>: visible colours as mixture of basic colours



- **<u>Propagation of colours as waves</u> : different colour → different wavelength**



The coloured waves emulate the mechanism of n oscillation:
in Quantum Mechanics <u>particles</u> are represented by <u>waves</u>
*(and their **<u>wavelength</u>** depends on **<u>mass</u>** !)*

Coupled pendulums’ analogy

Production

Classical physics

Movement of pendulum ν_μ
(“visible” eigenstate)

Quantum mechanics

ν_μ production
(Weak Int. eigenstate)

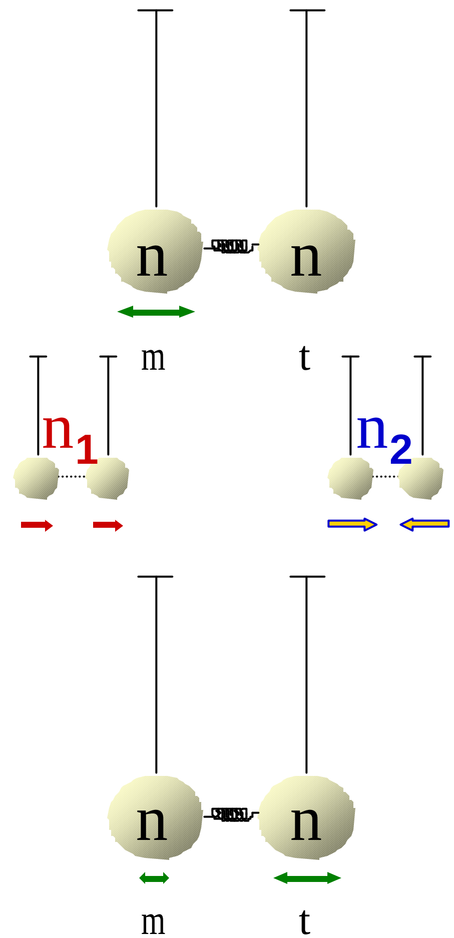
Propagation based on

Coupling

Principal modes of oscillation n_1 n_2
(with different time evolution)

Mixing

Mass eigenstates n_1 n_2
(with different space-time evolution)



Observation

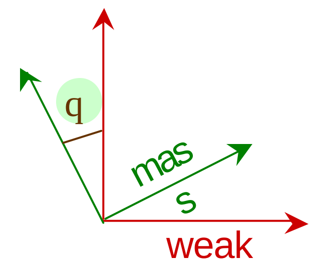
energy “**oscillates**”
from ν_μ to ν_τ and back

ν_μ and ν_τ
disappear and appear
with space-time

How $m_\nu > 0$ results in ν oscillation

"Mixed" eigen-states

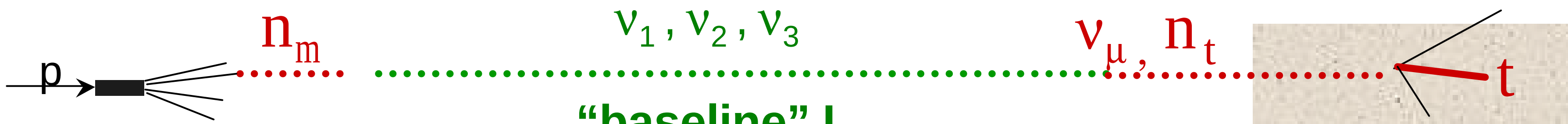
interactions: **"weak" e.s.** (n_e, n_m, n_t) — *"visible"*
propagation : **"mass" e.s.** (n_1, n_2, n_3) — *"invisible"*



Quantum Mechanics

weak → mass → weak

n_m ν_1, ν_2, ν_3 ν_μ, n_t



"baseline" L

n_m **production**

Dm^2_{ij}

® **different** propagation of ν_i waves
® **different** ν_i mixture at detector
® **not only** ν_μ at detector !

n_t **detected**, although ν_μ was produced !

***M.C. Escher, Metamorphose** III (1967-68), part of a "**long baseline**" xylograph (**19 cm x 680 cm**)*

The intergenerational mixing

- ✓ 1957 B. Pontecorvo: concept of neutrino oscillations
- ✓ 1962 Maki-Nakagawa-Sakata: charged-current weak interactions have lepton flavour mixings*

$$\begin{pmatrix} \text{weak} \\ \text{eigenstates} \end{pmatrix} = U_{PMNS} \begin{pmatrix} \text{mass} \\ \text{eigenstates} \end{pmatrix}$$

*1963 Cabibbo hypothesis
1973 CKM matrix for quarks was introduced

Standard Theory of Neutrino Oscillations in Vacuum

[Bilenky, Pontecorvo, Phys. Rep. 41 (1978) 225]

Neutrino Production: $j_\rho^{CC} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \gamma_\rho \ell_{\alpha L}$ $\qquad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$ **Fields**

$$\langle 0|\nu_{\alpha L}|\nu_\beta\rangle = \sum_{k,j} U_{\alpha k} U_{\beta j}^* \underbrace{\langle 0|\nu_{kL}|\nu_j\rangle}_{\propto \delta_{kj}} \propto \sum_k U_{\alpha k} U_{\beta k}^* = \delta_{\alpha\beta}$$

$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$ **States**

$$\mathcal{H}|\nu_k\rangle = E_k|\nu_k\rangle \quad \Rightarrow \quad |\nu_k(t)\rangle = e^{-iE_k t}|\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t}|\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k}|\nu_\beta\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t} U_{\beta k}\right)}_{\mathcal{A}_{\nu_\alpha\to\nu_\beta}(t)} |\nu_\beta\rangle$$

Transition Probability: $P_{\nu_\alpha\to\nu_\beta}(t) = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = \left|\mathcal{A}_{\nu_\alpha\to\nu_\beta}(t)\right|^2 = \left|\sum_k U_{\alpha k}^* e^{-iE_k t} U_{\beta k}\right|^2$

$$P_{\nu_\alpha\to\nu_\beta}(t) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-i\left(E_k - E_j\right)t\right]$$

Relativistic Approximation + Assumption $p_k = p = E$

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} = E + \frac{m_k^2}{2E} \Longrightarrow E_k - E_j \simeq \frac{\Delta m_{kj}^2}{2E}$$

$$\boxed{\Delta m_{kj}^2 \equiv m_k^2 - m_j^2}$$

Approximation $t \simeq L \Longrightarrow$

$$\boxed{P_{\nu_\alpha\to\nu_\beta}(L) \simeq \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-i\frac{\Delta m_{kj}^2 L}{2E}\right]}$$

$$P_{\nu_\alpha\to\nu_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 \quad \Leftarrow \text{constant term}$$

$$+ 2\mathrm{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right) \quad \Leftarrow \text{oscillating term}$$

$\Updownarrow$

COHERENCE

NEUTRINO OSCILLATIONS

The most promising way to verify if $m_n > 0$

(Pontecorvo 1958; Maki, Nakagawa, Sakata 1962)

Basic assumption: neutrino mixing

n_e, n_m, n_t **are not mass eigenstates but linear superpositions of mass eigenstates** n_1, n_2, n_3 **with masses** m_1, m_2, m_3, **respectively:**

$$\left|n_a\right\rangle = \sum_i U_{ai} \left|n_i\right\rangle$$

α = e, μ, τ ("flavour" index)

i = 1, 2, 3 (mass index)

U_{ai}: **unitary mixing matrix**

$$\left|n_i\right\rangle = \sum_a V_{ia} \left|n_a\right\rangle$$

$$V_{ia} = (U_{ai})^*$$

Time evolution of a neutrino state of momentum p created as n$_a$ at time t=0

$$|n(t)\rangle = e^{i\mathbf{p}\cdot\mathbf{r}} \sum_k U_{ak} e^{-iE_k t} |n_k\rangle$$

Note: $|n(0)\rangle = |n_a\rangle$

$$E_k = \sqrt{p^2 + m_k^2}$$ ⟶ phases $e^{-iE_k t}$ are different if $m_j \neq m_k$

⟶ **appearance of neutrino flavour n$_b$ ¹ n$_a$ at t > 0**

Case of two-neutrino mixing

$$|n_a\rangle = \cos q\,|n_1\rangle + \sin q\,|n_2\rangle$$
$$|n_b\rangle = -\sin q\,|n_1\rangle + \cos q\,|n_2\rangle$$

q ° **mixing angle**

For n=n$_a$ at production (t = 0):

$$|n(t)\rangle = e^{i(\mathbf{p}\cdot\mathbf{r} - E_1 t)}\left[\cos q\,|n_1\rangle + e^{-i(E_2 - E_1)t}\sin q\,|n_2\rangle\right]$$

Probability to detect n_b at time t if pure n_a was produced at t = 0

$$P_{\alpha\beta}(t) = \left|\langle \nu_\beta | \nu(t) \rangle\right|^2 = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 t}{4E}\right)$$

Natural units: $\hbar = c = 1$

Dm^2 º $m_2^2 - m_1^2$

Note: for *m* << p $E = \sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p}$ **(in vacuum!)**

$$\Rightarrow \quad E_2 - E_1 \approx \frac{m_2^2 - m_1^2}{2p} \approx \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m^2}{2E}$$

Use more familiar units:

$$P_{\alpha\beta}(L) = \sin^2(2\theta)\sin^2\left(1.267\,\Delta m^2 \frac{L}{E}\right)$$

$L = ct$ distance between neutrino source and detector

Units: Δm^2 [eV^2]; L [km]; E [GeV] (or L [m]; E [MeV])

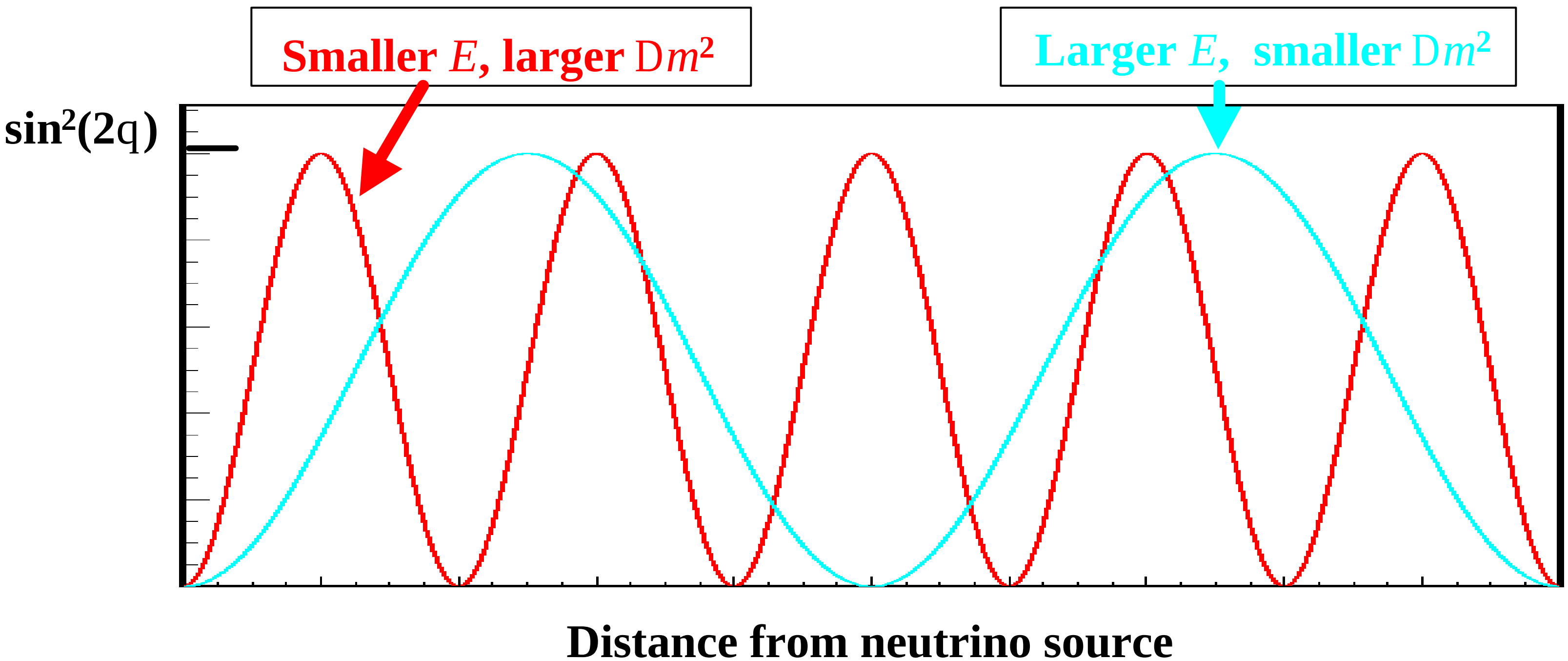
NOTE: $P_{a\,b}$ depends on Dm^2 and not on *m*. However, if $m_1 << m_2$ (as for charged leptons and quarks), then Dm^2 º $m_2^2 - m_1^2$ » m_2^2

Define oscillation length l :

$$l = 2.48\frac{E}{\Delta m^2}$$

Units: λ [km]; *E* [GeV]; Δm^2 [eV^2]
(or λ [m]; *E* [MeV])

$$P_{ab}(L) = \sin^2(2q)\sin^2(p\frac{L}{l})$$



Two Generations: $k = 1, 2$

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition ($\alpha \neq \beta$):

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival $\alpha = \beta$: $[P_{\nu_\alpha \to \nu_\alpha}(L) = 1 - P_{\nu_\alpha \to \nu_\beta}(L)]$

$$P_{\nu_\alpha \to \nu_\alpha}(L) = 1 - \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

General formula for any number of generations

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

Main Assumptions of Standard Theory

(A1) Neutrinos are extremely relativistic particles OK!

(A2) Neutrinos produced in CC weak interaction processes together with charged leptons α^+ are described by the flavor state $|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle$

Correct approximation for ultrarelativistic ν's [Giunti, Kim, Lee, PRD 45 (1992) 2414]

(A3) Massive neutrino states $|\nu_k\rangle$ have the same momentum $p_k = p$ ("Equal Momentum Assumption") and different energies: $E_k \simeq E + \frac{m_k^2}{2E}$

Unrealistic assumption, forbidden by energy-momentum conservation and Lorentz invariance, but gives correct result (as well as the "Equal Energy Assumption")

[Winter, LNC 30 (1981) 101], [Giunti, Kim, FPL 14 (2001) 213], [Giunti, MPLA 16 (2001) 2363], [Giunti, hep-ph/0302026]

(A4) Propagation Time $T \simeq L$ Source-Detector Distance OK!

⇕

WAVE PACKETS

Easy Example of Neutrino Production: $\pi^+ \to \mu^+ + \nu_\mu \quad \pi^- \to \mu^- + \bar{\nu}_\mu$

two-body decay $\Longrightarrow$ fixed kinematics

$$E_k^2 = p_k^2 + m_k^2$$

π at rest:

$$\begin{cases} p_k^2 = \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_k^2}{2}\left(1+\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4\,m_\pi^2} \\ E_k^2 = \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_k^2}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4\,m_\pi^2} \end{cases}$$

0^{th} order: $m_k = 0 \Rightarrow p_k = E_k = E = \frac{m_\pi}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$

1^{st} order:

$$E_k \simeq E + \xi\,\frac{m_k^2}{2E} \qquad p_k \simeq E - (1-\xi)\,\frac{m_k^2}{2E} \qquad \xi = \frac{1}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

↖ ↗

general!

Plane Wave Approximation

LORENTZ INVARIANT

OSCILLATION PROBABILITY

$$P_{\nu_\alpha \to \nu_\beta}(L,T) = \left| \sum_k U_{\alpha k}^* \, e^{ip_k L - iE_k t} \, U_{\beta k} \right|^2$$

[Dolgov et al., NPB 502 (1997) 3], [Dolgov, hep-ph/0004032], [Giunti, Kim, FPL 14 (2001) 213], [Bilenky, Giunti, IJMPA 16 (2001) 3931]

$$P_{\nu_\alpha \to \nu_\beta}(L,T) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \, e^{i(p_k - p_j)L - i(E_k - E_j)T}$$

relativistic approximation: $p_k - p_j \simeq -(1-\xi)\,\dfrac{\Delta m_{kj}^2}{2E}$ $\qquad E_k - E_j \simeq \xi\,\dfrac{\Delta m_{kj}^2}{2E}$

$$P_{\nu_\alpha \to \nu_\beta}(L,T) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* e^{-i(1-\xi)\frac{\Delta m_{kj}^2}{2E} L - i\xi \frac{\Delta m_{kj}^2}{2E} T}$$

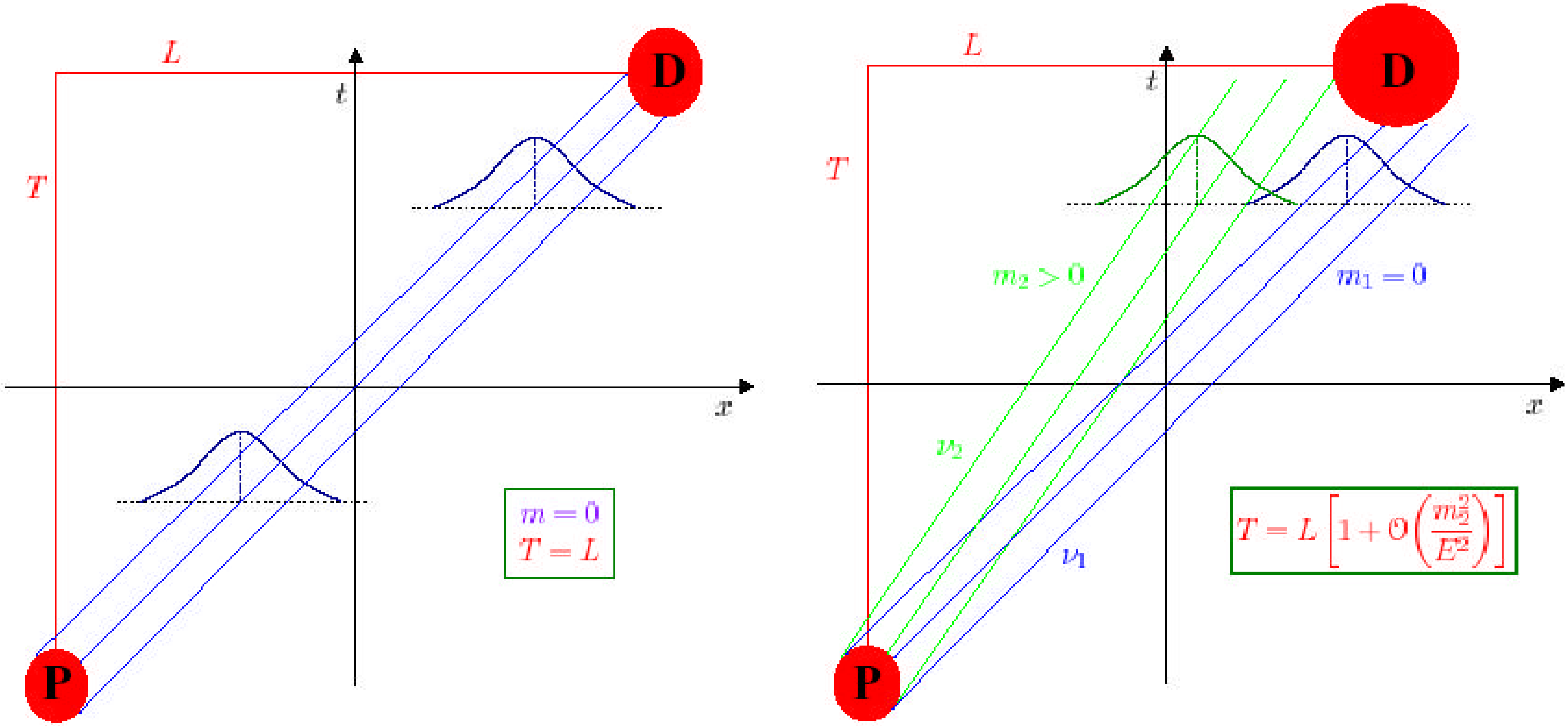
$$T = L \implies P_{\nu_\alpha \to \nu_\beta}(L,T) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* e^{-i \frac{\Delta m_{kj}^2}{2E} L}$$

$T = L \Longleftrightarrow$ WAVE PACKETS

Other Motivations:

[Kayser, PRD 24 (1981) 110]

- ⇝ Localization of Production and Detection Processes
- ⇝ Exact Energy-Momentum conservation would imply creation and detection of only one massive neutrino (neutrino mass measurement)



Coherence Length

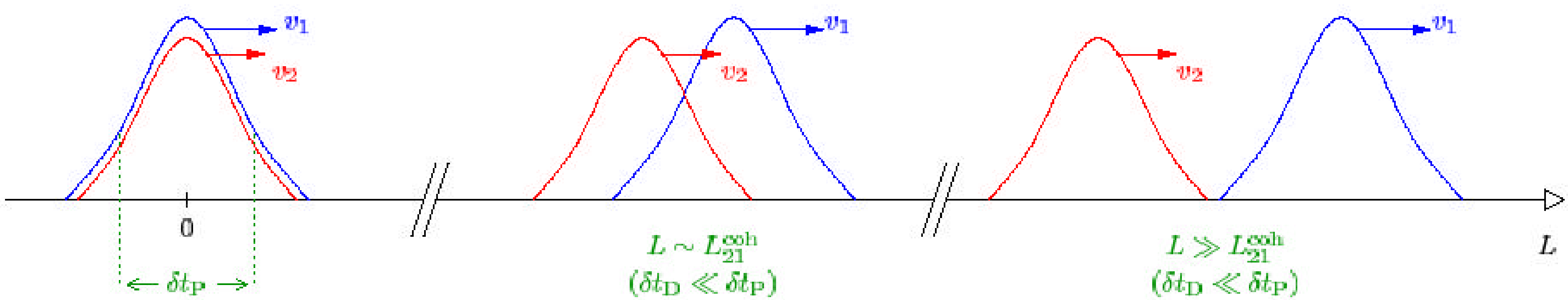
[Nussinov, PLB 63 (1976) 201], [Kiers, Nussinov, Weiss, PRD 53 (1996) 537]

Wave Packets have different velocities and separate

Different massive neutrinos can interfere
if and only if
wave packets arrive with $\delta t_{kj} < \delta t_{\mathrm{D}}$

$\Longrightarrow \quad L \lesssim L^{\mathrm{coh}}_{kj}$

$$|\delta t_{kj}| \simeq |v_k - v_j|\, T \simeq \frac{|\Delta m^2_{kj}|}{2E^2} L \quad \Longrightarrow \quad L^{\mathrm{coh}}_{kj} \sim \frac{2E^2}{|\Delta m^2_{kj}|} \sqrt{\delta t_{\mathrm{P}}^2 + \delta t_{\mathrm{D}}^2}$$



Estimates of Coherence Length

$$L^{\rm osc} = \frac{4\pi E}{|\Delta m^2|} = 2.5 \frac{(E/{\rm MeV})}{(|\Delta m^2|/{\rm eV}^2)} {\rm m} \qquad L^{\rm coh} = \frac{4\sqrt{2}\omega E^2}{|\Delta m^2|}\sigma_x \sim 10^{12} \frac{(E^2/{\rm MeV}^2)}{(|\Delta m^2|/{\rm eV}^2)} \left(\frac{\sigma_x}{\rm m}\right) {\rm m}$$

Process	$|\Delta m^2|$	$L^{\rm osc}$	σ_x	$L^{\rm coh}$
$\pi \to \mu + \nu$ at rest in vacuum: $E \simeq 30\,{\rm MeV}$ natural linewidth	$2.5 \times 10^{-3}\,{\rm eV}^2$	$30\,{\rm km}$	$\tau_\pi \sim 10\,{\rm m}$	$\sim 10^{16}\,{\rm km}$
$\pi \to \mu + \nu$ at rest in matter: $E \simeq 30\,{\rm MeV}$ collision broadening	$2.5 \times 10^{-3}\,{\rm eV}^2$	$30\,{\rm km}$	$\tau_{\rm col} \sim 10^{-5}\,{\rm m}$	$\sim 10^{10}\,{\rm km}$
$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ at rest in matter: $E \lesssim 50\,{\rm MeV}$ collision broadening	$1\,{\rm eV}^2$	$\lesssim 125\,{\rm m}$	$\tau_{\rm col} \sim 10^{-10}\,{\rm m}$	$\lesssim 10^2\,{\rm km}$
${}^7{\rm Be} + e^- \to {}^7{\rm Li} + \nu_e$ in solar core: $E \simeq 0.86\,{\rm MeV}$ collision broadening	$7 \times 10^{-5}\,{\rm eV}^2$	$31\,{\rm km}$	$\tau_{\rm col} \sim 10^{-9}\,{\rm m}$	$\sim 10^4\,{\rm km}$

Comments

Flavors, masses, mixing

Flavor neutrino states:

ν_e ν_μ ν_τ

e μ τ

correspond to certain charged leptons

interact in pairs

eigenstates of the CC weak interactions

ν_s Sterile neutrinos?

Mass eigenstates

ν_1 ν_2 ν_3

m_1 m_2 m_3

Mixing

Flavor states $\neq$ Mass eigenstates

Two aspects of mixing

vacuum mixing angle

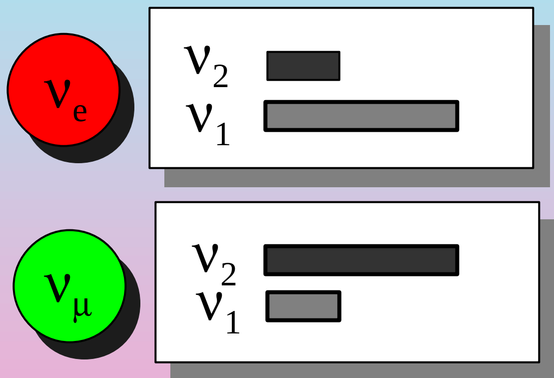
$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2$$

coherent mixtures of mass eigenstates

inversely

$$\nu_2 = \sin\theta\, \nu_e + \cos\theta\, \nu_\mu$$
$$\nu_1 = \cos\theta\, \nu_e - \sin\theta\, \nu_\mu$$

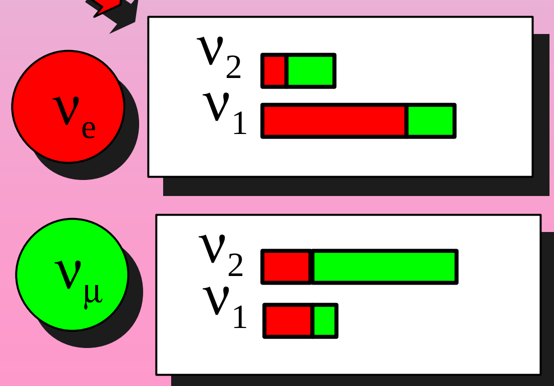
flavor composition of the mass eigenstates



wave packets

Flavors of eigenstates

inserting



The relative phases of the mass states in ν_e and ν_μ are opposite

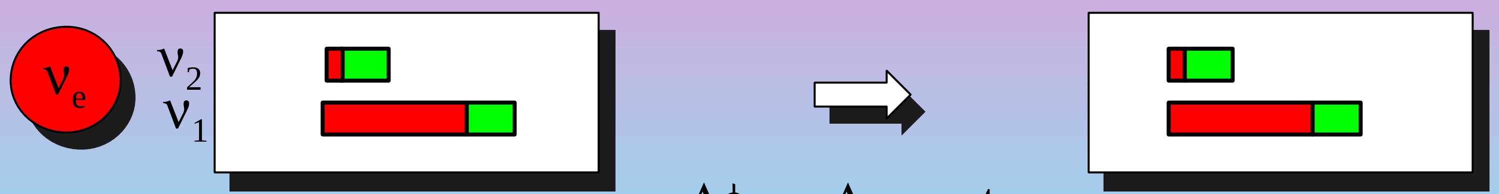
Interference of the parts of wave packets with the same flavor depends on the phase difference $\Delta\phi$ between ν_1 and ν_2

Vacuum oscillations

Propagation in vacuum:

- Flavors of mass eigenstates do not change
- Admixtures of mass eigenstates do not change: no $\nu_1 <\!\!-\!\!> \nu_2$ transitions

Determined by θ



$\Delta\phi = 0$

$\Delta\phi = \Delta v_{phase}\, t$

- Due to difference of masses ν_1 and ν_2 have different phase velocities:

$$\Delta v_{phase} = \frac{\Delta m^2}{2E} \qquad \Delta m^2 = m_2^2 - m_1^2$$

oscillations:

effects of the phase difference increase which changes the interference pattern

Oscillation length:

$$l_\nu = 2\pi/\Delta v_{phase} = 4\pi E/\Delta m^2$$

Amplitude (depth) of oscillations:

$$A = \sin^2 2\theta$$

NEUTRINOS AND ANTINEUTRINOS

$$P_{\nu_\alpha\to\nu_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re}\sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m^2_{kj} L}{2E}\right)$$

Antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathrm{CP}} = \gamma^0 \mathcal{C} \overline{\nu}^T = -\mathcal{C}\nu^*$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Righ-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\mathrm{CP}} \nu_{\alpha L}^{\mathrm{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathrm{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathrm{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\bar{\nu}_\alpha\to\bar{\nu}_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re}\sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i\frac{\Delta m^2_{kj} L}{2E}\right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

Survival : $P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}$

Reactor oscillation results for ν_e are valid also for $\bar{\nu}_e$!

CPT Asymmetries:

$$A_{\alpha\beta}^{(\mathrm{CPT})} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

Quantum Field Theory $\Rightarrow$ $A_{\alpha\beta}^{(\mathrm{CPT})} = 0$

Indeed:

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

is invariant under CPT:

$$U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$$

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries:

$$A^{(\mathrm{CP})}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

$$\begin{aligned} A^{(\mathrm{CP})}_{\alpha\beta}(L) \quad &= 2\mathrm{Re} \sum_{k>j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right) \\ &- 2\mathrm{Re} \sum_{k>j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right) \end{aligned}$$

$$A^{(\mathrm{CP})}_{\alpha\beta}(L) = 4 \sum_{k>j} J_{\alpha\beta;kj} \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

Jarlskog rephasing invariants:

$$U_{\alpha k} \to e^{i\lambda_\alpha} U_{\alpha k} e^{i\eta_k}$$

$$J_{\alpha\beta;kj} = \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right]$$

$\rightsquigarrow$ Violation of CP symmetry depends on Jarlskog rephasing invariants

$\rightsquigarrow$ $\left\langle A^{(\mathrm{CP})}_{\alpha\beta} \right\rangle = 0 \Rightarrow$ Measurement of CP violation needs measurement of oscillations.

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} = P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries:

$$A^{(\mathrm{T})}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$$

$$\begin{aligned} A^{(\mathrm{T})}_{\alpha\beta}(L) \quad &= 2\mathrm{Re} \sum_{k>j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E} \right) \\ &- 2\mathrm{Re} \sum_{k>j} U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E} \right) \end{aligned}$$

$$A^{(\mathrm{T})}_{\alpha\beta}(L) = 4 \sum_{k>j} J_{\alpha\beta;kj} \sin\left(\frac{\Delta m^2_{kj} L}{2E} \right)$$

⇝ $A^{(\mathrm{T})}_{\alpha\beta}(L) = A^{(\mathrm{CP})}_{\alpha\beta}(L)$

⇝ Violation of T symmetry depends on Jarlskog rephasing invariants

⇝ $\left\langle A^{(\mathrm{T})}_{\alpha\beta} \right\rangle = 0 \Rightarrow$ Measurement of T violation needs measurement of oscillations.

If neutrinos have mass: $|n_l\rangle = \sum U_{li} |n_i\rangle$

For three neutrinos:

$$U_{li} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{t1} & U_{t2} & U_{t3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-id} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$$\textit{where } c_{ij} = \cos q_{ij}, \textit{ and } s_{ij} = \sin q_{ij}$$

$$P(?_\mu \rightarrow ?_e) = \sin^2 2? \sin^2 (1.27 \frac{?m^2 L}{E})$$

If neutrinos have mass: $|n_l\rangle = \sum U_{li} |n_i\rangle$

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where $c_{ij} = \cos q_{ij}$, *and* $s_{ij} = \sin q_{ij}$

Three Angles

$$P(?_\mu \rightarrow ?_e) = \sin^2 2? \sin^2 (1.27 \frac{?m^2 L}{E})$$

If neutrinos have mass: $|n_l\rangle = \sum U_{li} |n_i\rangle$

For three neutrinos:

$$U_{li} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{t1} & U_{t2} & U_{t3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-id} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

where $c_{ij} = \cos q_{ij}$, ***and*** $s_{ij} = \sin q_{ij}$

Two mass differences

$$P(?_\mu \to ?_e) = \sin^2 2? \sin^2 (1.27 \frac{?m^2 L}{E})$$

If neutrinos have mass: $|n_l\rangle = \sum U_{li} |n_i\rangle$

For three neutrinos:

$$U_{li} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{t1} & U_{t2} & U_{t3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-id} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

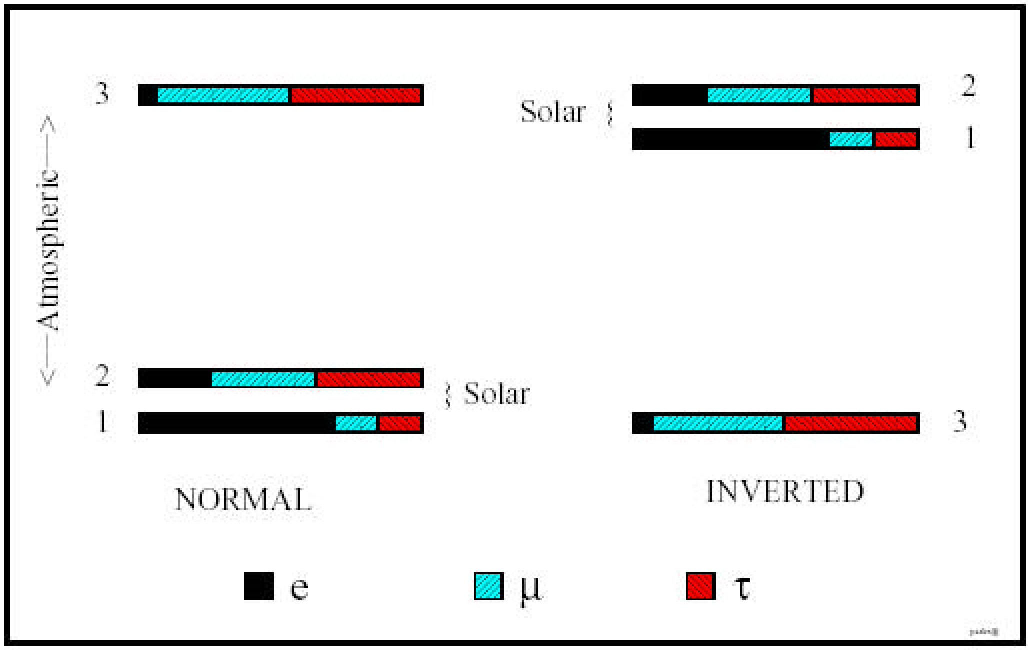
where $c_{ij} = \cos q_{ij}$, *and* $s_{ij} = \sin q_{ij}$

CP violating phase!

$$P(?_\mu \rightarrow ?_e) = \sin^2 2? \sin^2 (1.27 \frac{?m^2 L}{E})$$

Notation

- **Mixing parameters:** $U = U(q_{12}, q_{13}, q_{23}, d)$ **as for CKM matrix**
- **Mass-gap parameters:** $M^2 = Dm^2_{12}\ ,\ \pm Dm^2_{23}$



The absolute neutrino mass scale should be set by direct mass measurements:

- b-decay
- 0n2b-decay
- "W-MAP"

So what do we have to measure?

- ✓ Three angles (θ_{12}, θ_{13}, θ_{23})
- ✓ Two mass differences (δm^2, Δm^2)
- ✓ The sign of the mass difference Δm^2 ($\pm\Delta m^2$)
- ✓ One CP phase (δ)
- ✓ The source of atmospheric oscillations (detect τ appearance)
- ✓ The absolute masse scale
- ✓ Are neutrino Dirac or Majorana particles (or both)?
- ✓ Are there more - sterile - neutrinos?

The MSW effect

MSW is such a beautiful phenomenon that Nature would be well advised to use it. After all, it may eventually give us the unambiguous, incontrovertible, uncontestable, clear and definitive evidence we so eagerly seek that the neutrino has mass

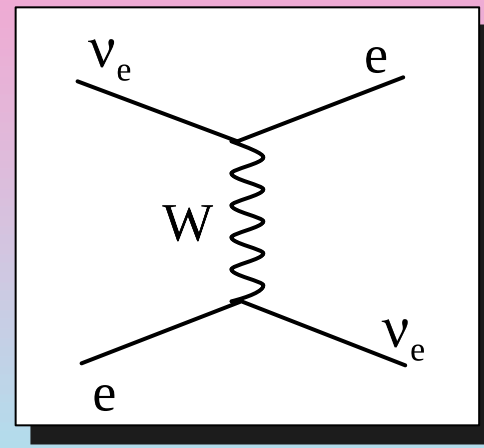
S.P. Rose, 1986

Matter Effect: Refraction

L. Wolfenstein, 1978

Elastic forward scattering → Potentials V_e, V_μ

- $V \sim 10^{-13}$ eV inside the Earth for E = 10 MeV
- Difference of potentials is important → for ν_e ν_μ:



$$V = V_e - V_\mu = \sqrt{2}\, G_F n_e$$

- Refraction index:

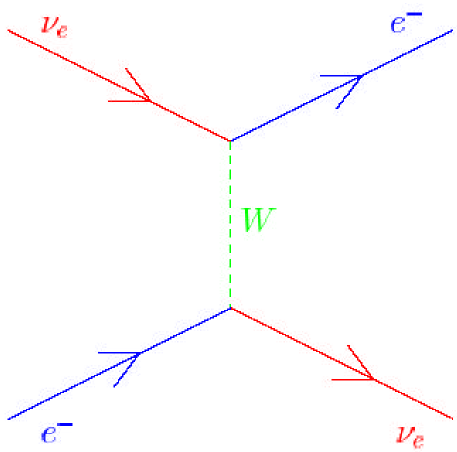
$$n - 1 = V / p$$

- $$n - 1 \begin{cases} \sim 10^{-20} & \text{inside the Earth} \\ < 10^{-18} & \text{inside the Sun} \\ \sim 10^{-6} & \text{inside the neutron star} \end{cases}$$

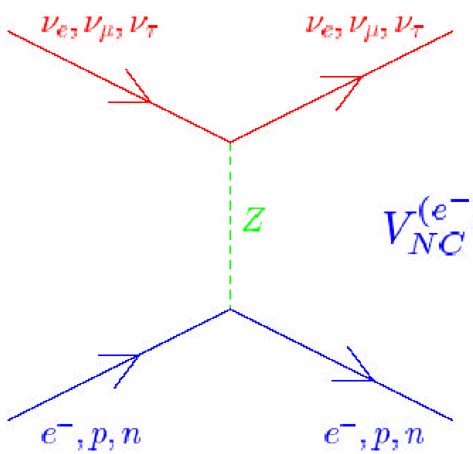
- Refraction length (oscillation length in matter):

$$l_0 = 2\pi / (V_e - V_\mu) = \sqrt{2}\, \pi / G_F n_e$$

EFFECTIVE POTENTIAL IN MATTER



$$V_{CC} = \sqrt{2} G_F N_e$$



$$V_{NC}^{(e^-)} = -V_{NC}^{(p)} \Rightarrow V_{NC} = -\frac{\sqrt{2}}{2} G_F N_n$$

$$V_e = V_{CC} + V_{NC}\,, \quad V_\mu = V_\tau = V_{NC}$$

$$\overline{V}_{CC} = -V_{CC}\,, \quad \overline{V}_{NC} = -V_{NC}$$

Neutrino eigenstates in matter

	in vacuum:		in matter:
Effective Hamiltonian	H_0	➡	$H = H_0 + V$
Eigenstates	$\nu_1, \ \nu_2$	➡	$\nu_{1m}, \ \nu_{2m}$
Eigenvalues	$m_1, \ m_2$ $m_1^2/2E, \ m_2^2/2E$	➡	$m_{1m}, \ m_{2m}$ $H_{1m}, \ H_{2m}$

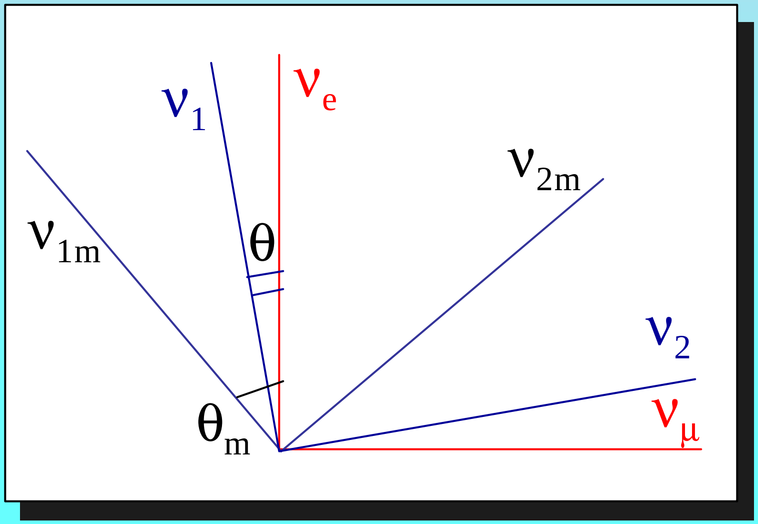
$V = V_e - V_\mu$

depend on n_e, E

Mixing in matter

is determined with respect to eigenstates in matter

θ_m is the mixing angle in matter



mass-squared matrix in vacuum $M^2 \xrightarrow{MATTER} M^2 + 2EV_{CC} = M^2_{\text{eff}}$ effective mass-squared matrix in matter

(arrow to V_{CC}:) potential due to coherent forward CC elastic scattering

$$V_{CC} = \sqrt{2} G_F N_e \qquad A_{CC} \equiv 2EV_{CC} = 2\sqrt{2} E G_F N_e$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{M^2}{2E} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \xrightarrow{MATTER} i \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{M^2_{\text{eff}}}{2E} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 c_{2\vartheta} + 2A_{CC} & \Delta m^2 s_{2\vartheta} \\ \Delta m^2 s_{2\vartheta} & \Delta m^2 c_{2\vartheta} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

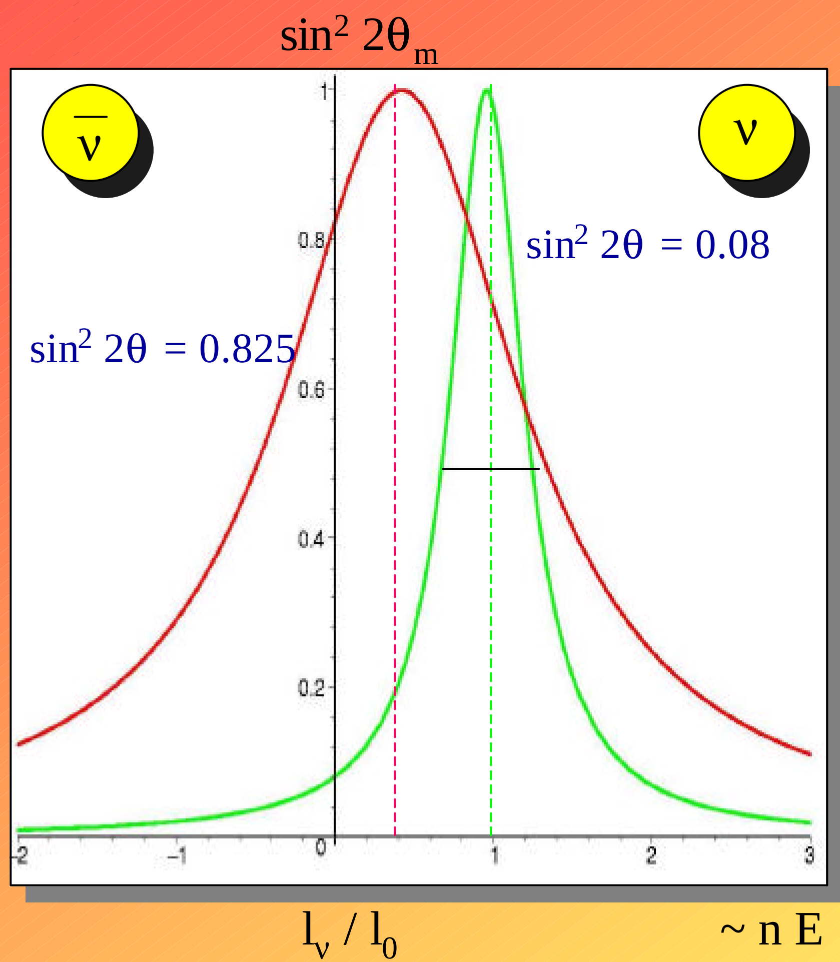
Effective Mixing Angle in Matter: $\tan 2\vartheta_M = \dfrac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$

Resonance: $A^R_{CC} = \Delta m^2 \cos 2\vartheta \implies N^R_e = \dfrac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_F}$

Effective Squared-Mass Difference:

$$\Delta m^2_{\text{eff}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance



- Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$
- Resonance layer: $n = n_R \pm \Delta n_R$

In resonance:

$$\sin^2 2\theta_m = 1$$

- Mixing in matter is maximal
- Level split is minimal (Δm2eff$\rightarrow$0)

$$l_\nu = l_0 \cos 2\theta$$

Vacuum oscillation length $\approx$ Refraction length

For large mixing: $\cos 2\theta = 0.4 - 0.5$
the equality is broken
the case of strongly coupled system
shift of frequencies

Level crossing

Dependence of the neutrino eigenvalues
on the matter potential (density)

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

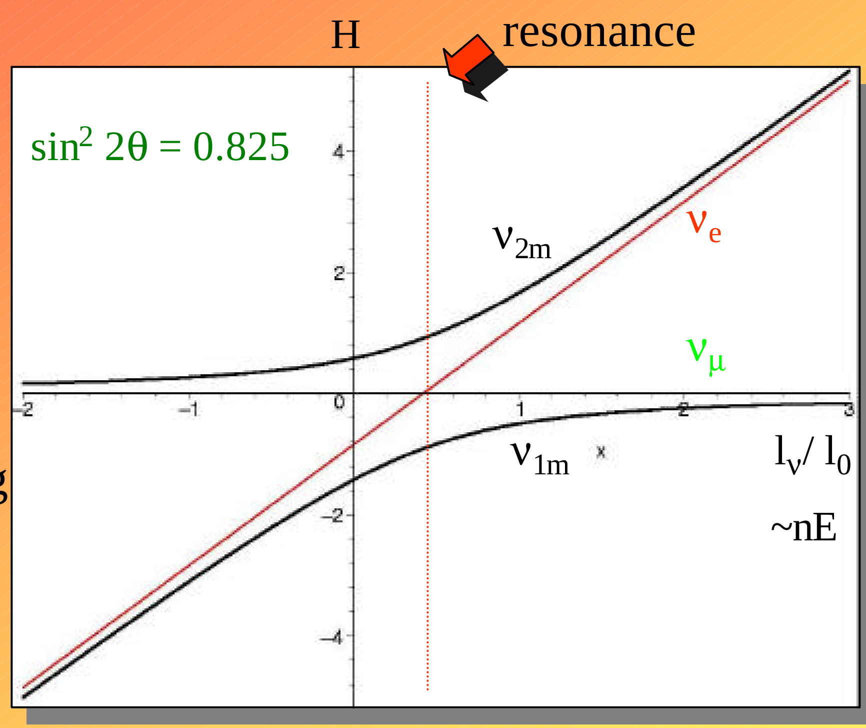
$$\frac{l_\nu}{l_0} = \cos 2\theta$$

Crossing point - resonance

- the level split in minimal
- the oscillation length is maximal

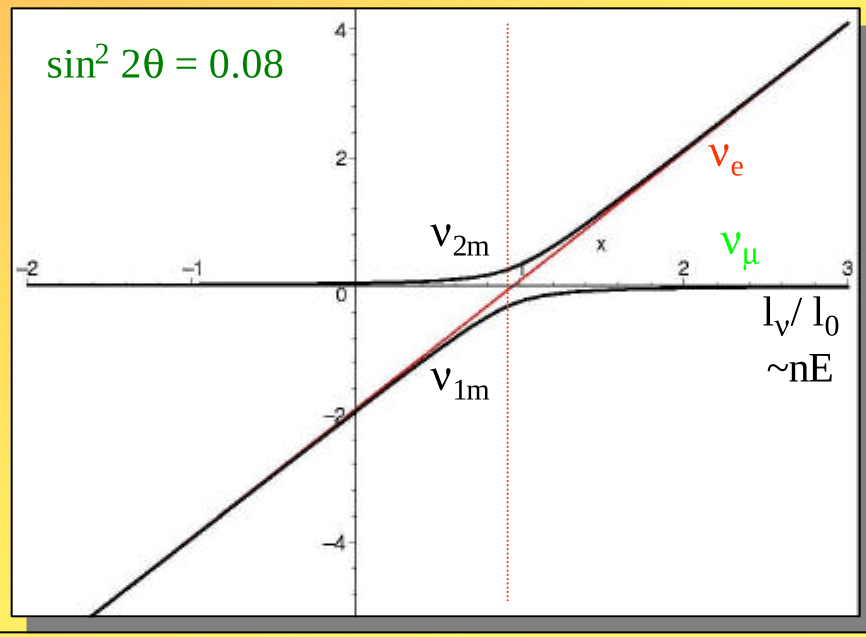
For maximal mixing: $n_R = 0$

Large mixing



ν

Small mixing



Comments

- ✓ In absence of mixing the energy levels cross at the MSW resonance point
- ✓ With non vanishing mixing the levels “repel” each other
- ✓ If the probability of the transition between the two matter eigenstates is small, neutrinos produced as ν_e at high densities and propagating towards smaller densities follow the upper (ν_{2m}) branch and end up on the level that corresponds to ν_μ at small densities
- ✓ NB: The previous point looks paradoxical: the smaller the vacuum mixing angle, the larger the probability that the initially produced ν_e will be converted into ν_μ or viceversa. Does this mean that in the limit of vanishing θ one can still have strong neutrino conversion?
  - The answer is of course no!
  - The reason is simple: the applied approximations do not hold anymore

Two effects

Resonance enhancement of neutrino oscillations

Adiabatic (partially adiabatic) neutrino conversion

Density profiles:	Constant density	Variable density
Degrees of freedom:	Change of the phase difference between neutrino eigenstates	Change of mixing, or flavor of the neutrino eigenstates

In general: *Interplay of oscillations and adiabatic conversion*

MSW

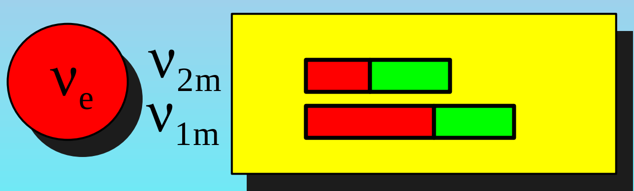
Oscillations in matter

In uniform matter (constant density)
mixing is constant

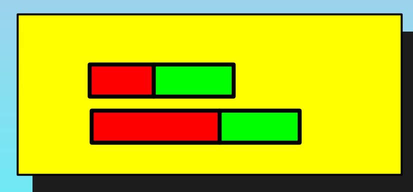
$\theta_m(E, n)$ = constant

- Flavors of the eigenstates do not change
- Admixtures of matter eigenstates do not change: no $\nu_{1m} <-> \nu_{2m}$ transitions
- Monotonous increase of the phase difference between the eigenstates $\Delta\phi_m$

→ **Oscillations** as in vacuum



$\Delta\phi_m = 0$

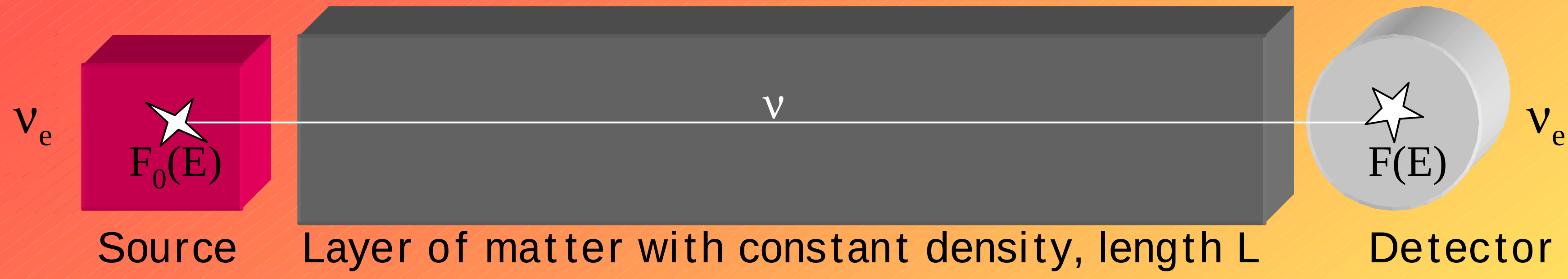


$\Delta\phi_m = (H_2 - H_1) L$

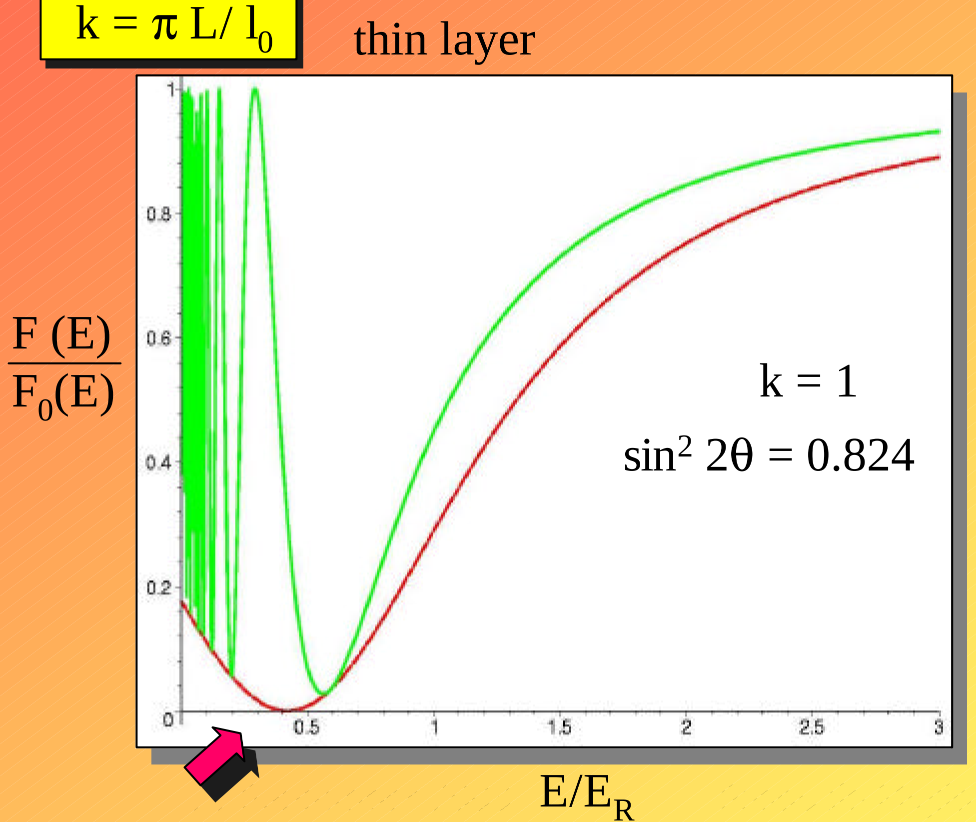
Parameters of oscillations (depth and length)
are determined by mixing in matter
and by effective energy split in matter

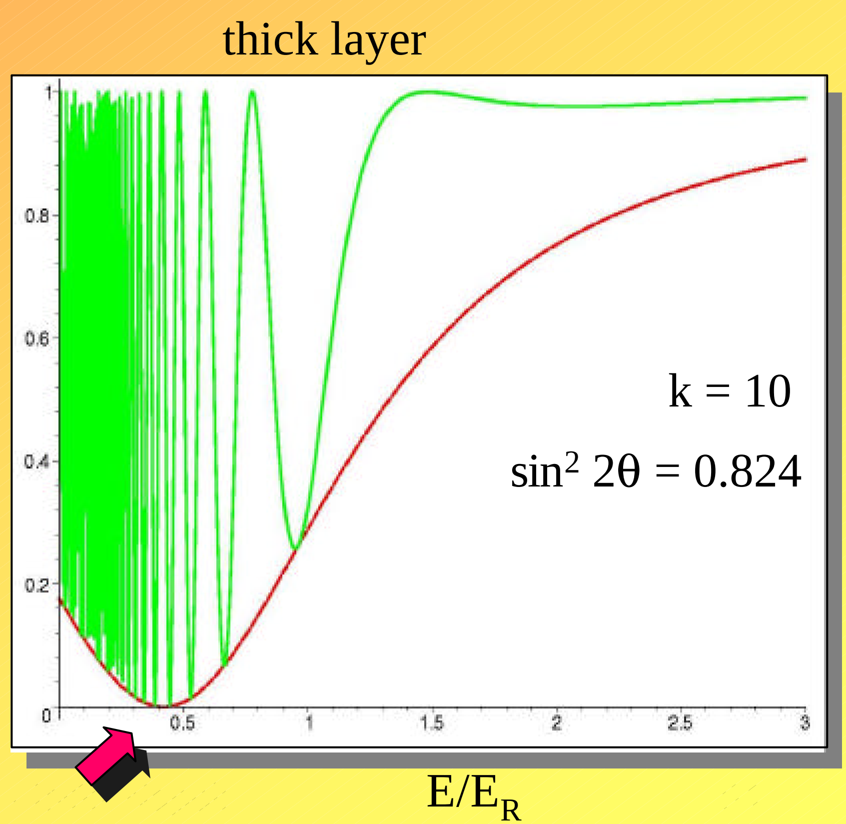
$\sin^2 2\theta, \ l_\nu$ → $\sin^2 2\theta_m, l_m$

Resonance enhancement of oscillations

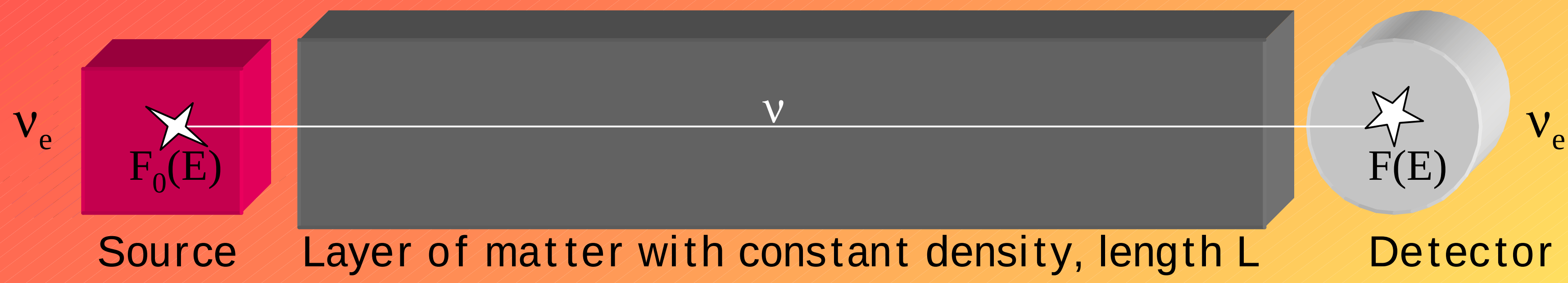


$k = \pi L / l_0$

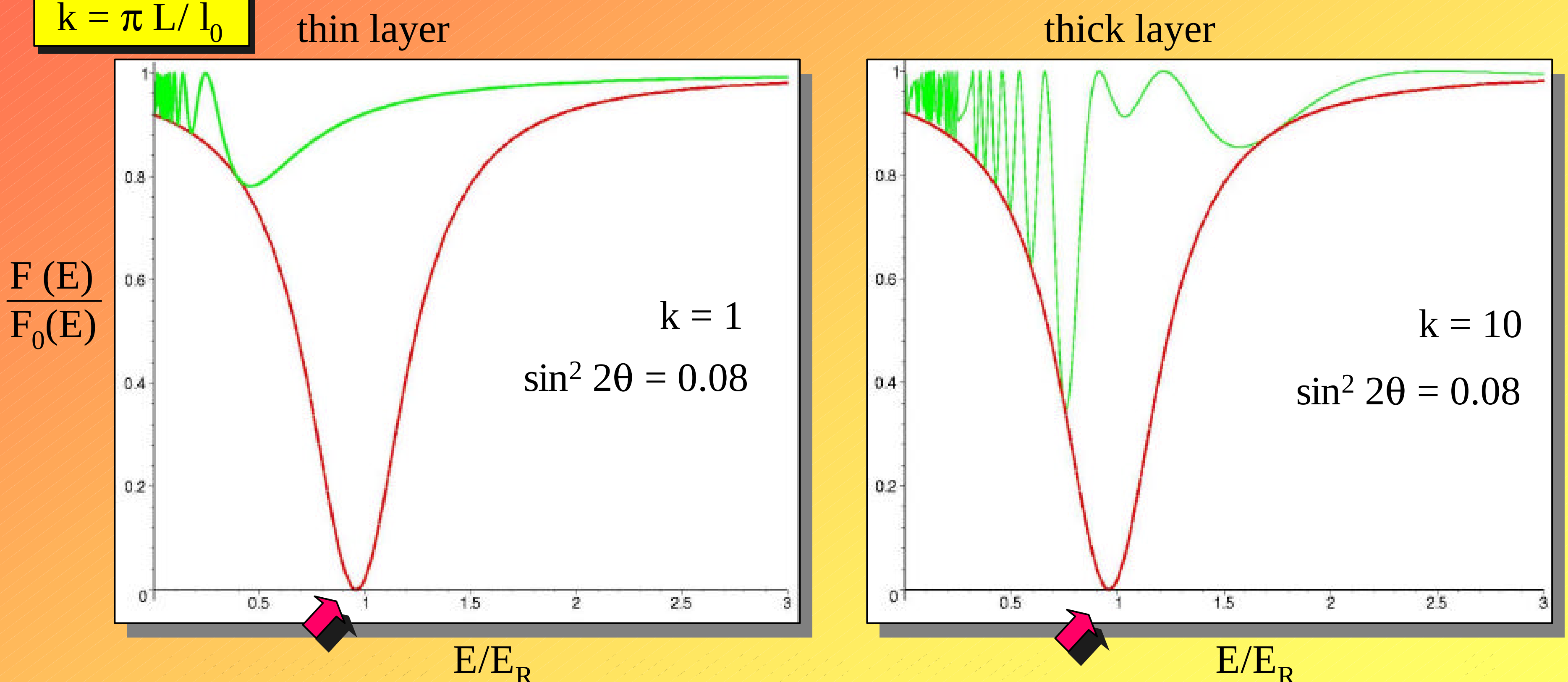




Resonance enhancement of oscillations



$k = \pi L / l_0$



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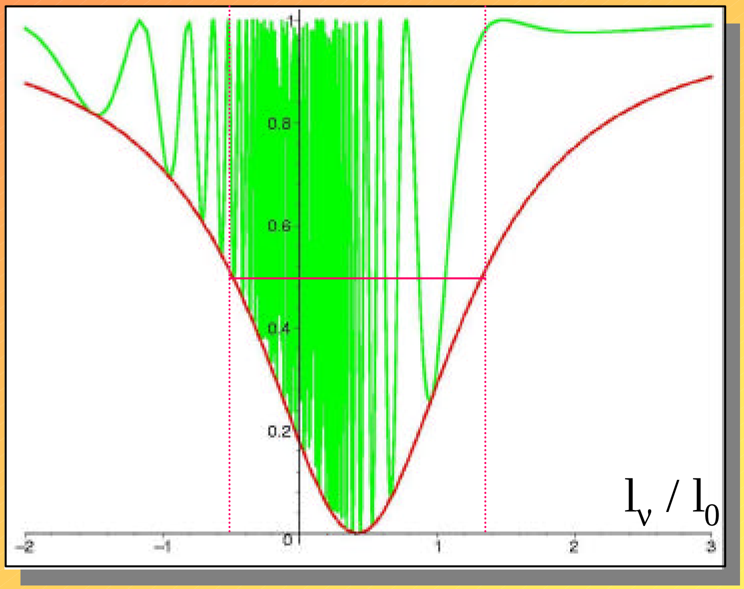
Resonance enhancement of oscillations

resonance layer

- Continuity:
  neutrino and antineutrino semiplanes
  normal and inverted hierarchy

P

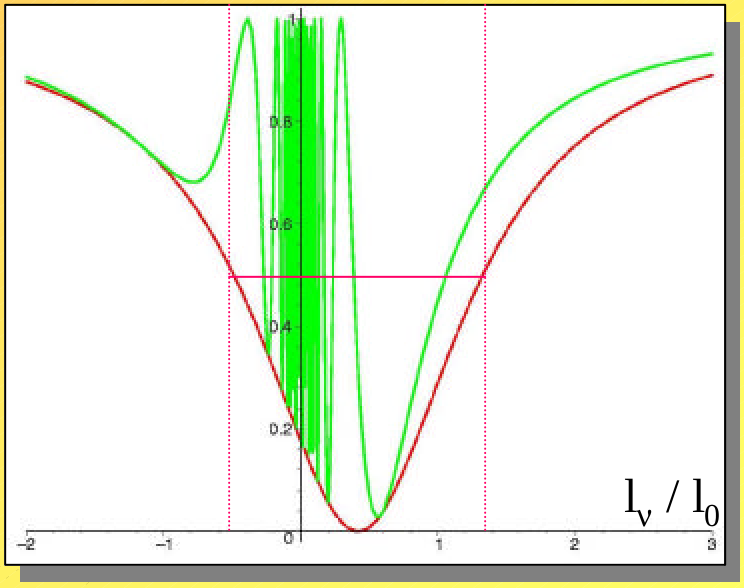


- Oscillations (amplitude of oscillations) are enhanced in the resonance layer

$$E = (E_R - \Delta E_R) \;\text{--}\; (E_R + \Delta E_R)$$
$$\Delta E_R = E_R \tan 2\theta = E_R^0 \sin 2\theta$$

$$E_R^0 = \Delta m^2 / 2V$$

P



- With increase of mixing: $\theta \to \pi/4$

$$E_R \to 0$$
$$\Delta E_R \to E_R^0$$

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MSW: adiabatic conversion

Non-uniform matter density changes on the way of neutrinos: $n_e = n_e(t)$

H = H(t) depends on time

$\theta_m = \theta_m(n_e(t))$ mixing changes in the course of propagation

ν_{1m} ν_{2m} are no more the eigenstates of propagation -> ν_{1m} <-> ν_{2m} transitions

However

if the density changes slowly enough (adiabaticity condition) ν_{1m} <-> ν_{2m} transitions can be neglected

- Flavors of eigenstates change according to the density change → determined by θ_m
- Admixtures of the eigenstates, ν_{1m} ν_{2m}, do not change → fixed by mixing in the production point
- Phase difference increases → according to the level split which changes with density

MSW

Effect is related to the change of flavors of the neutrino eigenstates in matter with varying density

Adiabaticity

- External conditions (density) change slowly so the system has time to adjust itself

$$\frac{\left|\frac{d\theta_m}{dt}\right|}{H_2 - H_1} \ll 1$$

Adiabaticity condition

➡ transitions between the neutrino eigenstates can be neglected

$$\nu_{1m} <\!\!\not\!\!-\!\!> \nu_{2m}$$

➡ The eigenstates propagate independently

- Crucial in the resonance layer:
  - the mixing angle changes fast
  - level splitting is minimal

$$\Delta r_R > l_R$$

if vacuum mixing is small

$l_R = l_\nu/\sin 2\theta$ is the oscillation width in resonance

$\Delta r_R = n_R / (dn/dx)_R \tan 2\theta$ is the width of the resonance layer

- If vacuum mixing is large the point of maximal adiabaticity violation is shifted to larger dencities

$$n(a.v.) \to n_R^0 > n_R$$

$$n_R^0 = \Delta m^2 / 2\sqrt{2}\, G_F E$$

Adiabatic conversion and initial condition

The picture of conversion depends on how far from the resonance layer in the density scale the neutrino is produced

$n_0 > n_R$

$n_0 - n_R >> \Delta n_R$

Non-oscillatory conversion

$n_0 \sim n_R$

Interplay of conversion and oscillations

$n_0 < n_R$

$n_R - n_0 >> \Delta n_R$

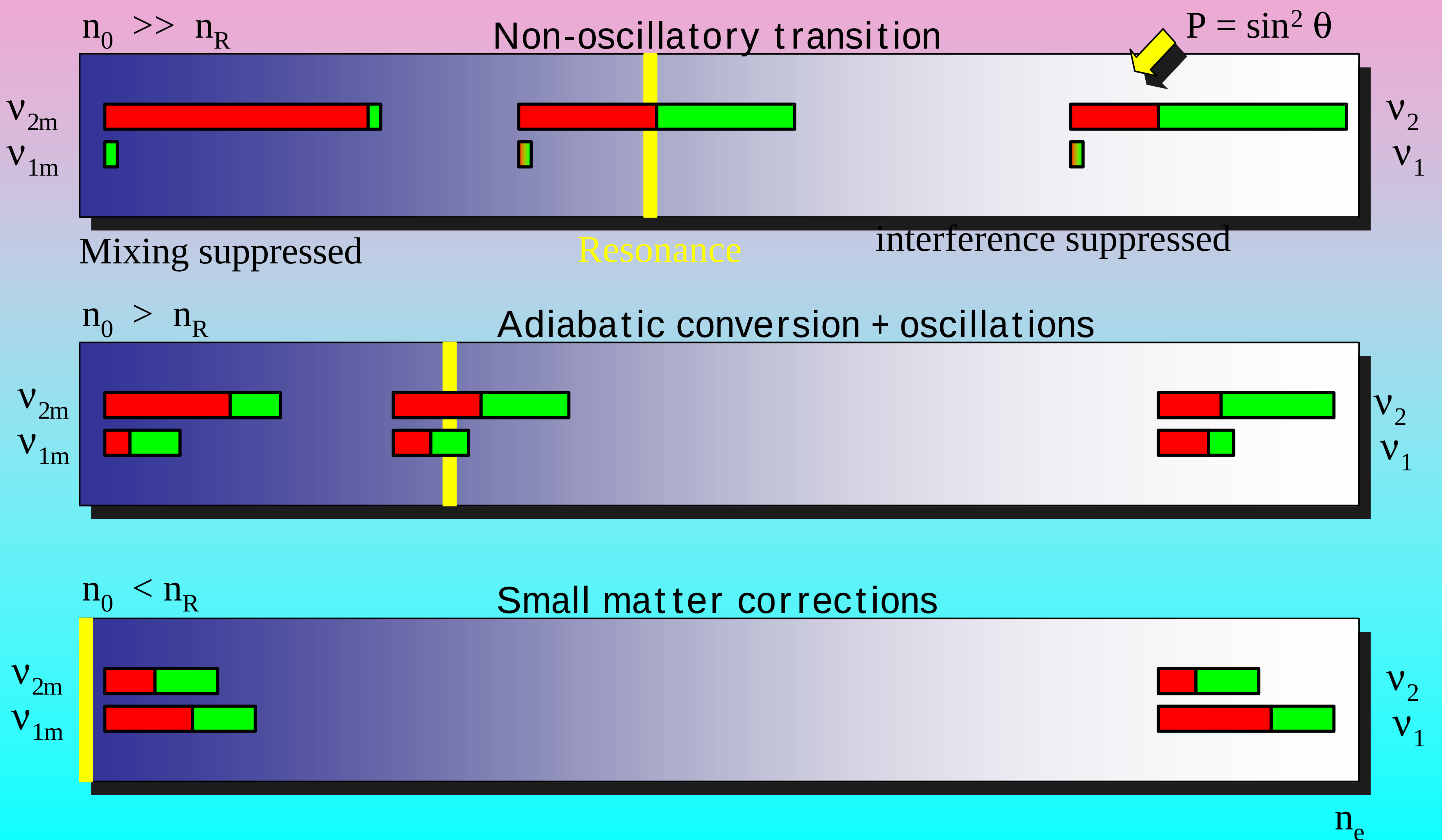
Oscillations with small matter effect

$n_R \sim 1/E$

All three possibilities are realized for the solar neutrinos in different energy ranges

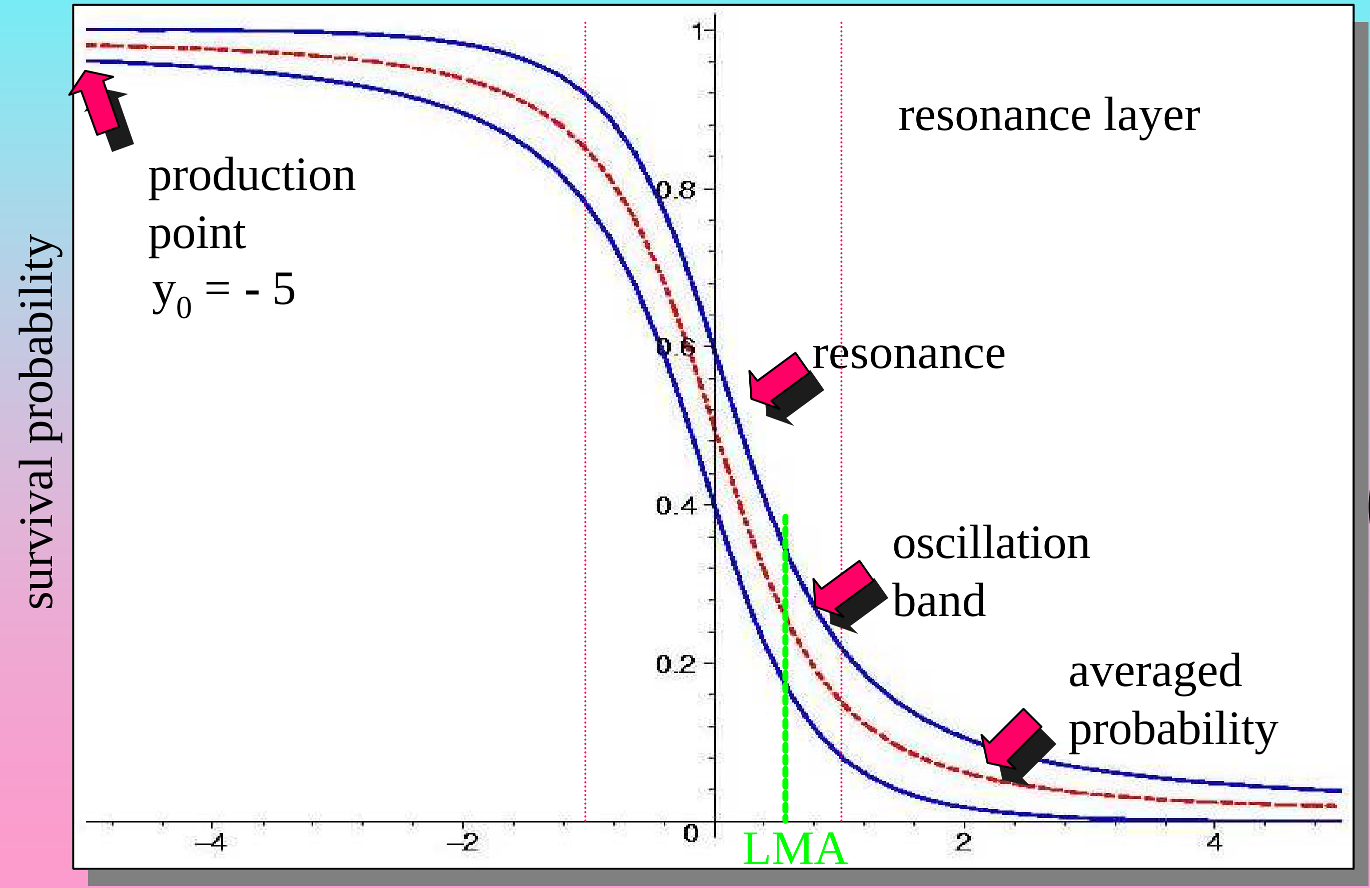
Adiabatic conversion

$\nu_{1m} <\!\!\!\not\!\!\!-\!\!\!> \nu_{2m}$



The MSW effect

The picture of adiabatic conversion is universal in terms of variable $y = (n_R - n) / \Delta n_R$
(no explicit dependence on oscillation parameters density distribution, etc.)
Only initial value y_0 matters.



$y = (n_R - n) / \Delta n_R$ (distance)

For zero final density:
$y = 1/\tan 2\theta$

A Yu Smirnov

Disappearance experiments

Use a beam of n_a and measure n_a flux at distance L from source

Measure $$P_{aa} = 1 - \sum_{b \neq a} P_{ab}$$

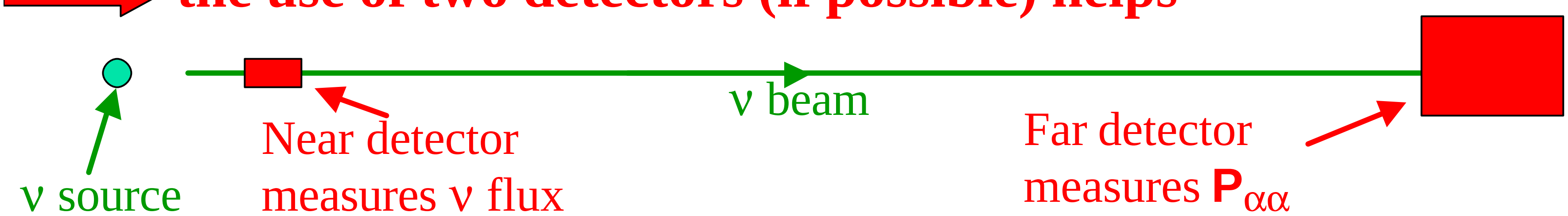
<u>Examples:</u>

- **Oscillation experiments using $\bar{n}_e$ from nuclear reactors**
  ($E_\nu \approx$ few MeV: under threshold for μ or τ production)

- n_m **detection at accelerators or from cosmic rays**
  (to search for $\nu_\mu \to \nu_\tau$ oscillations if E_ν is under threshold for τ production)

<u>Main uncertainty</u>: knowledge of the neutrino flux for no oscillation

the use of two detectors (if possible) helps



Appearance experiments

Use a beam of ν_α and detect ν_β ($\beta \neq \alpha$) at distance L from source

<u>Examples:</u>

- **Detect** n_e **+ Nucleon** ® **e**$^-$ **+ hadrons in a** n_m **beam**
- **Detect** n_t **+ Nucleon** ® t $^-$ **+ hadrons in a** n_m **beam**
  (Energy threshold ≈ 3.5 GeV)

NOTES

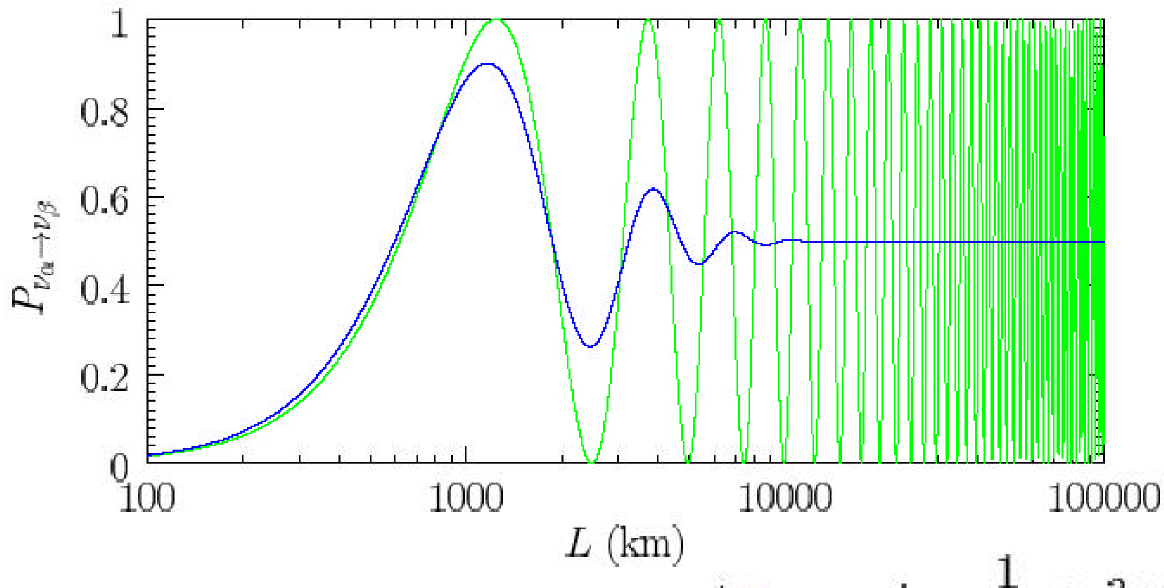
- n_b **contamination in beam must be precisely known**
  ($\nu_e/\nu_\mu \approx 1\%$ in ν_μ beams from high-energy accelerators)
- **Most neutrino sources are not mono-energetic but have wide energy spectra. Oscillation probabilities must be averaged over neutrino energy spectrum.**

Average over neutrino spectrum

$$P_{\nu_\alpha \to \nu_\beta}(L) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E} \right) \right] \qquad (\alpha \neq \beta)$$

$$\Delta m^2 = 10^{-3}\,\mathrm{eV}\,, \quad \sin^2 2\vartheta = 1$$

$$\langle E \rangle = 1\,\mathrm{GeV}\,, \quad \Delta E = 0.2\,\mathrm{GeV}$$



$$\langle P_{\nu_\alpha \to \nu_\beta} \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E)\,\mathrm{d}E \right]$$

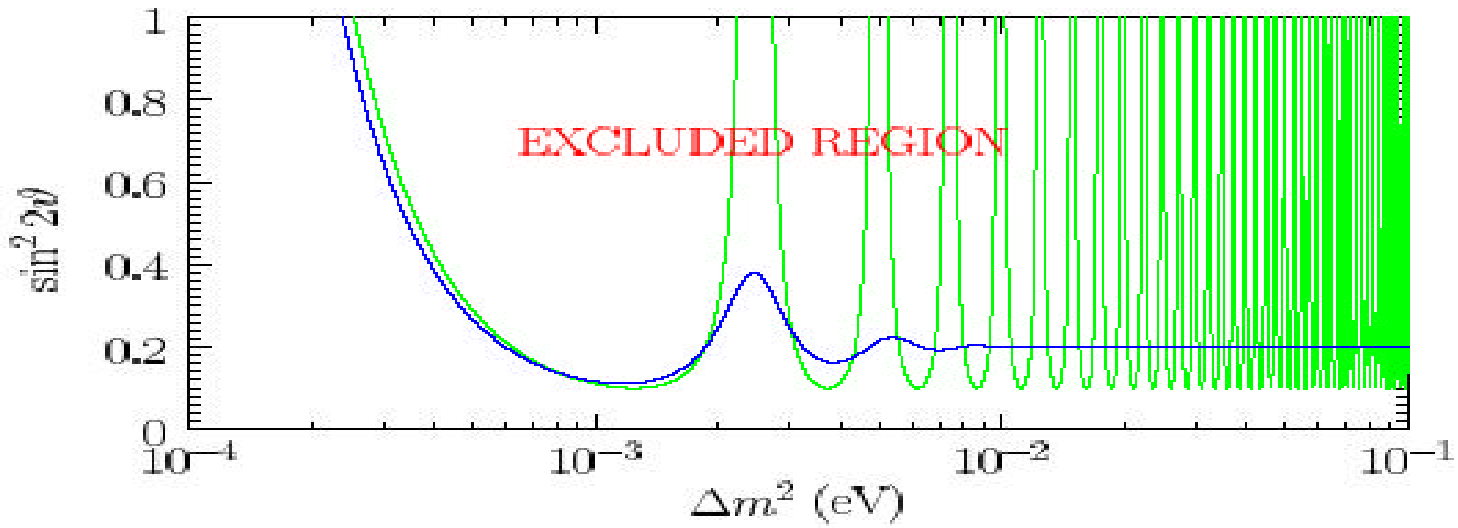
$$\text{Experiment}: P_{\nu_\alpha \to \nu_\beta} \leq P^0_{\nu_\alpha \to \nu_\beta}$$

$$\Downarrow$$

$$\frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E} \right) \right] \leq P^0_{\nu_\alpha \to \nu_\beta}$$

$$\Downarrow$$

$$\sin^2 2\vartheta \leq \frac{2 P^0_{\nu_\alpha \to \nu_\beta}}{1 - \cos\left(\frac{\Delta m^2 L}{2E} \right)}$$



$$L = 10^3\,\text{km}\,, \quad P^0_{\nu_\alpha \to \nu_\beta} = 10^{-1}$$

$$\sin^2 2\vartheta \leq \frac{2 P^0_{\nu_\alpha \to \nu_\beta}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E) \, \mathrm{d}E}$$

Exclusion curves

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$(\Delta m^2)_{\text{min}}$

$$\frac{\Delta m^2 L}{4E} \ll 1 \quad \Rightarrow \quad P \simeq \sin^2 2\vartheta \left(\frac{\Delta m^2 L}{4E}\right)^2$$

$$\Delta m^2 \simeq \frac{\sqrt{P}}{\frac{L}{4E} \sin 2\vartheta} \quad \Rightarrow \quad (\Delta m^2)_{\text{min}} \simeq \frac{4\sqrt{\langle P \rangle}}{\langle L/E \rangle}$$

$(\sin^2 2\vartheta)_{\text{min}}$

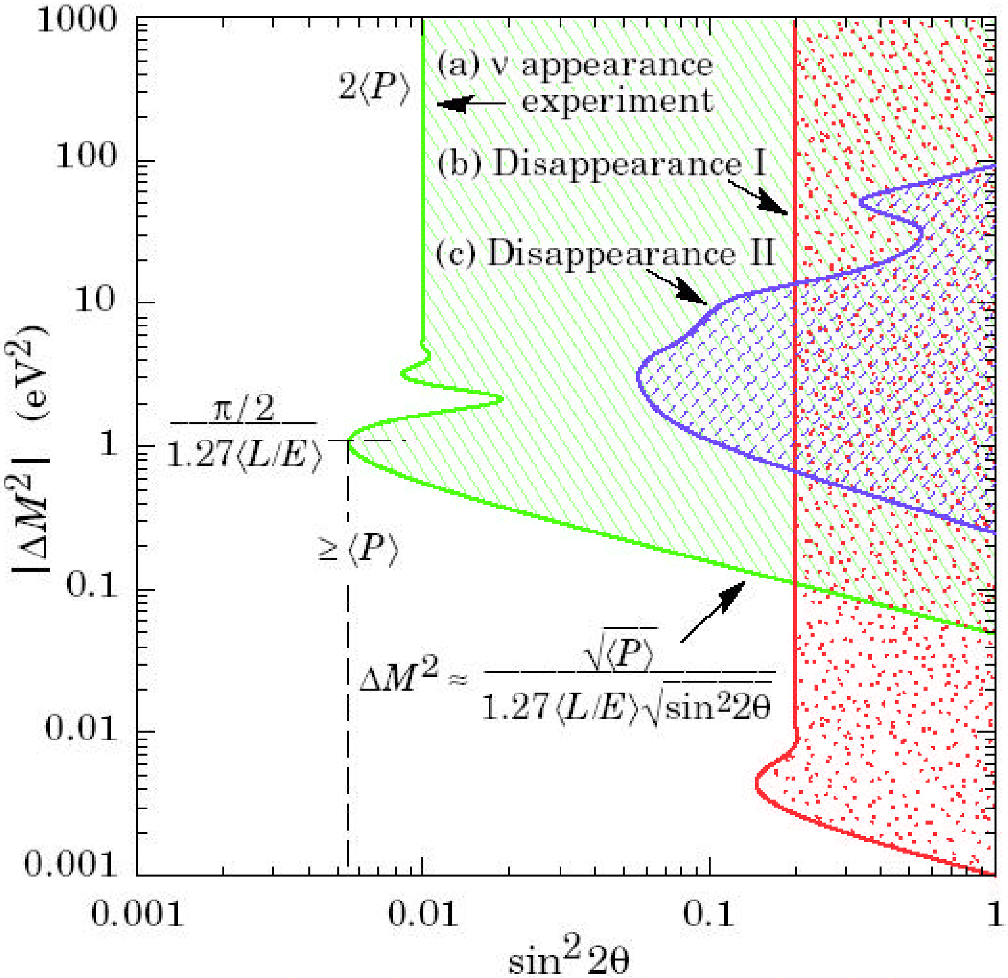
$$\Delta m^2 \simeq \frac{2\pi}{\langle L/E \rangle} \quad \Rightarrow \quad \frac{\Delta m^2 L}{4E} \simeq \frac{\pi}{2}$$

$$(\sin^2 2\vartheta)_{\text{min}} \gtrsim \langle P \rangle$$

$\sin^2 2\vartheta$ for Large Δm^2

$$\frac{\Delta m^2 L}{4E} \gg 1 \quad \Rightarrow \quad \langle P \rangle = \frac{1}{2} \sin^2 2\vartheta \quad \Rightarrow \quad \sin^2 2\vartheta = 2\langle P \rangle$$

$\sin^2 2\theta - \Delta m^2$ plots



Particle Data Group, The Review of Particle Physics, http://pdg.ge.infn.it

$$\frac{\Delta m^2 L}{4E} = 1.27\,\frac{\Delta m^2(\mathrm{eV}^2)\,L(\mathrm{m})}{4E(\mathrm{MeV})} = 1.27\,\frac{\Delta m^2(\mathrm{eV}^2)\,L(\mathrm{km})}{4E(\mathrm{GeV})}$$

PARAMETERS OF OSCILLATION SEARCH EXPERIMENTS

Neutrino source	Flavour	Baseline L	Energy	Minimum Δm^2
Sun	ν_e	$\sim 1.5 \times 10^8$ km	0.2 - 15 MeV	$\sim 10^{-11}$ eV^2
Cosmic rays	ν_μ ν_e $\bar{\nu}_\mu$ $\bar{\nu}_e$	10 km - 13000 km	0.2 GeV - 100 GeV	$\sim 10^{-4}$ eV^2
Nuclear reactors	$\bar{\nu}_e$	20 m - 250 km	$\langle E \rangle \approx 3$ MeV	$\sim 10^{-1}$ - 10^{-6} eV^2
Accelerators	ν_μ ν_e $\bar{\nu}_\mu$ $\bar{\nu}_e$	15 m - 730 km	20 MeV - 100 GeV	$\sim 10^{-3}$ - 10 eV^2

EVIDENCE/HINTS FOR NEUTRINO OSCILLATIONS

- **Solar Neutrino Deficit: ν_e disappearance between Sun and Earth**
- **Atmospheric neutrino problem: deficit of ν_μ coming from the other side of the Earth**
- **LSND Experiment at Los Alamos: excess of $\bar{\nu}_e$ in a beam consisting mainly of ν_μ, ν_e and $\bar{\nu}_\mu$**